

lets say our training samples are $(x_1, y_1), (x_2, y_2), \dots (x_m, y_m)$ where $x_i \in X$ and $y_i \in$

So total "m" samples.

we will have T iterations,

$t = 1, \dots, T$

weights will always follow a distribution, that means $\sum D_t(i) = 1$, where $i = i^{th}$ sample

for 1st iteration $D_1(i) = \frac{1}{m} = \frac{1}{10}$

1) Train the weak learner using weights $D_1(i)$

2) Get the weak hypothesis $h_t : X \rightarrow (-1, +1)$

error rate $\epsilon_t = \Pr[h_t(x_i) \neq y_i] = \frac{3}{10} = \sum_{h_t(x_i) \neq y_i} D_t(i)$

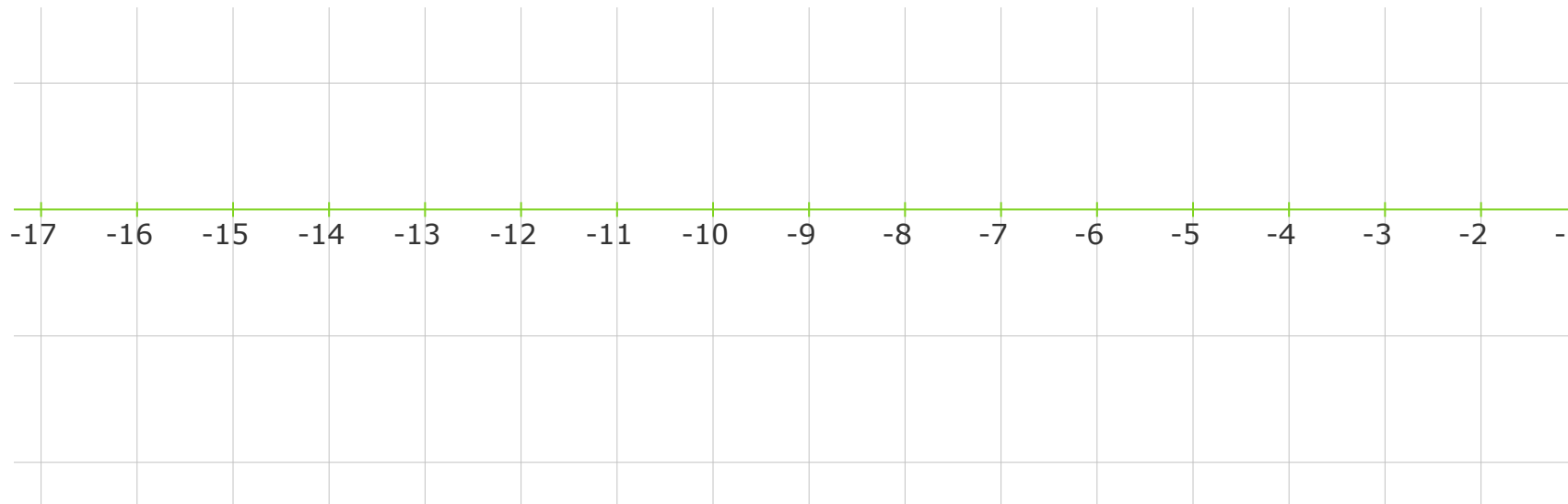
$1 - \epsilon_t = \sum_{h_t(x_i) = y_i} D_t(i)$

$$3) \ \alpha_t = \frac{1}{2} \ln \left(\frac{1 - \epsilon_t}{\epsilon_t} \right)$$

$$\alpha_t = \ln \left(\sqrt{\frac{1 - \epsilon_t}{\epsilon_t}} \right)$$

$$e^{\alpha_t} = \sqrt{\frac{1 - \epsilon_t}{\epsilon_t}}$$

$$e^{-\alpha_t} = \sqrt{\frac{\epsilon_t}{1 - \epsilon_t}}$$



4) *Update* :

$$D_{t+1}(i) = \frac{D_t(i)}{Z_t} * \begin{cases} e^{-\alpha_t} & \text{if } h_t(x_i) = y_i \\ e^{\alpha_t} & \text{if } h_t(x_i) \neq y_i \end{cases}$$

$$\begin{aligned}
&= \frac{D_t(i) e^{-\alpha_t y_i h_t(x_i)}}{Z_t} \\
&= \frac{D_t(i)}{Z_t} * \begin{cases} \sqrt{\frac{\epsilon_t}{1-\epsilon_t}} & \text{if } h_t(x_i) = y_i \\ \sqrt{\frac{1-\epsilon_t}{\epsilon_t}} & \text{if } h_t(x_i) \neq y_i \end{cases}
\end{aligned}$$

Now what will be the value of Z_t ,

$$\begin{aligned}
&\sum_{h_t(x_i)=y_i} \frac{D_t(i) \sqrt{\frac{\epsilon_t}{1-\epsilon_t}}}{Z_t} + \sum_{h_t(x_i) \neq y_i} \frac{D_t(i) \sqrt{\frac{1-\epsilon_t}{\epsilon_t}}}{Z_t} = 1 \\
&\Rightarrow \sqrt{\frac{\epsilon_t}{1-\epsilon_t}} \sum_{h_t(x_i)=y_i} D_t(i) + \sqrt{\frac{1-\epsilon_t}{\epsilon_t}} \sum_{h_t(x_i) \neq y_i} D_t(i) = Z_t
\end{aligned}$$

$$\Rightarrow \sqrt{\frac{\epsilon_t}{1-\epsilon_t}} (1-\epsilon_t) + \sqrt{\frac{1-\epsilon_t}{\epsilon_t}} \epsilon_t = Z_t$$

$$\Rightarrow \sqrt{\epsilon_t(1-\epsilon_t)} + \sqrt{\epsilon_t(1-\epsilon_t)} = Z_t$$

$$\Rightarrow 2\sqrt{\epsilon_t(1-\epsilon_t)} = Z_t$$

$$D_{t+1}(i) = \frac{D_t(i) \sqrt{\frac{\epsilon_t}{1-\epsilon_t}}}{2\sqrt{\epsilon_t(1-\epsilon_t)}} = \frac{1}{2(1-\epsilon_t)} D_t(i) \text{ where } h_t(x_i) = y_i$$

$$D_{t+1}(i) = \frac{D_t(i) \sqrt{\frac{1-\epsilon_t}{\epsilon_t}}}{2\sqrt{\epsilon_t(1-\epsilon_t)}} = \frac{1}{2(\epsilon_t)} D_t(i) \text{ where } h_t(x_i) \neq y_i$$

$$\sum_{h_t(x_i) = y_i} D_{t+1}(i) = \frac{1}{2(1 - \epsilon_t)} \sum_{h_t(x_i) = y_i} D_t(i) = \frac{1}{2}$$

$$\sum_{h_t(x_i) \neq y_i} D_{t+1}(i) = \frac{1}{2(\epsilon_t)} \sum_{h_t(x_i) \neq y_i} D_t(i) = \frac{1}{2}$$

$$\textit{Final Hypothesis } H(x) = \textit{sign} \left(\sum_{t=1}^T \alpha_t h_t(x) \right)$$

