3-7

> 利用
>
> $$\bar{\psi}'\gamma_\alpha\gamma_\beta\cdots\gamma_\gamma\psi' = kA_{\alpha\mu}A_{\beta\nu}\cdots A_{\gamma\rho}\bar{\psi}\gamma_\mu\gamma_\nu\cdots\gamma_\rho\psi$$
>
> $$k = \begin{cases} +1, \Lambda = S, P \\ -1, \Lambda = T \end{cases}$$
>
> $$\bar{\psi}\gamma_5\psi = \frac{1}{4!}\varepsilon_{\alpha\beta\gamma\delta}\bar{\psi}\gamma_\alpha\gamma_\beta\gamma_\gamma\gamma_\delta\psi$$
>
> 导出，对于 S 和 P 变换
>
> $\bar{\psi}\gamma_5\psi$ 是赝标量，$\bar{\psi}\gamma_\mu\gamma_5\psi$ 是赝矢量，$\bar{\psi}\gamma_\mu\gamma_\nu\gamma_5\psi$ 是赝张量。

当时空坐标进行广义洛伦兹变换，$\bar{\psi}$ 和 ψ 的变换规律为：

$$\bar{\psi}' = k\bar{\psi}\Lambda^{-1}, \quad \psi' = \Lambda\psi$$

$$k = \begin{cases} +1, \Lambda = S, P \\ -1, \Lambda = T \end{cases}$$

$$\begin{aligned} \bar{\psi}'\gamma_\alpha\gamma_\beta\cdots\gamma_\delta\psi' &= k\bar{\psi}\Lambda^{-1}\gamma_\alpha\gamma_\beta\cdots\gamma_\delta\Lambda\psi \\ &= k\bar{\psi}\Lambda^{-1}\gamma_\alpha\Lambda\Lambda^{-1}\gamma_\beta\Lambda\cdots\Lambda^{-1}\gamma_\delta\Lambda\psi \\ &= k\bar{\psi}A_{\alpha\mu}\gamma_\mu A_{\beta\nu}\gamma_\nu\cdots A_{\delta\rho}\Lambda_\rho\psi \\ &= kA_{\alpha\mu}A_{\beta\nu}\cdots A_{\delta\rho}\bar{\psi}\gamma_\mu\gamma_\nu\cdots\gamma_\rho\psi \end{aligned}$$

特别地，若进行 S 或 P 变换，则 $k = 1$，于是 $\bar{\psi}\gamma_\alpha\gamma_\beta\cdots\gamma_\delta\psi$ 是个张量。

利用

$$\Lambda^{-1}\gamma_5\Lambda = |A|\,\gamma_5$$

则对于 S 和 P 变换，有：

$$\begin{aligned} \bar{\psi}'\gamma_5\psi' &= k\bar{\psi}\Lambda^{-1}\gamma_5\Lambda\psi \\ &= \bar{\psi}\Lambda^{-1}\gamma_5\Lambda\psi \\ &= |A|\,\bar{\psi}\gamma_5\psi \end{aligned}$$

即 $\bar{\psi}\gamma_5\psi$ 服从赝标量的变换规律，因此 $\bar{\psi}\gamma_5\psi$ 是赝标量。

$$
\begin{aligned}
\bar{\psi}'\gamma_\mu\gamma_5\psi' &= k\bar{\psi}\Lambda^{-1}\gamma_\mu\gamma_5\Lambda\psi \\
&= \bar{\psi}\Lambda^{-1}\gamma_\mu\gamma_5\Lambda\psi \\
&= \bar{\psi}\Lambda^{-1}\gamma_\mu\Lambda\Lambda^{-1}\gamma_5\Lambda\psi \\
&= \bar{\psi}A_{\mu\nu}\gamma_\nu\left|A\right|\gamma_5\psi \\
&= \left|A\right|A_{\mu\nu}\bar{\psi}\gamma_\nu\gamma_5\psi
\end{aligned}
$$

即 $\bar{\psi}\gamma_\mu\gamma_5\psi$ 服从赝矢量的变换规律，因此 $\bar{\psi}\gamma_\mu\gamma_5\psi$ 是赝矢量。

$$
\begin{aligned}
\bar{\psi}'\gamma_\mu\gamma_\nu\gamma_5\psi' &= k\bar{\psi}\Lambda^{-1}\gamma_\mu\gamma_\nu\gamma_5\Lambda\psi \\
&= \bar{\psi}\Lambda^{-1}\gamma_\mu\gamma_\nu\gamma_5\Lambda\psi \\
&= \bar{\psi}\Lambda^{-1}\gamma_\mu\Lambda\Lambda^{-1}\gamma_\nu\Lambda\Lambda^{-1}\gamma_5\Lambda\psi \\
&= \bar{\psi}A_{\mu\alpha}\gamma_\alpha A_{\nu\beta}\gamma_\beta\left|A\right|\gamma_5\psi \\
&= \left|A\right|A_{\mu\alpha}A_{\nu\beta}\bar{\psi}\gamma_\alpha\gamma_\beta\gamma_5\psi
\end{aligned}
$$

即 $\bar{\psi}\gamma_\mu\gamma_\nu\gamma_5\psi$ 服从赝张量的变换规律，因此 $\bar{\psi}'\gamma_\mu\gamma_\nu\gamma_5\psi'$ 是赝张量。