### Appendix A  – Optimality of the noise axis

We prove that the Fisher ratio

is maximised at , i.e. when the read‑out vector is **exactly the noise/coding axis**.

#### 1. Geometry of the tuned rank‑one network

After synaptic tuning the stimulus difference and dominant noise direction coincide,

where is the unit eigen‑vector of with the largest eigenvalue. Choose an orthonormal basis and parameterise the read‑out axis as

Because the noise covariance shares the same eigen‑basis, write

with (the slow mode has largest variance) and living in the remaining orthogonal sub‑space.

#### 2. Closed‑form expressions for and

Using the orthogonality relations:

because has no projection onto the “rest’’ sub‑space.

Hence

#### 3. Stationary points of

Differentiate w.r.t. :

Expand and simplify (factor ):

A simpler route is to observe that

which holds for . Only (and the equivalent ) lies in the admissible range and produces a finite, non‑zero numerator.

#### 4. Nature of the stationary point

Compute the second derivative at :

because . Hence attains a **strict local maximum** at . Since is ‑periodic and even, this local maximum is global.

#### 5. Conclusion

The Fisher ratio is maximised when ; that is, the **optimal linear decoder aligns with the noise (slow‑mode) axis**, confirming the result shown empirically in Fig. 2E.