Contents

Abstract

In order to rank the "best college sports coaches"[1] for the previous century, we will use linear regression, exponential regression, normalization, analytic hierarchy process, and other techniques to construct a ranking model. The universality of this model would be discussed, and three application of this ranking model would be presented at the Part V, VI, and VII of this paper. Assumptions we made are in Part VIII.

Part I

Introduction

While more and more college students are cheering for their universities in sports stadiums, only few of them have noticed the importance of coaches. Sports audiences always focus their attention on valiant sports players and always forget the intelligent coaches who were standing at the corner of the ground. However, enthusiastic sports fans and university administrators know the importance of coaches. Sports enthusiasts love to discuss coaches after each game, and school officers spend lots of time and funds to invest on sports coaches.

It is easy for us to compare contemporary coaches by using recent game data. Nevertheless, the audiences like us, enthusiastic and loyal sports fans are not satisfied with the ranking of contemporary coaches. It is a hard task to compare coaches from different times, for instance, we can hardly say whether Fielding H. Yost[2] was better than Pete Carroll[3] or not.

Just like us, *Sports Illustrated*, a sport magazine, also wants to name the "best college coaches" for the previous century. So, we will discuss how to get the ranking by using accessible data and information.

Coach ranking is completely different from team ranking. People usually rank a team by the team's scores and game results. However, we could not merely rank coaches by their winning percentage. A great coach does

[1] Best All Time College Coach: this is the Solution to Problem B for 2014 MCM.

[2] Fielding H. Yost: football coach in University of Michigan from 1901 to 1926, except 1924

[3] Pete Carroll: football coach in USC from 2000 to 2009

improve the performance of a team significantly, even if the team does not get the championship; so, we will place considerable weight on the improvements of a team's performance. We will also consider the number of games played for each coach. The methodology of our ranking will be discussed in Part II of this paper.

Although gender equality has become more and more widely accepted during the past century, there are only a few famous female coaches. Female coaches received less complimentary than male coaches with the same ability. Because society has ignored female coaches' talent and ability for a long time, Male coaches even dominate in college women's sports. The pool sizes for male coaches and female coaches are different. Moreover, coaching style for males and females is quite different. So, it is unfair to rank male coaches and female coaches together. Due to these differences, we have to modify our ranking method in order to apply the model to female coach ranking. The details of gender difference among coaches are discussed in Part III of this paper.

Our ranking strategy could apply to most college games, for instance, football, basket ball, and baseball. However, our ranking method is not universal to all kinds of sports. The universality of our methodology is discussed in Part IV.

In Part V, VI, and VII, we will apply our ranking model to college football, basketball, and baseball coaches.

In Part VIII, we will discuss the assumptions we made when constructing the model.

The article for *Sports Illustrated* is at the beginning of the whole file.

Part II

Methodology

Players' performances in sports is crucial to the game result. This fact is especially true for one-player sports, such as tennis. However, for sports played by more than three players, coach usually can stimulate players to collaborate better and provide useful gaming strategies.

Across-sports ranking is not feasible because many factors should be taken into consideration, the following ranking model is invented for a single sport. In order to verify our model, we will apply it to three sports in Part V, VI, and VII.

In this part, we will first discuss the influence of each factor and then clarify the methods to build the ranking model.

1 Factors in Ranking

Ranking sports coaches may involve many factors, for instance, the time span a coach worked, the average game per year the coach instructed, the winning percentage under the coach's leading, the number of awards and championships the coach won, the improvements the coach made, and the fame of the coach. The influence of each factor is discussed below.

1.1 Winning Percentage

A coach's winning percentage is the most significant and intuitive data people would like to talk about. We will take winning percentage as an important indicator to evaluate coaches.

1.2 Term of Office

A good coach usually works for a long time. For instance, Bear Bryant[4] served for more than 38 years, and Robert Neyland[5] for 26 years. Colleges always value experienced coaches, so do sports enthusiasts. Time span of a coach plays an important role in the ranking model.

1.3 Average Game

[4]Bear Bryant: started coaching in 1945, and served as the head football coach at the University of Alabama from 1958 to 1982.

[5]Robert Neyland: head football coach at the University of Tennessee from 1926 to 1952

Everyone gets tired after intense work. Even robust athletes and brilliant coaches may make huge mistakes after playing games week after week. Average games a team played per year is a barometer of athletes' stamina and resistance. The more games the coach instructed per year, the greater the probability the team under-performs, which means the coach should have a higher evaluation. Furthermore, instructing more games per year is positively related to the coach's experience.

1.4 Awards and Championships

Better coaches always receive more awards and gain more championships than normal coaches. Championships and awards are also objective measurements of the coach's experience, the coach's instructional skills, and the preference of the media and the audience.

To calculate the score (index) for championships, we put together the championships each coach won, and divide the number by the total number of available championships during the career of the coach, calculated using a step function and including the start and end years.

1.5 Improvements (Delta)

A good coach could train weak athletes to form a victorious team, because the best coach knows how to stimulate athletes and raise their potential. We can measure how much the coach have improved a team by comparing the team's change in performance.

We will use the rankings of a team as the indicator of its performance. Rankings can be divided into reasonable ordinal categories, such as $[1, 5]$, which indicates the ranking from the first position to the fifth position. We divide the rankings into eight parts: $[1, 5]$, $(5, 10]$, $(10, 20]$, $(20-30]$, $(30-50]$, $(50-70]$, $(70, 100]$, $(100, +\infty)$, which we indicate by $8, 7, 6, 5, 4, 3, 2, 1$.

For schools where the coach had stayed for at least five years, we will find the category of the average ranking (two years) of a team before the coach entered the school as $rank_0$, and then compare it with that after the coach serves, called $rank_1$. Then, we will calculate the category of the average ranking in the last two years the coach served (excluding the year the coach left), namely $rank_2$, and then compare it with that after the coach's leaving, denoted by $rank_3$.

Then the index of improvement, which we will call the Delta Index, can be computed by $\frac{1}{2}\left(rank_1 - rank_0 + rank_2 - rank_3\right)$. If the coach has served more than one school, we take the average of the Delta Indices for each school. If the coach is still currently in the position, we only consider $(rank_1 - rank_0)$.

1.6 Fame and Time

As we discussed above, it is hard for us to compare two coaches who were coaching in different times. Sports rules are changing, and the funds spent on college sports are also changing. We can compare their winning percentages and terms of offices, but these factors are not enough. Luckily, we find an useful indicator to compare coaches in different times. Fame, an important factor to rank coaches, will help us to deal with the problem of time.

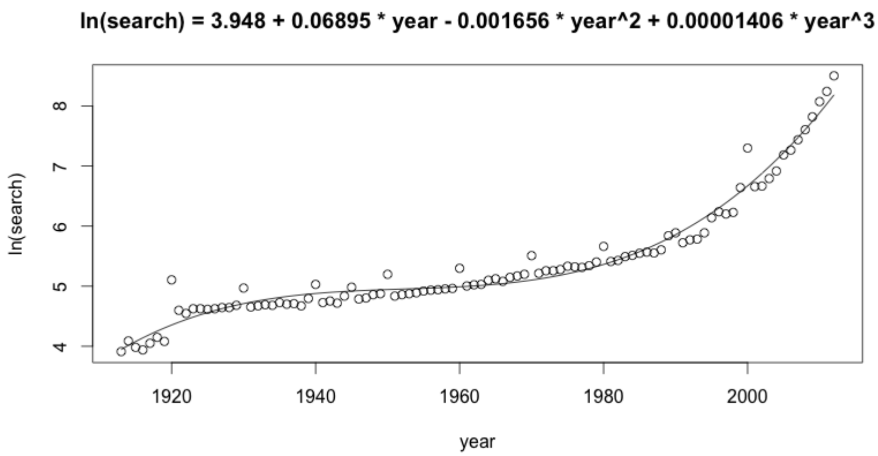
We can estimate the fame of a coach by counting how many times newspapers, magazines, and websites mentioned the coach. An accessible method to get the counting data is Google Search. For instance, there are 1,020,000 searching results for Johnny Vaught[6] on Google.

However, we cannot use the raw data from Google Search directly. For instance, recent coaches would be mentioned much more frequently than older coaches on the website. Obviously, search results is heavily related to the time, and we should find a function to deal with the search results "inflation" over time. So, we tried to find the relationship by using the searching result of year number; for instance, there are 131,000,000 results for searching "1953" and 890,000,000 results for searching "2003".

Looking at the plot between search result and year, the relationship is somewhat exponential. To find a more accurate quantitative relationship, we regress the log of the result number on year, year squared, and year cubed to add more linearity in our model without much loss of degrees of freedom. Under homoskedastic t-test, the largest p-value for all coefficients

[6] Johnny Vaught: head football coach for University of Mississippi from 1947 to 1973

is 1.388607×10^{-20}, and using heteroscedasticity-consistent robust standard errors that is 1.716700×10^{-32}. Therefore, we can see that all of the slope coefficients and the intercept are statistically significant with p-values far less than any conventional significance level.



So, the relatioship between year and searching result numbers can be characterized by:

$f(t) = \text{Exp}(3.948+0.06895(t-1913)-0.001656(t-1913)^2+0.00001406(t-1913)^3)$, where the t represents the year number.

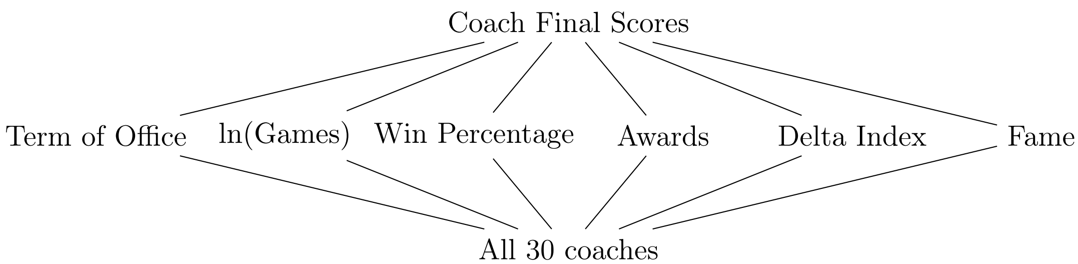
By using the "time deflator" $f(t)$, we can find the fame of a coach by using the function:

$\text{Fame} = \ln\left(\dfrac{n}{\int_a^b f(t)dt}\right)$, where n, where a and b are the start and end year of the coach's career respectively, and n represents the number of results of this coach.

Thus, we have created a nice indicator for fame.

2 Ranking Methods

There are thousands of college sport coaches in the previous century for each sport, and let us call the list that contains thousands of college sports coaches the *original list*. In order to find the top five coaches, we should first exclude unskilled and inexperienced coaches from our ranking list, and let us call this list the *professional list*. Then, we should make a list of 30 skillful and famous coaches by using Analytic Hierarchy Process (AHP hereafter), and let us call this list the *top* 30 *list*. Finally, we should choose five coaches out of the 30 by using a more comprehensive AHP.



Here we would introduce the method of Analytic Hierarchy Process. The intuition is that while it is psychologically proven to be hard to decide the weights of several factors simultaneously, human can easily compare them pairwise. Suppose we have k factors, we will create a $k \times k$ pairwise comparison matrix, namely M, with $M(i,j)$ being the ratio between the influences of the i^{th} and j^{th} factors. Note that by definition we must have $M(i,j) \times M(j,i) = 1$. For example, if the 2^{nd} factor is the log of total games instructed by the coach, and the 3^{rd} factor is the winning percentage, and we believe that the percentage factor is 3 times as important as the game factor, then we have $M(3,2) = 3$, and accordingly $M(2,3) = 1/3$. For the entries, we will use the integers from 1 to 9 and their reciprocals as the intensity of importance levels, a rule of thumb suggested by Thomas L. Saaty, the developer of the AHP method.

If the measurement of each factor is observable and absolute, then apparently $M(i,j) \times M(j,k) = M(i,k)$, and we do not need to use the pairwise comparison matrices, since we already know how their weights should be determined. In that case M is called *consistent*. Therefore, usually when we are using the matrix to determine the weight for the factors, the matrix is inconsistent since by subjectivity $M(i,j) \times M(j,k)$ might not equal to $M(i,k)$, and thus we need to test if the matrix is consistent enough, or if its inconsistency is acceptable.

Suppose that we are finding the weight for factors in the $(n+1)^{st}$ level with respect to the factor in the n^{th} level that they are subordinate to, and there are k factors in the $(n+1)^{st}$ level. For example, in the graph above, when $n = 1$, we are determining the weight for all the six factors in the second level with respect to the "Coach Final Scores" in the first level, and thus $k = 6$. To test the inconsistency of the pairwise comparison matrix M, we first need to find the largest possible eigenvalue λ of M, and calculate the Consistency Index (CI) of $M = \dfrac{\lambda - k}{k-1}$. Then by dividing CI by the Random Consistency Index (RI), a fixed constant for every k, we get the Consistency Rate (CR) $= \dfrac{\mathrm{CI}}{\mathrm{RI}}$. The rule of thumb is that if CR < 0.1, the inconsistency of M is acceptable, and we can use the normalized positive eigenvector corresponding to λ, called $w_k^{(n+1)}$, as the weight vector for the factors in the $(n+1)^{st}$ level with respect to the factor in the n^{th} level that they are subordinate to.

Denote the number of coaches in our professional list by N. For the influence of the factors in the third level with respect to the second level, i.e. the "influence" of each coach on each of the k factors, since we already have an absolute index for each of the coaches, we no longer need to use the pairwise comparison matrix. Therefore, the combined weight vector for the third level with respect to the second level is just the $N \times k$ score matrix for the k factors with column vectors normalized to have norm 1. Denote that combined weight vector by $W_k^{(3)}$, and for simplicity we shall call it *the* $N \times k$ *Score Matrix*. Note that $k = 6$ in the example illustrated by the graph, which we would use in our final decision of the Top 5 list.

Then by left-multiplying $w^{(2)}$ by $W^{(3)}$, we get the final $N \times 1$ weight vector of all coaches with respect to "Coach Final Scores" $w_{30}^{(3)} = W_k^{(3)} w_k^{(2)}$. Then $w_{30}^{(3)}$ is just the final scores for the 30 coaches, which we can use to determine the Top 5 coaches by sorting our Top 30 list.

(All the column vectors in the *score matrices* are normalized. Normalization means subtracting the mean of them column vector from each entry and divide it by the standard deviation. The weight vectors are already normalized by definition.)

2.1 Excluding Inexperienced Coaches

In order to get our professional coach lists, we need to exclude inexperienced coaches out of our original list.

Step 1: Find the average term of office of all the coaches. Exclude the coaches who served below average.

Step 2: Find the average number of games played per year for all the coaches using linear regression. Exclude the coaches who played below the average years times the average number of games.

Step 3: Find the third quartile (or other reasonable quantiles) of Winning Percentage for all the coaches in our list. Exclude the coaches below the that quantile.

Now, we get the *professional list* of coaches.

2.2 Rank the Top 30

It is difficult to gather data for Delta Index and Fame for all hundreds of coaches in the professional list. Thus, we should first run a simplified AHP to make a Top 30 list by considering only four major factors: term of office (Y), log of total games played (G), winning percentage (P), and Championship rewards (C).

By carefully deciding the pairwise relative weight of each factor, we get the 4×4 pairwise comparison matrix M:

	Y	G	P	C
Y	1	1	1/4	1/4
G	1	1	1/3	1/5
P	4	3	1	1/3
C	4	5	3	1

, so then $M = \begin{bmatrix} 1 & 1 & 1/4 & 1/4 \\ 1 & 1 & 1/3 & 1/5 \\ 4 & 3 & 1 & 1/3 \\ 4 & 5 & 3 & 1 \end{bmatrix}$.

The largest positive eigenvalue of M is 4.107, and its corresponding normalized positive eigenvector, which we would use for the weights, is $w_4^{(2)} = (0.1548, 0.1531, 0.4405, 0.8709)^\top$. To test its consistency, the consistency index is $CI = \dfrac{\lambda - 4}{4 - 1} = 0.0358$. Random Consistency Index for $(n = 4) = 0.90$, and thus its consistency rate is $CR = \dfrac{CI}{RI} = 0.0398 < 0.1$. Therefore, we passed the consistency test.

Recall that we denote the $N \times 4$ *Score Matrix* by $W_4^{(3)}$, and by left-multiplying $w_4^{(2)}$ by $W_4^{(3)}$ we get the 30×1 matrix of raw scores $w_N^{(3)} = W_4^{(3)} \times w_4^{(2)}$ to decide the Top 30 list, where N is the number of coaches in the *professional list*.

2.3 Five Best Coaches

This step is crucial in our ranking strategy. We will again use the Analytic Hierarchy Process to evaluate different coaches' performances, but with all six factors this time. Denote the 6×6 pairwise comparison matrix by M.

Let Y (short for "year") denote the term of office of the coach, G the log of total games instructed by the coach, P the winning percentage of this coach, C the awards the coach received or the championships the coach achieved, D the Delta Index, and F his/her fame.

	Y	G	P	C	D	F
Y	1	1	1/4	1/4	4	1/2
G	1	1	1/3	1/5	4	1/2
P	4	3	1	1/3	9	2
C	4	5	3	1	9	2
D	1/4	1/4	1/9	1/9	1	1/8
F	2	2	1/2	1/2	8	1

, then $M = \begin{bmatrix} 1 & 1 & 1/4 & 1/4 & 4 & 1/2 \\ 1 & 1 & 1/3 & 1/5 & 4 & 1/2 \\ 4 & 3 & 1 & 1/3 & 9 & 2 \\ 4 & 5 & 3 & 1 & 9 & 2 \\ 1/4 & 1/4 & 1/9 & 1/9 & 1 & 1/8 \\ 2 & 2 & 1/2 & 1/2 & 8 & 1 \end{bmatrix}$.

For the pairwise comparison matrix M, its largest positive eigenvalue is 6.1807 with its corresponding normalized positive eigenvector $w_6^{(2)} = (0.1629, 0.1631, 0.4839,$
$0.7772, 0.0506, 0.3258)^\top$. Consistency Index $= \dfrac{\lambda - 6}{6 - 1} = 0.0361$. Random Consistency Index for $(n = 6) = 1.24$. Consistency Rate $\dfrac{CI}{RI} = 0.0291 < 0.1$, which indicates that we passed the consistency test.

Then by left-multiplying $w_6^{(2)}$ by the corresponding 30×6 *Score Matrix* $W_6^{(3)}$, we get the final scores $w_{30}^{(3)} = W_6^{(3)} \times w_6^{(2)}$ to rank the Top 30 coaches.

After applying the AHP, we can finally get the best five collge coaches for each sport.

Part III
Gender Difference

As we discussed in Part I, gender differences are significant for coaches. Female coaches received less complimentary than male coaches with the same ability. "Vigorous, powerful, and strong were masculine role identifiers. Women could not assume these character traits, or if they did, they were violating the rules of nature"[7]. While male coaches dominated sports for a long time, females began to emerge on the stage of coaching. Rene Portland[8] is one example of a successful female coach.

Usually, male coaches are better at motivating athletes, but female coaches are much better at making improvements on details. Because of these coaching style differences, the team led by female coaches would always change more slowly than the team led by male coaches. In the statistic viewpoint, the pool for female coaches is too small, and we have to modify our models to apply to female coaches.

First, the term of office of female coaches is always shorter than male coaches, and we should weight this factor less in the pairwise comparison matrix. Second, the more games played by female coaches and the more rewards she received, the more significant she is a better coach; so, we should weight more for the number rewards she received and the average number of games played per year. Third, female coaches always influence athletes in a broader but slower way; so, we should measure the improvements of the team by eight years rather than four years. At last, we should not try to make a professional list (discussed in Part II 2.1) for female coaches, because the pool for female coaches is not big enough.

Thus, by applying these modifications, we can apply our model to female coaches.

[7] *Women College Basketball Coaches*, Page 13

[8] Rene Portland: head women college basketball coach in Penn State University for 27 years

Part IV
Universality of the Methodology

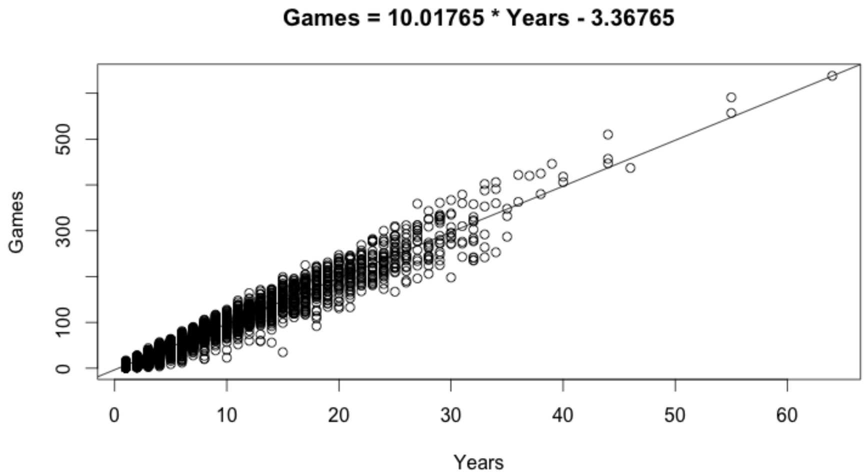
For instance, we can not apply our ranking method for sports like table-tennis that player's personal skills are more important than coach's training.

Our ranking method is not universal to all the sports. As we discussed above, there are two major requirements for our ranking strategy. First, we need lots of data, i.e. game result, coach's historical record, team ranking, and some alternative data such as Hall of Fame, to analyze the performance of each coach; second, coach should play an important role in the sports. We applied our model to football, basketball, and baseball. The three sports satisfied both of the requirements.

For instance, we can not apply our ranking model for marathon, because player's personal skills are more important than coach's training in marathon. We could not apply our model to table-tennis either, because the data for table-tennis team is not consistent and easy-accessible.

Part V
Model for College Football

For football, the average term of office for a coach is 5.5. We plot the total game numbers against years the coaches served. It is already obvious that they have a linear relationship, with years being the independent variable. Running a regression of games on year, we get a slope coefficient 10.01765 with p-value 8.679353×10^{-63}. Then we will exclude the coaches who played fewer than five years and 5×10.01765 games.



The third quartile of the winning percentage of the football coaches is 0.6071. We would use that as a cut-off to filter out the under-performing coaches. Now we have reduced our *original list* with 7014 coaches to our *professional list* with 675 coaches.

Now we use the method indicated in the "Rank the Top 30" section, i.e. use the normalized positive eigenvector corresponding to the largest positive eigenvalue as the weight vector $w_4^{(2)} = (0.1548, 0.1531, 0.4405, 0.8709)^\top$ of our four factors to make our Top 30 list.

For the championship factor, we consider the "College Football National Championships in NCAA Division I FBS" (1869-), the "Rose Bowl" (1902, 1916-), the "Sugar Bowl" (1935-), the "Orange Bowl" (1935-), the "Cotton Bowl" (1937-), the "NCAA Division II Football Championship" (1973-), and the "NCAA Division I Football Championship" (1978-). Here our assumptions are: 1. Roughly assume that all teams led by the coaches before 1935, when Sugar, Orange, and Cotton Bowls came to presence, were able to attend Rose Bowl. 2. Since Cotton Bowl dates back to only two years after Sugar and Orange Bowls do, and likewise NCAA I (1978-) lags only five years behind NCAA II (1973-), we assume they existed from the same year as their earlier counterparts. Then we put all the championships together, and divide the number by the total number of available championships during the career of the coach to obtain the scores (indices) for this factor.

By left-multiplying $w_4^{(2)}$ by the 675×4 *Score Matrix* $W_4^{(3)}$, we get the scores for choosing 30 from 675 $w_{675}^{(3)} = W_4^{(3)} \times w_4^{(2)}$. The resulting list is attached in the Appendix.

To do a quick verification of our list, we look up their names in the *College Football Hall of Fame* for coaches. Surprisingly, 28 coaches in our Top 30 list are in the "Hall of Fame". Pete Carrol, a talented coach who helped USC achieve five Rose Bowl championships in six consecutive years, is not in the "Hall of Fame", but the reason is that he only had a recent and short experience in college coaching.

Next, we find the Delta Indices and Fame for each of the 30 coaches, and thus completing the 30×6 *score matrix* $W_6^{(3)}$. By left-multiplying the weight $w_6^{(2)} = (0.1629, 0.1631, 0.4839, 0.7772, 0.0506, 0.3258)^\top$ by the *score matrix*, we get the final weight, or final scores for all the 30 coaches $w_{30}^{(3)} = W_6^{(3)} \times w_6^{(2)}$. Their final scores are also indicated in the Appendix.

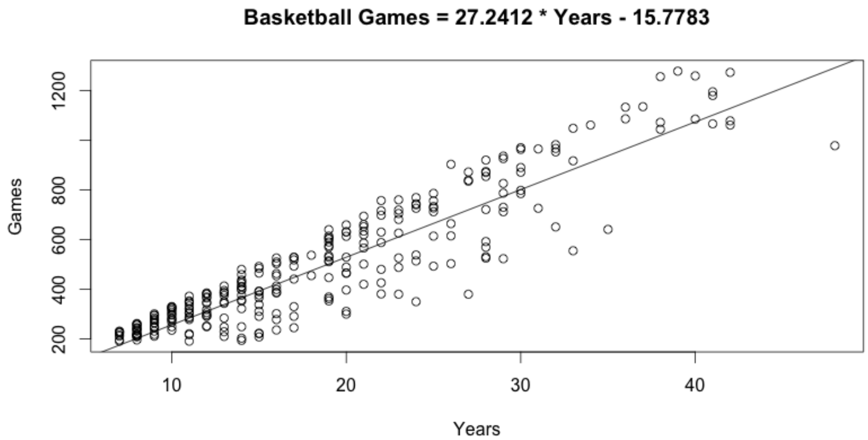
Thus, by applying AHP, we get the five best college football coaches in the previous century.

Rank	Name
1	Pete Carroll
2	Charles B. Wilkinson
3	Paul W. Bryant
4	Tom Osborne
5	Knute Kenneth Rockne

Part VI

Model for College Basketball

For basketball, the average term of office for a coach is 6.654. We plot the total game numbers against years the coach served. It is already obvious that they have a linear relationship, with years being the independent variable. Running a regression of games on year, we get a slope coefficient 27.2412 with a p-value not different from 0 within the numerical precision of the R language. Then we will exclude the coaches who played fewer than seven years and 7×27.2412 games.



The third quartile of the winning percentage of the basketball coaches is 0.6070. We would use that as a cut-off to filter out the under-performing coaches. Now we have reduced our *original list* with 3501 coaches to our *professional list* with 259 coaches.

Now we use the weight vector $w_4^{(2)} = (0.1548, 0.1531, 0.4405, 0.8709)^\top$ of our four factors to make our Top 30 list.

For the championship factor, we consider the NCAA Division I Championship and the National Invitation Tournament (NIT).

For the NCAA part, we consider the number of times being chosen into the NCAA, getting in the Final Four (FF), and achieving the National Championship (NC). Since the number of teams has been changing over time, we use the following method to calculate the scores for NCAA. Assume that the probability of being chosen into the NCAA is proportional to the number of teams chosen. Assume also that the probability of getting into the First Four is the same for all NCAA teams, and that the probability of achieving the championship is the same for all First Four teams. Suppose with k teams chosen into NCAA, the probability of being chosen is pk for some constant p. Then the probability of getting into the Final Four is $\mathbb{P}(FF) = \mathbb{P}(FF|NCAA)\mathbb{P}(NCAA) = \dfrac{4}{k}pk = 4p$, and that of achieving the National Championship is $\mathbb{P}(NC) = \mathbb{P}(NC|FF)\mathbb{P}(FF) = \dfrac{1}{4}4p = p$. Since the ratio among the three probabilities is independent of p, we can simply set $p = 1$. The scores for each achievement can be naturally determined by taking the reciprocal of the probabilities: $\frac{1}{k}$ points for NCAA, $\frac{1}{4}$ for FF, and 1 for NC.

We first calculate the average number of teams in NCAA during the coach's career, $\overline{k}$. If the team has entered n_{NCAA} times NCAA, got into Final Four for n_{FF} times, and finally achieved n_{NC} National Championships. Then we get the raw points for NCAA Championship $p_{NCAA} = \frac{n_{NCAA} - n_{FF}}{\overline{k}} + \frac{n_{FF} - n_{NC}}{4} + n_{NC}$. Suppose the coach has won n_{NIT} NIT championships and denote the number of years NCAA and NIT were held during the coach's career by Y_{NCAA} and Y_{NIT}, respectively. Then the total score for the championship factor is just $p_{Champs} = \frac{2}{3}\frac{p_{NCAA}}{\mathrm{Y_{NCAA}}} + \frac{1}{3}\frac{n_{NIT}}{\mathrm{Y_{NIT}}}$.

By left-multiplying $w_4^{(2)}$ by the 675×4 *Score Matrix* $W_4^{(3)}$, we get the scores for choosing 30 from 675 $w_{675}^{(3)} = W_4^{(3)} \times w_4^{(2)}$. The resulting list is attached in the Appendix.

After getting the list of top 30 college basketball coaches. It is noticeable that about 15 coaches in our list are in the Hall of Fame[9]. Although this seems to be a smaller number than that for football, it can be resulted from the fact that the basketball Hall of Fame has fewer inductees than the football one does.

Next, we find the Delta Indices and Fame for each of the 30 coaches, and thus completing the 30×6 *score matrix* $W_6^{(3)}$. By left-multiplying the weight $w_6^{(2)} = (0.1629, 0.1631, 0.4839, 0.7772, 0.0506, 0.3258)^\top$ by the *score matrix*, we get the final weight, or final scores for all the 30 coaches $w_{30}^{(3)} = W_6^{(3)} \times w_6^{(2)}$. Their final scores are also indicated in the Appendix.

Thus, by applying AHP, we get the five best college basketball coaches in the previous century.

[9]Hall of Fame: National Collegiate Basketball Hall of Fame

Rank	Name
1	Mike Krzyzewski
2	Dean Smith
3	Roy Williams
4	John Wooden
5	John Calipari

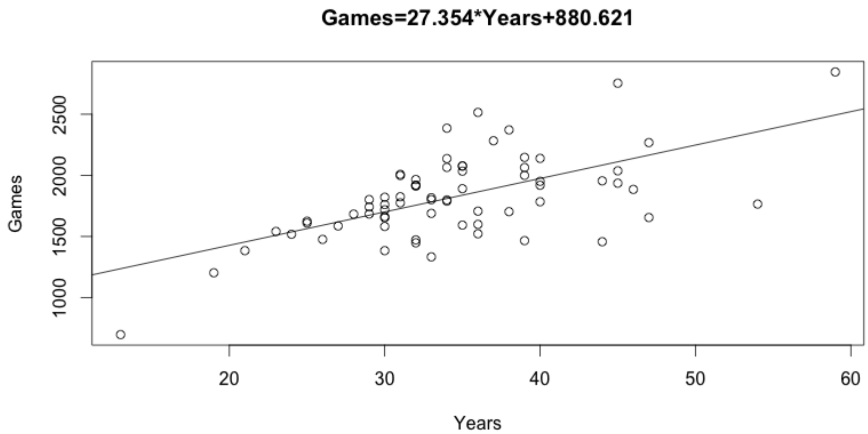
One noticeable characteristic for these five best coaches is that all of them began serving as basketball coach after 1940. This phenomena is due to the change of popularity of basketball; talent coaches preferred to work in football and other sports in early days because basketball was not so popular at that time.

Part VII

Model for College Baseball

For College Baseball, since we only have data for 69 college coaches, we progress directly from the *original list* to the Top 30 list, without an intermediate *professional list*.

The average term of office for a coach is 34.35. We plot the total game numbers against years the coach served. It is obvious that they have a linear relationship, with years being the independent variable. Running a regression of games on year, we get a slope coefficient 27.354 with p-value far smaller than 2.257934×10^{-8}. Then we will exclude the coaches who played fewer than 34 years and 34×27.354 games.



Note that the standard deviation of the winning percentage for the 69 coaches is really small (0.06127381), and the difference between the first decile and the mean is as small as 0.0699. Therefore, in order to choose as many as 30 coaches from the list, we only filter out those coaches whose win percentages are below the first decile.

In the next step we would directly use all the six factors to make our Top 5 list as we did for Football and Basketball. For the championship part, we use number of championships for College World Series (n_{CWS}) and NAIA World Series (n_{NAIA}). Denote the years CWS and NAIA were held during the coach's career by Y_{CWS} and Y_{NAIA}. Then the score we would use for the championship factor is simply $p_{Champs} = \dfrac{n_{CWS} + n_{NAIA}}{Y_{CWS} + Y_{NAIA}}$.

For the Delta Index, since it is hard to find an annual ranking of each college baseball team, we would use the induction into the National College Baseball Hall of Fame as an alternative approach to evaluate the influence of the coach.

Since we changed the fifth factor from the Delta Index to Hall of Fame, we should change the pairwise comparison matrix accordingly. Believing that the Hall of Fame is twice as important as the Delta Index, we change our pairwise comparison matrix to

	Y	G	P	C	D	F
Y	1	1	1/4	1/4	2	1/2
G	1	1	1/3	1/5	2	1/2
P	4	3	1	1/3	4	2
C	4	5	3	1	4	2
D	1/2	1/2	1/4	1/4	1	1/4
F	2	2	1/2	1/2	4	1

, then $M' = \begin{bmatrix} 1 & 1 & 1/4 & 1/4 & 2 & 1/2 \\ 1 & 1 & 1/3 & 1/5 & 2 & 1/2 \\ 4 & 3 & 1 & 1/3 & 4 & 2 \\ 4 & 5 & 3 & 1 & 4 & 2 \\ 1/2 & 1/2 & 1/4 & 1/4 & 1 & 1/4 \\ 2 & 2 & 1/2 & 1/2 & 4 & 1 \end{bmatrix}$.

For the pairwise comparison matrix M', it's largest positive eigenvalue is 6.2061 with its corresponding normalized positive eigenvector $w_6'^{(2)} = (0.1648, 0.1650, 0.4803, 0.7712, 0.1076, 0.3296)^\top$. Consistency Index $= \dfrac{\lambda' - 6}{6 - 1} = 0.0412$. Random Consistency Index for $(k = 6) = 1.24$. Consistency Rate $\dfrac{CI}{RI} = 0.0332 < 0.1$, which indicates that we passed the consistency test.

Then using this adjusted weight vector $w'^{(2)_6}$, we get the final scores for the coaches in the Top 30 list, and thus we have determined the five best college baseballl coaches in the previous century.

Rank	Name
1	Ed Cheff
2	Rod Dedeaux
3	Mike Martin
4	Frank Vieira
5	Augie Garrido

Part VIII
Basic Assumptions

1. Google Search number can be a proxy variable of the unobservable fame of the coach.

2. Roughly assume that all teams led by the coaches before 1935, when Sugar, Orange, and Cotton Bowls came to presence, were able to attend Rose Bowl (1902, 1916-).

3. Since Cotton Bowl dates back to only two years after Sugar and Orange Bowls do, and NCAA I (1978-) lags only five years behind NCAA II (1973-), we assume they existed from the same year as their earlier counterparts.

4. The probability of being chosen into the NCAA is proportional to the number of teams chosen.

5. The probability of getting into the First Four is the same for all NCAA teams, and that of achieving the championship is the same for all First Four teams.

Part IX
Appendix

Final Ranking for the Top 30 College Sport Coaches with Final Scores (scaled for presentation)

Football

Rank	Name	Final Score	Rank	Name	Final Score
1	Pete Carroll	81.04	2	Charles B. Wilkinson	79.88
3	Paul W. Bryant	74.20	4	Tom Osborne	72.64
5	Knute Kenneth Rockne	69.84	6	Frank W. Leahy	66.31
7	Frank W. Thomas	63.91	8	Fielding H. Yost	62.97
9	Barry Switzer	62.60	10	Wayne Hayes	60.86
11	Howard H. Jones	60.49	12	John McKay	59.71
13	Darrell Royal	58.86	14	Robert S. Devaney	58.65
15	Andrew L. Smith	58.03	16	Robert R. Neyland	48.95
17	Robert L. Dodd	47.62	18	John H. Vaught	47.20
19	Bobby Bowden	42.09	20	Glenn Scobey Warner	41.20
21	Dana X. Bible	36.28	22	Ara R. Parseghian	36.06
23	John Gagliardi	32.33	24	Eddie G. Robinson	31.92
25	Joe Paterno	31.61	26	Jim Tressel	31.33
27	Roy Kidd	25.35	28	Forrest Westering	23.23
29	Lou Holtz	20.11	30	Amos Alonzo Stagg Sr.	14.73

Basketball

Rank	Name	Final Score	Rank	Name	Final Score
1	Mike Krzyzewski	93.96	2	Dean Smith	81.75
3	Roy Williams	81.27	4	John Wooden	79.65
5	John Calipari	70.62	6	Bob Knight	65.22
7	Bill Self	63.27	8	Adolph Rupp	59.55
9	Jerry Tarkanian	58.68	10	Rick Pitino	57.51
11	John Thompson	56.64	12	Jim Boeheim	55.35
13	Jim Calhoun	53.82	14	Mark Few	53.49
15	Nolan Richardson	53.25	16	Billy Donovan	52.14
17	Tom Izzo	51.54	18	Lute Olson	47.16
19	Eddie Sutton	44.46	20	Denny Crum	43.74
21	Thad Matta	43.26	22	Al McGuire	41.46
23	Gary Williams	38.73	24	Steve Fisher	38.46
25	Sam Burton	37.68	26	Larry Brown	34.59
27	Bob Huggins	30.42	28	Tubby Smith	27.45
29	Jim Harrick	20.61	30	Ben Howland	18.81

Baseball

Rank	Name	Final Score	Rank	Name	Final Score
1	Ed Cheff	98.476	2	Rod Dedeaux	86.508
3	Mike Martin	75.62	4	Frank Vieira	75.052
5	Augie Garrido	74.752	6	Gordie Gillespie	74.444
7	Gene Stephenson	74.012	8	Don Brandon	73.616
9	Jack Coffey	72.428	10	Don Schaly	71.728
11	Bill Wilhelm	69.328	12	Mark Marquess	69.2
13	Larry Hays	69.004	14	Chuck Hartman	68.68
15	Bob Hannah	68.236	16	Ron Polk	67.732
17	Bill Holowaty	67.576	18	Jim Mallon	67
19	Tommy Thomas	66.512	20	Bob Bennett	66.312
21	John Winkin	66.072	22	Itch Jones	65.612
23	Al Ogletree	64.82	24	Larry Cochell	64.748
25	Jack Stallings	64.544	26	Gary Grob	64.448
27	Dean Bowyer	64.164	28	Fred Hill	63.804
29	George Valesente	62.784	30	Jim Gilligan	62.776

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