Application of graph matching in Computer Vision

Master Seminar

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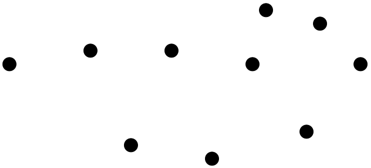
Agenda

1. Graph matching
2. 2LevelGM
3. Evaluation

Attributed undirected graph I

Attributed undirected graph $G = (V,$

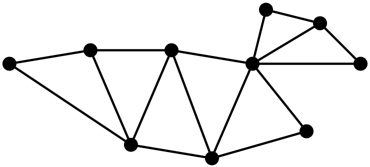
- set of nodes $V = \{v_i\}_{i=1}^n$



Attributed undirected graph II

Attributed undirected graph $G = (V, E,$

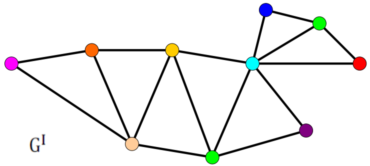
- set of nodes $V = \{v_i\}_{i=1}^{n}$
- set of edges $E \subseteq \{\{u, v\} | u, v \in V\}$



Attributed undirected graph

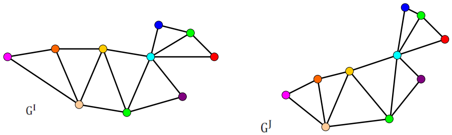
Attributed undirected graph $G = (V, E, D)$

- set of nodes $V = \{v_i\}_{i=1}^{n}$
- set of edges $E \subseteq \{\{u, v\} | u, v \in V\}$
- node attributes $D = \{d_i\}_{i=1}^{n}$

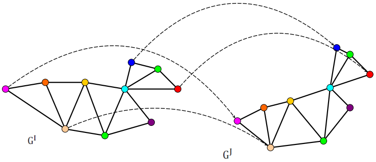


Graph matching

Let us consider two undirected attributed graphs $G^I = (V^I, E^I, D^I)$ and $G^J = (V^J, E^J, D^J)$:



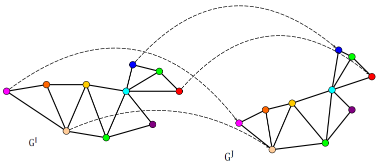
Graph matching



A matching function between G^I and G^J is a mapping

$$m : V^I \to V^J$$

Graph matching

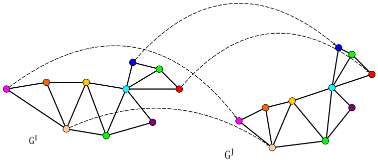


A matching function between G^I and G^J is a mapping

$$m : V^I \to V^J$$

not unique!

Graph matching



A matching function between G^I and G^J is a mapping

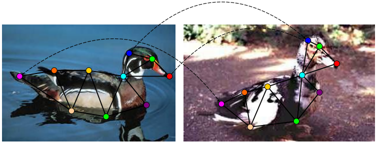
$$m : V^I \rightarrow V^J$$

Define a function $S(G^I, G^J, m)$ to measure the quality of matching m that fulfills some conditions

$\Rightarrow$ Graph matching problem between G^I and G^J

$$m = \underset{\hat{m}}{\operatorname{argmax}}\, S(G^I, G^J, \hat{m})$$

Graph matching in computer vision

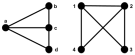


- image matching
- shape matching
- object detection
- object tracking
- ...

Exact graph matching I

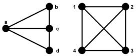
Edge preserving mapping m: $\{v_i, v_{i'}\} \in E^I \Rightarrow \{m(v), m(v_{i'})\} \in E^J$

- mapping m is bijective $\rightarrow$ graph isomorphism (GI)



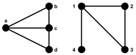
$a \mapsto 1 \quad b \mapsto 4 \quad c \mapsto 2 \quad d \mapsto 3$

- mapping m is injective $\rightarrow$ graph monomorphism



$a \mapsto 1 \quad b \mapsto 4 \quad c \mapsto 2 \quad d \mapsto 3$

- mapping m is total $\rightarrow$ graph homomorphism



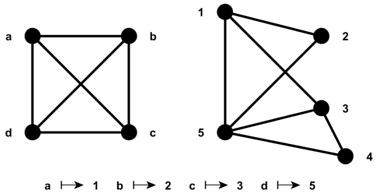
$a \mapsto 1 \quad b \mapsto 3 \quad c \mapsto 2 \quad d \mapsto 3$

NP complete (except GI) [9]

Exact graph matching II

Exact graph matching:

- too strict
- time/memory consuming
- cannot handle object deformation



Inexact graph matching I

$$m = \operatorname*{argmax}_{\hat{m}} S(G^I, G^J, \hat{m})$$

- second-order (edge) similarity $s_E(e_{ii'}, e_{jj'})$, $e_{ii'} \in E^I, e_{jj'} \in E^J$
- first-order (node) similarity $s_V(v_i, v_j)$, $v_i \in V^I, v_j \in V^J$

$$S(G^I, G^J, m) = \sum_{\substack{m(v_i)=v_j \\ m(v_i')=v_j'}} s_E(e_{ii'}, e_{jj'}) + \sum_{m(v_i)=v_j} s_V(v_i, v_j)$$

- Assignment matrix $x \in \{0,1\}^{n_1 \times n_2}$, $x_{ij} = 1 \iff m(v_i) = v_j$

Inexact graph matching II

The most common problem formulation:

Quadratic Assignment Problem (NP complete) [3]

$$x^* = \arg\max \sum_{\substack{x_{ij}=1 \\ x_{i'j'}=1}} s_E(e_{ii'}, e_{jj'}) + \sum_{x_{ij}=1} s_V(v_i, v_j)$$

$$s.t. \begin{cases} x \in \{0,1\}^{n_1 n_2} \\ \sum_{i=1}^{n_1} x_{ij} \leq 1 \\ \sum_{j=1}^{n_2} x_{ij} \leq 1 \end{cases}$$

Using matrix notation : $\arg\max_x x^T S x$, $S-$similarity (or affinity) matrix

Inexact graph matching III

Solution techniques [8]

- discrete optimization
  - tree search [2, 21, 22, 25]
  - simulated annealing [11]
- continuous optimization
  - constraint relaxation [10, 14, 15, 24, 26]
  - spectral methods [13, 23]
  - probabilistic frameworks [1, 12, 16, 20]
  - clustering [4, 6, 19, 17]

Drawback of the existing algorithms

- most of them were developed for matching relative small graphs ($\sim$ 100 nodes)
- scale badly due to the polynomial increase of time and storage demand
- algorithms for the big graphs use another formulation of the graph matching optimization problem

  $P = \mathrm{argmin}_{\hat{P} \in \Pi_n} \|A^I - \hat{P} A^J \hat{P}^T\|^2 + \|D^I - \hat{P} D^J\|_2^2$

Complexity reduction

$$x^* = \arg\max x^T S x$$

$$s.t. \begin{cases} x \in \{0,1\}^{n_1 n_2} \\ \sum_{i=1}^{n_1} x_{ij} \leq 1 \\ \sum_{j=1}^{n_2} x_{ij} \leq 1 \end{cases}$$

- set of candidate correspondences
- sparse affinity matrix
- subdivide problem into a set of smaller subproblems

Complexity reduction

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Complexity reduction

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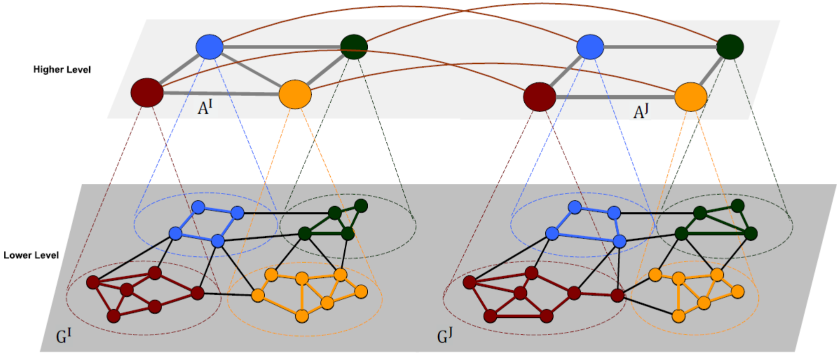
- set of candidate correspondences
- sparse affinity matrix
- subdivide problem into a set of smaller subproblems $\leftarrow$
  Similar works:
  - semisupervised case [17]
  - another objective function [4, 19]
  - special kind of subproblem [19, 18]

Two level graph matching framework

Lower level: initial graphs G^I, G^J

Higher level: simplified graphs (anchor graphs A^I, A^J)



Anchor graph construction

Goal: $G^I = (V^I, E^I, D^I) \rightarrow A^I = (V^{Ia}, E^{Ia}, U^{Ia})$
Equivalent: partitioning of $G^I \supset (G^I_1 \cup \cdots \cup G^I_{|V^{Ia}|})$
Done by:

- grid with r rows and c columns
- graph coarsening algorithms: Heavy Edge Matching (HEM) and Light Edge Matching (LEM)

Anchor graph and subgraph matching

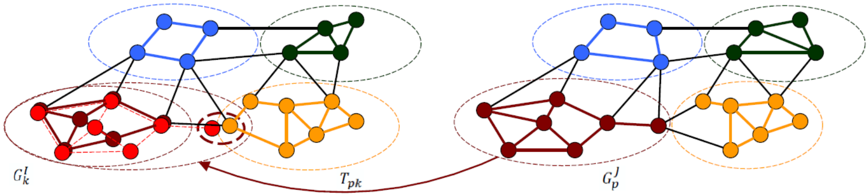
Find correspondences between two anchor graphs $A^I = (V^{Ia}, E^{Ia}, U^{Ia})$ and $A^J = (V^{Ja}, E^{Ja}, U^{Ja})$

- edge similarity: compare length of the edges beween anchors
- node similarity:
  - score of the matching of G_k^I and G_p^J
  - define anchor attributes based on the D^I, D^J and/or on the geometry of G^I, G^J

Match anchor graphs and subgraph using some existing algorithm (e.g. RRWM [7])

Graph partition update

1. estimate an affine transformation between matched subgraphs based on the provided local correspondences (point set registration problem)
2. let nodes "vote" to which subgraphs they should belong to

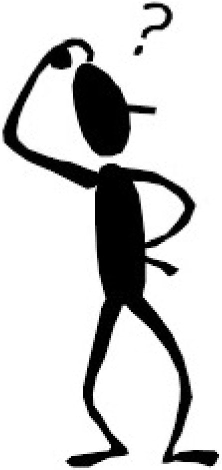


Complexity

- size of the anchor graphs and subgraphs
- number of iterations

The end

Thank you for your attention!



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