## 3-6

证明正反粒子单位旋量正交关系

$$egin{aligned} u_i^\dagger(ec p)u_j(ec p) &= \delta_{ij} \ v_i^\dagger(ec p)v_j(ec p) &= \delta_{ij} \ u_i^\dagger(ec p)v_j(-ec p) &= 0 \ v_i^\dagger(-ec p)u_j(ec p) &= 0 \ u_i^\dagger(-ec p)v_j(ec p) &= 0 \ v_i^\dagger(ec p)u_j(-ec p) &= 0 \end{aligned}$$

证明:

由于  $u_a(\vec{p}), a=1,2,3,4$  是力学量完全集  $\{H, \vec{\sigma}\cdot\vec{n}\}$  属于不同本征值的本征态,因此它们正交。若进一步要求正交归一,则有:

$$u_a^\dagger(ec{p})u_b(ec{p})=\delta_{ab}, \quad a,b=1,2,3,4$$

因此:

$$u_i^\dagger(ec{p})u_j(ec{p})=\delta_{ij}, \quad i,j=1,2$$

 $v_a(\vec{p})$  的正交性:

$$egin{aligned} v_a^\dagger v_b &= \left(u_a^C
ight)^\dagger u_b^C \ &= \left(Car{u}_a^{
m T}
ight)^\dagger \left(Car{u}_b^{
m T}
ight) \ &= \left(ar{u}_a^{
m T}
ight)^\dagger C^\dagger Car{u}_b^\dagger \ &= \left(ar{u}_a^{
m T}
ight)^\dagger ar{u}_b^{
m T} \ &= \left(\left(u_a^\dagger \gamma_4
ight)^{
m T}
ight)^\dagger \left(u_b^\dagger \gamma_4
ight)^{
m T} \ &= \left(\gamma_4 u_a
ight)^{
m T} \gamma_4^{
m T} \left(u_b^\dagger
ight)^{
m T} \ &= \left(u_b^\dagger \gamma_4 \gamma_4 u_a
ight)^{
m T} \ &= \left(u_b^\dagger u_a
ight)^{
m T} \ &= \delta_{ba} \ &= \delta_{ab} \end{aligned}$$

因此

$$v_i^\dagger v_j = \delta_{ij}, \quad i,j=1,2$$

由于:

$$egin{align} v_1(ec p) &= lpha_1 u_4(-ec p) \ v_2(ec p) &= lpha_2 u_3(-ec p) \ v_3(ec p) &= lpha_3 u_2(-ec p) \ v_4(ec p) &= lpha_4 u_1(-ec p) \ ec a_aer = 1 \ \end{split}$$

因此,由  $u_a(\vec{p})$  的正交性可得:

$$u_i^{\dagger}(\vec{p})v_j(-\vec{p}) = 0$$
  $v_i^{\dagger}(-\vec{p})u_i(\vec{p}) = 0$ 

由  $v_a(\vec{p})$  的正交性可得:

$$u_i^\dagger(-ec p)v_j(ec p)=0$$
  $v_i^\dagger(ec p)u_j(-ec p)=0$