Application of graph matching in Computer Vision

Master Seminar

Ekaterina Tikhoncheva

University of Heidelberg
Faculty of Mathematics and Computer Science
Computer Vision group
at
Heidelberg Collaboratory for Image Processing

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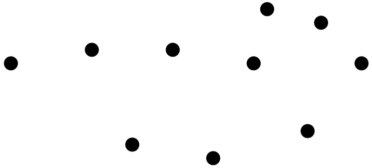
Agenda

Attributed undirected graph I

Attributed undirected graph $G = (V,$

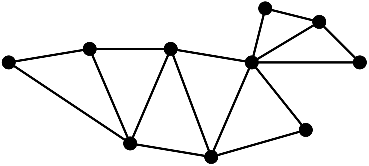
- set of nodes $V = \{v_i\}_{i=1}^{n}$



Attributed undirected graph II

Attributed undirected graph $G = (V, E,$

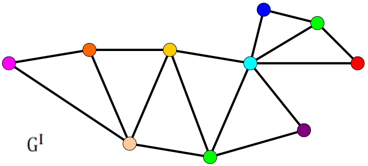
- set of nodes $V = \{v_i\}_{i=1}^n$
- set of edges $E \subseteq \{\{u, v\} | u, v \in V\}$



Attributed undirected graph

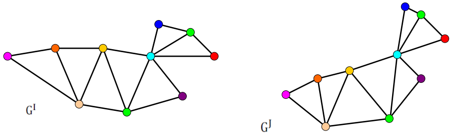
Attributed undirected graph $G = (V, E, D)$

- set of nodes $V = \{v_i\}_{i=1}^n$
- set of edges $E \subseteq \{\{u, v\} | u, v \in V\}$
- node attributes $D = \{d_i\}_{i=1}^n$, $D \subset \mathbb{R}^r$

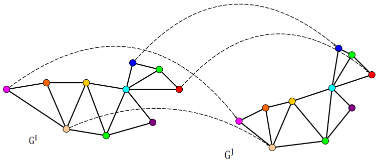


Graph matching

Let us consider two undirected attributed graphs $G^I = (V^I, E^I, D^I)$ and $G^J = (V^J, E^J, D^J)$:



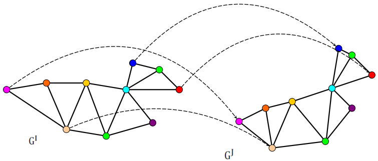
Graph matching



A matching function between G^I and G^J is a mapping

$$m : V^I \to V^J$$

Graph matching

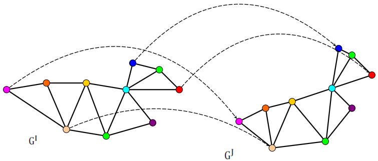


A matching function between G^I and G^J is a mapping

$$m : V^I \to V^J$$

not unique!

Graph matching



A matching function between G^I and G^J is a mapping

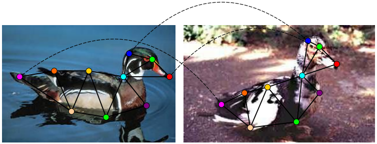
$$m : V^I \to V^J$$

Define a function $S(G^I, G^J, m)$ to measure the quality of matching m that fulfills some constraints

$\Rightarrow$ Graph matching problem between G^I and G^J

$$m = \operatorname*{argmax}_{\hat{m}} S(G^I, G^J, \hat{m})$$

Graph matching in computer vision



- image matching
- shape matching
- object detection
- object tracking
- ...

Graph matching

A matching function between G^I and G^J is a mapping

$$m : V^I \to V^J$$

Graph matching problem between G^I and G^J

$$m = \operatorname*{argmax}_{\hat{m}} S(G^I, G^J, \hat{m})$$

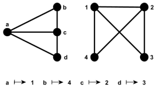
Depending on the required properties of a matching one distinguishes

- exact graph matching
- inexact graph matching

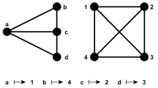
Exact graph matching I

Edge preserving mapping m: $\{v_i, v_{i'}\} \in E^I \Rightarrow \{m(v), m(v_{i'})\} \in E^J$

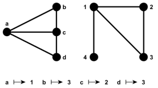
- mapping m is bijective $\rightarrow$ graph isomorphism (GI)



- mapping m is injective $\rightarrow$ graph monomorphism



- mapping m is total $\rightarrow$ graph homomorphism

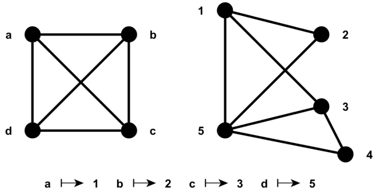


NP complete (except GI) [Garey and Johnson, 1979]

Exact graph matching II

Exact graph matching:

- too strict
- cannot handle object deformation
- time/memory consuming



Inexact graph matching I

Introduce similarity measure between nodes/edges in the graphs

$$m = \operatorname*{argmax}_{\hat{m}} S(G^I, G^J, \hat{m})$$

$$S(G^I, G^J, m) = \sum_{\substack{m(v_i)=v_j \\ m(v_i')=v_j'}} s_E(e_{ii'}, e_{jj'}) + \sum_{m(v_i)=v_j} s_V(v_i, v_j)$$

- second-order (edge) similarity $s_E(e_{ii'}, e_{jj'})$, $e_{ii'} \in E^I, e_{jj'} \in E^J$
- first-order (node) similarity $s_V(v_i, v_j)$, $v_i \in V^I, v_j \in V^J$
- assignment matrix $X \in \{0,1\}^{n_1 \times n_2}$, $X_{ij} = 1 \iff m(v_i) = v_j$, $x = \mathrm{vec}(X)$

Inexact graph matching II

The most common problem formulation:

Quadratic Assignment Problem (NP complete) [Burkard et al., 1998]

$$x^* = \arg\max \sum_{\substack{x_{ij}=1 \\ x_{i'j'}=1}} s_E(e_{ii'}, e_{jj'}) + \sum_{x_{ij}=1} s_V(v_i, v_j)$$

$$s.t. \begin{cases} x \in \{0,1\}^{n_1 n_2} \\ \sum_{i=1}^{n_1} x_{ij} \leq 1 \\ \sum_{j=1}^{n_2} x_{ij} \leq 1 \end{cases}$$

Using matrix notation : $\arg\max_x x^T S x$, S is a similarity (or affinity) matrix

Inexact graph matching III

Solution techniques [Conte et al., 2004]

- discrete optimization
  - tree search [Bunke and Allermann, 1983, Shapiro and Haralick, 1981, Tsai and Fu, 1979, Wang et al., 1995]
  - simulated annealing [Hérault et al., 1990]
- continuous optimization
  - constraint relaxation [Gold and Rangarajan, 1996, Leordeanu et al., 2009, Lu et al., 2012, Vogelstein et al., 2011/14, Zaslavskiy et al., 2009]
  - spectral methods [Leordeanu and Hebert, 2005, Umeyam, 1988]
  - probabilistic frameworks [Armiti and Gertz, 2014, Kittler and Hancock, 1989, Luo and Hancock, 2001, Sanromà et al., 2012]
  - clustering [Carcassoni and Hancock, 2003, Cho et al., 2009, Qiu and Hancock, 2006, Lyzinski et al., 2011/14]

Drawback of the existing algorithms

- most of the algorithms are developed for matching relatively small graphs (~ 150 nodes)
- scale badly due to the polynomial increase of time and storage demand
- algorithms for the big graphs use another formulation of the graph matching optimization problem

$$X^* = \operatorname{argmin}_X \|A^I - XA^J X^T\|^2 + \|D^I - XD^J\|_2^2$$

Aim of the master's thesis

- a novel framework that should help to extend the usability of existing graph matching algorithms to bigger graphs

Idea:
subdividing initial problem into a set of smaller problems, which can be easily handled with existing algorithms
$\Rightarrow$ a variant of the well-known divide-and-conquer paradigm

Complexity reduction

$$x^* = \arg\max x^T S x$$

$$s.t. \begin{cases} x \in \{0,1\}^{n_1 n_2} \\ \sum_{i=1}^{n_1} x_{ij} \leq 1 \\ \sum_{j=1}^{n_2} x_{ij} \leq 1 \end{cases}$$

- set of candidate correspondences [Cho and Lee, 2012]
- sparse affinity matrix
- subdivide problem into a set of smaller subproblems

Complexity reduction

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- subdivide problem into a set of smaller subproblems ←

Complexity reduction

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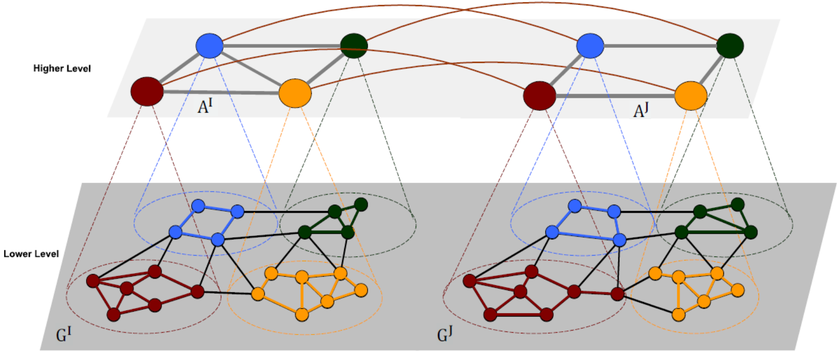
- set of candidate correspondences [Cho and Lee, 2012]
- sparse affinity matrix
- subdivide problem into a set of smaller subproblems $\leftarrow$
  Similar works:
  - semisupervised case [Lyzinski et al., 2011/14]
  - another objective function [Lyzinski et al., 2011/14, Carcassoni and Hancock, 2003, Qiu and Hancock, 2006]
  - special kind of subproblem [Qiu and Hancock, 2006, Nie et al., 2015]

Two level graph matching framework

Lower level: initial graphs G^I, G^J
Higher level: simplified graphs (anchor graphs A^I, A^J)



Anchor graph construction

Goal: $G^I = (V^I, E^I, D^I) \rightarrow A^I = (V^{Ia}, E^{Ia}, U^{Ia})$
Equivalent: partitioning of $G^I \supset (G_1^I \cup \cdots \cup G_{|V^{Ia}|}^I)$
Done by:

- grid with r rows and c columns
- graph coarsening algorithms: Heavy Edge Matching (HEM) and Light Edge Matching (LEM)

Anchor graph and subgraph matching

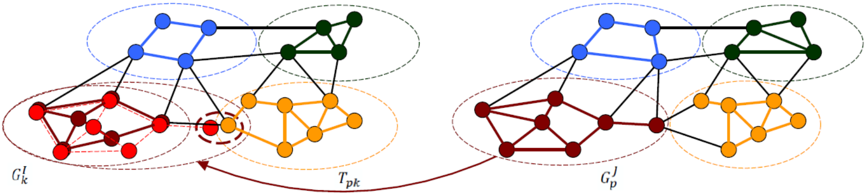
Goal: find correspondences between two anchor graphs $A^I = (V^{Ia}, E^{Ia}, U^{Ia})$ and $A^J = (V^{Ja}, E^{Ja}, U^{Ja})$

- edge similarity: compare length of the edges beween anchors
- node similarity:
  - score of the matching of G_k^I and G_p^J
  - define anchor attributes based on the D^I, D^J and/or on the geometry of G^I, G^J

Match anchor graphs and subgraph using some existing algorithm (e.g. RRWM [Cho et al., 2010])

Graph partition update

1. estimate an affine transformation between matched subgraphs (point set registration problem)
2. let nodes "vote" to which subgraphs they should belong to



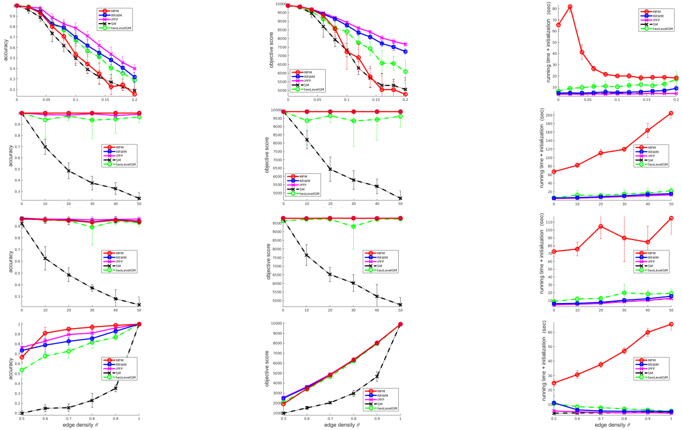
Evaluation

Two ways of evaluation have been used

- synthetic data
- real data

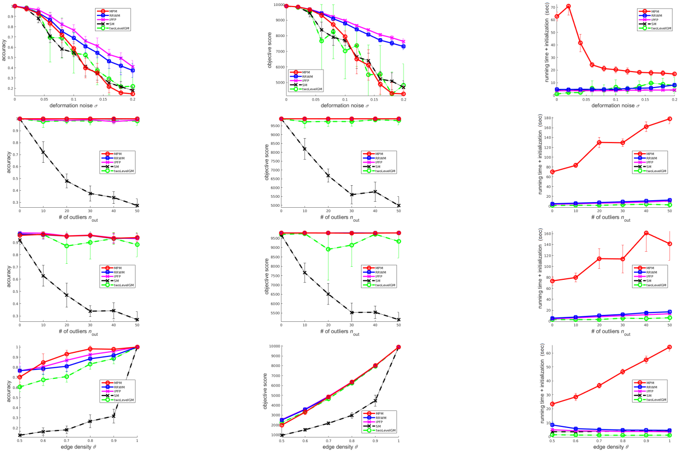
The quality is measured by the matching score and accuracy of an obtained solution together with the running time in seconds needed to find it.

Synthetic data I



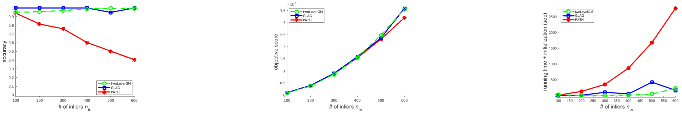
Performance of the 2LevelGM with non-attributed anchor graphs

Synthetic data II

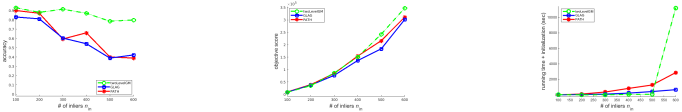


Performance of the 2LevelGM with attributed anchor graphs

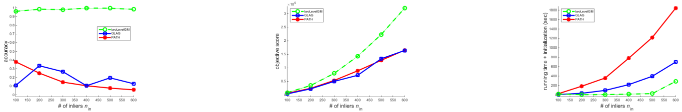
Synthetic data III



graph isomorphism



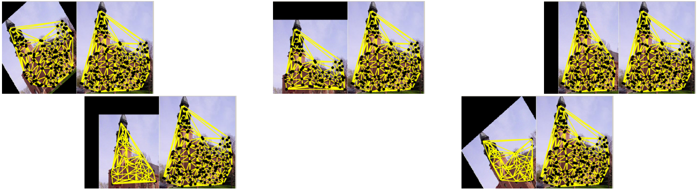
deformation noise $\sim \mathcal{N}(0, 0.03)$

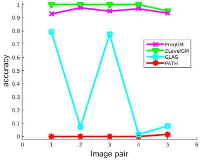


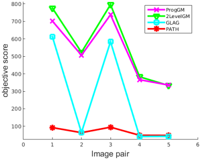
omit 90% of edges

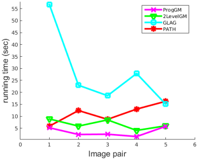
Comparison of 2LevelGM, GLAG and PATH on bigger graphs

Image affine transformation I



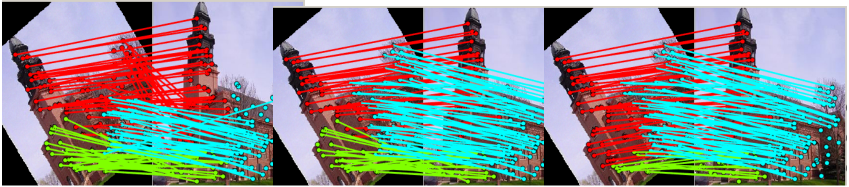




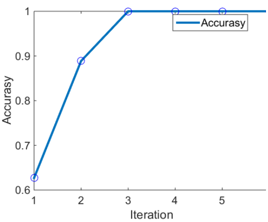


Evaluation of 2LevelGM on the synthetic image dataset

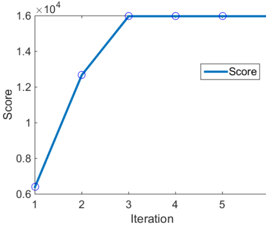
Image affine transformation II



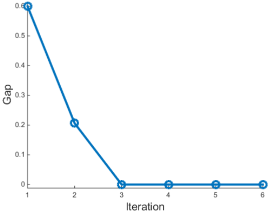
Iteration 1 | Iteration 2 | Iteration 3



Accuracy

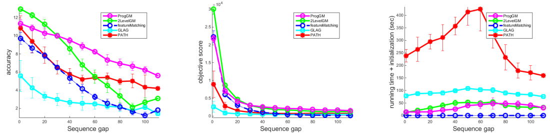


Matching score

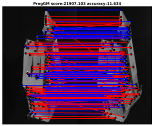


Gap between current and optimal solutions

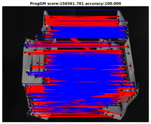
House data set



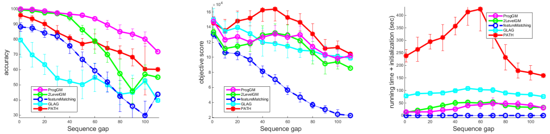
Evaluation of 2LevelGM on the CMU House sequence: not extrapolated solution



accuracy 11.63%



accuracy 100%



Evaluation of 2LevelGM on the CMU House sequence: extrapolated solution

Conclusions

The developed algorithm

- solves inexact graph matching problem
- is fast
- allows application of existing algorithms to bigger graphs
- shows very good results for (sub)graph isomorphism problems
- handles reasonably the affine deformations in graph structure

It has following properties:

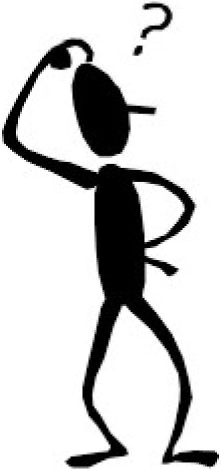
- it is sensible to non-affine deformations in graph structure
- it's complexity depends on the number of iterations and on the size of the anchor graphs and subgraphs
- anchor attributes for anchor graph matching are preferred

Future work

- more sophisticated graph partitioning techniques
- improvement of anchor attributes
- further improvement of the update rule
- probabilistic matching framework
- hierarchical method

The end

Thank you for your attention!



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