Kernel-Based Object Tracking
Additional Slides with formulas

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16.07.2014

Similarity Function

A distance function between two distributions :

$$d(\hat{p}_u, \hat{q}_u) = \sqrt{1 - \hat{\rho}(\hat{p}_u, \hat{q}_u)}$$

where $\hat{\rho}(\hat{p}_u, \hat{q}_u) = \sum_{u=1}^{m} \sqrt{\hat{p}_u \hat{q}_u}$ is the sample estimate of Bhattacharrya coefficient.

$d(y)$ is a metric I

$$d(\hat{p}_u, \hat{q}_u) = \sqrt{1 - \hat{\rho}(\hat{p}_u, \hat{q}_u)}$$

1. $\rho(\hat{p}, \hat{q}) = \sum_{u=1}^{m} \sqrt{\hat{p}_u \hat{q}_u} = \sum_{u=1}^{m} \hat{p}_u \sqrt{\frac{\hat{q}_u}{\hat{p}_u}} \leq \sqrt{\sum_{u=1}^{m} \hat{q}_u} = 1$

   Equality holds iff $\hat{p} = \hat{q}$. Therefore, $d(\hat{p}_u, \hat{q}_u)$ exist for all discrete distributions $\hat{p}_u$ and $\hat{q}_u$, is positive, symmetric and equal to zero iff $\hat{p} = \hat{q}$.

$d(y)$ is a metric II

2. Triangular inequality.
Consider three discrete distributions $\hat{p}$,$\hat{q}$ and $\hat{r}$, associated with the points $\xi_p = (\sqrt{\hat{p}_1}, \ldots \sqrt{\hat{p}_m})^T$, $\xi_q = (\sqrt{\hat{q}_1}, \ldots \sqrt{\hat{q}_m})^T$ and $\xi_r = (\sqrt{\hat{r}_1}, \ldots \sqrt{\hat{r}_m})^T$ on the unit hypersphere.

$$d(\hat{p}, \hat{r}) + d(\hat{r}, \hat{q}) \geq d(\hat{p}, \hat{q})$$

$$\sqrt{1 - \cos(\xi_p, \xi_r)} + \sqrt{1 - \cos(\xi_r, \xi_q)} \geq \sqrt{1 - \cos(\xi_p, \xi_q)}$$

If we fix points ξ_p and $\xi_{q'}$ and angle between ξ_p and ξ_r, the left side of inequality is minimized when the vectors ξ_p, ξ_q and ξ_r lie in the same plane. So we reduce the problem to a 2-dimensional problem, where triangular inequality is easy to be proofed.

Target Localization I

The Problem of Target Localization is equal to Minimization of $d(\hat{p}_u(y), \hat{q}_u)$ as function of y OR Maximization of Bhattacharrya coefficient $\hat{\rho}(\hat{p}_u(y), \hat{q}_u)$.

Let $\hat{y}_0$ be a target location in the previous frame. To localize target in current frame first calculate $\hat{p}_u(\hat{y}_0)$.

$$\hat{p}_u(\hat{y}_0) = C_h \sum_{i=1}^{n_h} k\left(\left\|\frac{\hat{y}_0 - x_i}{h}\right\|^2\right)\delta[b(x_i) - u]$$

Target Localization II

Taylor expansion of Bhattacharrya coefficient around $\hat{p}_u(\hat{y}_0)$:
$\Rightarrow$ Linear approximation of Bhattacharrya coefficient:

$$\hat{\rho}[\hat{p}(y), \hat{q}] \approx \frac{1}{2} \sum_{u=1}^{m} \sqrt{\hat{p}_u(\hat{y}_0)\hat{q}_u} + \frac{1}{2} \sum_{u=1}^{m} \hat{p}_u(y) \sqrt{\frac{\hat{q}_u}{\hat{p}_u(\hat{y}_0)}}$$

Target Localization III

$$\hat{\rho}[\hat{p}(y), \hat{q}] \approx \frac{1}{2}\sum_{u=1}^{m}\sqrt{\hat{p}_u(\hat{y}_0)\hat{q}_u} + \frac{1}{2}\sum_{u=1}^{m}\hat{p}_u(y)\sqrt{\frac{\hat{q}_u}{\hat{p}_u(\hat{y}_0)}}$$

$+$

$$\hat{p}_u(y) = C_h \sum_{i=1}^{n_h} k\left(\left\|\frac{y - x_i}{h}\right\|^2\right)\delta[b(x_i) - u]$$

$$\hat{\rho}[\hat{p}(y), \hat{q}] \approx \frac{1}{2}\sum_{u=1}^{m}\sqrt{\hat{p}_u(\hat{y}_0)\hat{q}_u} + \frac{C_h}{2}\sum_{u=1}^{m} w_i k\left(\left\|\frac{y - x_i}{h}\right\|^2\right)$$

where $w_i = \sum_{u=1}^{m}\sqrt{\frac{\hat{q}_u}{\hat{p}_u(\hat{y}_0)}}\delta[b(x_i) - u]$.

Target Localization IV

$$\hat{\rho}[\hat{p}(y), \hat{q}] \approx \frac{1}{2} \sum_{u=1}^{m} \sqrt{\hat{p}_u(\hat{y}_0)\hat{q}_u} + \frac{C_h}{2} \sum_{u=1}^{m} w_i k\left(\left\| \frac{y - x_i}{h} \right\|^2\right)$$

where $w_i = \sum_{u=1}^{m} \sqrt{\frac{\hat{q}_u}{\hat{p}_u(\hat{y}_0)}} \delta[b(x_i) - u]$.
Maximize the second term, because the first term is independent on y:

$$\hat{y}_1 = \frac{\sum_{i=1}^{n_h} x_i w_i g\left(\left\| \frac{y - x_i}{h} \right\|^2\right)}{\sum_{i=1}^{n_h} w_i g\left(\left\| \frac{y - x_i}{h} \right\|^2\right)}$$

where $g(x) = -k'(x)$, assumed that $k'(x)$ exist almost everywhere.

References I

[1] D. Comaniciu, V. Ramesh, and P. Meer. Kernel-based object tracking. *IEEE Computer Society*, 25:564–577, 2003.