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> 请证明广义洛伦兹变换 $X' = AX + b$ 和庞加莱变换 $X' = aX + b$ 分别构成群。

广义洛伦兹变换群

$$X' = AX + b$$

若定义抽象算符 $g(A, b)$ 满足:

$$g(A, b)X = AX + b$$

则可证明 $g(A, b)$ 具有群的性质:

（1）恒元:

$$g(I, 0)X = IX = X \Longrightarrow e = g(I, 0)$$

（2）封闭性:

$$g(A', b')g(A, b)X = g(A', b')\left(AX + b\right) = A'AX + A'b + b = g(A'A, A'b + b)X$$

（3）结合律:

$$\left[g(A_3, b_3)g(A_2, b_2)\right]g(A_1, b_1)X = g(A_3, b_3)\left[g(A_2, b_2)g(A_1, b_1)\right]X$$

（4）逆元:

$$g\left(A^{-1}, -A^{-1}b\right)g(A, b)X = g\left(A^{-1}, -A^{-1}b\right)\left(AX + b\right) = X + A^{-1}b - A^{-1}b = X$$
$$g^{-1}(A, b) = g\left(A^{-1}, -A^{-1}b\right)$$

庞加莱变换群

$$X' = aX + b$$

若定义抽象算符 $g(a, b)$ 满足:

$$g(a, b)X = aX + b$$

则可证明 $g(A, b)$ 具有群的性质:

（1）恒元:

$$g(I,0)X = IX = X \Longrightarrow e = g(I,0)$$

(2) 封闭性:

$$g(a',b')g(a,b)X = g(a',b')\left(aX+b\right) = a'aX + a'b + b = g(a'a, a'b+b)X$$

(3) 结合律:

$$\left[g(a_3,b_3)g(a_2,b_2)\right]g(a_1,b_1)X = g(a_3,b_3)\left[g(a_2,b_2)g(a_1,b_1)\right]X$$

(4) 逆元:

$$g\left(a^{-1},-a^{-1}b\right)g(a,b)X = g\left(a^{-1},-a^{-1}b\right)\left(aX+b\right) = X + a^{-1}b - a^{-1}b = X$$

$$g^{-1}(a,b) = g\left(a^{-1},-a^{-1}b\right)$$