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> 证明：$\operatorname{Tr}\left(\gamma_\mu\gamma_\nu\gamma_\lambda\gamma_\rho\gamma_5\right)=4\varepsilon_{\mu\nu\lambda\rho}, \operatorname{Tr}\left(\gamma_\mu\gamma_\nu\gamma_5\right)=0.$

证明 $\operatorname{Tr}\left(\gamma_\mu\gamma_\nu\gamma_\lambda\gamma_\rho\gamma_5\right)=4\varepsilon_{\mu\nu\lambda\rho}$

$$\begin{aligned}
\operatorname{Tr}\left(\gamma_\mu\gamma_\nu\gamma_\lambda\gamma_\rho\gamma_5\right) &= \operatorname{Tr}\left(\gamma_\mu\gamma_\nu\gamma_\lambda\gamma_\rho\gamma_1\gamma_2\gamma_3\gamma_4\right) \\
&= 4\sum_p \delta_p\delta_{\nu_1\nu_2}\delta_{\nu_3\nu_4}\delta_{\nu_5\nu_6}\delta_{\nu_7\nu_8}
\end{aligned}$$

由于在求和中 $(\nu_1,\nu_2,\nu_3,\nu_4,\nu_5,\nu_6,\nu_7,\nu_8)$ 所有取法中使得 $\delta_{\nu_1\nu_2}\delta_{\nu_3\nu_3}\delta_{\nu_5\nu_6}\delta_{\nu_7\nu_8}$ 不为零的取法只有 $(\nu_1,\nu_2,\nu_3,\nu_4,\nu_5,\nu_6,\nu_7,\nu_8)=(\mu,i_1,\nu,i_2,\lambda,i_3,\rho,i_4)$，其中 $(i_1,i_2,i_3,i_4)\equiv(i)$ 是 $(1,2,3,4)$ 的一个排列。注意到，通过 4 次置换可以把 $(\mu,i_1,\nu,i_2,\lambda,i_3,\rho,i_4)$ 还原为 $(\mu,\nu,\lambda,\rho,i_1,i_2,i_3,i_4)$ ，设从 (i_1,i_2,i_3,i_4) 还原到 $(1,2,3,4)$ 需要置换 m 次，则 $\varepsilon_{i_1i_2i_3i_4}=(-1)^m$

于是：

$$\begin{aligned}
\operatorname{Tr}\left(\gamma_\mu\gamma_\nu\gamma_\lambda\gamma_\rho\gamma_5\right) &= \operatorname{Tr}\left(\gamma_\mu\gamma_\nu\gamma_\lambda\gamma_\rho\gamma_1\gamma_2\gamma_3\gamma_4\right) \\
&= 4\sum_p \delta_p\delta_{\nu_1\nu_2}\delta_{\nu_3\nu_4}\delta_{\nu_5\nu_6}\delta_{\nu_7\nu_8} \\
&= 4\sum_{(i)} \varepsilon_{i_1i_2i_3i_4}\delta_{\mu i_1}\delta_{\nu i_2}\delta_{\lambda i_3}\delta_{\rho i_4} \\
&= 4\varepsilon_{i_1i_2i_3i_4}\delta_{\mu i_1}\delta_{\nu i_2}\delta_{\lambda i_3}\delta_{\rho i_4} \\
&= 4\varepsilon_{\mu\nu\lambda\rho}
\end{aligned}$$

证明 $\operatorname{Tr}\left(\gamma_\mu\gamma_\nu\gamma_5\right)=0$

$$\begin{aligned}
\operatorname{Tr}\left(\gamma_\mu\gamma_\nu\gamma_5\right) &= \operatorname{Tr}(\gamma_\mu\gamma_\nu\gamma_1\gamma_2\gamma_3\gamma_4) \\
&= \sum_p \delta_p\delta_{\nu_1\nu_2}\delta_{\nu_3\nu_3}\delta_{\nu_5\nu_6}
\end{aligned}$$

由于在求和中 $(\nu_1,\nu_2,\nu_3,\nu_4,\nu_5,\nu_6)$ 一种可能的取法必定是 $(\mu,\nu,1,2,3,4)$ 的一个排列，而所有排列都有 $\delta_{\nu_1\nu_2}\delta_{\nu_3\nu_3}\delta_{\nu_5\nu_6}=0$，因此：

$$\begin{aligned}
\operatorname{Tr}\left(\gamma_\mu\gamma_\nu\gamma_5\right) &= \operatorname{Tr}(\gamma_\mu\gamma_\nu\gamma_1\gamma_2\gamma_3\gamma_4) \\
&= \sum_p \delta_p\delta_{\nu_1\nu_2}\delta_{\nu_3\nu_3}\delta_{\nu_5\nu_6} \\
&= 0
\end{aligned}$$