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> 由旋量场的三维自旋矢量
>
> $$\vec{S}=\frac{1}{2}\int\psi^{\dagger}\vec{\sigma}\psi\mathrm{d}V$$
>
> 利用 Dirac 方程
>
> $$\mathrm{i}\frac{\partial\psi}{\partial t}=\left(-\mathrm{i}\vec{\alpha}\cdot\nabla+\beta m\right)\psi$$
>
> 求自旋矢量随时间的变化规律
>
> $$\frac{\partial\vec{S}}{\partial t}=\frac{1}{2\mathrm{i}}\int\psi^{\dagger}\left[\vec{\sigma}H-H\vec{\sigma}\right]\psi\mathrm{d}V$$

Dirac 方程:

$$\mathrm{i}\frac{\partial\psi}{\partial t}=\left(-\mathrm{i}\vec{\alpha}\cdot\nabla+\beta m\right)\psi=H\psi$$

取厄米共轭，并考虑 $H^{\dagger}=H$，有:

$$-\mathrm{i}\frac{\partial\psi^{\dagger}}{\partial t}=\psi^{\dagger}H^{\dagger}=\psi^{\dagger}H$$

于是自旋随时间变化:

$$\begin{aligned}\frac{\partial\vec{S}}{\partial t}&=\frac{\partial}{\partial t}\left[\frac{1}{2}\int\psi^{\dagger}\vec{\sigma}\psi\mathrm{d}V\right]\\&=\frac{1}{2}\int\left[\frac{\partial\psi^{\dagger}}{\partial t}\vec{\sigma}\psi+\psi^{\dagger}\vec{\sigma}\frac{\partial\psi}{\partial t}\right]\mathrm{d}V\\&=\frac{1}{2}\int\left[\left(\frac{-\psi^{\dagger}H}{\mathrm{i}}\right)\vec{\sigma}\psi+\psi^{\dagger}\vec{\sigma}\left(\frac{H\psi}{\mathrm{i}}\right)\right]\mathrm{d}V\\&=\frac{1}{2\mathrm{i}}\int\psi^{\dagger}\left(\vec{\sigma}H-H\vec{\sigma}\right)\psi\mathrm{d}V\end{aligned}$$