利用

$$egin{aligned} ar{\psi}'\gamma_lpha\gamma_eta\cdots\gamma_\gamma\psi' &= kA_{lpha\mu}A_{eta
u}\cdots A_{\gamma
ho}ar{\psi}\gamma_\mu\gamma_
u\cdots\gamma_
ho\psi \ k &= egin{cases} +1,\Lambda=S,P \ -1,\Lambda=T \end{cases} \ ar{\psi}\gamma_5\psi &= rac{1}{4!}arepsilon_{lphaeta\gamma\delta}ar{\psi}\gamma_lpha\gamma_eta\gamma_\gamma\gamma_\delta\psi \end{aligned}$$

导出,对于S和P变换

 $\bar{\psi}\gamma_5\psi$ 是赝标量, $\bar{\psi}\gamma_\mu\gamma_5\psi$ 是赝矢量, $\bar{\psi}\gamma_\mu\gamma_\nu\gamma_5\psi$ 是赝张量。

当时空坐标进行广义洛伦兹变换, $\bar{\psi}$ 和 ψ 的变换规律为:

$$egin{aligned} ar{\psi}' &= kar{\psi}\Lambda^{-1}, \quad \psi' = \Lambda\psi \ k &= egin{cases} +1, \Lambda &= S, P \ -1, \Lambda &= T \end{cases} \ ar{\psi}'\gamma_lpha\gamma_eta \cdots \gamma_\delta\psi' &= kar{\psi}\Lambda^{-1}\gamma_lpha\gamma_eta \cdots \gamma_\delta\Lambda\psi \ &= kar{\psi}\Lambda^{-1}\gamma_lpha\Lambda\Lambda^{-1}\gamma_eta\Lambda \cdots \Lambda^{-1}\gamma_\delta\Lambda\psi \ &= kar{\psi}A_{lpha\mu}\gamma_\mu A_{eta
u}\gamma_
u \cdots A_{\delta
ho}\Lambda_
ho\psi \ &= kA_{lpha\mu}A_{eta
u} \cdots A_{\delta
ho}ar{\psi}\gamma_\mu\gamma_
u \cdots \gamma_
ho\psi \end{aligned}$$

特别地,若进行 S 或 P 变换,则 k=1,于是 $\bar{\psi}\gamma_{\alpha}\gamma_{\beta}\cdots\gamma_{\delta}\psi$ 是个张量。

利用

$$\Lambda^{-1}\gamma_5\Lambda=|A|\,\gamma_5$$

则对于 S 和 P 变换, 有:

$$egin{aligned} ar{\psi}'\gamma_5\psi' &= kar{\psi}\Lambda^{-1}\gamma_5\Lambda\psi \ &= ar{\psi}\Lambda^{-1}\gamma_5\Lambda\psi \ &= |A|\,ar{\psi}\gamma_5\psi \end{aligned}$$

即 $ar{\psi}\gamma_5\psi$ 服从赝标量的变换规律,因此 $ar{\psi}\gamma_5\psi$ 是赝标量。

$$egin{aligned} ar{\psi}'\gamma_{\mu}\gamma_{5}\psi' &= kar{\psi}\Lambda^{-1}\gamma_{\mu}\gamma_{5}\Lambda\psi \ &= ar{\psi}\Lambda^{-1}\gamma_{\mu}\gamma_{5}\Lambda\psi \ &= ar{\psi}\Lambda^{-1}\gamma_{\mu}\Lambda\Lambda^{-1}\gamma_{5}\Lambda\psi \ &= ar{\psi}A_{\mu
u}\gamma_{
u}\left|A\right|\gamma_{5}\psi \ &= \left|A\right|A_{\mu
u}ar{\psi}\gamma_{
u}\gamma_{5}\psi \end{aligned}$$

即 $\bar{\psi}\gamma_{\mu}\gamma_{5}\psi$ 服从赝矢量的变换规律,因此 $\bar{\psi}\gamma_{\mu}\gamma_{5}\psi$ 是赝矢量。

$$\begin{split} \bar{\psi}'\gamma_{\mu}\gamma_{\nu}\gamma_{5}\psi' &= k\bar{\psi}\Lambda^{-1}\gamma_{\mu}\gamma_{\nu}\gamma_{5}\Lambda\psi \\ &= \bar{\psi}\Lambda^{-1}\gamma_{\mu}\gamma_{\nu}\gamma_{5}\Lambda\psi \\ &= \bar{\psi}\Lambda^{-1}\gamma_{\mu}\Lambda\Lambda^{-1}\gamma_{\nu}\Lambda\Lambda^{-1}\gamma_{5}\Lambda\psi \\ &= \bar{\psi}A_{\mu\alpha}\gamma_{\alpha}A_{\nu\beta}\gamma_{\beta} |A|\gamma_{5}\psi \\ &= |A|A_{\mu\alpha}A_{\nu\beta}\bar{\psi}\gamma_{\alpha}\gamma_{\beta}\gamma_{5}\psi \end{split}$$

即 $\bar{\psi}\gamma_{\mu}\gamma_{\nu}\gamma_{5}\psi$ 服从赝张量的变换规律,因此 $\bar{\psi}'\gamma_{\mu}\gamma_{\nu}\gamma_{5}\psi'$ 是赝张量。