标准规范 $\{u_{ij}=+1\}$ 下，哈密顿量

$$
\begin{aligned}
H(0) &= \frac{1}{2}\begin{pmatrix} a^\dagger & a^\top \end{pmatrix} h(0) \begin{pmatrix} a \\ (a^\dagger)^\top \end{pmatrix} \\
&= \frac{1}{2}\begin{pmatrix} a^\dagger & a^\top \end{pmatrix} U(0)D(0)U^\dagger(0) \begin{pmatrix} a \\ (a^\dagger)^\top \end{pmatrix} \\
&= \frac{1}{2}\begin{pmatrix} \alpha^\dagger(0) & \alpha^\top(0) \end{pmatrix} D(0) \begin{pmatrix} \alpha(0) \\ (\alpha^\dagger(0))^\top \end{pmatrix}
\end{aligned} \tag{1}
$$

$$
U(0)=\begin{pmatrix} W(0) & V^*(0) \\ V(0) & W^*(0) \end{pmatrix}, \quad D(0)=\operatorname{diag}(E_1(0),\cdots,E_N(0),-E_1(0),\cdots,-E_N(0)) \tag{2}
$$

$$
\begin{pmatrix} \alpha(0) \\ (\alpha^\dagger(0))^\top \end{pmatrix} = U^\dagger(0)\begin{pmatrix} a \\ (a^\dagger)^\top \end{pmatrix}, \quad \begin{pmatrix} \alpha^\dagger(0) & \alpha^\top(0) \end{pmatrix} = \begin{pmatrix} a^\dagger & a^\top \end{pmatrix} U(0) \tag{3}
$$

$$
\begin{pmatrix} a \\ (a^\dagger)^\top \end{pmatrix} = U(0)\begin{pmatrix} \alpha(0) \\ (\alpha^\dagger(0))^\top \end{pmatrix}, \quad \begin{pmatrix} a^\dagger & a^\top \end{pmatrix} = \begin{pmatrix} \alpha^\dagger(0) & \alpha^\top(0) \end{pmatrix} U^\dagger(0) \tag{4}
$$

则 $H(0)$ 的基态是 $\alpha(0)$ 的真空态 $\left|0_{\alpha(0)}\right\rangle$，可以由 a 的真空 $|0_a\rangle$ 生成:

$$
\left|0_{\alpha(0)}\right\rangle = \mathcal{N}(0)\exp\left(\frac{1}{2}\sum_{i,j} F_{i,j}(0)a_i^\dagger a_j^\dagger\right)|0_a\rangle \tag{5}
$$

$$
F(0)=V(0)^*\left[W^*(0)\right]^{-1} \tag{6}
$$

> 好像还要反对称化一下?

$$
\mathcal{N}(0)=\det{}^{-1/4}\left(I+F^\dagger(0)F(0)\right) \tag{7}
$$

考虑构型 $\{u_{ij}\}$ 下，哈密顿量

$$
\begin{aligned}
H &= \frac{1}{2}\begin{pmatrix} a^\dagger & a^\top \end{pmatrix} h \begin{pmatrix} a \\ (a^\dagger)^\top \end{pmatrix} \\
&= \frac{1}{2}\begin{pmatrix} a^\dagger & a^\top \end{pmatrix} UDU^\dagger \begin{pmatrix} a \\ (a^\dagger)^\top \end{pmatrix} \\
&= \frac{1}{2}\begin{pmatrix} \alpha^\dagger(0) & \alpha^\top(0) \end{pmatrix} U^\dagger(0)UDU^\dagger U(0) \begin{pmatrix} \alpha(0) \\ (\alpha^\dagger(0))^\top \end{pmatrix} \\
&\equiv \frac{1}{2}\begin{pmatrix} \alpha^\dagger(0) & \alpha^\top(0) \end{pmatrix} \widetilde{U}D\widetilde{U}^\dagger \begin{pmatrix} \alpha(0) \\ (\alpha^\dagger(0))^\top \end{pmatrix} \\
&\equiv \frac{1}{2}\begin{pmatrix} \alpha^\dagger & \alpha^\top \end{pmatrix} D \begin{pmatrix} \alpha \\ (\alpha^\dagger)^\top \end{pmatrix}
\end{aligned} \tag{8}
$$

$$
\widetilde{U}\equiv U^\dagger(0)U, \quad \widetilde{U}^\dagger = U^\dagger U(0) \tag{9}
$$

$$
\widetilde{U}=\begin{pmatrix} \widetilde{W} & \widetilde{V}^* \\ \widetilde{V} & \widetilde{W}^* \end{pmatrix}, \quad \widetilde{U}^\dagger=\begin{pmatrix} \widetilde{W}^\dagger & \widetilde{V}^\dagger \\ \widetilde{V}^\top & \widetilde{W}^\top \end{pmatrix} \tag{10}
$$

$$
\begin{pmatrix} \alpha^\dagger & \alpha^\top \end{pmatrix} = \begin{pmatrix} \alpha^\dagger(0) & \alpha^\top(0) \end{pmatrix}\widetilde{U} = \begin{pmatrix} \alpha^\dagger(0) & \alpha^\top(0) \end{pmatrix}\begin{pmatrix} \widetilde{W} & \widetilde{V}^* \\ \widetilde{V} & \widetilde{W}^* \end{pmatrix}, \tag{11}
$$

$$
\begin{pmatrix} \alpha \\ (\alpha^\dagger)^\top \end{pmatrix} = \widetilde{U}^\dagger \begin{pmatrix} \alpha(0) \\ (\alpha^\dagger(0))^\top \end{pmatrix} = \begin{pmatrix} \widetilde{W}^\dagger & \widetilde{V}^\dagger \\ \widetilde{V}^\top & \widetilde{W}^\top \end{pmatrix}\begin{pmatrix} \alpha(0) \\ (\alpha^\dagger(0))^\top \end{pmatrix} \tag{12}
$$

H 的基态是 α 的真空态 $|0_\alpha\rangle$，可以由 $\alpha(0)$ 的真空态 $\left|0_{\alpha(0)}\right\rangle$ 生成：

$$|0_\alpha\rangle = \widetilde{\mathcal{N}} \exp\left(\frac{1}{2}\sum_{i,j}\widetilde{F}_{i,j}\alpha_i^\dagger(0)\alpha_j^\dagger(0)\right)\left|0_{\alpha(0)}\right\rangle \tag{13}$$

$$\widetilde{F} = \widetilde{V}^*\left(\widetilde{W}^*\right)^{-1} \tag{14}$$

$$\widetilde{\mathcal{N}} = \det{}^{-1/4}\left(I + \widetilde{F}^\dagger\widetilde{F}\right) \tag{15}$$

实际上 $\widetilde{\mathcal{N}}$ 可以化简。

化简 $\widetilde{\mathcal{N}}$

$$\widetilde{N} = \left|\det\left(\widetilde{W}\right)\right|^{1/2} \tag{16}$$

c-Majorana如何换成 $\alpha(0)$

$$c_{i,A} = a_i + a_i^\dagger, \quad c_{i,B} = \frac{1}{\mathrm{i}}\left(a_i - a_i^\dagger\right), \tag{17}$$

$$\begin{pmatrix} a \\ \left(a^\dagger\right)^\top \end{pmatrix} = U(0)\begin{pmatrix} \alpha(0) \\ \left(\alpha^\dagger(0)\right)^\top \end{pmatrix} \tag{18}$$

$$U(0) = \begin{pmatrix} \mathbf{W}(0) & \mathbf{V}^*(0) \\ \mathbf{V}(0) & \mathbf{W}^*(0) \end{pmatrix} \tag{19}$$

$$a_i = \frac{1}{2}\left(c_{i,A} + \mathrm{i}c_{i,B}\right), \quad a_i^\dagger = \frac{1}{2}\left(c_{i,A} - \mathrm{i}c_{i,B}\right) \tag{20}$$

$$a \equiv \begin{pmatrix} a_1 \\ \vdots \\ a_N \end{pmatrix} = \frac{1}{2}c_A + \frac{\mathrm{i}}{2}c_B \tag{21}$$

$$\left(a^\dagger\right)^\top = \frac{1}{2}c_A - \frac{\mathrm{i}}{2}c_B \tag{22}$$

一方面

$$\begin{pmatrix} a \\ \left(a^\dagger\right)^\top \end{pmatrix} = \begin{pmatrix} \frac{1}{2}I & \frac{\mathrm{i}}{2}I \\ \frac{1}{2}I & -\frac{\mathrm{i}}{2}I \end{pmatrix}\begin{pmatrix} c_A \\ c_B \end{pmatrix} \tag{23}$$

另一方面

$$\begin{pmatrix} a \\ \left(a^\dagger\right)^\top \end{pmatrix} = U(0)\begin{pmatrix} \alpha(0) \\ \left(\alpha^\dagger(0)\right)^\top \end{pmatrix} \tag{24}$$

$$U(0) = \begin{pmatrix} \mathbf{W}(0) & \mathbf{V}^*(0) \\ \mathbf{V}(0) & \mathbf{W}^*(0) \end{pmatrix} \tag{25}$$

于是

$$
\begin{aligned}
\begin{pmatrix} c_A \\ c_B \end{pmatrix} &= \begin{pmatrix} \frac{1}{2}I & \frac{\mathrm{i}}{2}I \\ \frac{1}{2}I & -\frac{\mathrm{i}}{2}I \end{pmatrix}^{-1} \begin{pmatrix} \mathbf{W}(0) & \mathbf{V}^*(0) \\ \mathbf{V}(0) & \mathbf{W}^*(0) \end{pmatrix} \begin{pmatrix} \alpha(0) \\ \left(\alpha^\dagger(0)\right)^\top \end{pmatrix} = \begin{pmatrix} I & I \\ -\mathrm{i}I & \mathrm{i}I \end{pmatrix} \begin{pmatrix} \mathbf{W}(0) & \mathbf{V}^*(0) \\ \mathbf{V}(0) & \mathbf{W}^*(0) \end{pmatrix} \begin{pmatrix} \alpha(0) \\ \left(\alpha^\dagger(0)\right)^\top \end{pmatrix} \\
&\equiv U'(0) \begin{pmatrix} \alpha(0) \\ \left(\alpha^\dagger(0)\right)^\top \end{pmatrix} = \begin{pmatrix} W(0)+V(0) & V^*(0)+W^*(0) \\ \mathrm{i}\left(-W(0)+V(0)\right) & \mathrm{i}\left(-V^*(0)+W^*(0)\right) \end{pmatrix} \begin{pmatrix} \alpha(0) \\ \left(\alpha^\dagger(0)\right)^\top \end{pmatrix} \\
&\equiv \begin{pmatrix} \mathbf{U}'_{11}(0) & \mathbf{U}'_{12}(0) \\ \mathbf{U}'_{21}(0) & \mathbf{U}'_{22}(0) \end{pmatrix} \begin{pmatrix} \alpha(0) \\ \left(\alpha^\dagger(0)\right)^\top \end{pmatrix}
\end{aligned} \tag{26}
$$

$$
\boxed{c_{i,A} = U'_{i,}(0) \begin{pmatrix} \alpha(0) \\ \left(\alpha^\dagger(0)\right)^\top \end{pmatrix} = \mathbf{U}'_{11}(0)_{i,}\alpha(0) + \mathbf{U}'_{12}(0)_{i,} \left(\alpha^\dagger(0)\right)^\top} \tag{27}
$$

$$
\boxed{c_{i,A} = c_{i,A}^\dagger = \alpha^\dagger(0)\left[\mathbf{U}'_{11}(0)_{i,}\right]^\dagger + \alpha^\top(0)\left[\mathbf{U}'_{12}(0)_{i,}\right]^\dagger} \tag{28}
$$

$$
\boxed{c_{j,B} = U'_{j+N,}(0) \begin{pmatrix} \alpha(0) \\ \left(\alpha^\dagger(0)\right)^\top \end{pmatrix} = \mathbf{U}'_{21}(0)_{j,}\alpha(0) + \mathbf{U}'_{22}(0)_{j,} \left(\alpha^\dagger(0)\right)^\top} \tag{29}
$$

$$
\boxed{c_{j,B} = c_{j,B}^\dagger = \alpha^\dagger(0)\left[\mathbf{U}'_{21}(0)_{j,}\right]^\dagger + \alpha^\top(0)\left[\mathbf{U}'_{22}(0)_{j,}\right]^\dagger} \tag{30}
$$

跃迁振幅

$$
\begin{aligned}
&\left\langle 0_\chi, 0_{\alpha(1)} \,\middle|\, d_{\boldsymbol{r},x}(1) H_h d^\dagger_{\boldsymbol{r},y}(2) \,\middle|\, 0_\chi, 0_{\alpha(2)} \right\rangle \\
=&\left\langle \chi(\boldsymbol{r},x), 0_{\alpha(1)} \,\middle|\, \alpha(1)\left[-h_z\left(\sigma^z_{\boldsymbol{r}} + \sigma^z_{\boldsymbol{r}+\boldsymbol{\delta}_z}\right)\right]\alpha^\dagger(2) \,\middle|\, \chi(\boldsymbol{r},y), 0_{\alpha(2)} \right\rangle \\
=&-h_z\left\langle \chi(\boldsymbol{r},x), 0_{\alpha(1)} \,\middle|\, \alpha(1)\left(\sigma^z_{\boldsymbol{r}} + \sigma^z_{\boldsymbol{r}+\boldsymbol{\delta}_z}\right)\alpha^\dagger(2) \,\middle|\, \chi(\boldsymbol{r},y), 0_{\alpha(2)} \right\rangle \\
=&-h_z\left\langle \chi, 0_{\alpha(1)} \,\middle|\, \alpha(1)\left[-\mathrm{i}\left(1+\mathrm{i}c_{i,A}c_{j,B}\right)\right]\alpha^\dagger(2) \,\middle|\, \chi, 0_{\alpha(2)} \right\rangle \\
=&\mathrm{i}h_z\left\langle \chi, 0_{\alpha(1)} \,\middle|\, \alpha(1)\left(1+\mathrm{i}c_{i,A}c_{j,B}\right)\alpha^\dagger(2) \,\middle|\, \chi, 0_{\alpha(2)} \right\rangle \\
=&\mathrm{i}h_z\widetilde{\mathcal{N}}(1)\widetilde{\mathcal{N}}(2)\left\langle \chi, 0_{\alpha(0)} \,\middle|\, \exp\left(\frac{1}{2}F^*_{i,j}(1)\alpha_i(0)\alpha_j(0)\right)\alpha(1)\left(1+\mathrm{i}c_{i,A}c_{j,B}\right)\alpha^\dagger(2)\exp\left(\frac{1}{2}\sum_{i,j}F_{i,j}(2)\alpha_i^\dagger(0)\alpha_j^\dagger(0)\right) \,\middle|\, \chi, 0_{\alpha(2)} \right\rangle \\
\equiv&\mathrm{i}h_z\left\langle \Psi(1) \,\middle|\, \alpha(1)\left(1+\mathrm{i}c_{i,A}c_{j,B}\right)\alpha^\dagger(2) \,\middle|\, \Psi(2) \right\rangle \\
=&\mathrm{i}h_z\left\langle \Psi(1) \,|\, \Psi(2) \right\rangle \left\langle \alpha(1)\left(1+\mathrm{i}c_{i,A}c_{j,B}\right)\alpha^\dagger(2) \right\rangle \\
=&h_z\langle \Psi(1)|\Psi(2)\rangle\left[\mathrm{i}\langle\alpha(1)\alpha^\dagger(2)\rangle - \langle\alpha(1)c_{i,A}c_{j,B}\alpha^\dagger(2)\rangle\right]
\end{aligned} \tag{31}
$$

- $\langle \Psi(1) \,|\, \Psi(2) \rangle$

$$
\langle \Psi(1) \,|\, \Psi(2) \rangle = \widetilde{N}(1)\widetilde{N}(2)\left(-1\right)^{N(N+1)/2}\mathrm{Pf}(M) \tag{32}
$$

$$
M \equiv \begin{pmatrix} \widetilde{F}(2) & -I \\ I & -\widetilde{F}^*(1) \end{pmatrix} \tag{33}
$$

$$
\widetilde{N} = \left|\det\left(\widetilde{W}\right)\right|^{1/2} \tag{34}
$$

$$
\widetilde{F} = \widetilde{V}^*\left(\widetilde{W}^*\right)^{-1} \tag{35}
$$

$$
\widetilde{U} \equiv U^\dagger(0)U, \quad \widetilde{U}^\dagger = U^\dagger U(0) \tag{36}
$$

$$
\widetilde{U} = \begin{pmatrix} \widetilde{W} & \widetilde{V}^* \\ \widetilde{V} & \widetilde{W}^* \end{pmatrix}, \quad \widetilde{U}^\dagger = \begin{pmatrix} \widetilde{W}^\dagger & \widetilde{V}^\dagger \\ \widetilde{V}^\top & \widetilde{W}^\top \end{pmatrix} \tag{37}
$$

- $\langle \alpha(1)\alpha^\dagger(2) \rangle$

$$\begin{pmatrix}\alpha(1)\\ \left(\alpha^\dagger(1)\right)^\top\end{pmatrix}=\widetilde{U}^\dagger(1)\begin{pmatrix}\alpha(0)\\ \left(\alpha^\dagger(0)\right)^\top\end{pmatrix}=\begin{pmatrix}\widetilde{W}^\dagger(1) & \widetilde{V}^\dagger(1)\\ \widetilde{V}^\top(1) & \widetilde{W}^\top(1)\end{pmatrix}\begin{pmatrix}\alpha(0)\\ \left(\alpha^\dagger(0)\right)^\top\end{pmatrix} \tag{38}$$

$$\boxed{\alpha(1)=\widetilde{W}^\dagger(1)\alpha(0)+\widetilde{V}^\dagger(1)\left(\alpha^\dagger(0)\right)^\top} \tag{39}$$

$$\begin{pmatrix}\alpha^\dagger(2) & \alpha^\top(2)\end{pmatrix}=\begin{pmatrix}\alpha^\dagger(0) & \alpha^\top(0)\end{pmatrix}\widetilde{U}(2)=\begin{pmatrix}\alpha^\dagger(0) & \alpha^\top(0)\end{pmatrix}\begin{pmatrix}\widetilde{W}(2) & \widetilde{V}^*(2)\\ \widetilde{V}(2) & \widetilde{W}^*(2)\end{pmatrix}, \tag{40}$$

$$\boxed{\alpha^\dagger(2)=\alpha^\dagger(0)\widetilde{W}(2)+\alpha^\top(0)\widetilde{V}(2)} \tag{41}$$

$$\begin{aligned}
&\left\langle\alpha(1)\alpha^\dagger(2)\right\rangle\\
&=\left\langle\left[\widetilde{W}^\dagger(1)\alpha(0)+\widetilde{V}^\dagger(1)\left(\alpha^\dagger(0)\right)^\top\right]\left[\alpha^\dagger(0)\widetilde{W}(2)+\alpha^\top(0)\widetilde{V}(2)\right]\right\rangle\\
&=\left\langle\begin{pmatrix}\widetilde{V}^\dagger(1) & \widetilde{W}^\dagger(1)\end{pmatrix}\begin{pmatrix}\left(\alpha^\dagger(0)\right)^\top\\ \alpha(0)\end{pmatrix}\begin{pmatrix}\alpha^\dagger(0) & \alpha^\top(0)\end{pmatrix}\begin{pmatrix}\widetilde{W}(2)\\ \widetilde{V}(2)\end{pmatrix}\right\rangle\\
&=\begin{pmatrix}\widetilde{V}^\dagger(1) & \widetilde{W}^\dagger(1)\end{pmatrix}\left\langle\begin{pmatrix}\left(\alpha^\dagger(0)\right)^\top\alpha^\dagger(0) & \left(\alpha^\dagger(0)\right)^\top\alpha^\top(0)\\ \alpha(0)\alpha^\dagger(0) & \alpha(0)\alpha^\top(0)\end{pmatrix}\right\rangle\begin{pmatrix}\widetilde{W}(2)\\ \widetilde{V}(2)\end{pmatrix}\\
&=\begin{pmatrix}\widetilde{V}^\dagger(1) & \widetilde{W}^\dagger(1)\end{pmatrix}\left(-M^{-1}+\begin{pmatrix}0 & I\\ 0 & 0\end{pmatrix}\right)\begin{pmatrix}\widetilde{W}(2)\\ \widetilde{V}(2)\end{pmatrix}
\end{aligned} \tag{42}$$

- $\left\langle\alpha(1)c_{i,A}c_{j,B}\alpha^\dagger(2)\right\rangle$

Wick定理给出

$$\begin{aligned}
&\left\langle\alpha(1)c_{i,A}c_{j,B}\alpha^\dagger(2)\right\rangle\\
&=\left\langle\alpha(1)c_{i,A}\right\rangle\left\langle c_{j,B}\alpha^\dagger(2)\right\rangle-\left\langle\alpha(1)c_{j,B}\right\rangle\left\langle c_{i,A}\alpha^\dagger(2)\right\rangle+\left\langle\alpha(1)\alpha^\dagger(2)\right\rangle\left\langle c_{i,A}c_{j,B}\right\rangle
\end{aligned} \tag{43}$$

- - $\left\langle\alpha(1)c_{i,A}\right\rangle$

$$\boxed{\alpha(1)=\widetilde{W}^\dagger(1)\alpha(0)+\widetilde{V}^\dagger(1)\left(\alpha^\dagger(0)\right)^\top} \tag{44}$$

$$\boxed{c_{i,A}=c_{i,A}^\dagger=\alpha^\dagger(0)\left[\mathbf{U}'_{11}(0)_{i,}\right]^\dagger+\alpha^\top(0)\left[\mathbf{U}'_{12}(0)_{i,}\right]^\dagger} \tag{45}$$

$$\begin{aligned}
&\left\langle\alpha(1)c_{i,A}\right\rangle\\
&=\left\langle\left\{\widetilde{W}^\dagger(1)\alpha(0)+\widetilde{V}^\dagger(1)\left(\alpha^\dagger(0)\right)^\top\right\}\left\{\alpha^\dagger(0)\left[\mathbf{U}'_{11}(0)_{i,}\right]^\dagger+\alpha(0)^\top\left[\mathbf{U}'_{12}(0)_{i,}\right]^\dagger\right\}\right\rangle\\
&=\begin{pmatrix}\widetilde{V}^\dagger(1) & \widetilde{W}^\dagger(1)\end{pmatrix}\left\langle\begin{pmatrix}\left(\alpha^\dagger(0)\right)^\top\\ \alpha(0)\end{pmatrix}\begin{pmatrix}\alpha^\dagger(0) & \alpha^\top(0)\end{pmatrix}\right\rangle\begin{pmatrix}\left[\mathbf{U}'_{11}(0)_{i,}\right]^\dagger\\ \left[\mathbf{U}'_{12}(0)_{i,}\right]^\dagger\end{pmatrix}\\
&=\begin{pmatrix}\widetilde{V}^\dagger(1) & \widetilde{W}^\dagger(1)\end{pmatrix}\left(-M^{-1}+\begin{pmatrix}0 & I\\ 0 & 0\end{pmatrix}\right)\begin{pmatrix}\left[\mathbf{U}'_{11}(0)_{i,}\right]^\dagger\\ \left[\mathbf{U}'_{12}(0)_{i,}\right]^\dagger\end{pmatrix}
\end{aligned} \tag{46}$$

- - $\left\langle\alpha(1)c_{j,B}\right\rangle$

$$\boxed{\alpha(1)=\widetilde{W}^\dagger(1)\alpha(0)+\widetilde{V}^\dagger(1)\left(\alpha^\dagger(0)\right)^\top} \tag{47}$$

$$\boxed{c_{j,B}=c_{j,B}^\dagger=\alpha^\dagger(0)\left[\mathbf{U}'_{21}(0)_{j,}\right]^\dagger+\alpha^\top(0)\left[\mathbf{U}'_{22}(0)_{j,}\right]^\dagger} \tag{48}$$

$$
\begin{aligned}
&\langle\alpha(1)c_{j,B}\rangle \\
&=\left\langle\left\{\widetilde{W}^{\dagger}(1)\alpha(0)+\widetilde{V}^{\dagger}(1)\left(\alpha^{\dagger}(0)\right)^{\top}\right\}\left\{\alpha^{\dagger}(0)\left[\mathbf{U}_{21}'(0)_{j,}\right]^{\dagger}+\alpha^{\top}(0)\left[\mathbf{U}_{22}'(0)_{j,}\right]^{\dagger}\right\}\right\rangle \\
&=\begin{pmatrix}\widetilde{V}^{\dagger}(1) & \widetilde{W}^{\dagger}(1)\end{pmatrix}\left\langle\begin{pmatrix}\left(\alpha^{\dagger}(0)\right)^{\top}\\ \alpha(0)\end{pmatrix}\begin{pmatrix}\alpha^{\dagger}(0) & \alpha^{\top}(0)\end{pmatrix}\right\rangle\begin{pmatrix}\left[\mathbf{U}_{21}'(0)_{j,}\right]^{\dagger}\\ \left[\mathbf{U}_{22}'(0)_{j,}\right]^{\dagger}\end{pmatrix} \\
&=\begin{pmatrix}\widetilde{V}^{\dagger}(1) & \widetilde{W}^{\dagger}(1)\end{pmatrix}\left(-M^{-1}+\begin{pmatrix}0 & I\\ 0 & 0\end{pmatrix}\right)\begin{pmatrix}\left[\mathbf{U}_{21}'(0)_{j,}\right]^{\dagger}\\ \left[\mathbf{U}_{22}'(0)_{j,}\right]^{\dagger}\end{pmatrix}
\end{aligned} \tag{49}
$$

- ◦ $\left\langle c_{i,A}\alpha^{\dagger}(2)\right\rangle$

$$
\boxed{\alpha^{\dagger}(2)=\alpha^{\dagger}(0)\widetilde{W}(2)+\alpha^{\top}(0)\widetilde{V}(2)} \tag{50}
$$

$$
\boxed{c_{i,A}=U_{i,}'(0)\begin{pmatrix}\alpha(0)\\ \left(\alpha^{\dagger}(0)\right)^{\top}\end{pmatrix}=\mathbf{U}_{11}'(0)_{i,}\alpha(0)+\mathbf{U}_{12}'(0)_{i,}\left(\alpha^{\dagger}(0)\right)^{\top}} \tag{51}
$$

$$
\begin{aligned}
&\left\langle c_{i,A}\alpha^{\dagger}(2)\right\rangle \\
&=\left\langle\left\{\mathbf{U}_{11}'(0)_{i,}\alpha(0)+\mathbf{U}_{12}'(0)_{i,}\left(\alpha^{\dagger}(0)\right)^{\top}\right\}\left\{\alpha^{\dagger}(0)\widetilde{W}(2)+\alpha^{\top}(0)\widetilde{V}(2)\right\}\right\rangle \\
&=\begin{pmatrix}\mathbf{U}_{12}'(0)_{i,} & \mathbf{U}_{11}'(0)_{i,}\end{pmatrix}\left\langle\begin{pmatrix}\left(\alpha^{\dagger}(0)\right)^{\top}\\ \alpha(0)\end{pmatrix}\begin{pmatrix}\alpha^{\dagger}(0) & \alpha^{\top}(0)\end{pmatrix}\right\rangle\begin{pmatrix}\widetilde{W}(2)\\ \widetilde{V}(2)\end{pmatrix} \\
&=\begin{pmatrix}\mathbf{U}_{12}'(0)_{i,} & \mathbf{U}_{11}'(0)_{i,}\end{pmatrix}\left(-M^{-1}+\begin{pmatrix}0 & I\\ 0 & 0\end{pmatrix}\right)\begin{pmatrix}\widetilde{W}(2)\\ \widetilde{V}(2)\end{pmatrix}
\end{aligned} \tag{52}
$$

- ◦ $\left\langle c_{j,B}\alpha^{\dagger}(2)\right\rangle$

$$
\boxed{\alpha^{\dagger}(2)=\alpha^{\dagger}(0)\widetilde{W}(2)+\alpha^{\top}(0)\widetilde{V}(2)} \tag{53}
$$

$$
\boxed{c_{j,B}=U_{j+N,}'(0)\begin{pmatrix}\alpha(0)\\ \left(\alpha^{\dagger}(0)\right)^{\top}\end{pmatrix}=\mathbf{U}_{21}'(0)_{j,}\alpha(0)+\mathbf{U}_{22}'(0)_{j,}\left(\alpha^{\dagger}(0)\right)^{\top}} \tag{54}
$$

$$
\begin{aligned}
&\left\langle c_{j,B}\alpha^{\dagger}(2)\right\rangle \\
&=\left\langle\left\{\mathbf{U}_{21}'(0)_{j,}\alpha(0)+\mathbf{U}_{22}'(0)_{j,}\left(\alpha^{\dagger}(0)\right)^{\top}\right\}\left\{\alpha^{\dagger}(0)\widetilde{W}(2)+\alpha^{\top}(0)\widetilde{V}(2)\right\}\right\rangle \\
&=\begin{pmatrix}\mathbf{U}_{22}'(0)_{j,} & \mathbf{U}_{21}'(0)_{j,}\end{pmatrix}\left\langle\begin{pmatrix}\left(\alpha^{\dagger}(0)\right)^{\top}\\ \alpha(0)\end{pmatrix}\begin{pmatrix}\alpha^{\dagger}(0) & \alpha^{\top}(0)\end{pmatrix}\right\rangle\begin{pmatrix}\widetilde{W}(2)\\ \widetilde{V}(2)\end{pmatrix} \\
&=\begin{pmatrix}\mathbf{U}_{22}'(0)_{j,} & \mathbf{U}_{21}'(0)_{j,}\end{pmatrix}\left(-M^{-1}+\begin{pmatrix}0 & I\\ 0 & 0\end{pmatrix}\right)\begin{pmatrix}\widetilde{W}(2)\\ \widetilde{V}(2)\end{pmatrix}
\end{aligned} \tag{55}
$$

- ◦ $\langle c_{i,A}c_{j,B}\rangle$

$$
\boxed{c_{i,A}=U_{i,}'(0)\begin{pmatrix}\alpha(0)\\ \left(\alpha^{\dagger}(0)\right)^{\top}\end{pmatrix}=\mathbf{U}_{11}'(0)_{i,}\alpha(0)+\mathbf{U}_{12}'(0)_{i,}\left(\alpha^{\dagger}(0)\right)^{\top}} \tag{56}
$$

$$
\boxed{c_{j,B}=c_{j,B}^{\dagger}=\alpha^{\dagger}(0)\left[\mathbf{U}_{21}'(0)_{j,}\right]^{\dagger}+\alpha^{\top}(0)\left[\mathbf{U}_{22}'(0)_{j,}\right]^{\dagger}} \tag{57}
$$

$$
\begin{aligned}
&\langle c_{i,A} c_{j,B} \rangle \\
&= \left\langle \left\{ \mathbf{U}'_{11}(0)_{i,} \alpha(0) + \mathbf{U}'_{12}(0)_{i,} \left(\alpha^\dagger(0)\right)^\top \right\} \left\{ \alpha^\dagger(0) \left[\mathbf{U}'_{21}(0)_{j,}\right]^\dagger + \alpha^\top(0) \left[\mathbf{U}'_{22}(0)_{j,}\right]^\dagger \right\} \right\rangle \\
&= \begin{pmatrix} \mathbf{U}'_{12}(0)_{i,} & \mathbf{U}'_{11}(0)_{i,} \end{pmatrix} \left\langle \begin{pmatrix} \left(\alpha^\dagger(0)\right)^\top \alpha^\dagger(0) & \left(\alpha^\dagger(0)\right)^\top \alpha^\top(0) \\ \alpha(0)\alpha^\dagger(0) & \alpha(0)\alpha^\top(0) \end{pmatrix} \right\rangle \begin{pmatrix} \left[\mathbf{U}'_{21}(0)_{j,}\right]^\dagger \\ \left[\mathbf{U}'_{22}(0)_{j,}\right]^\dagger \end{pmatrix} \\
&= \begin{pmatrix} \mathbf{U}'_{12}(0)_{i,} & \mathbf{U}'_{11}(0)_{i,} \end{pmatrix} \left(-M^{-1} + \begin{pmatrix} 0 & I \\ 0 & 0 \end{pmatrix} \right) \begin{pmatrix} \left[\mathbf{U}'_{21}(0)_{j,}\right]^\dagger \\ \left[\mathbf{U}'_{22}(0)_{j,}\right]^\dagger \end{pmatrix}
\end{aligned} \tag{58}
$$