**A diagram of a function

AI-generated content may be incorrect.**

**Figure 2. Alignment of stimulus information with the axis of correlated variability makes the “noise axis’’ the optimal read-out direction.**

**A.** Recurrently connected network with rank-one feed-forward weights (blue), recurrent weights (grey), and linear read-out weights (orange). A stimulus enters through (; independent private noise is injected at each neuron and is shaped only by .

**B.** In a generic rank-one network the *stimulus axis* (blue arrow) can lie at an arbitrary angle to the principal component of baseline activity (PC1, grey dots). However, theory and data show that learning tunes the recurrent weights ​so that their slowest dynamical mode aligns with the feed-forward drive conveyed by (Chadwick et al., 2023). After this tuning, stimulus-evoked activity rotates onto PC1, making the *noise axis* and *coding axis* one and the same.

**C.** Once this alignment is achieved, fluctuations along PC1 decay much more slowly than along any orthogonal mode: power on the slow mode (black curve) persists, whereas power on PC2 (light grey) vanishes rapidly. Assigning the task-relevant input to the slowest decaying eigen-direction enables the circuit to integrate information over time, providing a normative rationale for the recurrent tuning described in B.

**D.** We examine a linear read-out whose axis (orange) forms an angle with the recurrent / noise axis (blue).

**E.** Normalized signal (blue) and noise (grey) variances delivered to the read-out as a function of . Signal power decreases more steeply than noise variance as the read-out is rotated away from the noise axis.

**F.** Fisher discriminability peaks at demonstrating that, once stimulus and noise axes are aligned by tuning, the optimal linear decoder is to read out along the noise axis.

### Rank‑one E–I network and decoding‑axis analysis

#### 

#### Network architecture

We modelled a linear rate network of neurons obeying Dale’s law (80 % excitatory, 20 % inhibitory).  
The population dynamics are

where is a binary stimulus, is private Gaussian noise (), and the time constant is absorbed into the unit‑time scaling ().  
Both the feed‑forward and recurrent weights are **rank‑one**:

, where enforcing Dale’s law.

The feed‑forward driver

ensures that each row of is non‑negative (E) or non‑positive (I).

Because is rank‑one, its spectrum contains a single non‑zero eigenvalue; the corresponding right eigenvector defines the slowest dynamical mode and, as we show below, the dominant noise direction.

#### Steady‑state statistics

For the linear system above the stimulus‑conditioned steady‑state mean and covariance are obtained in closed form:

The signal to be discriminated is the mean difference

### Alignment of stimulus and noise axes

We focus on the feedback operator

whose spectral radius governs the stability of the linear‑rate dynamics. Let

be its (right) eigen‑decomposition, with eigenvectors orthonormal in the inner product. The **slow‑mode axis** is the direction that relaxes back to baseline most slowly after a small perturbation; in discrete time its relaxation constant is

so the slowest mode is .

#### Noise axis

Private noise bypasses and therefore propagates only through the recurrent loop. Its steady‑state covariance is

Because this series weights as , variance is maximal along ; hence the **noise axis** coincides with the slow mode:

#### Stimulus (coding) axis

For a binary stimulus the mean network response is

The discriminant therefore reads

Expanding in the eigenbasis gives

Slow modes () are thus preferentially amplified.  
If, in addition, already points along (i.e.  and ) then

This tuning rule matches the adaptive‑dynamics principle of Chadwick *et al.* (2023): plasticity steers and so that high‑SNR feed‑forward drive excites the slowest recurrent mode.

#### Consequence for our rank‑one construction

In our model both and are rank‑one and chosen so that  
. Consequently is an eigenvector of with eigenvalue , and all of the above conditions are satisfied. Therefore

guaranteeing that the coding axis is perfectly aligned with the dominant noise direction—an essential prerequisite for the optimal‑decoding result in Fig.​ 2E. Appendix B lists the general algebraic conditions for this alignment and shows that arbitrary rank‑one pairs do **not** guarantee it.

#### One‑parameter family of read‑out directions

To quantify performance as a function of mis‑alignment we define

where is any unit vector orthogonal to .  
For each we compute

the signal power, noise variance, and Fisher ratio, respectively.

#### Optimality of the noise axis

Because and are even in , their Maclaurin expansions begin with quadratic and constant terms, respectively.  
Thus decreases faster than as soon as ; the ratio attains its global maximum at , i.e. on the noise/coding axis (Fig. 2E,F).  
A full algebraic proof is given in Appendix A.

#### Numerical implementation

All quantities were evaluated analytically on a uniform grid of 181 angles ().  
Code is provided in **Python;** a fixed random seed (42) renders every panel exactly reproducible.

#### Parameter summary

| Parameter | Value | Description |
| --- | --- | --- |
|  | 120 | network size |
|  | 0.8 | excitatory fraction |
|  | 0.4 | recurrent spectral radius |
|  | 4 | feed‑forward tuning width |
|  | 1 | private‑noise s.d. |
|  | 1 | homogeneous gain |

### 

### 

### Appendix A – When do the stimulus and noise axes coincide?

This appendix demonstrates that, in a linear‑rate network, the stimulus (coding) axis **need not** align with the dominant noise (slow‑mode) axis, and derives the algebraic conditions under which the two directions coincide.

## 1 Network recap and notation

The population dynamics (see Methods) are

where

| Symbol | Meaning |
| --- | --- |
|  | recurrent weight matrix (spectral radius < 1) |
|  | static neuronal gains |
|  | feed‑forward drive vector |
|  | stimulus |
|  | private noise |
|  | time constant (set to 1) |

This is equivalent to

As .

### 1.1 Feedback operator

with eigen-decomposition

* **Noise / slow‑mode axis:** .

### 1.2 Stimulus axis

The stimulus‑conditioned mean is

Hence the **stimulus axis** is

## 2 the Spectral expansion of the stimulus axis

Using the Neumann series ,

Expand in the eigenbasis :

Then

The factor amplifies slower modes ().

### 3 Necessary and sufficient condition for alignment

#### Proposition

The stimulus axis and the noise axis  are collinear  
iff

equivalently .

#### Proof

##### Necessity

Assume .  
Then for some .

From the spectral expansion,

Because , this equality implies for all .  
Thus has no component outside , i.e.

which is exactly (A1).

##### Sufficiency

Assume (A1): with .

* .
* for (orthogonality).

Therefore

∎

## 4 Illustrative two‑unit example

One finds

whose cosine is (angle ). Unequal gains tilt the stimulus axis toward the high‑gain neuron despite the feature‑aligned weight matrices.

## 5 Conclusion

Stimulus–recurrent alignment is **not automatic**.

Appendix B (optimality proof) then shows that, once alignment is achieved, projecting along the common axis maximizes single‑axis Fisher information.

### Appendix B  –  Optimality of the noise axis

We prove that the Fisher ratio

is maximized at , i.e. when the read‑out vector is **exactly the noise/coding axis**.

#### 1. Geometry of the tuned rank‑one network

After synaptic tuning the stimulus difference and dominant noise direction coincide,

where is the unit eigen‑vector of with the largest eigenvalue. Choose an orthonormal basis and parameterise the read‑out axis as

Because the noise covariance shares the same eigen‑basis, write

with (the slow mode has largest variance) and living in the remaining orthogonal sub‑space.

#### 2. Closed‑form expressions for and

Using the orthogonality relations:

because has no projection onto the “rest’’ sub‑space.

Hence

#### 3. Stationary points of

Differentiate w.r.t. :

Expand and simplify (factor ):

A simpler route is to observe that

which holds for . Only (and the equivalent ) lies in the admissible range and produces a finite, non‑zero numerator.

#### 4. Nature of the stationary point

Compute the second derivative at :

because . Hence attains a **strict local maximum** at . Since is ‑periodic and even, this local maximum is global.

#### 5. Conclusion

The Fisher ratio is maximised when ; that is, the **optimal linear decoder aligns with the noise (slow‑mode) axis**, confirming the result shown empirically in Fig. 2E.