2-2

> 设 $\phi_{\mu\nu}(z)$ 为二阶反对称张量，则赝张量的定义为 $\tilde{\phi}_{\mu\nu}=\frac{1}{2!}\varepsilon_{\mu\nu\lambda\rho}\phi_{\lambda\rho}$，证明它具有变换规律 $\tilde{\phi}'_{\mu\nu}(x')=|A|\,A_{\mu\alpha}A_{\nu\beta}\tilde{\phi}_{\alpha\beta}(x)$

$$
\begin{aligned}
\tilde{\phi}'_{\mu\nu}(x') &= \frac{1}{2!}\varepsilon_{\mu\nu\lambda\rho}\phi'_{\lambda\rho} \\
&= \frac{1}{2!}\varepsilon_{\mu\nu\lambda\rho}A_{\lambda\alpha}A_{\rho\beta}\phi_{\alpha\beta} \\
&= \frac{1}{2!}\delta_{\mu\gamma}\delta_{\nu\sigma}\varepsilon_{\gamma\sigma\lambda\rho}A_{\lambda\alpha}A_{\rho\beta}\phi_{\alpha\beta} \\
&= \frac{1}{2!}A_{\mu\xi}A_{\gamma\xi}A_{\nu\zeta}A_{\sigma\zeta}\varepsilon_{\gamma\sigma\lambda\rho}A_{\lambda\alpha}A_{\rho\beta}\phi_{\alpha\beta} \\
&= \frac{1}{2!}A_{\mu\xi}A_{\nu\zeta}\left(\varepsilon_{\gamma\sigma\lambda\rho}A_{\gamma\xi}A_{\sigma\zeta}A_{\lambda\alpha}A_{\rho\beta}\right)\phi_{\alpha\beta} \\
&= A_{\mu\xi}A_{\nu\zeta}\varepsilon_{\xi\zeta\alpha\beta}\,|A|\,\phi_{\alpha\beta} \\
&= \frac{1}{2!}\,|A|\,A_{\mu\xi}A_{\nu\zeta}\left(\frac{1}{2!}\varepsilon_{\xi\zeta\alpha\beta}\phi_{\alpha\beta}\right) \\
&= |A|\,A_{\mu\xi}A_{\nu\zeta}\tilde{\phi}_{\xi\zeta} \\
&= |A|\,A_{\mu\alpha}A_{\nu\beta}\tilde{\phi}_{\alpha\beta}(x)
\end{aligned}
$$