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> 已知 $\gamma_5 \equiv \gamma_1\gamma_2\gamma_3\gamma_4$，证明：$\gamma_5 = \dfrac{1}{4!}\varepsilon_{\mu\nu\alpha\beta}\gamma_\mu\gamma_\nu\gamma_\alpha\gamma_\beta$

$$\gamma_5 \equiv \gamma_1\gamma_2\gamma_3\gamma_4$$

上式两边同乘 $\varepsilon_{\mu\nu\lambda\rho}$：

$$\begin{aligned}\varepsilon_{\mu\nu\lambda\rho}\gamma_5 &= \varepsilon_{\mu\nu\lambda\rho}\gamma_1\gamma_2\gamma_3\gamma_4 \\ &= \gamma_\mu\gamma_\nu\gamma_\lambda\gamma_\rho\end{aligned}$$

上式继续两边同乘 $\varepsilon_{\mu\nu\lambda\rho}$：

$$\varepsilon_{\mu\nu\lambda\rho}\varepsilon_{\mu\nu\lambda\rho}\gamma_5 = \varepsilon_{\mu\nu\lambda\rho}\gamma_\mu\gamma_\nu\gamma_\lambda\gamma_\rho$$

利用 $\varepsilon_{\mu\nu\lambda\rho}\varepsilon_{\mu\nu\lambda\rho} = 4!$：

$$4!\gamma_5 = \varepsilon_{\mu\nu\lambda\rho}\gamma_\mu\gamma_\nu\gamma_\lambda\gamma_\rho$$

即：

$$\gamma_5 = \frac{1}{4!}\varepsilon_{\mu\nu\lambda\rho}\gamma_\mu\gamma_\nu\gamma_\lambda\gamma_\rho$$