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> 证明广义 Lorentz 变换正交关系 $A_\lambda^\mu A_\lambda^\nu = \delta^{\mu\nu}$

广义洛伦兹变换：

$$x'^\mu = A_\nu^\mu x^\nu + b^\mu$$

两边同乘 A_λ^μ：

$$A_\lambda^\mu x'^\mu = A_\lambda^\mu A_\nu^\mu x^\nu + A_\lambda^\mu b^\mu = \delta_{\lambda\nu} x^\nu + A_\lambda^\mu b^\mu = x^\lambda + A_\lambda^\mu b^\mu$$

即：

$$x^\lambda = A_\lambda^\mu x'^\mu - A_\lambda^\mu b^\mu$$

取微分：

$$\mathrm{d}x^\lambda = A_\lambda^\mu \mathrm{d}x'^\mu$$

线元：

$$\mathrm{d}s^2 = -\mathrm{d}x^\lambda \mathrm{d}x^\lambda = -\left(A_\lambda^\mu \mathrm{d}x'^\mu\right)\left(A_\lambda^\nu \mathrm{d}x'^\nu\right) = -A_\lambda^\mu A_\lambda^\nu \mathrm{d}x'^\mu \mathrm{d}x'^\nu$$

$$\mathrm{d}s'^2 = -\mathrm{d}x'^\mu \mathrm{d}x'^\mu = -\delta^{\mu\nu} \mathrm{d}x'^\mu \mathrm{d}x'^\nu$$

由 $\mathrm{d}s^2 = \mathrm{d}s'^2$，对比可得：

$$A_\lambda^\mu A_\lambda^\nu = \delta^{\mu\nu}$$