An Extended Research on Split Bregmanized - ATV Model for Image Restoration

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Beilin Ye 21053851
Lu Cao 20310032
Jiaqi Zhang 21009547

Introduction

This extended research is based on a literature review of the "Split Bregmanized anisotropic total variation model for image deblurring" written by Huasong Chen, Chunyong Wang, Yang Song, and Zhenhua Li, published in 2015.

The presented research addresses the challenge of image restoration, specifically in the context of deblurring and denoising, through an innovative approach that integrates Anisotropic Total Variation (ATV) with the Split Bregman Iteration technique. This integration holds the potential to tackle the complexities posed by L1-norm minimization problems frequently encountered in image processing tasks such as denoising and deblurring. The Split Bregman iteration, as an optimization tool, partitions the optimization problem into subproblems, optimizing the image and an auxiliary variable iteratively [Chen et al., 2015]. ATV, an advancement from traditional Total Variation (TV) and Anisotropic Diffusion, is devised to adeptly preserve edge details while mitigating the shortcomings of previous methods. By combining these strategies, the proposed approach endeavors to restore degraded images with enhanced edge clarity and nuanced textures while effectively addressing blurring and noisy image restoration.

In our study, we will precisely replicate this SB and ATV integration and its application to denoising tasks as outlined in the selected paper. Beyond replication, we will contrast the ATV regularization method with the Isotropic Total Variation (ITV) regularization method, employing SB iteration as a foundation. This endeavor intends to effectively evaluate the method's performance and its practical utility.

Introducing an innovative facet to our study, we will extend the application of SB-ATV and SB-ITV methodologies to inpainting scenarios with SB iteration. This novel perspective involves adapting the same techniques to incomplete images instead of noisy ones. We will employ a mask to delineate the regions covered to come up with degraded images and apply the methodologies for inpainting processes.

Image denoising is a crucial aspect of digital image processing with widespread practical significance. It plays a vital role in various fields such as individual recognition[Ge et al., 2021], remote sensing[Danan et al., 2022], and other applications where clean and accurate images are essential.

The task of image denoising involves removing noise from noisy images to restore their true content[Jebur et al., 2023]. However, this is a challenging endeavor as noise, edges, and textures share high-frequency characteristics, often leading to the loss of important details during denoising. Despite being a longstanding problem, image denoising continues to be an open and complex challenge, given its inherent mathematical intricacies and non-uniqueness of solutions. Modern techniques strive to strike a balance between noise removal and preserving image details[Tian et al., 2020], contributing to the pursuit of high-quality denoised images.

Recent advancements in image denoising have been driven by a convergence of classical techniques and cutting-edge deep learning methods. Classical methods, such as non-local means, sparse representation, and low-rank approaches, have been instrumental in addressing noise reduction challenges [Fan et al., 2019]. These techniques leverage mathematical formulations and structural priors to effectively preserve image details while suppressing noise. Meanwhile, the emergence of convolutional neural networks (CNNs) has revolutionized image denoising[Ilesanmi and Ilesanmi, 2021]. CNN-based methods harness the power of deep learning to automatically learn intricate noise patterns and image features, leading to remarkable denoising performance. Leveraging large-scale datasets and sophisticated architectures, these methods offer a data-driven approach to noise removal, enabling improved results even in complex noise scenarios. The synergy between classical and modern techniques represents the latest frontiers in image denoising, with researchers continually refining and integrating approaches to enhance the quality and fidelity of denoised images.

Literature Review

The paper applies Split Bregman Iteration on ATV (Anisotropic Total Variation) for image restoration.

1. Split Bregman Iteration

The Split Bregman iteration is an optimization technique used to solve L1-norm minimization problems, which implies a numerical optimization problem. L1-norm problems arise in various image processing tasks, including denoising and deblurring. Traditional optimization approaches can be challenging due to the nonlinearity and non-differentiability of L1-norm terms[Xiao et al., 2011]. The Split Bregman iteration provides an effective way to address these challenges. In the context of image deblurring and denoising, the Split Bregman iteration divides the optimization problem into two subproblems: one for the image and another for an auxiliary variable. By iteratively solving these subproblems, the algorithm can efficiently minimize the L1-norm-based cost function. This technique offers computational advantages and helps achieve robust and high-quality results.

2. ATV

The evolution from Total Variation (TV) to Anisotropic Diffusion and finally to Anisotropic Total Variation (ATV) reflects a progression in image restoration techniques. TV initially gained favor for its ability to preserve edges by minimizing pixel differences. However, it suffered from staircase artifacts and loss of texture. Anisotropic diffusion was then introduced to enhance edge details while addressing some issues, but it still exhibited unwanted smoothing and speckles. In response, ATV emerged as a solution, aiming to combine the strengths of both TV and anisotropic diffusion[Pang et al., 2019]. ATV considers the direction of pixel value changes, allowing adaptive processing for better edge preservation and artifact reduction. The development of ATV represents an innovative step towards restoring images with sharp edges and corners while mitigating the drawbacks seen in previous methods[Guo and Chen, 2021].

3. Combination of Split Bregman and ATV

The paper proposes an image restoration method that combines the benefits of Anisotropic Total Variation (ATV) and the Split Bregman iteration. This approach aims to restore the degraded images by minimizing the impact of blurring while preserving sharp edges and fine textures. The Split Bregman iteration efficiently handles the L1-norm problems that arise in the optimization process.

The paper further discusses the benefits of the proposed method in comparison to other image deblurring techniques, such as traditional total variation methods. The experimental results demonstrate that the proposed method effectively reduces artifacts like stair-casing, works well with various types of blur, and performs favorably on images with detailed textures.

In summary, Split Bregman is an iterative method, and ATV is the regularization method in the cost function which we want to optimize to denoise images. The paper presents a method that leverages Anisotropic Total Variation and the Split Bregman iteration to enhance image restoration. This approach aims to overcome the limitations of traditional methods and achieve superior results in terms of edge preservation and overall image quality.

Proposed Solution

The primary focus of this section entails the comprehensive elucidation of the proposed resolution to the chosen problem, primarily centered around the replication of methods as described in the paper. Specifically, the study presents the application of the Split Bregman iteration technique to the Anisotropic Total Variation (ATV) framework for the purpose of image restoration. This methodology's practical implementations are examined in two key domains: image denoising and image deblurring. However, our study concentrates solely on the denoising aspect.

1. Proposal Review and Algorithms

Within the scope of our investigation, we meticulously replicate the integration of Split Bregman iteration with ATV as delineated in the selected paper, and subsequently, we employ this integrated approach to tackle denoising tasks. To assess the efficacy of the chosen paper's methodology, a comparative analysis is conducted encompassing the following aspects:

a. The original image.
b. Noisy images characterized by noise deviations of $1 \times 10^{-1.5}$, $3 \times 10^{-1.5}$, and $5 \times 10^{-1.5}$.
c. Denoised images obtained through the utilization of Split Bregman iteration in conjunction with the ATV method.
d. Denoised images achieved via the implementation of Split Bregman iteration alongside the Isotropic Total Variation (ITV) method[Cai et al., 2010]. It is noteworthy that this specific approach is introduced as an innovation to comprehensively evaluate the outcomes of the chosen paper.

The provided code, which is detailed in the attached appendices of this report, has been implemented using the Matlab programming environment.

Please see the table below for an illustration of our proposal (Table 1), as well as a block diagram of the proposed algorithmic flow (Figure 1), which together summarizes the paragraphs above.

Proposed solution: Comparasion Analysis			
Combination of iteration and regularization	Regularization Method		
	ATV	ITV	Inpainting
Iteration Method: Split Bregman	Replicate	Innovate	Innovate

Table 1: Table of proposed solution

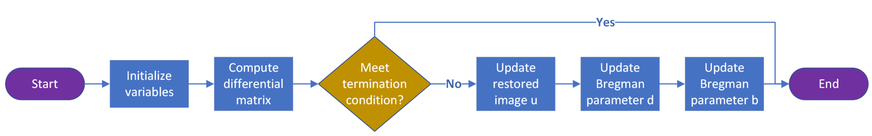

Figure 1: Flow chart of Split Bregman image restoration model

2. Comparative Analysis

In the subsequent sections of this paper, we outline the methodologies that form the basis of our comparative studies. To provide a concise overview of the methodologies under consideration, we shall begin by summarizing the key elements.

The Split Bregman Anisotropic Total Variation (SB-ATV) denoising approach, which has been implemented by our reference paper, serves as the benchmark against which our novel Split Bregman Isotropic Total Variation (SB-ITV) denoising method will be compared.

Anisotropic Total Variation (ATV) is an L1 norm-based technique that takes into account the direction of pixel value changes. This refined image restoration methodology has evolved from traditional Total Variation (TV) and Anisotropic Diffusion techniques, effectively preserving sharp edges while reducing artifacts.

On the other hand, Isotropic Total Variation (ITV) operates on an L2 norm basis, treating all variations equally across all directions, ultimately resulting in a smoother image output[Vishnevskiy et al., 2017].

For a more clear illustration, please see below for the formulas:

$$ATV : |y_{i+1,j} - y_{i,j}| + |y_{i,j+1} - y_{i,j}| \tag{1}$$

$$ITV : \sqrt{(y_{i+1,j} - y_{i,j})^2 + (y_{i,j+1} - y_{i,j})^2} \tag{2}$$

3. Scope Expansion

In addition to denoising, we introduce a distinct innovation in the form of inpainting. Inpainting is a process employed to seamlessly fill in missing portions of an image, thereby restoring or completing the visual content. To comprehensively analyze the effectiveness of our algorithm under varying scenarios, we utilized masks to cover 25%, 50%, and 75% of the original image, assessing the outcomes against different levels of data absence.

Furthermore, we briefly draw a comparison between inpainting and denoising methodologies. Rather than exclusively applying the SB - ATV approach to images with random noise, our innovative approach also tests the same methodology on images with missing portions. This expansion aims to evaluate the adaptability and robustness of the method across diverse scenarios.

An additional facet pertains to the parameter μ, a weight parameter that balances the regularizing term and the fidelity term in the optimization process. Through empirical experimentation, we conducted trials with different μ values ranging from 0.01 to 100, utilizing a step size of 0.01. Ultimately, by means of trial and error, we identified μ values that are approximately optimal for both denoising and inpainting tasks, thus enhancing the efficacy of our proposed methodologies.

Experimental Results

The Lena image was utilized as an illustrative example to showcase our experimental findings. Furthermore, we conducted extensive testing across a dataset consisting of 52 additional grayscale images. A detailed analysis of these outcomes will be presented in subsequent sections of this paper.

1. Replication of Denoising with SB-ATV method

The initial segment of our experimental results presents a faithful reproduction of the denoising outcomes as illustrated in the referenced paper. We meticulously replicated the methodology amalgamating SB - ATV, subsequently applying it across three distinct noise standard deviation levels: $1 \times 10^{-1.5}$, $3 \times 10^{-1.5}$, and $5 \times 10^{-1.5}$.

Each of the images in our study underwent an extensive process of refinement, with 500 iterations of the Split Bregman method applied. This substantial number of iterations was chosen deliberately to ensure the accuracy and robustness of our results. Through this meticulous iteration process, the algorithm continuously adjusted and optimized the denoised and inpainted images, allowing us to progressively refine the restoration outcomes. By subjecting our methodology to such a rigorous and repeated optimization procedure, we aimed to guarantee that our findings and conclusions are based on a solid foundation of well-defined results.

For a comprehensive visual representation, please refer to Figure 2 which offers a comparative display encompassing the original image, noisy images, and the restored outcomes achieved through the ATV methodologies.

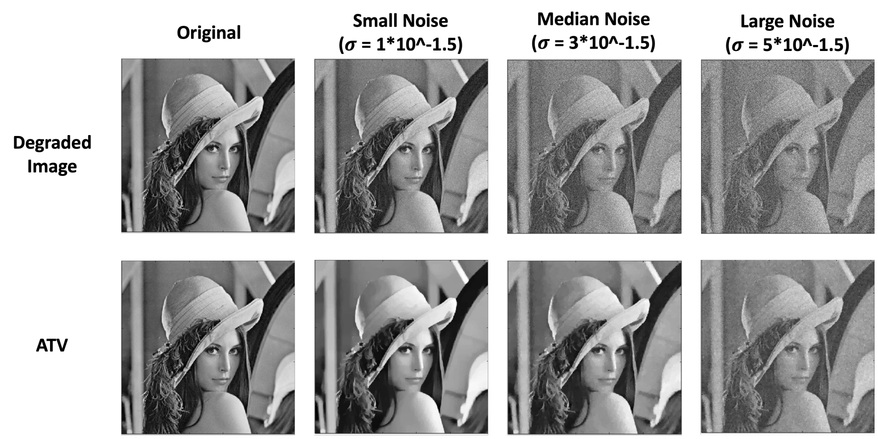

Figure 2: Evaluate - Denoising; ATV

As can be seen from Figure 2, SB-ATV does improve image quality. The improvement is more obvious in low and medium noisy levels, and less obvious in high noisy levels.

We have also computed visual and statistical figures of the Structural Similarity Index (SSIM) to evaluate the efficiency of the paper's method applied to denoising and compare with the results from the paper.

From a graphical standpoint, the Structural Similarity Index (SSIM) graph serves to quantify the degree of similarity between the denoised image and the original image[Wang et al., 2004]. Lighter shades on the graph correspond to higher degrees of similarity between the images. It is evident from the graph that the presented method effectively preserves the edges, while other regions appear to be more ”smoothed”.

Taking a more quantitative approach, the SSIM index furnishes a value within the range of -1 to 1, with a value of 1 signifying complete identity between the two compared images and 0 indicating no similarity.

As depicted in Figure 3 and Table 2, it is evident that images with small to medium noise levels exhibit similar and comparatively improved restoration scores. In contrast, images characterized by significant noise levels display considerably larger errors in the restoration process.

We also applied the SB-ATV methodology on the other 52 images, and below is the analysis of batch results (Figure 4).

It is evident that the performance of degraded images with minimal and moderate noise levels exhibits notable similarity. However, when subjected to substantial noise, distinct disparities emerge, with significantly higher error and lower SSIM values. This analysis drawn from the batch results parallels the conclusion we derived from the Lena image. The batch results serve to reinforce and provide robust substantiation for our earlier findings.

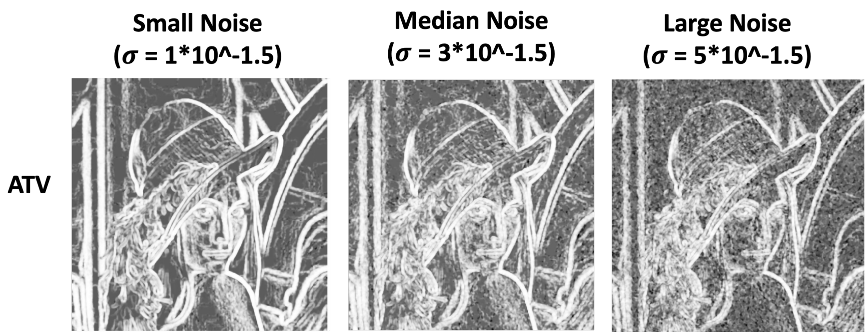

Figure 3: Evaluate - SSIM Graph; ATV

	Small Noise (σ=1*10^-1.5)	Medium Noise (σ=3*10^-1.5)	Large Noise (σ=5*10^-1.5)
ATV - SSIM Value	0.4243	0.4147	0.3308
ATV - Relative Error	0.0575	0.0613	0.0878

Table 2: Evaluate - SSIM Value and Relative Error; ATV

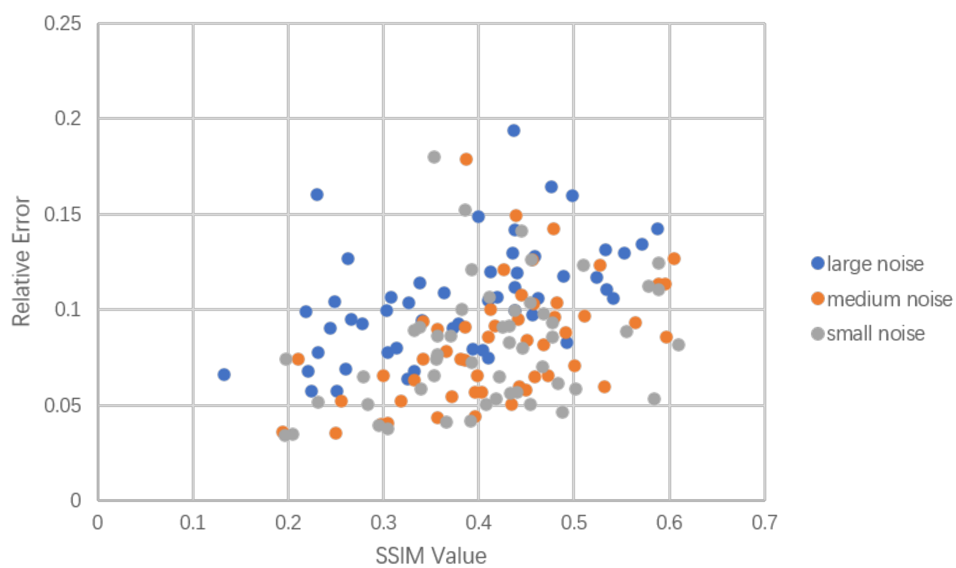

Figure 4: Evaluate - SSIM Value vs. Relative Error; ATV - Batch Result

2. Innovation - Compare ATV and ITV Regularization Methods

The second segment of our experiment involves an innovative comparison between ATV and ITV regularization techniques both using the Split Bregman iteration technique, maintaining the same noisy levels of $1 \times 10^{-1.5}$, $3 \times 10^{-1.5}$, and $5 \times 10^{-1.5}$ as utilized in the previous method for consistency (Figure 5).

Overall, ATV outperforms ITV for small and medium levels of noise, as indicated by both the SSIM and relative error metrics (Table 3 and Figure 6). However, the distinction is less definitive for large noise levels. This observation is consistent with both the analysis of the Lena image and the batch results.

The following image illustrates the comparative effectiveness of ATV and ITV in denoising, consider-

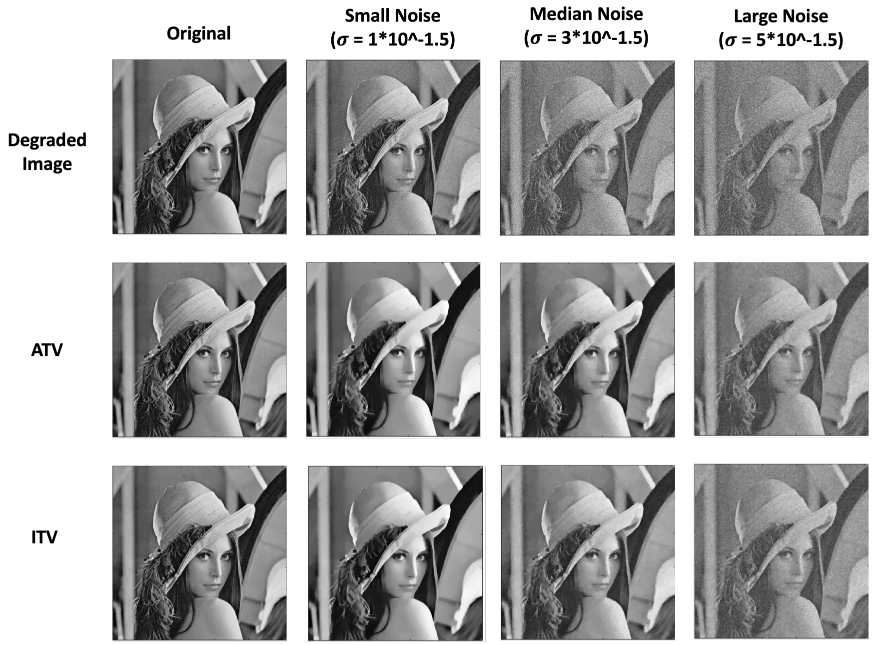

Figure 5: ATV vs. ITV - Denoising

	Small Noise (σ=1*10^-1.5)	Medium Noise (σ=3*10^-1.5)	Large Noise (σ=5*10^-1.5)
ATV - SSIM Value	0.4243	0.4147	0.3308
ITV - SSIM Value	0.4527	0.4445	0.2998
ATV - Relative Error	0.0575	0.0613	0.0878
ITV - Relative Error	0.0522	0.0580	0.1039

Table 3: ATV vs. ITV - SSIM Value and Relative Error

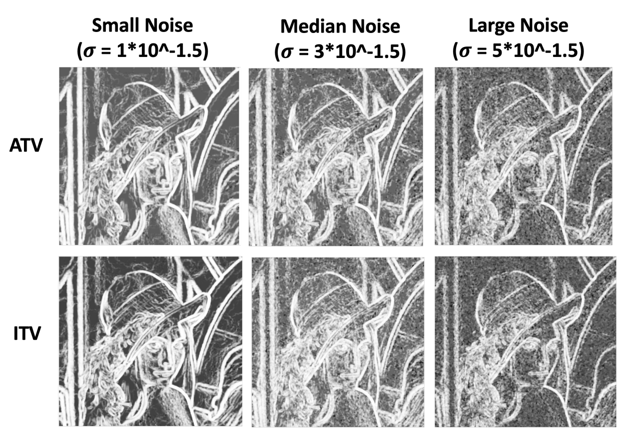

Figure 6: ATV vs. ITV - SSIM Graph

ing a noisy level of $1 \times 10^{-1.5}$ (Figure 7). It can be observed that, in general, ATV slightly outperforms ITV, although the difference is not substantial.

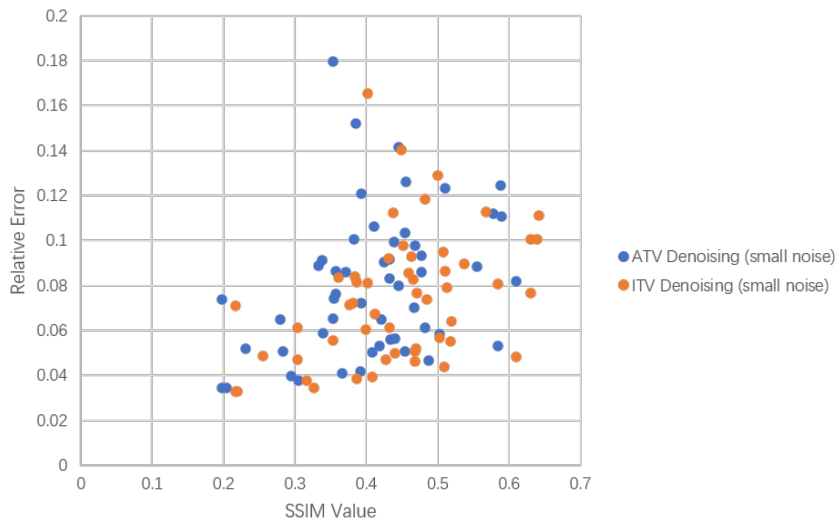

Figure 7: ATV vs. ITV - SSIM Value vs. Relative Error - Batch Result

3. Innocation - SB-ATV and SB-ITV on Inpainting

In the third section, we introduce an innovative extension of the SB-ATV methodology to inpainting, a domain beyond the original scope of denoising. Refer to Figure 8, 9, and Table 4 for the implementation of the ATV and ITV methods on inpainting. This visualization effectively illustrates the breadth of applications covered and demonstrates the versatility of our approach.

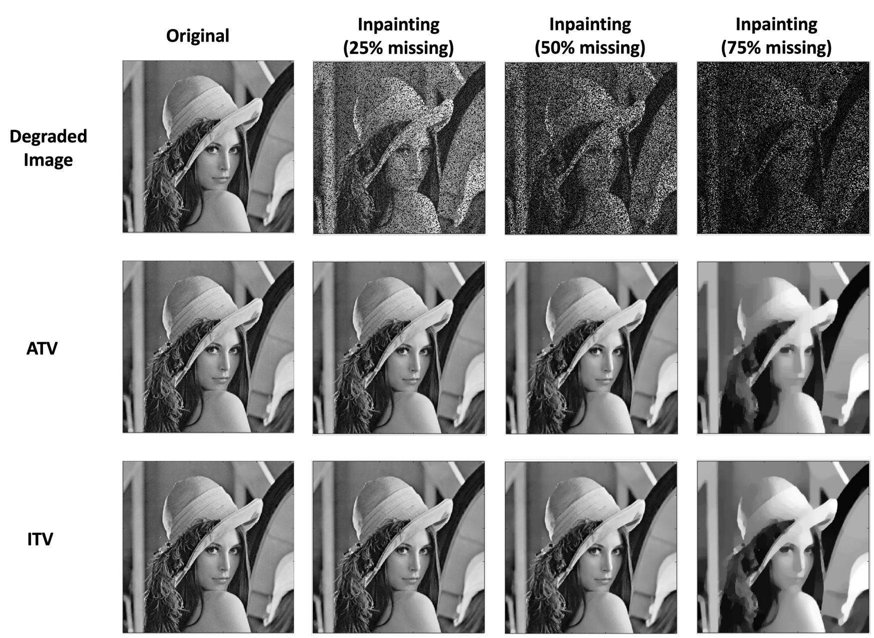

Figure 8: Inpainting - ATV and ITV

As discernible from the aforementioned figures, the restorative efficacy gradually diminishes with an increasing proportion of the image missing. Notably, the restoration effectiveness experiences an exponential decrease when 75% of the image is missing.

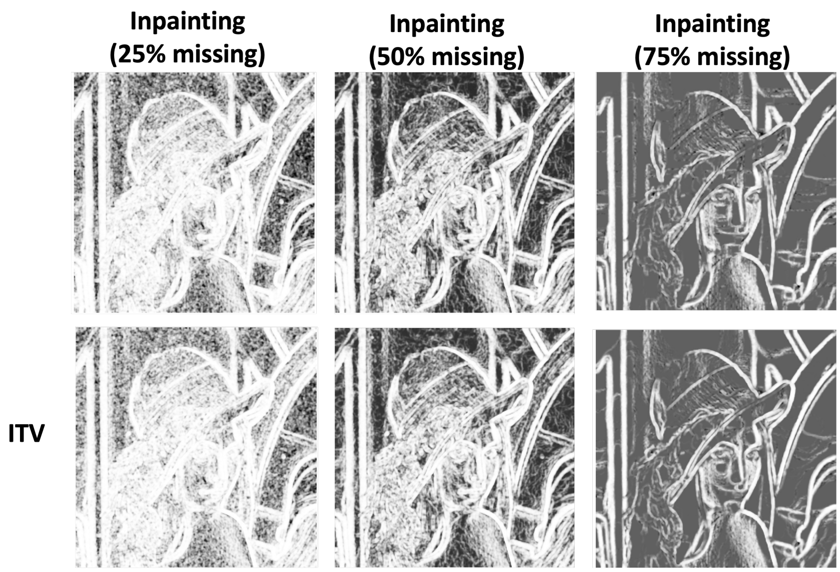

Figure 9: Inpainting - SSIM Graph; ATV and ITV

	Inpainting (25% missing)	Inpainting (50% missing)	Inpainting (75% missing)
ATV - SSIM Value	0.8150	0.6028	0.2445
ITV - SSIM Value	0.8445	0.6278	0.2652
ATV - Relative Error	0.0237	0.0425	0.1073
ITV - Relative Error	0.0221	0.0400	0.0989

Table 4: Inpainting - SSIM Value and Relative Error; ATV and ITV

To delve deeper into the evaluation of effectiveness, we will proceed to present and analyze batch results for an additional 52 images (Figure 10, 11, and 12).

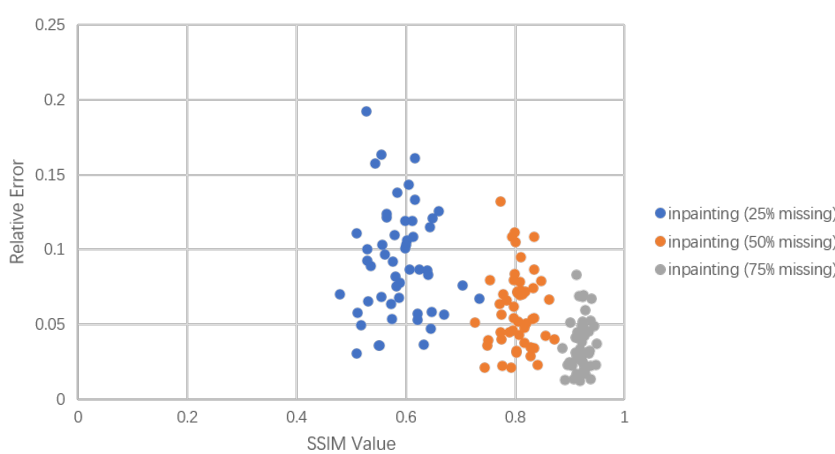

Figure 10: Inpainting - ATV - Batch Result

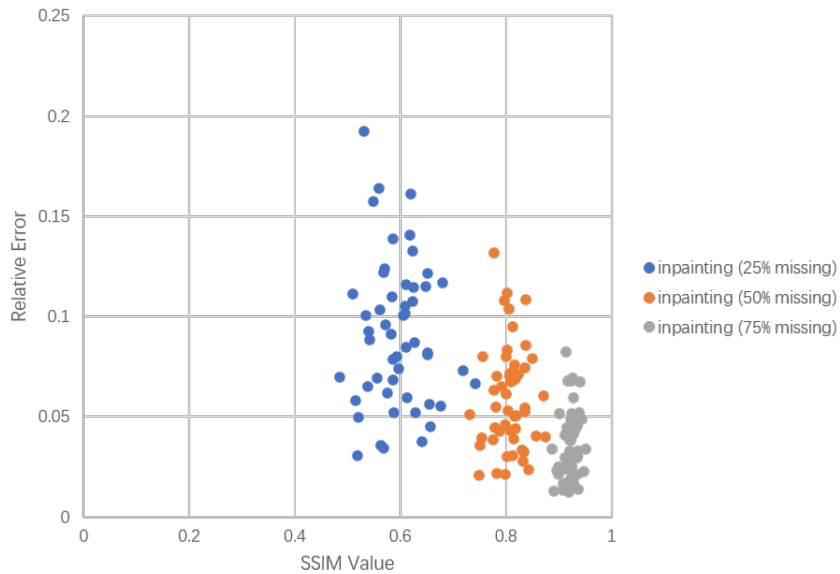

Figure 11: Inpainting - ITV - Batch Result

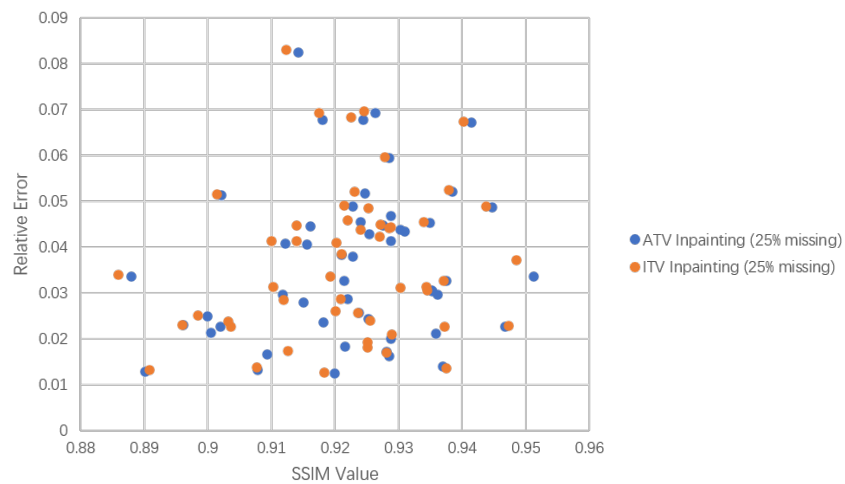

Figure 12: Inpainting - ATV vs. ITV - Batch Result

By looking at the batch results of the inpainting effectiveness, it's more grouped together, which reconfirms the results we derived from Lena image. The more missing in the degraded image, the less effective the ATV and ITV are in the context of inpainting.

The following image illustrates the comparative effectiveness of ATV and ITV in inpainting, considering a 25% missing level. It can be observed that, in general, ATV slightly outperforms ITV, although the difference is not substantial.

Discussion and Conclusions

Our study explored methods to restore noisy and incomplete images, revealing that the utilization of Anisotropic Total Variation (ATV) regularization in conjunction with the Split Bregman Iteration technique effectively achieves successful restoration for both noisy and incomplete images. The subsequent section provides a brief recap of our main findings and possible direction of future work.

The original solution to restore noisy images, as proposed in the chosen paper, is to use the ATV regularization combined with the Split Bregman Iteration technique, in our study we have successfully replicated it.

To assess the efficacy of ATV regularization in the context of denoising, we conducted a comparative analysis with the ITV regularization. A notable distinction between these two approaches lies in their regularization norms: ATV employs L1-norm and considers pixel value directions, while ITV employs L2-norm and treats all directions uniformly. The experimental results confirm our expectations, with ATV exhibiting a slight advantage owing to its ability to incorporate more information. Both methods perform comparably well for low and moderate noise levels, but their effectiveness diminishes for high noise levels.

As an innovation, we extended the scope of our study from denoising to inpainting, evaluating whether the SB-ATV and SB-ITV methods can effectively handle images with missing pixels. The results are encouraging, both methods successfully restore images with randomly scattered missing pixels. However, unlike their performance on noisy images, the quality of restoration (as measured by SSIM and Relative Error) gradually declines as the number of missing pixels increases. This aspect should be taken into consideration when applying SB-ATV or SB-ITV techniques to image restoration scenarios.

A limitation of our study resides in the exclusive focus on a single anisotropy approach within this investigation. To address this limitation, future research endeavors should aim to ascertain the generalizability of our experimental findings across diverse anisotropic methodologies. Furthermore, the Split Bregman Iteration is an iteration method that works particularly well for L1-norm. In our next study, we aim to investigate its applicability to ITV, which is an L2-norm regularization term.

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