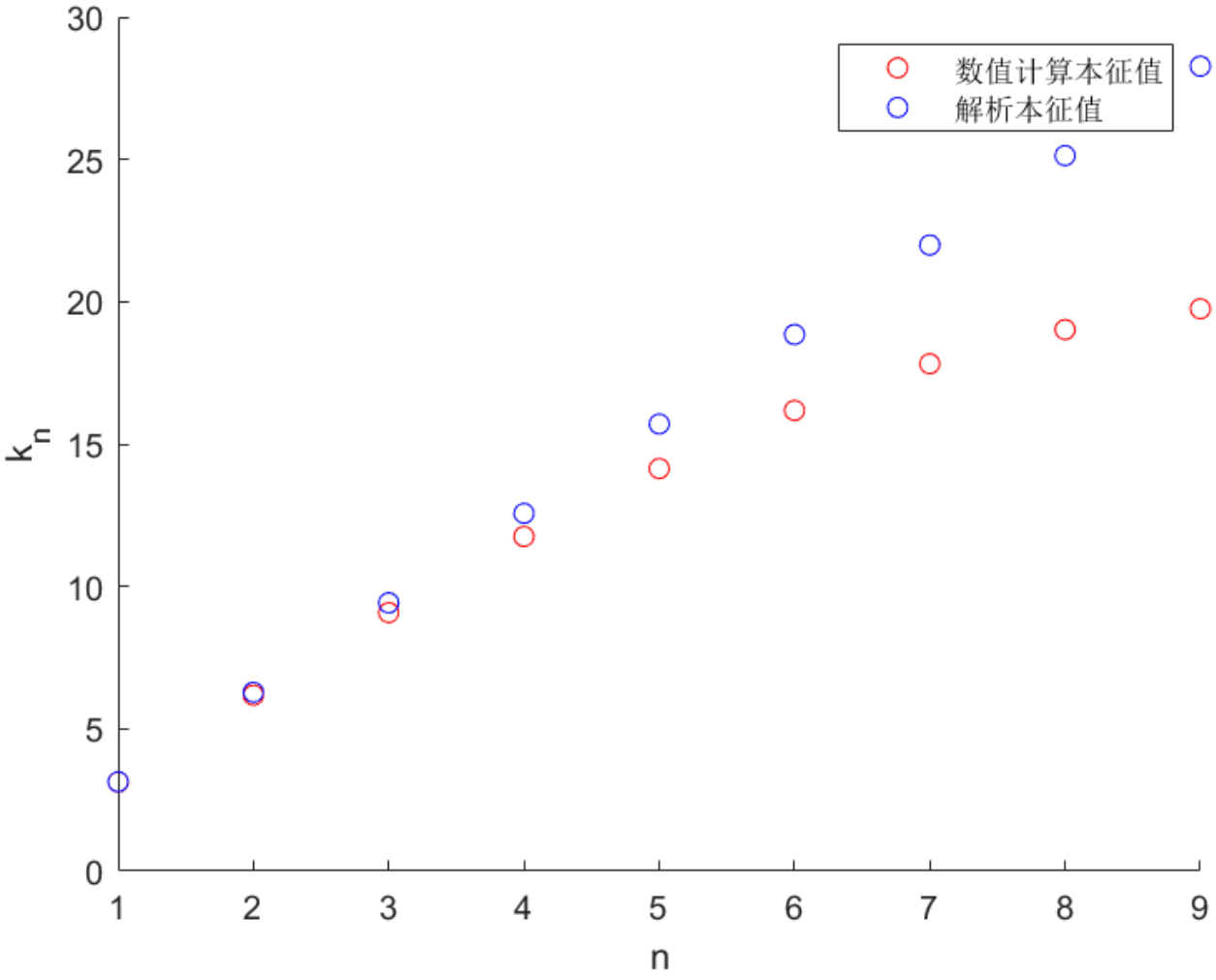
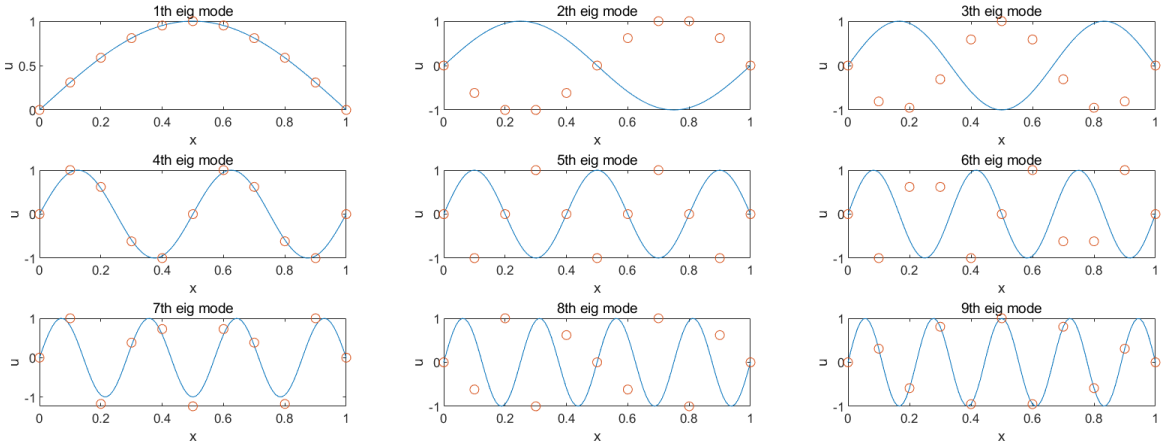
一维弦振动解析解为：

$$k_n = \frac{n\pi}{L},\ \ u_n(x) = \sin\frac{n\pi}{L}x$$

离散数值解（取 $L = 1$）：

$N = 10$，绘图如下：



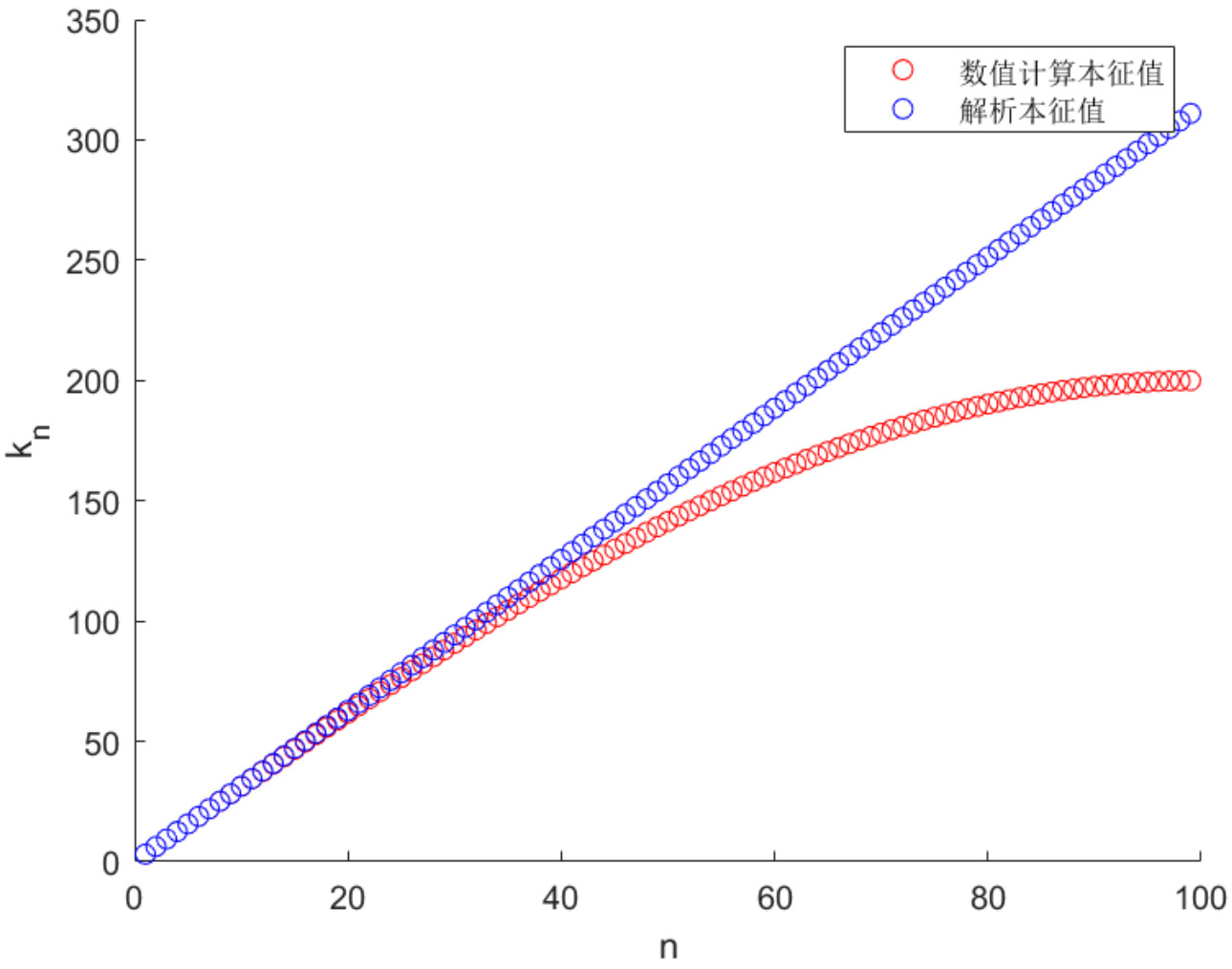

1th eig mode
2th eig mode
3th eig mode
4th eig mode
5th eig mode
6th eig mode
7th eig mode
8th eig mode
9th eig mode
u
x

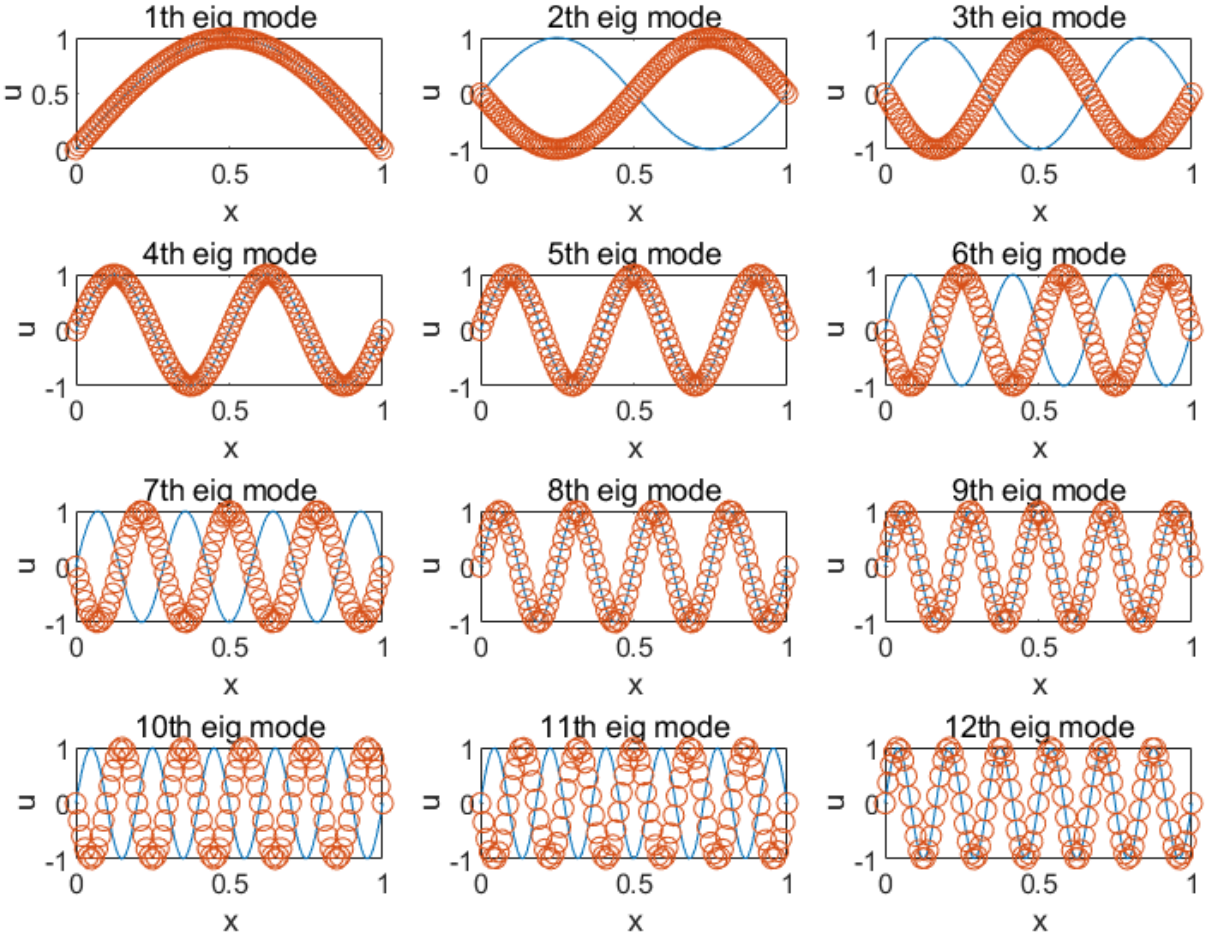
matlab 代码如下：

```
N = 10;
l = 1;
x = linspace(0,l,N);
h = l/N;

M = zeros(N-1, N-1);

for i = 1:N-1
    M(i, i) = -2;
end

for i=1:N-2
    M(i, i+1) = 1;
    M(i+1, i) = 1;
end

M = -M./h^2;

[V, D] = eig(M);

x_1 = 1:N-1;
y_1 = zeros(size(x_1));

for i = 1:N-1
    y_1(i) = sqrt(D(i, i));
end

figure(1)
scatter(x_1, y_1,'red', DisplayName='数值计算本征值');
hold on;
xlabel('n');
ylabel('k_n');
y_4 = x_1*pi/l;
scatter(x_1, y_4, 'blue', DisplayName='解析本征值');
legend;

x_2 = 0:h:l;

figure(2);

for n = 1:9
    subplot(3,3,n);
    x_3 = 0:0.01:l;
    y_3 = sin(n*pi/l*x_3);
    plot(x_3, y_3);
    hold on;
```

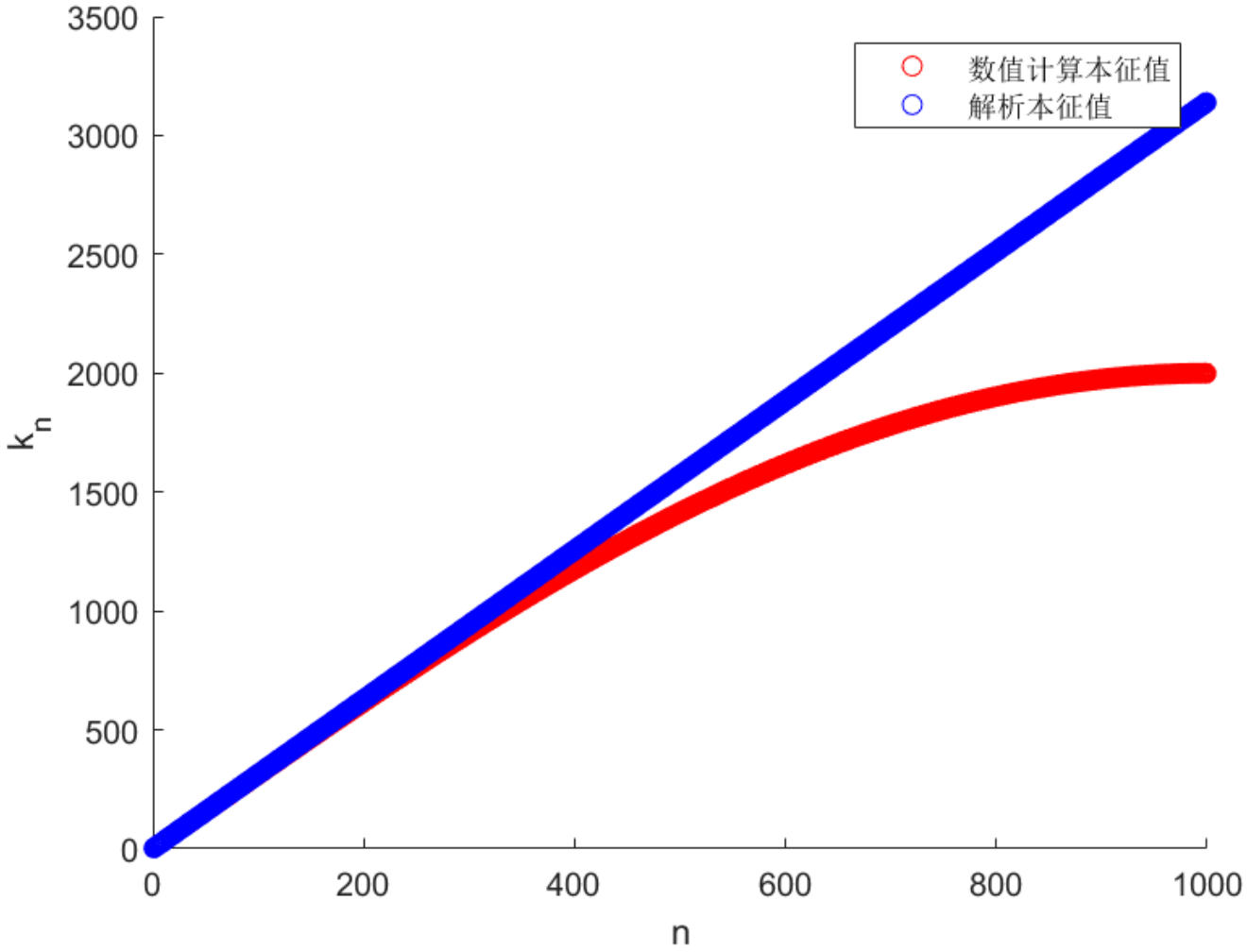
```
    y_2 = V(:,n);
    u = zeros(size(x_2));
    u(1) = 0;
    u(N+1) = 0;
    for i = 1:N-1
        u(i+1) = y_2(i);
    end
    % 整体缩放本征向量
    u = u.*(1/max(u));
    scatter(x_2, u);
    title([num2str(n), 'th eig mode']);
    xlabel('x');
    ylabel('u');
end
```

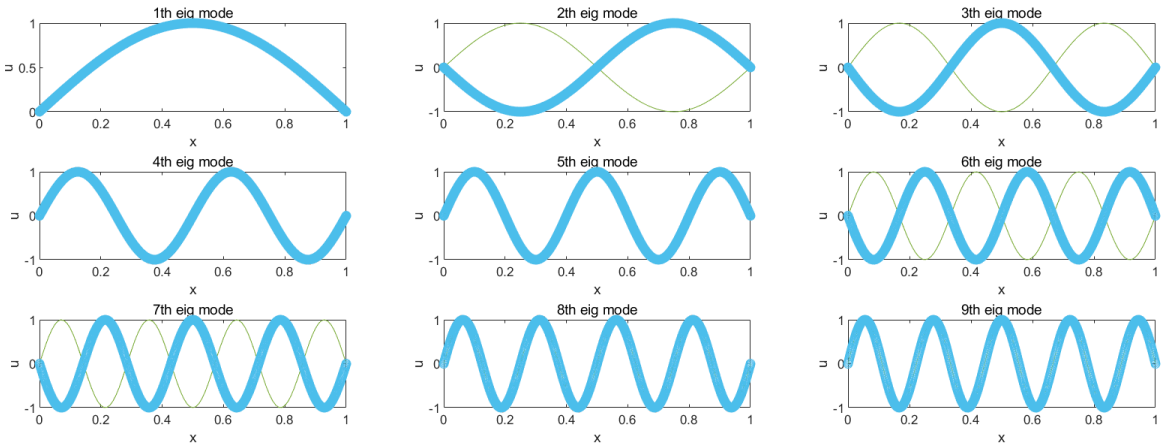
$N = 100$，绘图如下：




1th eig mode
2th eig mode
3th eig mode
4th eig mode
5th eig mode
6th eig mode
7th eig mode
8th eig mode
9th eig mode
10th eig mode
11th eig mode
12th eig mode
u
x

$N = 1000$，绘图如下：


数值计算本征值
解析本征值
k_n
n
0
500
1000
1500
2000
2500
3000
3500
0
200
400
600
800
1000


1th eig mode
2th eig mode
3th eig mode
4th eig mode
5th eig mode
6th eig mode
7th eig mode
8th eig mode
9th eig mode
u
x