CSC282-Main Homework 2

Draco (Yunlong) Xu
yxu103@u.rochester.edu
NetID:yxu103

Abstract

It is for the second Homework of CSC282.

1 2.1

Here we define x_i to be the volume of the mixture you sell to the i-th customer. Then, we could easily know that you sell $\frac{a_i}{a_i+b_i+c_i}x_i$ liters of A to to the i-th customer, $\frac{b_i}{a_i+b_i+c_i}x_i$ liters of B to to the i-th customer, and $\frac{c_i}{a_i+b_i+c_i}x_i$ liters of C to to the i-th customer.
Then the money you make, namely z, equals to $\sum_{i=1}^{n} p_i x_i$, and your mission is to maximize it. Thus, why don't we jump to the formal linear program now?

$$\textbf{Max}\, z = \sum_{i=1}^{n} p_i x_i$$

$$\sum_{i=1}^{n} \frac{a_i x_i}{a_i + b_i + c_i} \leqslant a$$

$$\sum_{i=1}^{n} \frac{b_i x_i}{a_i + b_i + c_i} \leqslant b$$

$$\sum_{i=1}^{n} \frac{c_i x_i}{a_i + b_i + c_i} \leqslant c$$

$$\forall i \in \{1, 2, ..., n\}, x_i \geqslant 0$$

2 2.2

It is really simple, we first have$x_1 = 1 - x_2$, then the constraits would be:
Min z

$$-x_2 - z \leqslant -1$$

$$2x_2 - z \leqslant 0$$

$$x_2, z \geqslant 0$$

Note that here we take z as a variable.
Or we could say that we want to minimize it to the $max\{x_1, 2x_2\} = max\{1 - x_2, 2x_2\}$. Then we solve itand find that $z^* = \frac{2}{3}$ and $x_1^* = \frac{2}{3}$,$x_2^* = \frac{1}{3}$. Here the z^* means the min value and x_1^*,x_2^* denote the coresponsending values of x_1 and x_2.

3 2.3

Something wrong with the question setting, it says that Zofka did around the track and n+1 numbers $a_0, a_1, \ldots, a_{n+1}$. But as you see, here are n+2 numbers.

Let p denote the real initial value of odometer, q denote real lenth of the track. Then the LP would be

Min q

$$\begin{array}{ll} p + iq \geqslant a_i & i \in \{0,1,2,...,n\} \\ p + iq \leqslant a_i + 1 & i \in \{0,1,2,...,n\} \\ p, q \geqslant 0 & \end{array}$$

For the example input, we could see that

Min q

$$\begin{array}{ll} p \geqslant 713 & i \in \{0,1,2,...,n\} \\ p \leqslant 714 & i \in \{0,1,2,...,n\} \\ p + q \geqslant 715 & i \in \{0,1,2,...,n\} \\ p + q \leqslant 716 & i \in \{0,1,2,...,n\} \\ p, q \geqslant 0 & \end{array}$$

After solving it, we see the $q^* = 1$, which shows the correctness.

4 2.4

4.1 A

Min z

$$\begin{array}{ll} z \geqslant ax_i + y_i - b & i \in \{1,2,...,n\} \\ z \geqslant -(ax_i + y_i - b) & i \in \{1,2,...,n\} \\ a, b, z \in R & \end{array}$$

4.2 B

Min $\sum_{i \in \{1,...,n\}} z_i$

$$\begin{array}{ll} z_i \geqslant ax_i + y_i - b & i \in \{1,2,...,n\} \\ z_i \geqslant -(ax_i + y_i - b) & i \in \{1,2,...,n\} \\ \forall i \in \{1,2,...,n\}, a, b, z_i \in R & \end{array}$$

4.3 C

$$\frac{\partial \sum_{i \in \{1,...,n\}} (ax_i + y_i - b)^2}{\partial a}$$
$$= 2a \sum_{i \in \{1,...,n\}} x_i^2 + 2 \sum_{i \in \{1,...,n\}} x_i y_i - 2b \sum_{i \in \{1,...,n\}} x_i$$
$$\frac{\partial \sum_{i \in \{1,...,n\}} (ax_i + y_i - b)^2}{\partial b}$$
$$= -2a \sum_{i \in \{1,...,n\}} x_i - 2 \sum_{i \in \{1,...,n\}} y_i + 2 \sum_{i \in \{1,...,n\}} b$$

We know the min requires that $\frac{\partial \sum_{i \in \{1,...,n\}} (ax_i + y_i - b)^2}{\partial a} = 0$ and $\frac{\partial \sum_{i \in \{1,...,n\}} (ax_i + y_i - b)^2}{\partial b} = 0$
To get a, we do:

$$\frac{\sum_{i \in \{1,...,n\}} x_i}{n} \frac{\partial \sum_{i \in \{1,...,n\}} (ax_i + y_i - b)^2}{\partial b} + \frac{\partial \sum_{i \in \{1,...,n\}} (ax_i + y_i - b)^2}{\partial a}$$

Since we know the sum would be 0, then we have:

$$(-\frac{2}{n} (\sum_{i \in \{1,...,n\}} x_i)^2 + 2 \sum_{i \in \{1,...,n\}} x_i^2) a + (2 \sum_{i \in \{1,...,n\}} x_i y_i - \frac{2}{n} \sum_{i \in \{1,...,n\}} x_i \sum_{i \in \{1,...,n\}} y_i) = 0$$

For the simplicity, let's use $\sum$ to denote $\sum_{i\in\{1,\dots,n\}}$, then we could see:

$$a = -\frac{\sum x_i y_i - \frac{1}{n}\sum x_i \sum y_i}{-\frac{1}{n}(\sum x_i)^2 + \sum x_i^2}$$

Then plug it into the $\frac{\partial \sum (a x_i + y_i - b)^2}{\partial b} = 0$, we would have

$$b = -\frac{1}{n}\sum y_i - \frac{\sum x_i}{n}\frac{\sum x_i y_i - \frac{1}{n}\sum x_i \sum y_i}{-\frac{1}{n}(\sum x_i)^2 + \sum x_i^2}$$