## 4-3

由旋量场的三维自旋矢量

$$ec{S}=rac{1}{2}\int\psi^{\dagger}ec{\sigma}\psi\mathrm{d}V$$

利用 Dirac 方程

$$\mathrm{i}rac{\partial \psi}{\partial t} = \left(-\mathrm{i}ec{lpha}\cdot
abla + eta m
ight)\psi$$

求自旋矢量随时间的变化规律

$$rac{\partial ec{S}}{\partial t} = rac{1}{2\mathrm{i}} \int \psi^\dagger \left[ ec{\sigma} H - H ec{\sigma} 
ight] \psi \mathrm{d}V$$

Dirac 方程:

$$\mathrm{i}rac{\partial \psi}{\partial t} = \left(-\mathrm{i}ec{lpha}\cdot
abla + eta m
ight)\psi = H\psi$$

取厄米共轭,并考虑  $H^{\dagger}=H$ ,有:

$$-\mathrm{i}rac{\partial \psi^\dagger}{\partial t}=\psi^\dagger H^\dagger=\psi^\dagger H$$

于是自旋随时间变化:

$$\begin{split} \frac{\partial \vec{S}}{\partial t} &= \frac{\partial}{\partial t} \left[ \frac{1}{2} \int \psi^{\dagger} \vec{\sigma} \psi dV \right] \\ &= \frac{1}{2} \int \left[ \frac{\partial \psi^{\dagger}}{\partial t} \vec{\sigma} \psi + \psi^{\dagger} \vec{\sigma} \frac{\partial \psi}{\partial t} \right] dV \\ &= \frac{1}{2} \int \left[ \left( \frac{-\psi^{\dagger} H}{i} \right) \vec{\sigma} \psi + \psi^{\dagger} \vec{\sigma} \left( \frac{H\psi}{i} \right) \right] dV \\ &= \frac{1}{2i} \int \psi^{\dagger} \left( \vec{\sigma} H - H \vec{\sigma} \right) \psi dV \end{split}$$