### Alignment of stimulus and noise axes

We focus on the feedback operator

whose spectral radius governs the stability of the linear‑rate dynamics. Let

be its (right) eigen‑decomposition, with eigenvectors orthonormal in the inner product. The **slow‑mode axis** is the direction that relaxes back to baseline most slowly after a small perturbation; in discrete time its relaxation constant is

so the slowest mode is .

#### Noise axis

Private noise bypasses and therefore propagates only through the recurrent loop. Its steady‑state covariance is

Because this series weights as , variance is maximal along ; hence the **noise axis** coincides with the slow mode:

#### Stimulus (coding) axis

For a binary stimulus the mean network response is

The discriminant therefore reads

Expanding in the eigenbasis gives

Slow modes () are thus preferentially amplified.  
If, in addition, already points along (i.e.  and ) then

This tuning rule matches the adaptive‑dynamics principle of Chadwick *et al.* (2023): plasticity steers and so that high‑SNR feed‑forward drive excites the slowest recurrent mode.

#### Consequence for our rank‑one construction

In our model both and are rank‑one and chosen so that  
. Consequently is an eigenvector of with eigenvalue , and all of the above conditions are satisfied. Therefore

guaranteeing that the coding axis is perfectly aligned with the dominant noise direction—an essential prerequisite for the optimal‑decoding result in Fig.​ 2E. Appendix B lists the general algebraic conditions for this alignment and shows that arbitrary rank‑one pairs do **not** guarantee it.