Graph Matching Framework

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This notes are a short description of graph matching model.

Contents

1 Problem statement

Consider two undirected weighted graphs $G^I = (V^I, E^I, A^I)$ and $G^J = (V^J, E^J, A^J)$, where V, E, A denote set of nodes, set of edges and set of node attributes respectively. We assume situation, where $|V^I| = n_1$, $|V^J| = n_2$ and n_1 is not necessary equal to n_2.
The aim of graph matching is to find a subset of possible node correspondences, which maximizes the similarity value between two graphs. Such subset can be represented by a binary vector $x \in \{0,1\}^{n_1 n_2}$, where $x_{(j-1)n_1+i} = 1$, if node $v_i \in V^I$ is matched to node $u_j \in V^J$, and $x_{(j-1)n_1+i} = 0$ otherwise. For simplification we will write further x_{ij} and meaning $x_{(j-1)n_1+i}$.
To measure similarity between graphs we define two similarity functions: *nodes similarity function* (first-order similarity) $s_V(v_i, u_j)$, $v_i \in V^I, u_j \in V^J$ and *edge similarity function* (second-order similarity) $s_E(e_{ii'}, e_{jj'})$, $e_{ii'} \in E^I, e_{jj'} \in E^J$ (see [1], [3], [2]). Both functions can be combined in one *similarity matrix* $S \in \mathbb{R}^{n_1 n_2 \times n_1 n_2}$, whose diagonal elements are $s_V(v_i, u_j)$ and non-diagonal elements are $s_E(e_{ii'}, e_{jj'})$.
Using this notation one can formulate *one-to-one graph matching problem* as an quadratic optimization problem:

$$\operatorname*{argmin}_x x^T S x \tag{1}$$

$$\text{s.t. } x \in \{0,1\}^{n_1 n_2} \tag{2}$$

$$\sum_{i=1 \ldots n_1} x_{ij} = 1 \tag{3}$$

$$\sum_{j=1 \ldots n_2} x_{ij} = 1 \tag{4}$$

In case when $n_1 \neq n_2$, we require one of the conditions (3) or (4) to be fulfilled according to, which graph is bigger.

2 Approach

2.1 Lower Level Graph Construction

2.2 Higher Level Graph Construction

2.3 Matching Algorithm

2.4 Connection between two levels

References

[1] Minsu Cho and Olivier Duchenne. Finding Matches in a Haystack : A Max-Pooling Strategy for Graph Matching in the Presence of Outliers. *Proceedings of the IEEE Computer Society Conference on Computer Vision and Pattern Recognition*, 2014.

[2] Minsu Cho, Jungmin Lee, and Kyoung Mu Lee. Reweighted Random Walks for Graph Matching. *ECCV*, 2010.

[3] Minsu Cho and Kyoung Mu Lee. Progressive graph matching: Making a move of graphs via probabilistic voting. *Proceedings of the IEEE Computer Society Conference on Computer Vision and Pattern Recognition*, 2012.