

6. Determination of the liquid viscosity by Stoke's Law

1. Objectives: Understand the concept of fluid viscosity and dimensionless parameters, determination of Reynolds number

2. Theory Viscosity is a property that provides an indication of the resistance to shear within a fluid

Newton's Law for viscous fluid; $F_d = (\mu dv/dy) S$
S - area; of contact Drag force;

$$F_d = C_D \rho A v^2$$

ρ - fluid dynamic velocity

v - velocity of the sphere relative to fluid

r - R of the sphere

There are 3 forces action to sphere into a fluid; F_d , F_b , and mg .
by summing them we have:

$$F_b + F_d = mg$$

$$F_b = \frac{4}{3} \pi R^3 \rho_{fluid} g$$

$$F_b + F_d = mg$$

$$\frac{4}{3} \pi R^3 \rho_{fluid} g + 6 \pi \mu v R = mg$$

Reynold's number: $R_e = \rho v d / \eta$

The buoyant force is defined by Archimedes principle: $p = \frac{4}{3} \pi r^3 \rho g$

ρ - density of liquid
By Newton's second law we have:

$$m(dv/dt) = p_i - p - F_d$$

Terminal velocity: $v_0 = \frac{2}{9} (\rho_1 - \rho) g r^2 / \eta$;

if we compare R (of vessel) with r ;

$$F_c = 6 \pi \eta R \left[1 + 2.4 (r/R) \right]$$

$$\text{Then } \Rightarrow v_0 = \frac{2}{9} g r^2 (\rho_1 - \rho) g r^2 / \eta \left[1 + 2.4 r/R \right]$$

By substitution all formulas we determined the final formula of liquid viscosity

$$\eta = \frac{2}{9} g r^2 (\rho_1 - \rho) g r^2 / v_0 \left[1 + 2.4 (r/R) \right]$$

Stoke's Law valid for $R_e < 0.5$;

$$\text{where } R_e = 2 v_0 r g / \eta$$

3 Equipment: Two transparent vessels with castor oil and glycerin for measurement we use steel spheres.

4. Experimental procedure:

N	1	2	3	4	5
D_1	1,52	1,58	1,16	1,45	1,54
D_2	1,51	1,63	1,17	1,46	1,56
D_3	1,5	1,59	1,15	1,48	1,54
D_4	1,49	1,61	1,13	1,51	1,57
D_5	1,52	1,6	1,16	1,49	1,56
D_6	1,52	1,62	1,14	1,47	1,55
D_7	1,49	1,61	1,13	1,48	1,55
\bar{D}_5	15,46	21,25	59,73	28,35	26,48

Table 1
D in mm

N	1	2	3	4	5
$\angle \Gamma$	0,7331	0,8028	0,5755	0,7396	0,7764
$\Delta \Gamma$	0,0069	0,0128	0,0045	0,0146	0,0039
U_0	0,0174	0,0207	0,0086	0,0155	0,0166
η	0,5402	0,5188	0,6258	0,584	0,6033

Table 2.

$\angle \Gamma$ and $\Delta \Gamma$ in mm

U_0 in m/s

η in Pa · s

$$\angle \Gamma_n = \frac{\angle D_n}{2} ; \Delta \Gamma = (\angle \Gamma - \Gamma_L) ; l = 44 \text{ cm}$$

$$\angle \Gamma_n = \frac{\sum D_n}{14} ; U_0 = \frac{L}{t} ; \eta = \frac{2}{9} g r^2 \frac{P_2 - P_1}{U_0} \left[1 + 2,4 \frac{\Gamma}{R} \right]$$

$$R = 2 \text{ cm} ; P_1 = 7,8 \text{ g/cm}^3 ; P_2 = 9,96 \text{ g/cm}^3$$

$$\langle \eta \rangle = \frac{0,5402 + 0,5188 + 0,6258 + 0,584 + 0,6033}{5} = 0,57442 \quad \rho_a \cdot S$$

Now we need to work out Reynold's number,

$$R_E = \frac{2U_0 r \cdot \rho}{\eta}$$

$$R_{E1} = \frac{2 \cdot 0,174 \cdot 0,7531 \cdot 10^{-3} \cdot 960}{0,5402} = 0,0465$$

$$R_{E2} = \frac{2 \cdot 0,0207 \cdot 0,8028 \cdot 10^{-3} \cdot 960}{0,5188} = 0,0615$$

$$R_{E3} = \frac{2 \cdot 0,0086 \cdot 0,5755 \cdot 10^{-3} \cdot 960}{0,6258} = 0,0151$$

$$R_{E4} = \frac{2 \cdot 0,0155 \cdot 0,7336 \cdot 10^{-3} \cdot 960}{0,584} = 0,0376$$

$$R_{E5} = \frac{2 \cdot 0,0166 \cdot 0,7764 \cdot 10^{-3} \cdot 960}{0,6033} = 0,041$$

5. Conclusion; In this Laboratory work, have determined the liquid viscosity by Stoke's law. Finally I get average value as 0,57442. from 5 measuring. Then, calculated the Reynold's number to finish the experiment. if we'll compare my result with $\eta = 0,985$ $\rho_a \cdot S$, had got much smaller value. They differ by 70% per cent from each other.