QUANTUM SERIES

For

B.Tech Students of Fourth Year
of All Engineering Colleges Affiliated to
Dr. A.P.J. Abdul Kalam Technical University,
Uttar Pradesh, Lucknow
(Formerly Uttar Pradesh Technical University)

Image Processing

By

Akhilesh Verma

Quantum
— Page —
QUANTUM PAGE PVT. LTD.
Ghaziabad ■ New Delhi

PUBLISHED BY : **Apram Singh**
Quantum Publications®
(A Unit of Quantum Page Pvt. Ltd.)
Plot No. 59/2/7, Site - 4, Industrial Area,
Sahibabad, Ghaziabad-201 010

Phone : 0120 - 4160479

Email : pagequantum@gmail.com **Website: www.quantumpage.co.in**

Delhi Office : 1/6590, East Rohtas Nagar, Shahdara, Delhi-110032

Image Processing (CS/IT : Sem-8)
1st Edition : *2011-12*
2nd Edition : *2012-13*
3rd Edition : *2013-14*
4th Edition : *2014-15*
5th Edition : *2015-16*
6th Edition : *2016-17*
7th Edition : *2017-18*
8th Edition : *2018-19*
9th Edition : *2019-20 (Throughly Revised Edition)*

Price: Rs. 110/- only

Printed at : Giriraj Printers, Delhi.

CONTENTS

RCS 082 : IMAGE PROCESSING

1 UNIT

Digital Image Fundamentals

CONTENTS

PART-1

Steps in Digital Image Processing, Components, Elements of Visual Perception, Image Sensing and Acquisition.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 1.1. **What is digital image processing ? Discuss some of its major applications.**

AKTU 2014-15, Marks 05

Answer

Digital image processing :

1. Digital image processing refers to the processing of digital images by means of digital computer.
2. Digital image processing requires a computer and an image digitizer to process image.
3. An image digitizer is used to convert image information into digital bits.
4. The physical image is divided into horizontal lines of adjacent pixels.
5. The number inserted into the digital image at each pixel location reflects the brightness of the image at the corresponding point.
6. The conversion process is called digitization and a common form is illustrated in Fig. 1.1.1.

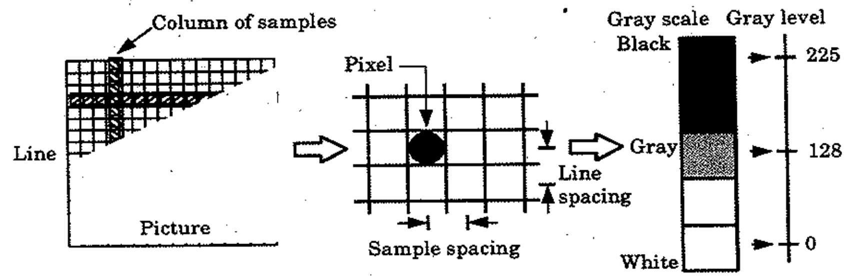


Fig. 1.1.1. Digitizing an image.

Applications of digital image processing :

1. **Gamma-ray imaging :** Major uses of imaging based on gamma rays include nuclear medicine and astronomical observations.

2. **X-ray imaging :** Use of X-ray is used in medical diagnostics, as well as in industry and other areas, like astronomy.
3. **Imaging in the ultraviolet band :** Applications of ultraviolet light include lithography, industrial inspection, microscopy, lasers, biological imaging, and astronomical observation.
4. **Imaging in the visible and infrared band :** Major area of visual processing is remote sensing and micro-inspection to material characterization.

Que 1.2. **What do you understand by digital image processing ? Explain the components of an image processing system.**

AKTU 2016-16, Marks 10

Answer

Digital image processing : Refer Q. 1.1, Page 1–2A, Unit-1.

Components of image processing system :

Fig. 1.2.1 shows the basic components comprising a typical general purpose system used for digital image processing.

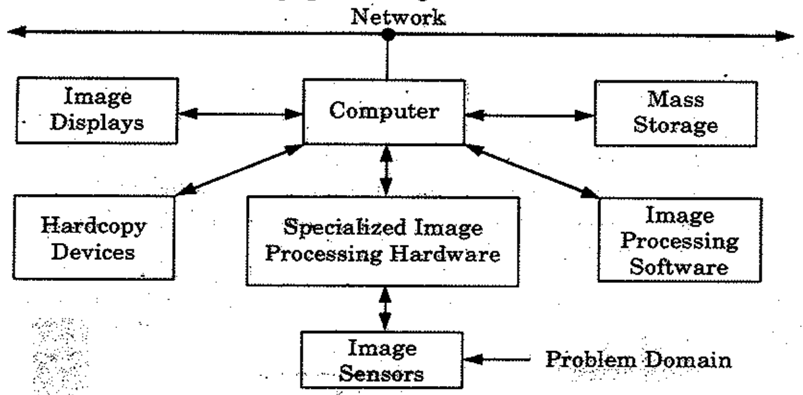


Fig. 1.2.1.

1. **Image sensors :**
   i. It refers to sensing.
   ii. The image sensor captures incoming light, convert it into an electric signal, measure that signal and output it to supporting electronics.
   iii. Image sensor is a 2D array of light-sensitive elements that convert photons to electrons.
   iv. CCD (Charged Coupled Device) and CMOS (Complementary Metal-oxide Semiconductor) image sensors are widely used in image-capturing devices like digital cameras.

   v. Image sensors have two elements that are required to capture digital images.
   vi. The first is a physical device (sensor) that is sensitive to the energy radiated by the object we wish to convert to image.
   vii. The second is a digitizer that is used for converting the output of the physical sensing device into digital form.
2. **Specialized image processing hardware :** It consists of the digitizer and hardware that performs primitive operations, such as an Arithmetic Logic Unit (ALU), which performs arithmetic and logical operations in parallel on entire images.
3. **Computer :** The computer in an image processing system is a general purpose computer and can range from a PC to a supercomputer.
4. **Image processing software :**
   i. Software for image processing consists of specialized modules that perform specific tasks.
   ii. A well-designed package also includes the capability for the user to write code.
5. **Mass storage :**
   i. Mass storage refers to the storage of large amount of data in persisting and machine-readable fashion.
   ii. Mass storage capability is must in image processing applications.
6. **Image displays :**
   i. Image display is the final link in the digital image processing chain.
   ii. Image displays are mainly coloured TV monitors.
7. **Hardcopy devices :** Various devices for recording images are : laser printers, film cameras, heat-sensitive devices, and digital units, such as optical and CD-ROM disks.
8. **Networking :**
   i. It is required component to transmit image information over networked computer.
   ii. Because of the large amount of data inherent in image processing applications, the key consideration in image transmission is bandwidth.

Que 1.3. **What do you mean by digital image representation ?**

AKTU 2017-18, Marks 10

Answer

1. An image can be defined as a 2D signal that varies over the spatial coordinates x and y and can be written mathematically as $f(x, y)$.

2. In general, the image can be written as a mathematical function $f(x, y)$ as follows :

$$f(x, y) = \begin{pmatrix} f(0,0) & f(0,1) & f(0,2) & \cdots & f(0, Y-1) \\ f(1,0) & f(1,1) & f(1,2) & \cdots & f(1, Y-1) \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ f(X-1,0) & f(X-1,1) & f(X-1,2) & \cdots & f(X-1, Y-1) \end{pmatrix}$$

3. The image $f(x, y)$ is divided into X rows and Y columns. Thus, the coordinate ranges are $x = \{0, 1, ..., X-1\}$ and $y = \{0, 1, ..., Y-1\}$.
4. The value of the function $f(x, y)$ at every point indexed by a row and a column is called gray value or intensity of the image. Generally, the value of the pixel is the intensity value of the image at that point.
5. The number of rows in a digital image is called vertical resolution. The number of columns is called horizontal resolution. The number of rows and columns describes the dimensions of the image.
6. The number of bits necessary to encode the pixel value is called bit depth.
7. The set of all colors that can be represented by the bit depth is called gamut or palette.
8. So, the total number of bits necessary to represent the image is
   Number of rows × Number of columns × Bit depth.

Que 1.4. **Briefly explain the elements of visual perception.**

Answer

The three elements that govern visual perception are :

1. **Structure of eyes :**
   i. Fig. 1.4.1 shows a simplified horizontal cross section of the human eye.
   ii. The shape of the eye is nearly spherical with a radius of 11 m. (approx). The outermost layer, called sclera, is 1 mm thick opaque membrane and merges into the transparent cornea.
   iii. At the rear, the optic nerve penetrates the sclera on the nasal side. The choroid, the membrane that lies directly below the sclera contains a network of blood vessels which provide nutrition to the eye.
   iv. The choroid being heavily pigmented reduces backscatter of light within the optical globe. It is divided into the ciliary body and the iris diaphragm.

   v. The iris is a nearly circular aperture which constitutes the pupil. It contracts and expands to control the amount of light entering the eye.
   vi. The innermost membrane is the retina. The retinal surface contains a mosaic of photoreceptor cells called rods and cones.

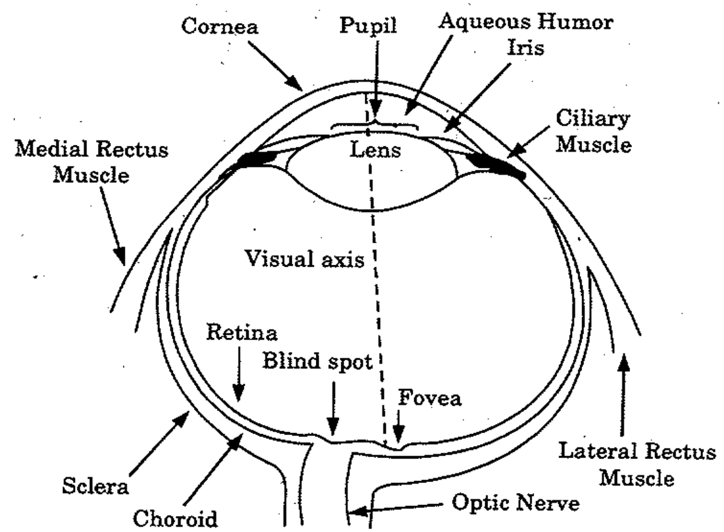


Fig. 1.4.1. Horizontal section of the human eye.

   vii. The number of cones in each eye is between 6 and 7 million and that of rods ranges from 75 to 150 million. Cones are primarily located at the centre of the retina, called fovea and are sensitive to colour.
   viii. The cones are also responsible for acute vision and the cone vision is known as photopic or bright-light vision.
   ix. Rods give an overall appearance of the scene, but are not involved in colour vision. They are sensitive to low levels of illumination. So, the rod vision is known as scotopic or dim-light vision.

2. **Image formation :**
   i. The principal difference between the lens of the eye and an ordinary optical lens is that the former is flexible.
   ii. In Fig. 1.4.2, the radius of the curvature of the anterior surface of the lens is greater than the radius of its posterior surface.

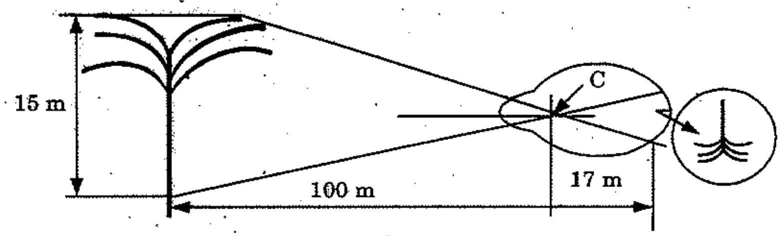


Fig. 1.4.2. Graphical representation of the eye looking at a tree. Point C is the optical center of the lens.

iii. The shape of the lens is controlled by the tension in the fibers of the ciliary body :

a. The focus on distant objects, the controlling muscles cause the lens to be relatively flattened. Similarly, these muscles allow the lens to become thicker in order to focus on objects near the eye.

b. The distance between the focal center of the lens and the retina varies from approximately 14 mm to about 17 mm, as the refractive power of the lens increases from minimum to maximum.

c. When the eye focuses on an object farther away than about 3 m, the lens exhibits its lowest refractive power, and when the eye focuses on a nearby object the lens is most strongly refractive.

3. **Brightness adaptation and discrimination :**

i. Brightness is a psycho-visual concept and is described as the sensation to the light intensity.

ii. The contrast is the difference in perceived brightness.

iii. Detection of a bright spot depends not only on the brightness, size (in space) and duration (in time), but also on the contrast between the spot and the background.

iv. The spot can be detected only when the contrast is greater than a threshold, depending on the average brightness of the surrounding. This dependence is known as brightness adaptation.

v. The range of intensity levels that can be adapted by the human visual system is enormous; the highest level (glare limit) is approximately 10^{10} times the lowest one (scotopic threshold).

vi. The experimental evidence shows that the brightness perceived by human visual system is a logarithmic function of intensity incident on the eye.

Que 1.5. **An image is 2400 pixels wide and 2400 pixels high. The image was scanned at 300 dpi. What is the physical size of the image ?**

Answer

The physical size is given by,

$$= \frac{\text{Number of pixels in width}}{\text{Resolution}} \times \frac{\text{Number of pixels in height}}{\text{Resolution}}$$

$$= \frac{2400}{300} \times \frac{2400}{300}$$

= 8 inches × 8 inches

Que 1.6. **Write short note on image acquisition.**

Answer

1. Image acquisition is the first step in any image processing system.
2. The general aim of image acquisition is to transform an optical image (real world data) into an array of numerical data which could be later manipulated on a computer.
3. Image acquisition is achieved by suitable cameras. We use different cameras for different applications.
4. If we need an X-ray image, we use a camera (film) that is sensitive to X-rays. If we want an infrared image, we use cameras which are sensitive to infrared radiations.
5. For normal images (family pictures etc.), we use cameras which are sensitive to the visual spectrum.

Que 1.7. **What are the different stages of digital image processing ? Explain each stage in detail.**

AKTU 2016-17, Marks 15

OR

What is digital image processing ? Draw a block diagram. And discuss some of its major applications.

AKTU 2016-17, Marks 10

Answer

Different stages in digital image processing :

1. **Image acquisition :**

a. This is the first stage of digital image processing.

b. Image acquisition stage involves preprocessing, such as scaling etc.

2. **Image enhancement :**
   a. Image enhancement is the simplest and most appealing areas of digital image processing.
   b. Basically, the idea behind enhancement techniques is to bring out detail that is obscured, or simply to highlight certain features of interest in an image such as, changing brightness and contrast etc.

3. **Image restoration :**
   a. Image restoration is an area that also deals with improving the appearance of an image.
   b. However, unlike enhancement, which is subjective, image restoration is objective, in the sense that restoration techniques tend to be based on mathematical or probabilistic models of image degradation.

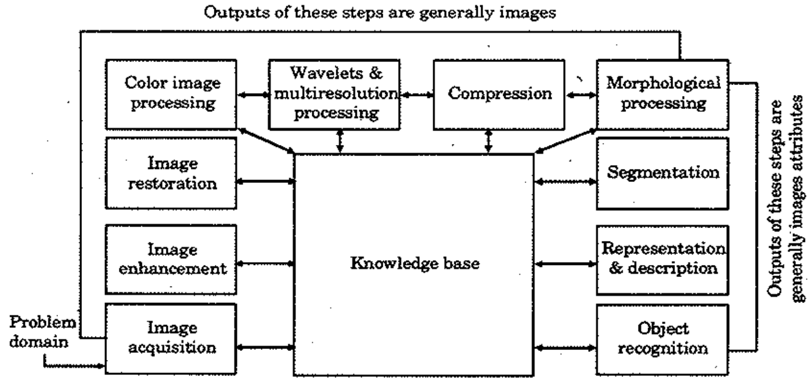


Fig. 1.7.1.

4. **Color image processing :**
   a. Color image processing is an area that has been gaining its importance because of the significant increase in the use of digital images over the Internet.
   b. This may include color modeling and processing in a digital domain etc.

5. **Wavelets and multi-resolution processing :**
   a. Wavelets are the foundation for representing images in various degrees of resolution.
   b. Image is subdivided into smaller regions for data compression and for pyramidal representation.

6. **Compression :**
   a. Compression deals with techniques for reducing the storage required to save an image or the bandwidth to transmit it.

   b. Particularly in the uses of internet it is very much necessary to compress data.

7. **Morphological processing :**
   a. Morphological processing deals with tools for extracting image components that are useful in the representation and description of shape.

8. **Segmentation :**
   a. Segmentation procedures partition an image into its constituent parts or objects.
   b. In general, autonomous segmentation is one of the most difficult tasks in digital image processing.
   c. A rough segmentation procedure brings the process a long way toward successful solution of imaging problems that require objects to be identified individually.

9. **Representation and description :**
   a. Representation and description always follow the output of a segmentation stage, which usually is raw pixel data, constituting either the boundary of a region or all the points in the region itself.
   b. Choosing a representation is only part of the solution for transforming raw data into a form suitable for subsequent computer processing.
   c. Description deals with extracting attributes that result in some quantitative information of interest or are basic for differentiating one class of objects from another.

10. **Object recognition :** Object recognition is the process that assigns a label, such as, "vehicle" to an object based on its descriptors.

11. **Knowledge base :**
   a. Knowledge may be as simple as detailing regions of an image where the information of interest is known to be located, thus limiting the search that has to be conducted in seeking that information.
   b. The knowledge base also can be quite complex, such as an interrelated list of all major possible defects in a materials inspection problem or an image database containing high-resolution satellite images of a region in connection with change-detection applications.

Digital image processing and its application : Refer Q. 1.1, Page 1–2A, Unit-1

PART-2

Image Sampling and Quantization, Relationship Between Pixels, Color Image Fundamentals, RGB, HSI Models, Two Dimensional Mathematical Preliminaries, 2D Transforms : DFT, DCT.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

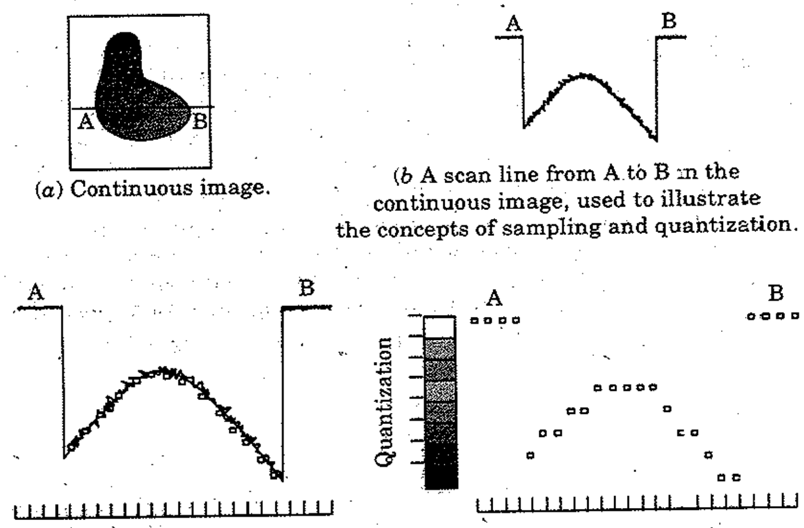
Que 1.8. Write short notes on sampling and quantization.

AKTU 2014-15, Marks 05

AKTU 2016-17, Marks 05

Answer

1. To create a digital image, we need to convert the continuous sensed data into digital form.
2. This involves two processes : sampling and quantization.
3. An image may be continuous with respect to x and y coordinates, and also in amplitude.
4. To convert it into digital form, we have to sample the function in both coordinates and in amplitude.
5. Digitizing the coordinate values is called sampling. Digitizing the amplitude values is called quantization.



(*a*) Continuous image.

(*b* A scan line from A to B in the continuous image, used to illustrate the concepts of sampling and quantization.

(*c*) Sampling and quantization.

(*d*) Digital scan line.

Fig. 1.8.1.

6. Fig. 1.8.1(*a*) shows a continuous image projected onto the plane.
7. Fig. 1.8.1(*b*) shows the image after sampling and quantization.

Que 1.9. Assume that a 10 m high structure is observed from a distance of 50 m. What is the size of the retinal image ?

Answer

Let the size of the retinal image be x.

$$\frac{10}{50} = \frac{x}{17}$$

We know that, the distance between the lens and retina is 17 mm.

Therefore, $x = \dfrac{17 \times 10}{50} = 3.4$ mm

Que 1.10. Explain low level, mid level and high level image processing. Also explain sampling and quantization process.

AKTU 2018-19, Marks 10

Answer

1. **Low level image processing :**
   a. Low-level processing operation involves tasks such as image preprocessing to reduce noise, contrast enhancement, image sharpening, etc.
   b. In the low-level process, both input and output are images.
2. **Mid level image processing :**
   a. Mid-level processing involves tasks such as image segmentation, description of images, object recognition, etc.
   b. In the mid-level process, inputs are generally images but its outputs are generally image attributes.
3. **High level image processing :**
   a. High-level processing involves making sense from a group of recognized objects.
   b. This process is normally associated with computer vision.

Sampling and quantization process : Refer Q. 1.8, Page 1–11A, Unit-1.

Que 1.11. How many minutes are required for a 512 × 512 image with 256 gray levels at 300 baud rate for transmission ? The transmission is accomplished using packets consisting of a start bit, a byte (8 bits) of information and a stop bit. Baud rate means number of bits per second.

Answer

The total amount of data (start bit + bits of data + stop bit) sent for the transmission of, 512 × 512 image is :

$$(512 \times 512) \times (8 + 2) \text{ bits}$$

The total time required to transmit this image over a 300 baud link is :

$$(512 \times 512) \times \frac{(8+2)}{300} = 262144 \times \frac{10}{300}$$

$$= 8738.13 \text{ second} \cong 145 \text{ minutes}$$

Que 1.12. **Explain types of sampling in image processing.**

Answer

There are three types of sampling methods in image processing :

i. **Ideal sampling :** In ideal sampling (Instantaneous sampling) pulses from the analog signal are sampled. This is an ideal sampling method and cannot be easily implemented.

ii. **Natural sampling :** Natural sampling is a practical method of sampling in which pulse have finite width equal to T. The result is a sequence of samples that retain the shape of the analog signal.

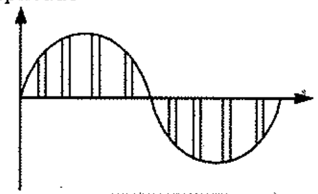


Fig. 1.12.1.

iii. **Flat top sampling :** In comparison to natural sampling, flat top sampling can be easily obtained. In this sampling technique, the top of the samples remains constant by using a circuit. This sampling method is commonly used.

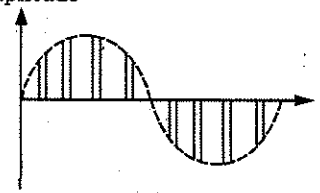


Fig. 1.12.2.

Que 1.13. **Write short note on pixels.**

Answer

1. Pixel is the smallest element of an image.
2. Each pixel corresponds to any one value. In an 8-bit gray scale image, the value of the pixel is between 0 and 255.
3. The value of a pixel at any point corresponds to the intensity of the light photons striking at that point.
4. Each pixel stores a value proportional to the light intensity at that particular location.
5. A pixel is also known as PEL.
6. Total number of pixels = Number of rows * Number of columns
7. The number of (x, y) coordinate pairs also make up the total number of pixels.

Que 1.14. **Define color model. Describe different color model used in digital image processing.**

Answer

Color model :

1. A color model is a system for creating a full range of colors from a small set of primary colors.
2. There are two types of color models :
   a. Additive color models which uses light to display color.
   b. Subtractive color models which uses printing inks.

Different color model used in image processing are :

A. RGB color model :

1. The RGB color model is an additive color model. In this case red, green and blue light are added together in various combinations to reproduce a wide spectrum of colors.
2. The primary purpose of the RGB color model is for the display of images in electronic systems, such as on television screens and computer monitors and it's also used in digital photography.
3. Cathode ray tube, LCD, plasma and LED displays all utilize the RGB model.
4. In order to create a color with RGB, three colored light beams (one red, one green, and one blue) must be superimposed.
5. With no intensity, each of the three colors is perceived as black, while full intensity leads to a perception of seeing white.
6. Differing intensities produce the hue of a color, while the difference between the most and least intense of the colors make the resulting color more or less saturated.

B. CMYK color model :

1. The CMYK color model (four-color process) is a subtractive color model.
2. CMYK works by partially or completely masking colors on a white background.
3. The printed ink reduces the light that would otherwise be reflected. That's why this model is called subtractive because inks 'subtract' brightness from a white background from four colors: cyan, magenta, yellow and black.
4. CMYK is able to produce the entire spectrum of visible colors due to the process of half-toning.
5. In this process, each color is assigned a saturation level and miniscule dots of each of the three colors are printed in tiny patterns.
6. This enables the human eye to perceive a specific color made from the combination. In order to improve print quality and reduce large-scale interference patterns, the screen for each color is set at a different angle.

Que 1.15. Explain HSI color model in details.

Answer

1. HSI stands for Hue, Saturation and Intensity. When humans view a color object it is described by its hue, saturation and brightness.
2. Hue is a color attribute that describes a pure color (yellow, orange or red)
3. Saturation gives a measure of the degree to which a pure color is diluted by white light.
4. Brightness depends upon color intensity, which is key factor in describing color sensation. The intensity is easily measurable and the results are also easily interpretable.
5. Thus the model that is used to describe a color object is the HSI model.

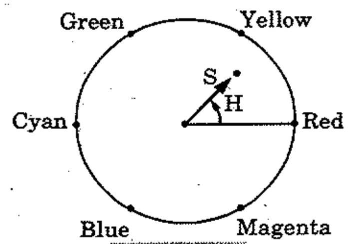


Fig. 1.15.1

6. The HSI model decouples the intensity from color-carrying information (hue and saturation) in a color image.
7. Fig. 1.15.1 shows the HSI model. The angle from the red axis gives the Hue, the length of the vector is the saturation and the intensity is given by the position of the plane on the vertical intensity axis.

Que 1.16. What are the advantages and disadvantages of RGB and CMYK color model ?

Answer

Advantages of RGB color model :

1. No transformation is required to display information on the screen, for this reason it considered as the base color space for various applications.
2. It is used in video display because of additive property.
3. It is computationally practical system.

Disadvantages of RGB color model :

1. It is non useful for objects specification and recognition of colors.
2. It is difficult to determine specific color in RGB model.
3. It is hardware oriented system.

Advantages of CMYK color model :

1. Commonly used for production printer color.

Disadvantages of CMYK color model :

1. It is a subtractive model, the components are pigments or inks not color.

Que 1.17. Explain two-dimensional mathematical preliminaries.

Answer

Two-dimensional mathematical preliminaries are :

1. **Matrix algebra :**
   i. An ordered set of M elements such as time series of some data is known as a vector and written as

$$g = \begin{bmatrix} g_0 \\ g_1 \\ \vdots \\ g_{M-1} \end{bmatrix}$$

   ii. A matrix G of size $M \times N$ is a double-indexed ordered set with MN elements :

$$G = \begin{bmatrix} G_{0,0} & G_{0,1} & \cdots & G_{0,N-1} \\ G_{1,0} & G_{1,1} & \cdots & G_{1,N-1} \\ \vdots & \ddots & & \vdots \\ G_{M-1,0} & G_{M-1,1} & \cdots & G_{M-1,N-1} \end{bmatrix}$$

iii. The matrix is said to consist of M rows and N columns. The first and second index of a matrix element denotes the row and column number, *i.e.*, the x and y coordinates, respectively.

2. **Fourier transformation :**

i. In one dimension, the Fourier transform of a complex-valued function $f(x)$ is defined as :

$$\hat{g}(k) = \frac{1}{2\pi}\int_{-\infty}^{\infty} dx\, g(x)\exp(-ikx)$$

where $k = 2\pi/\lambda$ is the wave number of the complex exponential $\exp(-ikx)$ with the wavelength λ. The back transformation is given by

$$g(x) = \int_{-\infty}^{\infty} dx\, \hat{g}(x)\exp(ikx)$$

ii. A function $g(x)$ and its Fourier transform $\hat{g}(k)$ form a Fourier transform pair denoted by

$$g(x) \circ\!\!-\!\!\!-\!\!\bullet\ \hat{g}(k)$$

iii. The complex exponentials, the kernel of the Fourier transform, constitute an orthonormal basis :

$$\int_{-\infty}^{\infty} dx \exp(-ik'x)\exp(ikx) = 2\pi\delta(k'-k)$$

iv. The n-dimensional Fourier transform is defined by

$$\hat{g}(k) = \frac{1}{(2\pi)^n}\int_{-\infty}^{\infty} d^n x\, g(x)\exp(-ikx)$$

v. The kernel of the multidimensional Fourier transform is separable :

$$\exp(-ikx) = \prod_{i=1}^{n}\exp(-ik_i x_i)$$

vi. Therefore the n-dimensional Fourier transform can be separated in n one-dimensional Fourier transforms. For example, the two-dimensional Fourier transform can be written as

$$\hat{g}(k) = \frac{1}{(2\pi)^2}\int_{-\infty}^{\infty} dx_2\left[\frac{1}{2\pi}\int_{-\infty}^{\infty} dx_1\, g(x)\exp(-ik_1x_1)\right]\exp(-ik_2x_2)$$

vii. The inverse Fourier transform is defined by

$$g(x) = \int_{-\infty}^{\infty} d^n k\, \hat{g}(k)\exp(ikx)$$

3. **Discrete Fourier Transform (DFT) :**

i. The one-dimensional DFT maps a complex-valued vector g onto another vector $\hat{g}$ of a vector space with the same dimension M :

$$\hat{g}_u = \frac{1}{M}\sum_{m=0}^{M-1} g_m \exp\left(-\frac{2\pi i\, mu}{M}\right) = \frac{1}{M}\sum_{m=0}^{M-1} g_m W_M^{-mu}$$

where $\quad W_M = \exp\left(\frac{2\pi i}{M}\right)$

ii. The back transformation is given by

$$g_m = \sum_{u=0}^{M-1} \hat{g}_u W_M^{mu}$$

iii. In two-dimensions, the DFT maps a complex-valued $M \times N$ matrix onto another matrix of the same size :

$$\hat{G}_{uv} = \frac{1}{MN}\sum_{m=0}^{M-1}\sum_{n=0}^{N-1} G_{mn}\exp\left(-\frac{2\pi i\, mu}{M}\right)\exp\left(-\frac{2\pi i\, nv}{N}\right)$$
$$= \frac{1}{MN}\sum_{m=0}^{M-1}\left(\sum_{n=0}^{N-1} G_{mn} W_N^{-nv}\right)W_N^{-mu}$$

The inverse two-dimensional DFT is given by

$$G_{uv} = \sum_{u=0}^{M-1}\sum_{v=0}^{N-1} \hat{G}_{uv} W_M^{mu} W_N^{nv}$$

Que 1.18. **Write short note on Discrete Cosine Transform (DCT).**

Answer

1. A Discrete Cosine Transform (DCT) expresses a finite sequence of data points in terms of a sum of cosine function oscillating at different frequencies.
2. Discrete Cosine Transform is used in lossy image compression because it has very strong energy compaction, *i.e.*, its large amount of information is stored in very low frequency component of a signal and other frequency having very small data which can be stored by using very less number of bits (usually, atmost 2 or 3 bit).

3. To perform DCT transformation on an image, first we have to fetch image file information (pixel value in term of integer having range 0 - 255) which we divides in block of 8 × 8 matrix and then we apply discrete cosine transform on that block of data.
4. For simplicity, we took a matrix of size 8 × 8 having all value as 255 (considering image to be completely white) and we are going to perform 2-D discrete cosine transform on that to observe the output.

Que 1.19. **Find the DFT of $f(x) = \{0, 1, 2, 1\}$.**

AKTU 2014-15, Marks 05

Answer

Given, $f(x) = \{0, 1, 2, 1\}$

We know, $F(k) = \sum_{x=0}^{N-1} f(x)\, e^{-j2\pi kx/N}$ $\quad k = 0, 1, \ldots N-1$

$F(0) = f(0)e^{-0} + f(1)e^{-0} + f(2)e^{-0} + f(3)e^{-0}$
$F(0) = [f(0) + f(1) + f(2) + f(3)] = [0 + 1 + 2 + 1] = 4$
$F(0) = 4$
$F(1) = [f(0)e^{-j2\pi/4} + f(1)e^{-j2\pi 1/4} + f(2)e^{-j2\pi 2/2} + f(3)e^{-j2\pi 3/4}]$
$F(1) = [f(0) + f(1)e^{-j2\pi/4} + f(2)e^{-j4\pi/4} + f(3)e^{-3\pi/2}]$

$$F(1) = \left\{0 + 1\left[\cos\frac{\pi}{2} - j\sin\frac{\pi}{2}\right] + 2[\cos\pi - j\sin\pi] + 1\left[\cos\frac{6\pi}{4} - j\sin\frac{6\pi}{4}\right]\right\}$$

$F(1) = 0 - j - 2 + j$
$F(1) = -2$
$F(2) = [f(0)e^{-j2\pi} + f(1)e^{-j\pi} + f(2)e^{-j2\pi} + f(3)e^{-j3\pi}]$
$F(2) = \{0 + 1[\cos\pi - j\sin\pi] + 2[\cos 2\pi - j\sin 2\pi] + [\cos 3\pi - j\sin 3\pi]\}$
$F(2) = [-1 + 2(1) - 1]$
$F(2) = 0$
$F(3) = [f(0)e^{-j2\pi} + f(1)e^{-j6\pi/4} + f(2)e^{-j3\pi} + f(3)e^{-j9\pi/2}]$

$$F(3) = \left[0 + 1\left[\cos\frac{6\pi}{4} - j\sin\frac{6\pi}{4}\right] + 2[\cos 3\pi - j\sin 3\pi] + 1\left[\cos\frac{9\pi}{2} - j\sin\frac{9\pi}{2}\right]\right]$$

$F(3) = 0 - j - 2 + j$
$F(3) = -2$
$F(k) = \{4, -2, 0, -2\}$

Que 1.20. **Explain 1-dimensional and 2-dimensional Fourier transform.**

Answer

1-dimensional Fourier transform :

1. Let $f(x)$ be a continuous function of x. The Fourier transform of $f(x)$ is,

$$F(u) = \int_{-\infty}^{\infty} f(x)\, e^{-j2\pi ux}\, dx$$

$$= \int_{-\infty}^{\infty} f(x)\, [\cos 2\pi ux - j\sin 2\pi ux]\, dx$$

2. Similarly, the inverse Fourier transform is,

$$f(x) = \int_{-\infty}^{\infty} F(u)\, e^{+j2\pi ux}\, du$$

3. Because $F(u)$ is complex, *i.e.*,

$$F(u) = R(u) + j\, I(u)$$

where R is real and I is imaginary.

4. $F(u)$ has a magnitude plot as well as a phase plot.

Mag $\rightarrow |F(u)| = [R^2(u) + I^2(u)]^{1/2}$

Phase $\rightarrow |F(u)| = \left[\frac{I(u)}{R(u)}\right]$

$|F(u)|$, when plotted is called the magnitude plot or the Fourier spectrum.

At times, we also plot the power spectrum

$$P(u) = |F(u)|^2 = R^2(u) + I^2(u)$$

5. It is the magnitude plot that gives us maximum information and hence we normally plot only the magnitude.

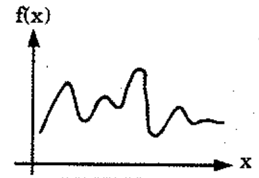


Fig. 1.20.1.

2-dimensional Fourier transform :

1. Images being 2-dimensional functions, we need to define a 2D Fourier transform.
2. The 1D Fourier transform can be easily extended to a function $f(x, y)$, of two variables.

$$F\{f(x, y)\} = F(u, v) = \iint_{-\infty}^{\infty} f(x, y)\, e^{-j2\pi(ux + vy)}\, dx\, dy \text{ and}$$

$$F^{-1}\{F(u, v)\} = f(x, y) = \iint_{-\infty}^{\infty} F(u, v)\, e^{j2\pi(ux + vy)}\, du\, dv$$

3. In 1D Fourier transform, the magnitude plot is given by,
$$|F(u, v)| = [R^2(u, v) + I^2(u, v)]^{1/2}$$
and the phase plot is given by,
$$F(u, v) = \tan^{-1}\left[\frac{I(u, v)}{R(u, v)}\right]$$

4. The power spectrum is given by,
$$P(u, v) = |F(u, v)|^2 = R^2(u, v) + I^2(u, v)$$

Que 1.21. **Give various properties of discrete Fourier transform.**

Answer

Properties of discrete Fourier transform :

1. **Separability :** The DFT can be represented as :
$$F(u, v) = \frac{1}{M}\sum_{x=1}^{M-1} \exp\left[-2\pi j\,\frac{ux}{M}\right]\sum_{y=1}^{M-1} f(x, y) \exp\left[-2\pi j\,\frac{vy}{M}\right]$$
for $u, v = 0, 1, \ldots\ldots, M-1$
The advantage of this property is that a 2D image can be conceived from two 1D transforms.

2. **Translation :** This property can be represented as :
$$f(x, y)\exp\left[j2\pi\frac{(u_0x + v_0y)}{M}\right] \Leftrightarrow F(u - u_0, v - v_0)$$
and
$$f(x - x_0, y - y_0) \Leftrightarrow F(u, v)\exp\left[-j2\pi\frac{(ux_0 + vy_0)}{M}\right]$$

3. **Periodicity :** The Fourier transform and its inverse are periodic. If M is the period,
$$\begin{aligned} F(u, v) &= F(u + M, v) \\ &= F(u, v + M) \\ &= F(u + M, v + M) \end{aligned}$$
If the values of $f(x, y)$ are real, the Fourier transform exhibits conjugate symmetry.
$$F(u, v) = |F(u, v)| = |F(-u, -v)|\ {}^*F(-u, -v)$$
and
$$|F(u, v)| = |F(-u, -v)|$$

4. **Rotation :** If the image is rotated by an angle, the Fourier transform is *rotated by the same angle.*
$$F(r, \theta + \theta_0) \Rightarrow F(w, \phi + \phi_0)$$

5. **Distribution :** The Fourier transform is distributive over addition. This is represented as :

$$F(f_1(x, y) + (f_2(x, y)) = F(f_1(x, y)) + F(f_2(x, y))$$

6. **Scaling :** Any scaling in the spatial domain is reflected in the frequency domain, *i.e.*,
$$af(x, y) \Rightarrow aF(u, v)$$
and
$$f(ax, by) \Rightarrow \frac{1}{|ab|}F\left(\frac{u}{a}, \frac{v}{b}\right)$$

7. **Average value :** This property can be represented as :
$F(x, y) = F(0, 0)$ is called the DC component.

8. **Laplacian :** It applies to the Fourier transform as :
$$F(\nabla^2 f(x, y)) \Leftrightarrow -(2\pi)^2(u^2 + v^2)F(u, v)$$

9. **Convolution and correlation :** The convolution of two functions $f(x)$ and $g(x)$ is given as :
$$f(x)^* g(x) = \int_{-\infty}^{+\infty} f(\alpha)^* g(x - \alpha)\, d\alpha$$
Here α is a dummy variable. Similarly, the correlation of two functions is given as,
$$f(x)^* g(x) = \int_{-\infty}^{+\infty} f(\alpha)^* g(x + \alpha)\, d\alpha$$

Que 1.22. **Prove that 2D continuous and discrete Fourier transforms are linear operations.**

Answer

2D continuous Fourier transform :

1. The Fourier transform of a continuous function $f(t)$ of a continuous variable, t, denoted by $\tau\{f(t)\}$, is defined by the equation
$$\tau\{f(t)\} = \int_{-\infty}^{\infty} f(t)\, e^{-j2\pi\mu t}\, dt \qquad \ldots(1.22.1)$$
where μ is also a continuous variable.
2. Because t is integrated out, $\tau[f(t)]$ is a function only of μ.
3. We can write the Fourier transform as $\tau\{f(t)\} = F(\mu)$, that is, the Fourier transform of $f(t)$ may be written as :
$$F(u) = \int_{-\infty}^{\infty} f(t)\, e^{-j2\pi\mu t}\, dt \qquad \ldots(1.22.2)$$
4. Conversely, given $F(u)$, we can obtain $f(t)$ back using the inverse.
5. Fourier transform, $f(t) = \tau^{-1}\{F(u)\}$ as :
$$f(t) = \int_{-\infty}^{\infty} F(u)\, e^{-j2\pi\mu t}\, du$$

6. Variable μ is integrated out in the inverse transform and written as $f(t)$, rather than $f(t) = \tau^{-1}\{F(\mu)\}$.
7. Equations (1.22.1) and (1.22.2) comprise Fourier transform pair.
8. This indicates that Fourier transform can be reconstructed completely via inverse process, with no loss of information.
9. This is one of the important characteristics of these representations because it allows to work in 'Fourier domain' and then return to original domain of the function without losing any information.
10. Hence, Fourier transform and its inverse is a linear process.

2D discrete Fourier transform :

1. Given the transform $F(u, v)$, we can obtain $f(x, y)$ by using the Inverse Discrete Fourier Transformer (IDFT) :

$$f(x, y) = \frac{1}{MN} \sum_{u=0}^{M-1} \sum_{v=0}^{N-1} F(u, v)\, e^{j2\pi(ux/M + vy/N)}$$

$$F(x, y) \underset{FT^{-1}}{\overset{FT}{\rightleftharpoons}} F(u, v)$$

2. Above expression is known as 2D-DFT pair.
   where,
   M, N : image size.
   x, y : image pixel position, where $x = 0, 1, 2, \ldots, M-1$ and $y = 0, 1, 2, \ldots, N-1$.
   u, v : spatial frequency, where $u = 0, 1, 2, \ldots, M-1$ and $v = 0, 1, 2, \ldots, N-1$.
3. Any signal $[f(x, y)]$ can be represented as a linear combination of set of basic component $[e^{j2\pi(ux/M + vy/N)}]$ also called as Fourier components.
4. Here $F(u, v)$ is weight factor assigned to the Fourier components.

$$\text{2D digital image} \longrightarrow \underbrace{f(x, y)}_{\text{2D digital image}}$$

$$= \frac{1}{MN} \sum_{u=0}^{M-1} \sum_{v=0}^{N-1} \underbrace{F(u, v)}_{\text{Weight}} \overbrace{e^{j2\pi(ux/M + vy/N)}}^{\text{Basic component}}$$

5. The basic component or Fourier component is called as spatial frequency, this is not to be confused with electromagnetic frequencies.
6. That is, it is the rate at which a spatial pattern is changing and it is regarded as spatial frequency.

Que 1.23. Apply DFT to the following sequence and verify whether it works

$$x = \{1\ 2\ 8\ 9\}$$

Answer

Here $M = 4$. Therefore,

$$F(u) = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{pmatrix} \times \begin{pmatrix} 1 \\ 2 \\ 8 \\ 9 \end{pmatrix} = \begin{pmatrix} 20 \\ -7+7j \\ -2 \\ -7-7j \end{pmatrix}$$

The inverse transform is given as

$$f(x) = \frac{1}{4}\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{pmatrix} \times F(u)$$

Take the conjugate of the kernel.

$$f(x) = \frac{1}{M}[W^* \times F(u)],\ W^* = \frac{1}{W_M}$$

$$f(x) = \frac{1}{4}\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{pmatrix} \times \begin{pmatrix} 20 \\ -7+7j \\ -2 \\ -7-7j \end{pmatrix}$$

$$= \frac{1}{4}\begin{pmatrix} 4 \\ 8 \\ 32 \\ 36 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 8 \\ 9 \end{pmatrix}$$

Hence, the original sequence is obtained without any loss of information.

Que 1.24. Apply the DCT to the following matrix :

$$F = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$$

Answer

The discrete cosine transform is given by

$$g = \begin{pmatrix} \frac{1}{\sqrt{2}} & \cos\frac{\pi}{4} \\ \frac{1}{\sqrt{2}} & \cos\frac{3\pi}{4} \end{pmatrix} \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} \frac{1}{\sqrt{2}} & \cos\frac{\pi}{4} \\ \frac{1}{\sqrt{2}} & \cos\frac{3\pi}{4} \end{pmatrix}$$

$$g = \begin{pmatrix} 3 & 0 \\ 0 & -1 \end{pmatrix}$$

It can be verified that $F = A \times g \times A$ gives back the original image.

VERY IMPORTANT QUESTIONS

Following questions are very important. These questions may be asked in your SESSIONALS as well as UNIVERSITY EXAMINATION.

Q. 1. What is digital image processing ? Discuss some of its major applications.

Ans. Refer Q. 1.1.

Q. 2. What do you understand by digital image processing ? Explain the components of an image processing system.

Ans. Refer Q. 1.2.

Q. 3. What are the different stages of digital image processing ? Explain each stage in detail.

Ans. Refer Q. 1.7.

Q. 4. Write short notes on sampling and quantization.

Ans. Refer Q. 1.8.

Q. 5. Explain low level, mid level and high level image processing. Also explain sampling and quantization process.

Ans. Refer Q. 1.10.

Q. 6. Find the DFT of $f(x) = \{0, 1, 2, 1\}$.

Ans. Refer Q. 1.19.

☺☺☺

2
UNIT

Image Enhancement

CONTENTS

PART-1

Spatial Domain : Gray Level Transformation, Histogram Processing, Basics of Spatial Filtering, Smoothing and Sharpening Spatial Filtering.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 2.1. What do you mean by enhancement in spatial domain ?

Answer

1. The term spatial domain means working in the given space, in this case, the image.
2. It implies working with the pixel values or working directly with the raw data.
3. Let $f(x, y)$ be the original image where f is the gray level value and (x, y) are the image coordinates.
4. For a 8-bit image, f can take values from 0 – 255 where 0 represents black, 255 represents white and all the intermediate values represent shades of gray.
5. In a image of size 256 × 256, x and y can take values from (0, 0) to (255, 255) as shown in the Fig. 2.1.1.

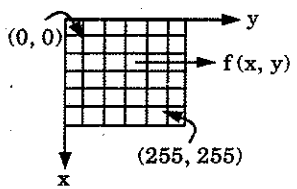


Fig. 2.1.1.

6. The modified image can be expressed as :

$$g(x, y) = T[f(x, y)]$$

7. Here $f(x, y)$ is the original image and T is the transformation applied to it, to get a new modified image $g(x, y)$.

Que 2.2. Distinguish between spatial domain techniques and frequency domain techniques of image enhancement.

Answer

S. No.	Spatial domain technique	Frequency domain technique
1.	In spatial domain technique, we operate directly on the individual pixels, composing an image.	In frequency domain technique, we process an image by changing the frequency components in an image.
2.	Spatial domain processing method includes point process, mask process and global process.	Frequency domain processing method includes only global process.
3.	Spatial domain image enhancement techniques are gray scale manipulation, histogram equalization, image smoothing and image sharpening.	Frequency domain image enhancement techniques are filtering and homomorphic filtering.
4.	In spatial domain image enhancement, some convolution operation is involved, hence computation cost is higher and the processes are complex.	Frequency domain processing does not involve any convolution operation, so computation cost becomes lesser and the processes become simpler.

Que 2.3. Define point processing. Give various gray level slicing techniques. What is contrast stretching ?

OR

Write short note on bit plane slicing. **AKTU 2018-19, Marks 05**

Answer

In point processing, we work with single pixels *i.e.*, T is 1 × 1 operator. It means that the new value $f(x, y)$ depends on the operator T and the present $f(x, y)$.

Various gray level slicing techniques are :

1. **Digital negative :**
   i. Digital negatives are used in displaying of an X-ray image.
   ii. Negative means inverting the gray levels *i.e.*, black in the original image will look white and vice versa.

iii. Fig. 2.3.1 is the digital negative transformation for a 8-bit image.

iv. The digital negative image can be obtained by using a simple transformation given by,

$$s = 255 - r \quad \text{where,} \quad r_{max} = 255$$

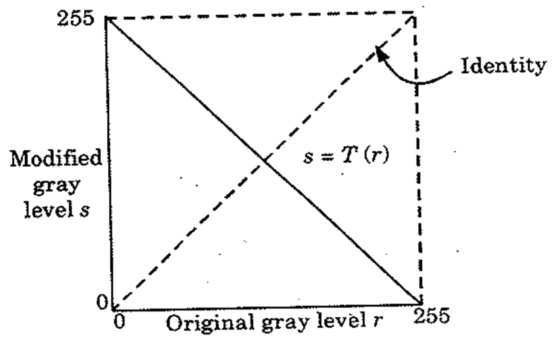


Fig. 2.3.1.

2. **Contrast stretching :**

i. Contrast stretching technique is used to stretch the dynamic range of an image.

ii. Dynamic range is the range between the minimum intensity value and the maximum intensity value of an image.

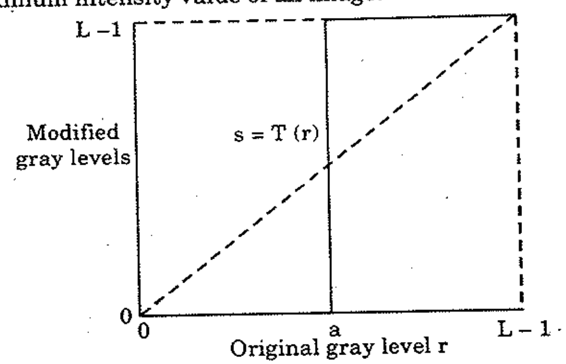


Fig. 2.3.2.

iii. Mathematically, contrast stretching is given by

$$l'(x, y) = \frac{d}{l_{max} - l_{min}} \times (l(x, y) - l_{min}) + l_0 \quad \text{... (2.3.1)}$$

iv. Where, $l'(x, y)$ is the new dynamic range image, d is the new dynamic range value, $l(x, y)$ is the input image, l_{min} is the minimum intensity value of the input image, l_{max} is the minimum intensity value of the input image and l_0 is the offset point of the new dynamic range for $l'(x, y)$.

v. This transformation will provide good visual representation of the original scene but some of the detail may be loss due to saturation and clipping as well as due to poor visibility in under-exposure regions of the image.

3. **Gray level slicing (Intensity slicing) :**

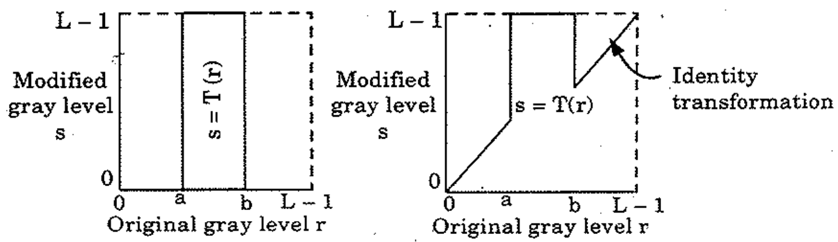
i. Thresholding splits the gray level images into two parts.

ii. When we need to highlight a specific range of gray values, for example enhancing the flaws in an X-ray.

iii. In such circumstances, we use a transformation known as gray level slicing.

iv. The transformation is shown in the Fig. 2.3.3(a).

v. It looks similar to the thresholding function except that here we select a band of gray level values.



(a) Slicing without background (b) Slicing with background

Fig. 2.3.3.

vi. This can be implemented using the formulation

$$s = L - 1 \; ; \; \text{if } a \le r \le b$$
$$s = 0 \quad ; \; \text{otherwise}$$

where L is the number of gray levels.

vii. This method is known as gray level slicing without background.

viii. This is because in this process, we have completely lost the background.

ix. In some applications, we not only need to enhance a band of gray levels but also need to retain the background.

x. This technique of retaining the background is called as gray level slicing with background. The transformation is as shown in the Fig. 2.3.3(b).

4. **Bit plane slicing :**

i. Bit plane slicing is the conversion of image into multilevel binary image.

ii. These binary images are then compressed using different algorithm.

iii. With this technique, the valid bits from gray scale images can be separated, and reduces the time complexity.

iv. Bit plane slicing is a method of representing an image with one or more bits of the byte used for each pixel.

v. One can use only MSB to represent the pixel, which reduces the original gray level to a binary image.

vi. The three main goals of bit plane slicing are :

a. Converting a gray level image to a binary image.

b. Representing an image with fewer bits and corresponding the image to a smaller size.

c. Enhancing the image by focusing.

5. Power law transformation :

i. The basic formula for power law transformation is,

$$g(x, y) = c \times f(x, y)^{\gamma}$$

ii. Here c and γ are positive constants.

iii. The transformation shown in Fig. 2.3.4 is for different values of γ which is also called the gamma correction factor.

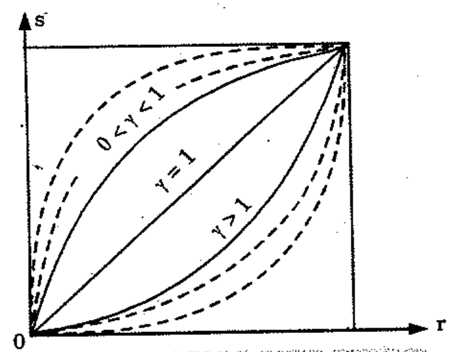


Fig. 2.3.4. Gamma correction factor.

iv. By changing the value of gamma, we can obtain a family of transformation curves.

v. Non-linearities encountered during image capturing, printing and displaying can be corrected using gamma correction.

vi. The power law transformation is used to improve the dynamic range of an image.

6. Log transformations :

i. The general form of the log transformation :

$$s = c \log (1 + r)$$

where c is a constant, and it is assumed that $r \geq 0$.

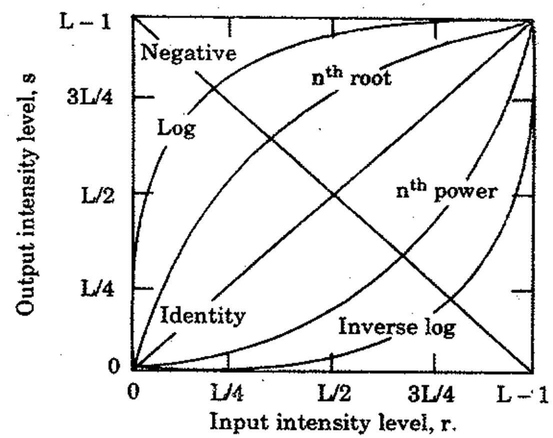


Fig. 2.3.5.

ii. The shape of the log curve in Fig. 2.3.5 shows that this transformation maps a narrow range of low intensity values in the input into a wider range of output levels.

iii. The opposite is true for higher values of input levels.

iv. We use a transformation of this type to expand the values of dark pixels in an image while compressing the higher-level values.

v. The opposite is true for the inverse log transformation.

vi. The log function has the important characteristic that it compresses the dynamic range of images with large variations in pixel values.

Que 2.4. **What is bit-plane slicing ? Given the following 3 × 3 image, find its bit planes.**

1	2	3
4	5	0
7	2	1

AKTU 2014-15, Marks 05

Answer

Bit plane slicing : Refer Q. 2.3, Page 2–3A, Unit-2.

Numerical : The given image is,

1	2	3
4	5	0
7	2	1

Fig. 2.4.1.

Dividing the gray level values into bits :

001	010	011
100	101	000
111	010	001

0	0	0
1	1	0
1	0	0

Bit plane MSB

0	1	1
0	0	0
1	1	0

Bit plane middle

1	0	1
0	1	0
1	0	1

Bit plane LSB

Fig. 2.4.2.

Que 2.5. Obtain the digital negative of the following 8-bit per pixel image :

139	**205**	**105**
141	**252**	**99**
201	**15**	**76**

Answer

For image negative, we have formula

$$S = L - 1 - r$$

where L is the range (0, 255),

$$S_0 = 255 - 1 - 139 = 115$$
$$S_1 = 255 - 1 - 205 = 49$$

Similarly for all other pixels,

Image negative =

115	49	149
113	2	155
53	239	178

Que 2.6. Explain the 4, 8 and m connectivity of pixels. Explain region, edge in context with connectivity of pixels.

Answer

1. **4-neighbours (direct neighbours) :** These are position at {1, 3, 5, 7} and denoted by $N_4(p_0)$ and (2, 4, 6, 8) denoted by $N_D(p_0)$.
2. **8-neighbours :** These are at position (1, 2, 3, 4, 5, 6, 7, 8) and denoted by $N_8(p_0)$.

3. **3 × 3 neighbour (3 × 3 window) are :** These are position at {0, 1, 2, 3, 4, 5, 6, 7, 8}.
4. Adjacency is defined for two pixels p and q by following types :
   a. **4-adjacency :** Two pixels p and q with values from gray level domain V are 4-adjacent if q is in the set $N_4(p)$.
   b. **8-adjacency :** Two pixels p and q with values from gray level domain V are 8-adjacent if q is in set $N_8(p)$.
   c. ***m*-adjacency (mixed adjacency) :** Two pixel p and q with values from gray level domain V are m-adjacent if [q is in $N_4(p)$] OR [q is in $N_D(p)$] AND the set $[N_4(p) \cap N_4(q) = \phi]$.
5. **Connectivity** is defined between two regions as :
   Suppose there are two regions S_1 and S_2 then,
   a. S_1 and S_2 are 4-connected if some pixel in S_1 is adjacent to some pixel in S_2 by 4-adjacency.
   b. S_1 and S_2 are 8-connected if some pixel in S_1 is adjacent to some pixel in S_2 by 8-adjacency.
   c. S_1 and S_2 are m-connected if some pixel in S_1 is adjacent to some pixel in S_2 by m-adjacency.
6. Regions are defined as subset of pixel in an image. Let two pixel p and q are in given region then there exists a digital path between p and q such that all the intermediate pixels in digital path are 4, 8 or m-adjacency away. Hence, regions are connected set of pixels.
7. Edges are defined as contour or border of regions, it is the set of pixel in the image that have one or more neighbours that are not in region R.

Que 2.7. An image segment is shown below. Let V be the set of gray level values used to define connectivity in the image. Compute D_4, D_8 and D_m distances between pixel p and q for :

i. $v = \{2, 3\}$

ii. $v = \{2, 6\}$

p

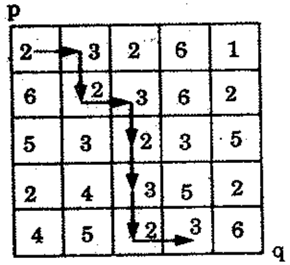
2	3	2	6	1
6	2	3	6	2
5	3	2	3	5
2	4	3	5	2
4	5	2	3	6

q

Answer

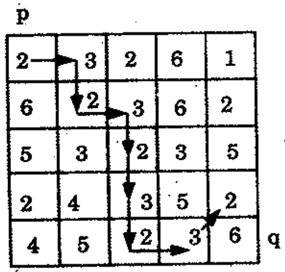
i. $v = \{2, 3\}$

D_4 :



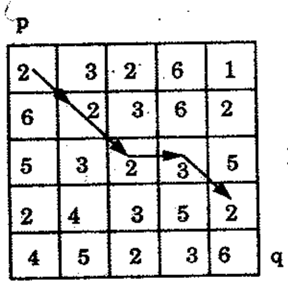
$D_4 = 7$

D_8:



$D_8 = 8$

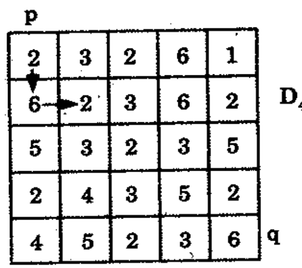
D_m :



$D_m = 4$

ii. $v = \{2, 6\}$

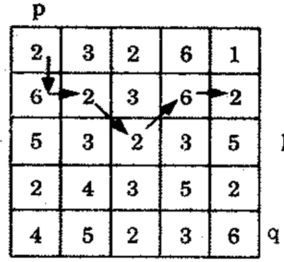
D_4:



D_4 = Not possible

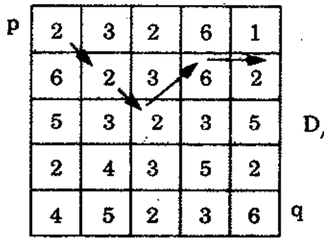
We cannot move further from 2 to 3 because 3 does not belong to the given set v and hence further moving is not possible.

D_8 :



$D_8 = 5$

D_m :



$D_m = 4$

Que 2.8. **Consider two image subsets S_1 and S_2 as shown in the Fig. 2.8.1. For $V = \{0\}$ determine whether the regions are :**

i. 4-Adjacent

ii. 8-Adjacent

iii. m-Adjacent

Give reasons for your answer.

S_1				S_2			
1	1	1	1	1	1	0	0
1	1	0	1	1	0	1	1
1	1	0	1	0	0	1	1
1	0	0	0	1	1	1	1

Fig. 2.8.1.

AKTU 2014-15, Marks 05

Answer

Let us assume there is a padding of 0's and there are two pixels p and q encircled in the Fig. 2.8.2.

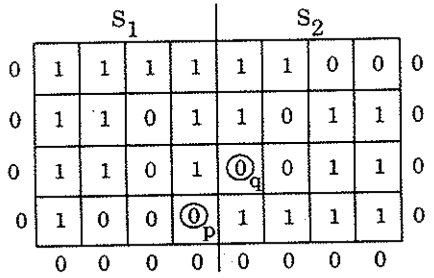


Fig. 2.8.2.

$N_4(p) = \{1, 1, 0, 0\}$ $\quad$ $N_4(p) \wedge N_4(q) = \text{NULL}$

$N_4(q) = \{0, 0, 1, 1\}$

$N_8(p) = \{0, 1, q, 1, 0, 0, 0, 0\}$

$N_8(q) = \{1, 1, 0, 0, 1, 1, 1, p, 1\}$

Pixels are m-connected as $N_4(p)$ is not in $N_4(q)$ but it is in $N_8(q)$.

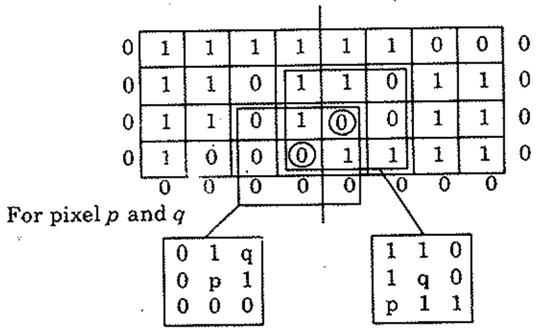


Fig. 2.8.3.

Que 2.9. **Write short notes on the following :**

i. Gamma correction

ii. Piecewise-linear transformation **AKTU 2014-15, Marks 05**

OR

Explain piecewise-linear transformations of image enhancement with suitable example. **AKTU 2018-19, Marks 10**

Answer

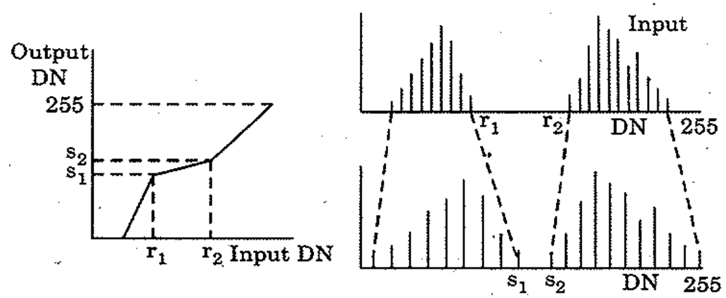
i. Gamma correction :

1. Gamma correction is a nonlinear operation used to encode and decode luminance values in video or still image systems.

2. Gamma correction function is a function that maps luminance levels to compensate the non-linear luminance effect of display devices (or sync it to human perceptive bias on brightness).
3. The luminance generated by a physical device is generally not a linear function of the applied signal.
4. Luminance produced in the display is approximately proportional to the applied voltage raised to the 2.5 power.
5. This numerical value of the exponent is known as gamma.
6. This non-linearity must be compensated in order to achieve correct reproduction of luminance.

ii. Piecewise-linear transformation :

1. Piecewise-linear transformation (Pipewise linear stretch) functions uses different linear functions to stretch different domain (DN) ranges of an input image.
2. Practical implementation of some important transformations can be formulated only as piecewise functions.
3. A disadvantage of piecewise function is that their specification requires a lot of user input.
4. In Fig. 2.9.1(a) a piecewise linear stretch function is shown, it stretches different parts of the original histogram by different amounts.
5. As it can be seen in Fig. 2.9.1(b) that the part from $r_1 - r_2$ has been compressed to $s_1 - s_2$.



(a) Piecewise linear stretch (b) Histogram before and after piecewise linear stretch.

Fig. 2.9.1. Piecewise linear stretch example.

6. Piecewise Linear Stretch (PLS) is a versatile point operation function. It can be used to simulate a non-linear function that cannot be easily defined by a mathematical function.
7. Most image processing software packages have interactive PLS functionality allowing users to configure PLS for optimized visualization.

Que 2.10.

3	1	2	1 (q)
2	2	0	2
1	2	1	1
(p) 1	0	1	2

i. Take $V = \{0, 1\}$ and compute the lengths of shortest 4, 8, and m-path between p and q. If a particular path does not exist between these two points, explain why,

ii. Repeat for $V = \{1, 2\}$

AKTU 2018-19, Marks 10

Answer

i. When $V = \{0,1\}$, 4-path does not exist between p and q because it is impossible to get from p to q by traveling along points that are both 4-adjacent and also have values from V. Fig. 2.10.1 (a) shows this condition; it is not possible to get to q.

The shortest 8-path is shown in Fig. 2.10.1 (b); its length is 4. The length of the shortest m-path (shown dashed) is 5. Both of these shortest paths are unique in this case.

ii. Repeat for V = {1, 2}

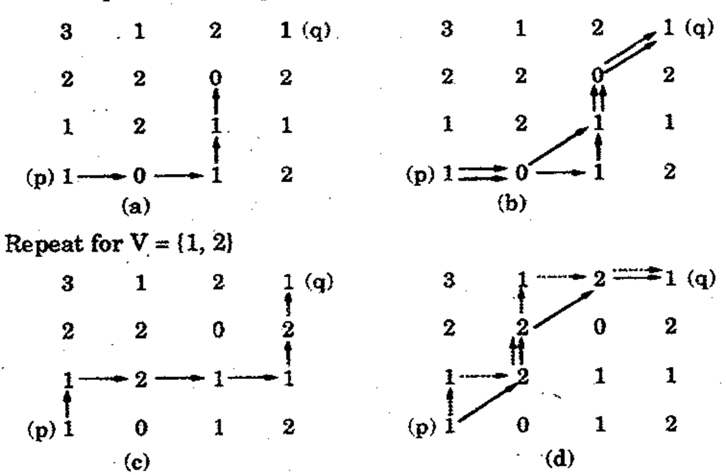


Fig. 2.10.1.

Que 2.11. **Consider the image segment :**

3	4	1	2	0
0	1	0	4	2(q)
2	2	3	1	4
3(p)	0	4	2	1
1	2	0	3	4

Let $V = \{2, 3, 4\}$. Compute the lengths of the shortest-8 and path between p and q. If a particular path does not exist between these two points, explain why ? What is the significance of 'm' path ?

AKTU 2015-16, Marks 10

Answer

One possibility for the shortest 4-path when $V = \{1, 2\}$ is shown in Fig. 2.11.1(a); its length is 6. It is easily verified that another 4-path of the same length exists between p and q. Another possibility for the shortest 8-path (it is not unique) is shown in Fig. 2.11.1(b); its length is 4. The length of a shortest m-path (shown dashed) is 6. This is not unique.

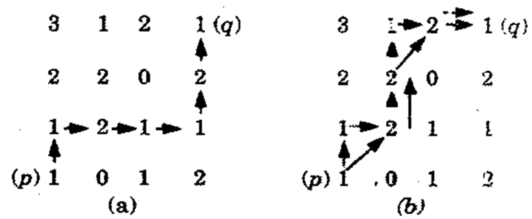


Fig. 2.11.1.

Significance of m-path : It is used to eliminate the double paths arise when we use 8-connectivity.

Que 2.12. **What do you understand by hit-miss transform and why they are used explain in brief ?**

AKTU 2017-18, Marks 10

Answer

1. The hit-or-miss transform is a transformation which is used for template matching.
2. The transformation involves two template sets, B and $(W - B)$, which are disjoint.

3. Template B is used to match the foreground image, while $(W-B)$ is used to match the background of the image.
4. The hit-or-miss transformation is the intersection of the erosion of the foreground with B and the erosion of the background with $(W-B)$.
5. The hit-or-miss transform is defined as

$$HM(X, B) = (X \ominus B) \cap (X^c \ominus (W-B)) \quad ...(2.12.1)$$

6. The small window W is assumed to have at least one pixel, thicker than B.
7. The input image X and the structuring element B are shown in Fig. 2.12.1.

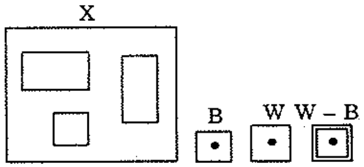


Fig. 2.12.1. Input image X, structuring element B, W, W – B.

8. Equation (2.12.1) describes the hit-or-miss transformation.
9. First, we have to find the erosion of the input X with the structuring element B as illustrated in Fig. 2.12.2.

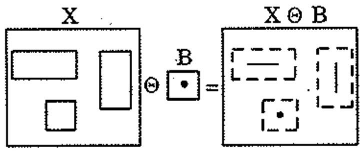


Fig. 2.12.2. Eroded image of X.

10. As per eq. (2.12.1), find the complement of the input image X, and then erode it with the structuring element $(W-B)$. The resultant image is shown in Fig. 2.12.3.

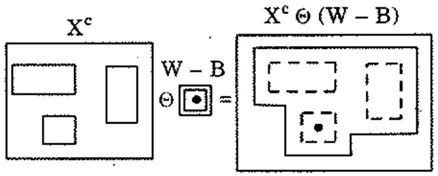


Fig. 2.12.3. Eroded image of X^c.

11. Now, find the intersection of the image shown in Fig. 2.12.3 and Fig. 2.12.4 which give the hit-or-miss transformation of the input image X. The hit-or-miss transformation of the input image is shown in Fig. 2.12.4.

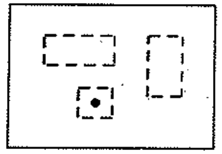


Fig. 2.12.4. Intersection result of above two results.

Use of Hit-or-Miss transformation :

1. It is used for shape detection.
2. It is used to thin and skeletonize a shape in a binary image.
3. It is used to detect pattern in a binary image, using a structuring element containing 1's, 0's and blank for don't cares.

Que 2.13. Explain the term histogram. How can histogram of an image be plotted ?

OR

Write short note on image histogram. **AKTU 2018-19, Marks 05**

Answer

Histogram : Histogram is an accurate representation of the distribution of numerical data.

Image histogram :

1. An image histogram is a type of histogram that acts as a graphical representation of the tonal distribution in a digital image.
2. It plots the number of pixels for each tonal value.
3. By observing the histogram for a specific image a viewer will be able to judge the entire tonal distribution at a glance.
4. Image histograms are present on many modern digital cameras. Photographers can use them as an aid to show the distribution of tones captured, and whether image detail has been lost to blown-out highlights or blacked-out shadows.
5. This is less useful when using a raw image format, as the dynamic range of the displayed image may only be an approximation to that in the raw file.
6. The horizontal axis of the graph represents the tonal variations, while the vertical axis represents the number of pixels in that particular tone.
7. The left side of the horizontal axis represents the black and dark areas, the middle represents medium grey and the right hand side represents light and pure white areas.
8. The vertical axis represents the size of the area that is captured in each one of these zones. Thus, the histogram for a very dark image will have most of its data points on the left side and center of the graph.

9. Conversely, the histogram for a very bright image with few dark areas and/or shadows will have most of its data points on the right side and center of the graph.

Histogram of an image can be plotted in two ways :

Method 1 : In the first method, the x-axis has the gray levels and the y-axis has the number of pixels in each gray level.

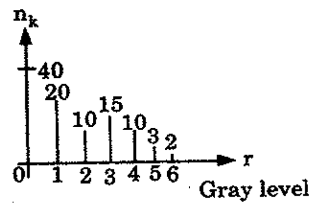


Fig. 2.13.1.

Method 2 (Normalised histogram) :

a. In this method, the x-axis represents the gray levels, while the y-axis represents the probability of the occurrence of that gray level.

b. In this method, instead of plotting the number of pixels directly, we plot their probability of occurrence *i.e.*,

$$p(r_k) = \frac{n_k}{n}$$

$r_k \rightarrow k^{th}$ gray level
$n_k \rightarrow$ Number of pixels in the k^{th} gray level
$n \rightarrow$ Total number of pixels in an image

Gray level	Number of pixels (n_k)	$p(r_k) = \frac{n_k}{n}$
0	40	0.4
1	20	0.2
2	10	0.1
3	15	0.15
4	10	0.1
5	3	0.03
6	2	0.02
	$n = 100$	

c. This is known as a normalised histogram.

d. The advantage of this is that the maximum value to be plotted will always be 1.

e. Generally black is considered as gray level 0 and white as the maximum.

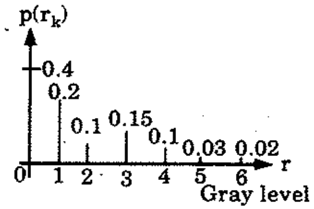


Fig. 2.13.2.

Que 2.14. **Explain the properties of images which can be described by histogram. Also explain normalized histogram.**

Answer

Properties of images described by histogram :

1. **Aspect ratio :** The horizontal-to-vertical dimensional ratio of a digital image is referred to as the aspect ratio of the image and can be calculated by dividing the horizontal width by the vertical height.
2. **Spatial resolution :**
   a. The quality of a digital image, often referred to as image resolution, is determined by the number of pixels and the range of brightness values available for each pixel utilized in the image.
   b. Resolution of the image is regarded as the capability of the digital image to reproduce fine details that were present in the original analog image.
   c. Spatial resolution is reserved to describe the number of pixels utilized in constructing and rendering a digital image
3. **Image brightness and bit depth :**
   a. The brightness of a digital image is a measure of relative intensity values across the pixel array after the image has been acquired with a digital camera or digitized by an analog-to-digital converter.
   b. Bit depth refers to the binary range of possible gray scale values utilized by the analog-to-digital converter to translate analog image information into discrete digital values capable of being read and analyzed by a computer.
4. **Distance :** The distance between two pixels in a digital image is a significant quantitative measure.

Normalized histogram : Refer Q. 2.13, Page 2–17A, Unit-2.

Que 2.15. **Explain histogram equalization. And equalize the given histogram.**

Gray level	0	1	2	3	4	5	6	7
Number of Pixel	790	1023	850	656	329	245	122	81

AKTU 2016-17, Marks 10

OR

What is histogram equalization ? **AKTU 2017-18, Marks 10**

Answer

Histogram equalization :

1. Histogram equalization is a computer image processing technique used to improve contrast in images.
2. It is accomplished by effectively stretching out the intensity range of the image.
3. This method usually increases the global contrast of images when its usable data is represented by close contrast values.
4. This allows for areas of lower local contrast to gain a higher contrast.
5. A colour histogram of an image represents the number of pixels in each type of colour component.
6. Histogram equalization cannot be applied separately to the red, green and blue components of the image as it leads to dramatic changes in the image's colour balance.
7. However, if the image is first converted to another colour space, like HSL/HSV colour space, then the algorithm can be applied to the luminance or value channel without resulting in changes to the saturation of the image.

Numerical :

Number of gray levels *i.e.*, $L = 8$

Now, plot the original histogram :

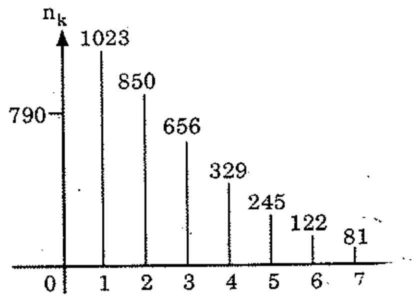


Fig. 2.15.1. Original dark

Gray level	n_k	PDF $p_r(r_k) = \frac{n_k}{n}$	CDF $S_k = \sum p_{r(r_k)}$	$(L-I) \times S_k$ $7 \times S_k$	Rounding off
0	790	0.19	0.19	1.33	1
1	1023	0.25	0.44	3.08	3
2	850	0.21	0.65	4.55	5
3	656	0.16	0.81	5.67	6
4	329	0.08	0.89	6.23	6
5	245	0.06	0.95	6.65	7
6	122	0.03	0.98	6.86	7
7	81	0.02	1	7	7
	N = 4096				

We take 1st, 2nd and the last column

Old gray level	Equalized gray level	New gray level
0	790	→ 1
1	1023	→ 3
2	850	→ 5
3	656	→ 6
4	329	→ 6
5	245	→ 7
6	122	→ 7
7	81	→ 7

We notice that the new gray levels have pixels only at 1, 3, 5, 6, 7. There are no pixels in gray levels 0, 2 and 4.

Equalized gray level	Number of pixels
0	0
1	790
2	0
3	1023
4	0
5	850
6	656 + 329 = 985
7	245 + 122 + 81 = 448

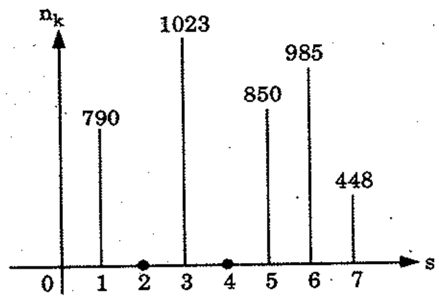


Fig. 2.15.2. Equalized histogram.

Que 2.16. **Perform histogram equalization on the following 8 × 8 image. The gray level distribution of the image is given below :**

Gray levels (r_k)	0	1	2	3	4	5	6	7
Number of pixels (p_k)	8	10	10	2	12	16	4	2

AKTU 2014-15, Marks 10

Answer

Gray levels (r_k)	0	1	2	3	4	5	6	7
Number of pixels (p_k)	8	10	10	2	12	16	4	2

Total number of pixels = 8 + 10 + 10 + 2 + 12 + 16 + 4 + 2 = 64

$p_k = 64$

Fig. 2.16.1 shows input image histogram.

Equalization process :

k	r_k	r_k/p_k	$\Sigma r_k/p_k$	$1 = 7 \times CDF$	Round off	No. of pixel
0	8	0.125	0.125	0.875	1	8
1	10	0.156	0.281	1.967	2	10
2	10	0.156	0.437	3.059	3	
3	2	0.031	0.468	3.276	3	12
4	12	0.187	0.655	4.585	5	12
5	16	0.250	0.905	6.335	6	16
6	4	0.062	0.967	6.769	7	
7	2	0.031	0.998 =1	7	7	6

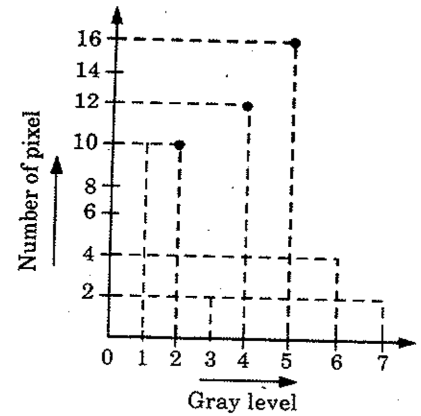


Fig. 2.16.1.

Original gray level	0	1	2	3	4	5	6	7
Histogram equalized value	1	2	3	3	5	6	7	7

Frequency distribution of equalized image :

Gray level	0	1	2	3	4	5	6	7
Number of pixels	8	10	0	12	12	16	0	6

Output image histogram is shown in Fig. 2.16.2.

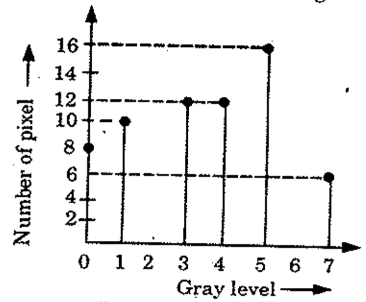


Fig. 2.16.2.

Que 2.17. **Perform the histogram equalization for 8×8 image shown below :**

Gray levels	0	1	2	3	4	5	6	7
No. of pixels	9	8	11	4	10	15	4	3

Answer

Gray level (r_k)	0	1	2	3	4	5	6	7
No. of pixels (P_k)	9	8	11	4	10	15	4	3

We see that $\sum_{k=0}^{7} P_k = 64$, which is again equal to 6 × 6 so probability

$P_m = \frac{P_k}{64}$ and cumulative probability $C_m = \sum_{k=0}^{m} P_l$. The output gray level is computed as $l = 7\ C_m$. The steps are revealed more clearly in the following table :

r_k	P_k	P_m	C_m	l
0	9	0.14062	0.14062	0.98434 ≈ 1
1	8	0.125	0.26562	1.85934 ≈ 2
2	11	0.17187	0.43749	3.06243 ≈ 3
3	4	0.0625	0.4999	3.4993 ≈ 3
4	10	0.15625	0.65624	4.59368 ≈ 5
5	15	0.234375	0.890615	6.234305 ≈ 6
6	4	0.0625	0.953115	6.671805 ≈ 7
7	3	0.046875	0.9999	6.993 ≈ 7

Finally, we have the following frequency table for the output gray level :

l	0	1	2	3	4	5	6	7
n_l	0	9	8	15	0	10	15	7

Que 2.18. **Two images $f(x, y)$ and $g(x, y)$, have histogram h_f and h_g. Give the conditions under which the histogram of $f(x, y) + g(x, y)$ and $f(x, y) \times g(x, y)$ can be determined in terms of h_f and h_g.**

Answer

1. Assume that the histograms are not normalize. So, $h_f(r_k)$ is the number of pixels in $f(x, y)$ having intensity level r_k.
2. Assume that all the pixels in $g(x, y)$ have constant value c.
3. The pixels of both images are assumed to be positive. Finally, let u_k denote the intensity levels of the pixels of the images formed by any of the arithmetic operations.
4. Under the preceding set of conditions, the histograms are determined as follows :

a. **$f(x, y) + g(x, y)$:**

i. The histogram $h_{sum}(u_k)$ of the sum is obtained by letting $u_k = r_k + c$, and also $h_{sum}(u_k) = h_f(r_k)$ for all k.

ii. The values (height) of the components of h_{sum} are the same as the components of h_f, but their locations on the intensity axis are shifted right by an amount c.

b. **$f(x, y) \times g(x, y)$:**

i. The values (heights) of the components of histogram $h_{prod}(u_k)$ of the product are the same as h_f, but their locations are at $u_k = c \times r_k$.

ii. Note that while the spacing between components of the resulting histograms in $f(x, y) + g(x, y)$ was not affected, the spacing between components of $h_{prod}(u_k)$ will be spread out by an amount c.

Que 2.19. **In an image the gray scale spans from black to near white in only three increments. A certain noise has corrupted the image. The image and its histogram are as follows. What type of mean filters can you use to eliminate the noise ? Explain.**

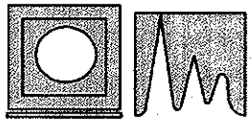


Fig. 2.19.1.

AKTU 2014-15, Marks 05

Answer

1. The histogram of the corrupted image is showing Gaussian shape.
2. The noise which has corrupted the image is Gaussian which is tractable in spatial as well as frequency domain.
3. The presence of the noise can be seen by the spikes of the histogram.
4. Arithmetic mean filters are used for removal of such noise.

PART-2

Frequency Domain : Introduction to Fourier Transform, Smoothing and Sharpening Frequency Domain Filters, Ideal, Butterworth and Gaussian Filters, Homomorphic Filtering, Color Image Enhancement.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 2.20. **Write short note on smoothing and sharpening frequency domain filter.**

Answer

Smoothing frequency domain filter :

a. Smoothing (blurring) is achieved in the frequency domain by high-frequency attenuation ; that is, by low pass filtering.

b. We consider four types of low pass filters : Ideal, Butterworth, and Gaussian, high pass filter.

c. These three categories cover the range from very sharp (Ideal) to very smooth (Gaussian) filtering.

d. The Butterworth filter has a parameter called the filter order.

Types of smoothing frequency domain filter :

1. **Ideal Low Pass Filter (ILPF) :**

   a. This filter cuts off all high frequency components of the Fourier transform that are at a distance greater than a specified distance D_0.

$$H(u, v) = 1 \text{ ; if } D(u, v) \le D_0$$
$$= 0 \text{ ; if } D(u, v) > D_0$$

   Here D_0 is the specified non-negative distance.

   b. $D(u, v)$ is the distance from the point (u, v) to the origin of the frequency rectangle for an $M \times N$ image.

$$D(u, v) = [(u - M/2)^2 + (v - N/2)^2]^{1/2}$$

$\therefore$ For an image, when $u = \frac{M}{2}, v = \frac{N}{2}$

$$D(u, v) = 0$$

2. **Butterworth Low Pass Filter (BLPF) :**

   a. The ringing effects are due to the sharp cut-offs in the ideal filter and to get rid of the ringing effects, we need to eliminate these sharp cut-offs.

   b. This is done with Butterworth low pass filters.

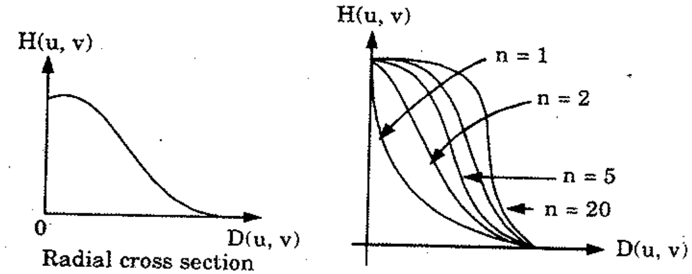


Fig. 2.20.1.

   c. The transfer function of the Butterworth low pass filter of order n and cut-off frequency at a distance D_0 from the origin is defined as,

$$H(u, v) = \frac{1}{1 + [D(u, v) / D_0]^{2n}}$$

   d. Unlike the ILPF, the BLPF does not have sharp discontinuities and hence there are no ringing effects present when a BLPF is used.

   e. But as the order of the filter goes on increasing, a small amount of ringing effect does creep in because the Butterworth low pass filter tends to be an ideal filter.

3. **Gaussian Low Pass Filter (GLPF) :**

   a. Gaussian LPF is given by,

$$H(u, v) = e^{-D^2(u, v)/2\sigma^2}$$

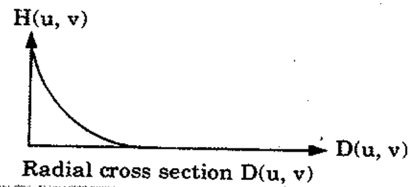


Fig. 2.20.2. Schematic of DNC system.

   Here σ is the standard deviation and is a measure of spread of the Gaussian curve.

   b. If we put $\sigma = D_0$ we get,

$$H(u, v) = e^{-D^2(u, v)/2D_0^2}$$

4. **High pass filter :**

   a. High pass filtering eliminates the low frequency regions while retaining or enhancing the high frequency components.

   b. An image, which is high-passed, would have no background and would have enhanced edges.

   c. Hence high pass filters are used to sharpen blurred images.

Sharpening frequency domain filter :

1. Image sharpening can be achieved in the frequency domain by high pass filtering, which attenuates the low-frequency components without disturbing high-frequency information in the Fourier transform, because edges and other abrupt changes in intensities are associated with high-frequency components.
2. A high pass filter is obtained from a given low pass filter using the equation :

$$H_{Hp}(u, v) = 1 - H_{LP}(u, v)$$

where $H_{LP}(u, v)$ is the transfer function of the low pass filter.

Types of sharpening frequency domain filter :

1. **Ideal High Pass Filter (IHPF) :**

a. A 2D ideal high pass filter is defined as,

$$H(u, v) = 0 \text{ ; if } D(u, v) \le D_0$$
$$= 1 \text{ ; if } D(u, v) > D_0$$

Where D_0 is called the cut-off.

b. $D(u, v)$ is the same as defined in the low pass filtering section.

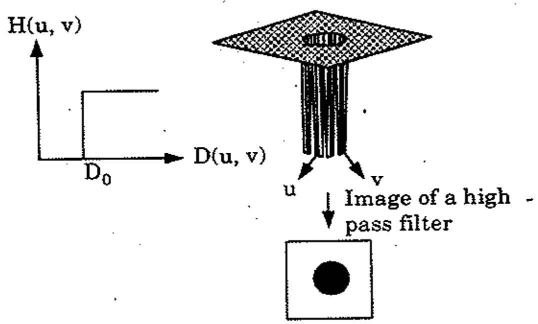


Fig. 2.20.3.

2. **Butterworth High Pass Filters (BHPF) :**

a. The general formula for any high pass filter is,

$$H_{HP}(u, v) = 1 - H_{LP}(u, v)$$
$$\therefore \quad H_{HBWF}(u, v) = 1 - H_{LBWF}(u, v)$$

(LBWF → Low pass Butterworth filter)
(HBWF → High pass Butterworth filter)

b. We know

$$H_{LBWF}(u, v) = \frac{1}{1 + \left[\frac{D(u, v)}{D_0}\right]^{2n}}$$

$$H_{HBWF}(u, v) = 1 - \frac{1}{1 + \left[\frac{D(u, v)}{D_0}\right]^{2n}}$$

c. Let $\left[\frac{D(u, v)}{D_0}\right]^{2n} = x$

$$\therefore \quad H_{HBWF}(u, v) = 1 - \frac{1}{1+[x]} = \frac{1+[x]-1}{1+[x]} = \frac{[x]}{1+[x]}$$
$$= \frac{1}{1+\frac{1}{[x]}} = \frac{1}{1+\frac{1}{\left[\frac{D(u, v)}{D_0}\right]^{2n}}} = \frac{1}{1+\frac{1}{\left[\frac{D_0}{D(u, v)}\right]^{2n}}}$$

d. Hence, Butterworth high pass filter is,

$$\therefore \quad H_{HBWF}(u, v) = \frac{1}{1+\frac{1}{\left[\frac{D_0}{D(u, v)}\right]^{2n}}}$$

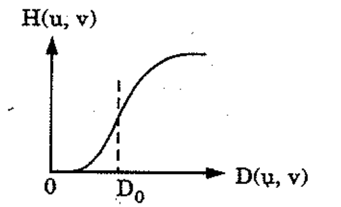


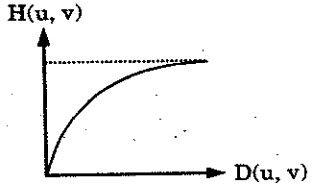
Fig. 2.20.4.

e. The high pass Butterworth filter does not have a sharp cut-off *i.e.*, the transition is smooth and hence there are no ringing effects.

3. **Gaussian High Pass filter (GHPH) :**

a. We again use the basic formula,

$$H_{HP}(u, v) = 1 - H_{LP}(u, v)$$
$$\therefore \quad H_{\text{Gaussian } HP}(u, v) = 1 - H_{\text{Gaussian } LP}(u, v)$$
$$H_{GHPF} = 1 - e^{-D^2(u, v)/2D_0^2}$$



Radial cross section

Fig. 2.20.5.

Que 2.21. **Compare and contrast the smoothing and sharpening filters.**

Answer

S. No.	Smoothing filters	Sharpening filters
1.	Smoothing filters (low pass filters) remove the high frequency content from the image.	Sharpening filters (high pass filters) remove the low frequency content from the image.
2.	Smoothing filters are used to remove noise present in the image.	Sharpening filters are used to enhance the edges of the image.
3.	The two types of smoothing filters are averaging filter and median filter.	The two types of sharpening filters are standard high pass filter and high-boost filter.
4.	Smoothing filter removes the noise but in the process, it also blurs the edges.	Sharpening filter sharpens the edges but end up removing most of the background information.
5.	An image processed using smoothing filters will always have positive values.	An image processed using sharpening filters will have positive as well as negative values.

Que 2.22. **Explain the action of the following spatial mask on an image.**

0	–1	0
–1	4	–1
0	–1	0

Answer

1. This mask represents an isotropic filter, as it is rotation invariant. It will give same result on either applying this filter directly to the image or applying this filter to the rotated image.
2. Since the sum of all the weights of the mask adds up to zero, therefore, it is a Laplacian operator *i.e.* double derivative operator, which is defined as :

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

But the mask given will be obtained by taking negatives of $\frac{\partial^2 f}{\partial x^2}$ and $\frac{\partial^2 f}{\partial y^2}$

$$-\frac{\partial^2 f}{\partial x^2} = -f(x+1, y) - f(x-1, y) + 2f(x, y)$$

$$-\frac{\partial^2 f}{\partial y^2} = -f(x, y+1) - f(x, y-1) + 2f(x, y)$$

$$\nabla^2 f = -f(x+1, y) - f(x-1, y) - f(x, y+1) - f(x, y-1) + 4f(x, y)$$

3. This Laplacian-derivative operator will highlight the intensity discontinuities in an image and de-emphasize the regions with slow varying intensity levels. This will tend to produce images that have grayish edge lines and other discontinuities all superimposed on a dark, featureless background.
4. This filter can be used to give image sharpening details as
$$g(x, y) = f(x, y) + c[\nabla^2 f(x, y)]$$
where $f(x, y)$ and $g(x, y)$ are input and sharpened images respectively. c is the constant and its value is 1 for this particular filter.

Que 2.23. **Briefly explain the working of a Laplacian mask. What will be the effect of applying the filter (a) on the image (b) ?**

1	1	1
1	–8	1
1	1	1

(a)

50	50	50	50	50	50
50	50	50	50	50	50
50	50	50	50	50	50
100	100	100	100	100	100
100	100	100	100	100	100
100	100	100	100	100	100

(b)

Fig. 2.23.1.

AKTU 2014-15, Marks 05

Answer

Working of Laplacian mask :

1. The Laplacian $L(x, y)$ of an image with pixel intensity values $I(x, y)$ is given by :

$$L(x, y) = \frac{\partial^2 I}{\partial x^2} + \frac{\partial^2 I}{\partial y^2}$$

2. This can be calculated using convolution filter.
3. Since the input image is represented as a set of discrete pixels, we have to find a discrete convolution kernel that can approximate the second derivatives in the definition of the Laplacian.
4. Two commonly used small kernels are shown in Fig. 2.23.2.

0	– 1	0
– 1	4	– 1
0	– 1	0

– 1	– 1	– 1
– 1	8	– 1
– 1	– 1	– 1

Fig. 2.23.2.

5. These kernels are approximating a second derivative measurement on the image, they are very sensitive to noise.
6. To counter this, the image is often Gaussian smoothed before applying the Laplacian filter.
7. This pre-processing step reduces the high frequency noise components prior to the differentiation step.
8. In fact, since the convolution operation is associative, we can convolve the Gaussian smoothing filter with the Laplacian filter first of all, and then convolve this hybrid filter with the image to achieve the required result.
9. Two advantages of Laplacian filter are :
   a. Since both the Gaussian and the Laplacian kernels are much smaller than the image, this method requires far fewer arithmetic operations.
   b. The LoG (Laplacian of Gaussian) kernel can be precalculated in advance so only one convolution needs to be performed at run time on the image.

Numerical :

$$g(x, y) = f(x, y).h(x,y)$$

	50	50	50	50	50	50	
50	50	50	50	50	50	50	50
50	50	50	50	50	50	50	50
50	50	50	50	50	50	50	50
100	100	100	100	100	100	100	100
100	100	100	100	100	100	100	100
100	100	100	100	100	100	100	100
	100	100	100	100	100	100	

(a) $f(x, y)$

1	1	1
1	–8	1
1	1	1

(b) $h(x, y)$

Fig. 2.23.3.

$$g(x, y) = f(x, y).h(x, y)$$

If we apply the filter on pixel (0, 0), we get

$$g(0, 0) = 50 \times 1 + 50 \times 1 + 50 \times 1 + 50 \times 1 + 50 \times 1 + 50 \times 1 + 50 \times 1 + 50 \times 1 + 50 \times -8 = 0$$

Similarly for row = 0, all pixels will be 0.

$$g(1,0) = 50 \times 1 + 50 \times 2 + 50 \times 1 + 50 \times 1 + 50 \times 1 + 50 \times -8 + 100 \times 1 + 100 \times 1 + 100 \times 1$$
$$= 50 \times 6 - 50 \times 8 + 100 \times 3 = 300 - 400 + 300 = 200$$
$$g(2, 0) = 50 \times 1 + 50 \times 1 + 50 \times 1 + 100 \times 1 + 100 \times -8 + 100 \times 1 + 100 \times 1 + 100 \times 1 + 100 \times 1$$
$$= 50 \times 3 - 100 \times 8 + 100 \times 5$$
$$= 150 + 500 - 800 = -150$$
$$g(3, 0) = 100 \times 1 + 100 \times 1 + 100 + 1 \times 100 + 1 \times 100 + 1 \times 100 + 1 \times 100 + 1 - 100 \times 8 + 100 \times 1$$
$$= 800 - 800 = 0$$

The resultant image will be,

$g(x, y) =$

0	0	0	0	0	0
200	150	150	150	150	150
–150	–150	–150	–150	–150	–150
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0

Fig. 2.23.4.

Que 2.24. **Explain the process of filtering in frequency domain. Discuss low pass and high pass frequency domain filters.**

AKTU 2018-19, Marks 10

Answer

Process of filtering in frequency domain :

1. Frequency filtering is based on the Fourier Transform.
2. The operator usually takes an image and a filter function in the Fourier domain.
3. This image is then multiplied with the filter function in a pixel-by-pixel fashion :

$$G(k, l) = F(k, l) \times H(k, l)$$

where $F(k, l)$ is the input image in the Fourier domain, $H(k, l)$ is the filter function and $G(k, l)$ is the filtered image.

4. To obtain the resulting image in the spatial domain, $G(k, l)$ has to be re-transformed using the inverse Fourier Transform.
5. Since the multiplication in the Fourier space is identical to convolution in the spatial domain, all frequency filters can be implemented as a spatial filter.
6. However, the Fourier domain filter function can only be approximated by the filtering kernel in spatial domain.
7. The form of the filter function determines the effects of the operator. There are three different kinds of filters :
   - **a. Low pass filters :**
     - i. A low pass filter attenuates high frequencies and retains low frequencies unchanged.
     - ii. The result in the spatial domain is equivalent to that of a smoothing filter as the blocked high frequencies correspond to sharp intensity changes, *i.e.*, to the fine-scale details and noise in the spatial domain image.
   - **b. High pass filters :** Refer Q. 2.20, Page 2–26A, Unit-2.
   - **c. Band pass filters :**
     - i. A band pass attenuates very low and very high frequencies, but retains a middle range band of frequencies.
     - ii. Band pass filtering can be used to enhance edges (suppressing low frequencies) while reducing the noise at the same time (attenuating high frequencies).

Que 2.25. **Given $H(u, v)$ as follows. Discuss its frequency response.**

	1/6	
1/6	1/3	1/6
	1/6	

Fig. 2.25.1.

AKTU 2014-15, Marks 05

Answer

Given, $H(u, v)$

	1/6	
1/6	1/3	1/6
	1/6	

Fig. 2.25.2.

The given filter is equivalent to Laplacian filter. Let us apply the filter on a smooth image.

$F(u, v) =$

8	0	1	2
7	3	0	9
5	8	9	6
9	9	9	4

Fig. 2.25.3.

If we apply the filter to the input image, the output image would be $G(u, v) = F(u, v)\, H(u, v)$

8	0	1	2
7	3	0	9
5	8	9	6
9	9	9	4

Fig. 2.25.4.

$$f(1, 0) = 7 \times \frac{1}{3} + 3 \times \frac{1}{6} + 5 \times \frac{1}{6} + 3 \times \frac{1}{6} = 5$$

$$f(1, 1) = 7 \times \frac{1}{6} + 3 \times \frac{1}{3} + 8 \times \frac{1}{6} = 3.5 \approx 4$$

$$f(1, 2) = 3 \times \frac{1}{6} + 1 \times \frac{1}{6} + 9 \times \frac{1}{6} + 9 \times \frac{1}{6} = 3.6 \approx 4$$

$G(u, v) =$

8	0	1	2
5	4	4	4
6	7	7	6
9	9	9	4

Fig. 2.25.5.

The output image is having darker pixels than the input image. So, the filter is a sharpening filter that enhances edges and other dark gray levels in the image.

Que 2.26. **Explain Laplacian filter.** **AKTU 2017-18, Marks 10**

Answer

1. It was shown that first derivative does enhance the edges of the image. We know,

$$\nabla F = \frac{\partial f}{\partial x} + \frac{\partial f}{\partial y}$$

$$\therefore \quad \frac{\partial f}{\partial x} = f(x+1, y) - f(x, y)$$

and $$\frac{\partial f}{\partial y} = f(x, y+1) - f(x, y)$$

2. The second derivative is given by,

$$\nabla^2 F = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

where $$\frac{\partial^2 f}{\partial x^2} = f(x+1, y) + f(x-1, y) - 2f(x, y)$$

and $$\frac{\partial^2 f}{\partial y^2} = f(x, y+1) + f(x, y-1) - 2f(x, y)$$

$$\nabla^2 F = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

$$\therefore \quad |\nabla^2 F| = [f(x+1, y) + f(x-1, y) + f(x, y+1) + f(x, y-1) - 4f(x, y)]$$

3. Considering the 3 × 3 neighbourhood

	$y-1$	y	$y+1$
$x-1$	Z_1	Z_2	Z_3
x	Z_4	Z_5	Z_6
$x+1$	Z_7	Z_8	Z_9

4. This equation in the discrete form reduces to

$$|\nabla^2 F| = [Z_8 + Z_2 + Z_6 + Z_4 - 4Z_5]$$

5. This equation can be implemented using a mask shown as :

0	1	0
1	–4	1
0	1	0

6. This is known as the Laplacian filter.

Que 2.27. **Derive the frequency domain transformation function $H(u, v)$ for the following spatial domain filter $h(x, y)$.**

0	–1	0
–1	8	–1
0	–1	0

How homomorphic filtering is implemented ?

AKTU 2015-16, Marks 10

OR

Write short note on Homomorphic filtering.

AKTU 2014-15, Marks 05

AKTU 2016-17, Marks 05

AKTU 2018-19, Marks 05

Answer

We proceed as :

$$8f(x, y) \xleftrightarrow{F} 8F(u, v)$$

$$-1\, f(x+1, y) \xleftrightarrow{F} -1\, e^{\frac{j2\pi u}{N}} F(u, v)$$

$$-1\, f(x-1, y) \xleftrightarrow{F} -1\, e^{\frac{-j2\pi u}{N}} F(u, v)$$

$$-1\, f(x, y+1) \xleftrightarrow{F} -1\, e^{\frac{j2\pi v}{N}} F(u, v)$$

$$-1\, f(x, y-1) \xleftrightarrow{F} -1\, e^{\frac{-j2\pi v}{N}} F(u, v)$$

$$G(u, v) = \left[8 - 1\left(e^{\frac{j2\pi u}{N}} + e^{\frac{-j2\pi u}{N}}\right) - 1\left(e^{\frac{j2\pi v}{N}} + e^{\frac{-j2\pi v}{N}}\right)\right] F(u, v)$$

$$G(u, v) = \left[8 - 2\cos\left(\frac{2\pi u}{N}\right) - 2\cos\left(\frac{2\pi v}{N}\right)\right] F(u, v)$$

$$\therefore \quad H(u, v) = \left[8 - 2\cos\left(\frac{2\pi u}{N}\right) - 2\cos\left(\frac{2\pi v}{N}\right)\right]$$

Homomorphic filtering :

1. Homomorphic filtering is used for image enhancement.
2. It simultaneously normalized the brightness across an image and increases the contrast.
3. Here, homomorphic filtering is used to remove multiplicative noise.
4. Illumination and reflectance are not separable, but their approximate locations in the frequency domain may be located.
5. Since illumination and reflectance are combined multiplicatively, the component are made additive by taking the logarithm of the image intensity, so that these linearly in the frequency domain.
6. Illumination variations can be thought of as a multiplicative noise, and can be reduced by filtering in the log domain.

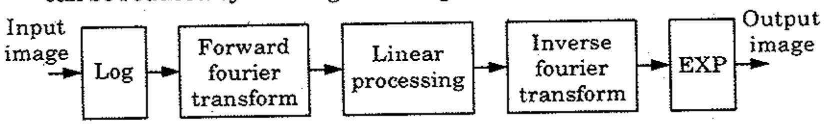


Fig. 2.27.1. Block diagram of homomorphic filtering.

Implementation of homomorphic filtering :

1. Homomorphic filtering is implemented using VHDL implementation *i.e.*, Very High Speed Integrated-Circuit Hardware-Description language.
2. VHDL implementation of the homomorphic filter is done in the spatial domain.
3. Using spatial domain, we can avoid the Fourier Transform computation and generate effective spatial domain filter kernel for the filtering.
4. In this implementation, the main components are the logarithm transformation components and the spatial domain filter.
5. By building each individual component separately, debugging and testing becomes easier.

Que 2.28. **What are the linear and non-linear smoothing filters in spatial domain ? Compute the new pixel values after applying the 3*3 box filter on the following 5*5 matrix of an 8-bit image.**

139	128	237	126	129
145	129	123	89	132
146	122	128	87	135
141	125	134	131	139
112	127	138	133	142

1/9*

1	1	1
1	1	1
1	1	1

AKTU 2015-16, Marks 15

Answer

Linear smoothing filter :

1. Smoothing linear spatial filter is the average of the pixels contained in the neighborhood of the filter mask. Linear smoothing filters are :
   a. **Mean filtering :** Mean filtering is simply to replace each pixel value in an image with the mean (average) value of its neighbours, including itself.
   b. **Gaussian filters :**
      i. Gaussian filters smoothens an image by calculating weighted averages in a filter box.
      ii. It is used to 'blur' images and remove detail and noise.
      iii. Gives more weight at the central pixels and less weights to the neighbours.
      iv. The farther away the neighbours, the smaller the weight.
      v. Gaussian blurs produce a pure smoothing effect without side effects.
2. **Non-linear smoothing filter :** Nonlinear spatial filters are order-statistics filters whose response is based on ordering (ranking) the pixels contained in the image area encompassed by the filter, and then replacing the value of the center pixel with the value determined by the ranking result. Non-linear smoothing filters are :
   i. **Minimum filter :** The minimum filter selects the smallest value within the pixel values and maximum filter selects the largest value within of pixel values.
   ii. **Maximum filter :** Maximum filter is useful for finding the brightest points in an image *i.e.*, it removes salt noise.
   iii. **Midpoint filter :**
      a. The midpoint filter blurs the image by replacing each pixel with the average of the highest pixel and the lowest pixel (with respect to intensity) within the specified window size.
      b. Midpoint = (darkest + lightest)/2.

Numerical : Given 8-bit image :

139	128	237	126	129
145	129	123	89	132
146	122	128	87	135
141	125	134	131	139
112	127	138	133	142

Given filter :

1/9

1	1	1
1	1	1
1	1	1

We place 3 × 3 filter on this image. We start from the left hand top corner. We cannot work with the borders and hence are normally left as they are. We then multiply each component of the image with corresponding value of the mask and put the resultant value at center pixel.

139	128	237	126	129
145	129	223	89	132
146	122	128	87	135
141	125	134	131	139
112	127	138	133	142

139	128	237	126	129
145	155.23	223	89	132
146	122	128	87	135
141	125	134	131	139
112	127	138	133	142

139	128	237	126	129
145	155.23	143.91	89	132
146	122	128	87	135
141	125	134	131	139
112	127	138	133	142

139	128	237	126	129
145	155.23	143.91	134.1	132
146	122	128	87	135
141	125	134	131	139
112	127	138	133	142

Similarly, final image is,

139	128	237	126	129
145	155.23	143.91	134.1	132
146	137.79	130.67	129.63	135
141	132.38	132.60	134.54	139
112	127	138	133	142

Que 2.29. **What are the linear and non-linear smoothing filters in spatial domain ? Compute the new pixel values after applying the 3 × 3 box filter on the following 5 × 5 matrix of a 3-bit image.**

2	3	7	0	7
6	4	4	1	5
1	3	2	2	3
4	5	0	6	4
6	1	7	6	4

AKTU 2018-19, Marks 10

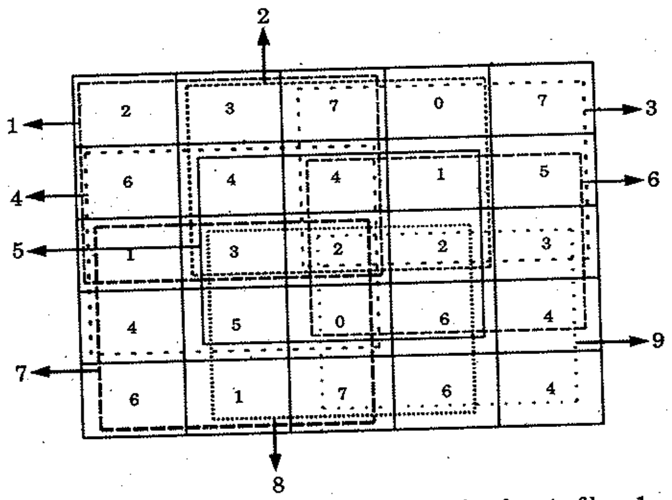
Answer

Linear and non-linear smoothing filters in spatial domain : Refer Q. 2.28, Page 2–38A, Unit-2.

Numerical :

Assumptions :

1. We are calculating the box filter values for the inner pixel values only.
2. If we have to calculate the border pixel value then we have to add the extra padding with zero values.



Step 1 : Applying 3 × 3 box filter to the pixel value 4 of box 1.

2	3	7
6	4	4
1	3	2

The pixel value '4' will be replaced by :

$$\frac{1}{9}[2+3+7+6+4+4+1+3+2] = \frac{32}{9} = 3.5 \approx 4$$

Step 2 : Applying 3 × 3 box filter to the pixel value 4 of box 2.

3	7	0
4	4	1
3	2	2

The pixel value '4' will be replaced by :

$$\frac{1}{9}[3+7+0+4+4+1+3+2+2] = \frac{26}{9} = 2.8 \approx 3$$

Step 3 : Applying 3 × 3 box filter to the pixel value 1 of box 3.

7	0	7
4	1	5
2	2	3

The pixel value '1' will be replaced by :

$$\frac{1}{9}[7+0+7+4+1+5+2+2+3] = \frac{31}{9} = 3.4 \approx 3$$

Step 4 : Applying 3 × 3 box filter to the pixel value 3 of box 4.

6	4	4
1	3	2
4	5	0

The pixel value '3' will be replaced by :

$$\frac{1}{9}[6+4+4+1+3+2+4+5+0] = \frac{29}{9} = 3.2 \approx 3$$

Step 5 : Applying 3 × 3 box filter to the pixel value 2 of box 5.

4	4	1
3	2	2
5	0	6

The pixel value '2' will be replaced by :

$$\frac{1}{9}[4+4+1+3+2+2+5+0+6] = \frac{30}{9} = 3.3 \approx 3$$

Step 6 : Applying 3 × 3 box filter to the pixel value 2 of box 6.

4	1	5
2	2	3
0	6	4

The pixel value '2' will be replaced by :

$$\frac{1}{9}[4+1+5+2+2+3+0+6+4] = \frac{27}{9} = 3$$

Step 7 : Applying 3 × 3 box filter to the pixel value 5 of box 7.

1	3	2
4	5	0
6	1	7

The pixel value '5' will be replaced by :

$$\frac{1}{9}[1+3+2+4+5+0+6+1+7] = \frac{29}{9} = 3.2 \approx 3$$

Step 8 : Applying 3 × 3 box filter to the pixel value 0 of box 8.

3	2	2
5	0	6
1	7	6

The pixel value '0' will be replaced by :

$$\frac{1}{9}[3+2+2+5+0+6+1+7+6] = \frac{32}{9} = 3$$

Step 9 : Applying 3 × 3 box filter to the pixel value 6 of box 9.

2	2	3
0	6	4
7	6	4

The pixel value '6' will be replaced by :

$$\frac{1}{9}[2+2+3+6+0+4+7+6+4] = \frac{34}{9} = 3.7 \approx 4$$

Hence, the new pixel values after applying 3 × 3 box filter on 5 × 5 matrix of a 3-bit image will be :

2	3	7	0	7
6	4	4	1	5
1	3	2	2	3
4	5	0	6	4
6	1	7	6	4

→

2	3	7	0	7
6	4	3	3	5
1	3	3	3	3
4	3	4	4	4
6	1	7	6	4

Que 2.30. **Compare and contrast between linear spatial filtering and non-linear spatial filtering.** **AKTU 2017-18, Marks 10**

Answer

S. No.	Linear spatial filtering	Non-linear spatial filtering
1.	In linear spatial filtering, operation performed on image pixels is linear.	In non-linear spatial filtering, operation performed on image pixels is non-linear.
2.	There is one-to-one correspondence between linear spatial filters.	In this filtering, filters are not in one-to-one correspondence.
3.	Linear spatial filtering focus on the odd size of filters.	Non-linear spatial filtering does not focus on the odd size of filters.
4.	Linear spatial filters have a direct equivalent Fourier or frequency domain implementation.	Non-linear spatial filtering does not have direct equivalent Fourier.
5.	Linear spatial filtering obey superposition and shift invariance property.	Non-linear spatial filtering does not obey superposition and shift invariance property.

Que 2.31. **Prove that Prewitt and Sobel operator act as a low pass and high pass filter.** **AKTU 2017-18, Marks 10**

Answer

1. High pass filters remove low frequency components from images and enhance high frequency components.
2. So when high pass filter is applied to an image, it will remove the background, as it is a low frequency region and enhances the edges, which are high frequency components.
3. The Sobel operator is an algorithm used for edge detection applications.
4. It is a 3 × 3 filter that approximates the gradient of the image intensity function.
5. It provides a separate filter to compute the gradient in a horizontal and vertical direction.
6. As Sobel operator is used in edge detection and enhancement and edges are high frequency components in digital images. Hence, according to point (2) we can say that Sobel operator acts as a high pass filter.

7. The Prewitt operator is similar to the Sobel operator (only different weighting factor {1, 1, 1} are used in the smoothing). Hence, Prewitt operator also acts as a high pass filter.

Que 2.32. Explain image enhancement with its techniques.

Answer

1. Image enhancement is a process of improving the interpretability or perception of information in images for human viewers and providing better input for other automated image processing techniques.
2. There exist many techniques that can enhance a digital image without spoiling it.
3. The enhancement methods can broadly be divided into the following two categories :
   a. **Spatial domain techniques :**
      i. We directly deal with the image pixels.
      ii. The pixel values are manipulated to achieve desired enhancement.
   b. **Frequency domain techniques :**
      i. The image is first transferred into frequency domain.
      ii. It means that, the Fourier Transform of the image is computed first.
4. All the enhancement operations are performed on the Fourier transform of the image and then the Inverse Fourier transform is performed to get the resultant image.
5. Image enhancement is applied in every field where images are ought to be understood and analyzed.
6. For example, medical image analysis, analysis of images from satellites etc.

VERY IMPORTANT QUESTIONS

Following questions are very important. These questions may be asked in your SESSIONALS as well as UNIVERSITY EXAMINATION.

Q. 1. Write short note on bit plane slicing.
Ans. Refer Q. 2.3.

Q. 2. Explain piecewise-linear transformations of image enhancement with suitable example.
Ans. Refer Q. 2.9.

Q. 3. What do you understand by hit-miss transform and why they are used explain in brief ?
Ans. Refer Q. 2.12.

Q. 4. Write short note on image histogram.
Ans. Refer Q. 2.13.

Q. 5. Explain the properties of images which can be described by histogram. Also explain normalized histogram.
Ans. Refer Q. 2.14.

Q. 6. Write short note on Homomorphic filtering.
Ans. Refer Q. 2.27.

Q. 7. Compare and contrast between linear spatial filtering and non-linear spatial filtering.
Ans. Refer Q. 2.30.

Q. 8. Prove that Prewitt and Sobel operator act as a low pass and high pass filter.
Ans. Refer Q. 2.31.

☺☺☺

3
UNIT

Image Restoration

CONTENTS

PART-1

Image Restoration, Degradation Model, Properties, Noise Models, Mean Filters, Order Statistics, Adaptive Filter, Band Reject Filter, Band Pass Filter, Notch Filter.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 3.1. **What is image restoration ? Draw and explain the basic block diagram of the restoration process. Give two areas where restoration process can be applied.**

AKTU 2016-17, Marks 10

AKTU 2017-18, Marks 10

OR

Give a model of image degradation/restoration process.

AKTU 2014-15, Marks 05

OR

What is image restoration ? Explain in detail the image restoration in presence of noise only.

AKTU 2018-19, Marks 10

Answer

Image restoration :

1. Image restoration can be defined as the process of removal or reduction of degradation in an image through linear or non-linear filtering.
2. Degradations are usually incurred during the acquisition of the image itself.
3. The aim of image restoration is to bring the image towards what it would have been if it had been recorded without degradation.
4. The ultimate goal in restoration is to improve an image.
5. Restoration is an objective process.
6. Restoration uses a priori knowledge of the degradation phenomena.
7. Restoration deals with getting an optimal estimate of the desired result.

Degradation model :

1. Fig. 3.1.1 shows a typical degradation model with additive noise present.

   Where,
   $f(x, y) \rightarrow$ original image before degradation
   $g(x, y) \rightarrow$ observed degraded image
   $n(x, y) \rightarrow$ additive noise
   $h(x, y) \rightarrow$ Spatially invariant point spread function (Degradation function)

$\tilde{f}(x, y) \rightarrow$ Restored image

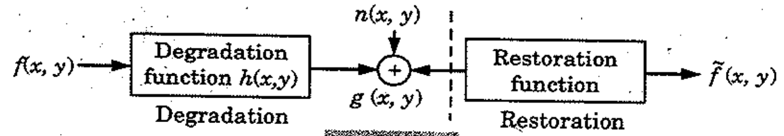


Fig. 3.1.1.

2. In degradation block the linear image model is given by,

$$g(x, y) = \int_{-\infty}^{+\infty}\int f(k, l) \cdot h(x-k, y-l)\, dk\, dl + n(x, y) \quad ...(3.1.1)$$

$$g(x, y) = f(x, y) * h(x, y) + n(x, y) \quad ...(3.1.2)$$

3. In other words, the original image gets convolved with the degradation function $h(x, y)$ and noise gets added to it.
4. Hence, to remove this degradation function (convolution) we need to apply inverse filtering (de-convolution) to the degraded image.
   In the discrete domain, eq. (3.1.2) is given by the formula,

$$g(x, y) = \sum_k \sum_l h(x-k, y-l) f(k, l) + n(x, y)$$

Two areas of restoration :

i. Quantum limiting imaging in X-ray.
ii. CT (Computed Tomography) in health care areas.

Que 3.2. **What is the difference between image enhancement and image restoration ? Mention some important causes of image degradation.** **AKTU 2014-15, Marks 05**

Answer

S. No.	Image enhancement	Image restoration
1.	It gives better visual representation.	It removes effects of sensing environment.
2.	No model required.	Mathematical model of degradation is needed.
3.	It is a subjective process.	It is an objective process.
4.	Contrast stretching, histogram equalization etc., are some enhancement techniques.	Inverse filtering, Wiener filtering, denoising are some restoration techniques.

Different causes of image degradation are :

1. Improper opening and closing of the shutter.
2. Atmospheric turbulence.
3. Misfocus of lens.
4. Relative motion between camera and object which causes motion blur.

Que 3.3. **Explain all noise models in details.**

OR

Explain any two noise models in detail. **AKTU 2014-15, Marks 05**

OR

What is noise ? Define any two noise models in detail.

AKTU 2016-17, Marks 10

Answer

Noise : Noise is a disturbance in the image during acquisition and transmission.

Various noise models are :

1. **Gaussian noise :**
   a. Gaussian noise provides a good model of noise because of its simplicity.
   b. They are used when all other noise models fail.
   c. The PDF (Probability Density Function) of Gaussian distribution is given by,

$$p(z) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-(z-\mu)^2/2\sigma^2}$$

Here, z represents the gray level, σ is the standard deviation and μ is the mean.

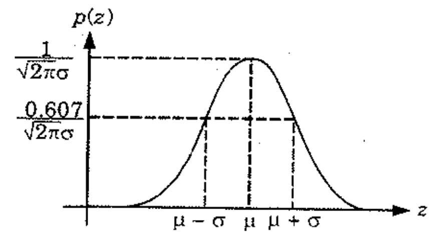


Fig. 3.3.1.

2. **Rayleigh noise :**
   a. Unlike the Gaussian distribution, the Rayleigh distribution is not symmetric.
   b. It is given by the formula,

$$p(z) = \begin{cases} \frac{2}{b}(z-a)e^{-(z-a)^2/b} & z \ge a \\ 0 & z < a \end{cases}$$

c. The plot of the Rayleigh distribution is shown in Fig. 3.3.2.

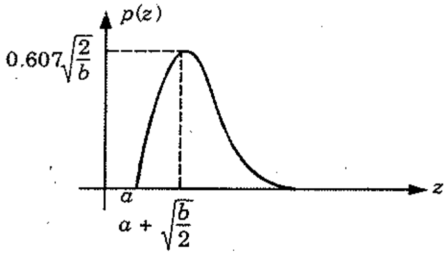


Fig. 3.3.2.

3. **Gamma noise (Erlang noise) :**

a. The shape of the Gamma noise is similar to the Rayleigh distribution.

b. The Gamma noise distribution starts from zero.

c. It is given by the formula,

$$p(z) = \begin{cases} \dfrac{a^b z^{b-1}}{(b-1)!} e^{-az} & \text{for } z \ge 0 \\ 0 & \text{for } z < 0 \end{cases}$$

d. Here, $a > 0$ and b is a positive integer.

e. The mean and the variance of this distribution are given by,

$\mu = b/a$, and
$\sigma^2 = b/a^2$

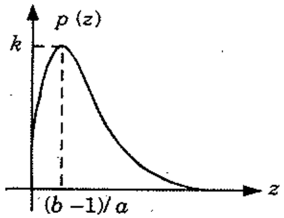


Fig. 3.3.3.

4. **Exponential noise :**

a. Exponential distribution has an exponential shape.

b. It is given by the equation,

$$p(z) = \begin{cases} ae^{-az} & \text{for } z \ge 0 \\ 0 & \text{for } z < 0 \end{cases}$$

c. The mean and the variance of the exponential noise is given by,

$$\mu = \frac{1}{a}$$

$$\sigma^2 = \frac{1}{a^2}$$

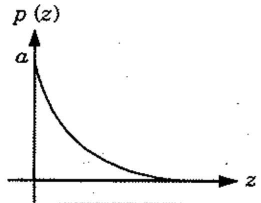


Fig. 3.3.4.

5. **Salt and Pepper noise (Impulse) :**

a. The PDF of salt and Pepper is given by the formula,

$$p(z) = \begin{cases} P_a & z = a \\ P_b & z = b \\ 0 & \text{otherwise} \end{cases}$$

b. This noise appears as black and white dots on the entire image.

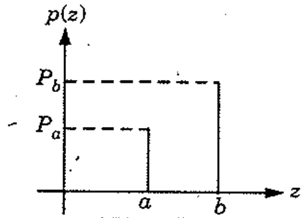


Fig. 3.3.5.

6. **Uniform noise :**

a. This noise is uniform over a certain band of gray levels.

b. The PDF of uniform noise is shown as :

$$p(z) = \begin{cases} \dfrac{1}{b-a} & \text{if } a \le z \le b \\ 0 & \text{otherwise} \end{cases}$$

c. The mean of the function is,

$$\mu = \frac{a+b}{2}$$

d. The variance of the function is,

$$\sigma^2 = \frac{(b-a)^2}{12}$$

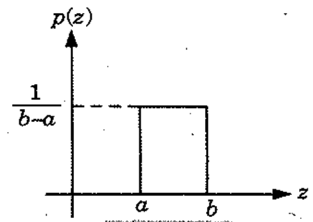


Fig. 3.3.6.

7. **Sinusoidal noise :**
   a. Sinusoidal noise is a special kind of noise that cannot be eliminated in the spatial domain.
   b. Images are usually corrupted by periodic noise due to electrical and electromechanical interface during acquisition.

Que 3.4. **What are order statistic filters ?**

AKTU 2014-15, Marks 05

OR

Discuss order statistics filters with suitable example.

AKTU 2018-19, Marks 10

Answer

1. Order statistic filters are non-linear spatial filters whose response is based on the ordering (ranking) of the pixels contained in the image area encompassed by the filter, and then replacing the value in the center pixel with the value determined by the ranking result.
2. The different types of order statistics filters are :
   i. **Median filtering :** It replaces the value of a pixel by the median of the pixel values in the neighborhood of that pixel.

   For example :

   Median filter : R = median $\{Z_k \mid k = 1, 2, ..., n \times n\}$

   Where, $n \times n$ is the size of the mask

   ii. **Max and min filtering :** The max filtering is achieved using the following equation :
   $$f(x, y) = \max g(s, t)$$
   The min filtering is achieved using the following equation :
   $$f(x, y) = \min g(s, t)$$
   For example :

   Max filter : $R = \max \{Z_k \mid k = 1, 2, ..., n \times n\}$

   Min filter : $R = \min \{Z_k \mid k = 1, 2, ..., n \times n\}$

   Where, $n \times n$ is the size of the mask

   iii. **Mid-point filtering :** It replaces the value of a pixel by the midpoint between the maximum and minimum pixels in a neighborhood.

Que 3.5. **Draw the diagram for image restoration/degradation process. Explain the linear position invariant property of degradation function.**

AKTU 2015-16, Marks 10

Answer

Diagram for image restoration/degradation process : Refer Q. 3.1, Page 3–2A, Unit-3.

Linear position invariant property : The input-output relationship before the restoration stage is expressed as
$$g(x, y) = H[f(x, y)] + \eta(x, y) \quad ...(3.5.1)$$
Let us assume that
$$\eta(x, y) = 0 \text{ so that } g(x, y) = H[f(x, y)].$$
H is linear if
$$H[af_1(x, y) + bf_2(x, y)] = aH[f_1(x, y)] + bH[f_2(x, y)] \quad ...(3.5.2)$$
where a and b are scalars and $f_1(x, y)$ and $f_2(x, y)$ are any two input images.

If $a = b = 1$, eq. (3.5.2) becomes
$$H[f_1(x, y) + f_2(x, y)] = H[f_1(x, y)] + H[f_2(x, y)] \quad ...(3.5.3)$$
which is called the property of additivity.

With $f_2(x, y) = 0$, eq. (3.5.2) becomes
$$H[af_1(x, y)] = aH[f_1(x, y)] \quad ...(3.5.4)$$
which is called the property of homogeneity. Thus a linear operator possesses both the property of additivity and the property of homogeneity.

An operator having the input-output relationship
$$g(x, y) = H[f(x, y)]$$
is said to be position (or space) invariant if
$$H[f(x - \alpha, y - \beta)] = g(x - \alpha, y - \beta) \quad ...(3.5.5)$$
for any $f(x, y)$ and (α, β).

With a slight (but equivalent) change in notation in the definition of the impulse $f(x, y)$ can be expressed as :
$$f(x, y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(\alpha, \beta)\, \delta(x - \alpha, y - \beta)\, d\alpha\, d\beta \quad ...(3.5.6)$$
Substitution of eq. (3.5.6) into eq. (3.5.5) results in the expression
$$g(x, y) = H[f(x, y)] = H\left[\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(\alpha, \beta)\, \delta(x - \alpha, y - \beta)\, d\alpha\, d\beta\right] \quad ...(3.5.7)$$

If H is a linear operator and we extend the additivity property to integrals, then

$$g(x, y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} H[f(\alpha, \beta)\, \delta(x - \alpha, y - \beta)]\, d\alpha\, d\beta \quad ...(3.5.8)$$

Because $f(\alpha, \beta)$ is independent of x and y, and using the homogeneity property it follows that

$$g(x, y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(\alpha, \beta)\, H[\delta(x - \alpha, y - \beta)]\, d\alpha\, d\beta \quad ...(3.5.9)$$

The term

$$h(x, \alpha, y, \beta) = H[\delta(x - \alpha, y - \beta)] \quad ....(3.5.10)$$

is called the impulse response of H. In other words, if $\eta(x, y) = 0$ in Eq. (3.5.1), then $h(x, a, y, b)$ is the response of H to an impulse at coordinates (x, y). In optic, the impulse becomes a point of light and $h(x, \alpha, y, \beta)$ is commonly referred to as the Point Spread Junction (PSJ).

Substituting eq. (3.5.10) into eq. (3.5.9) yields the expression

$$g(x, y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(\alpha, \beta)\, h(x, \alpha, y, \beta)\, d\alpha\, d\beta \quad ...(3.5.11)$$

which is called the superposition integral of the first kind. This expression is a fundamental result at the core of linear theory. It states that if the response of H to an impulse is known, the response to any input $f(\alpha, \beta)$ can be calculated by means of eq. (3.5.11). In other words, a linear system H is completely characterized by its impulse response.

If H is position invariant, then, from eq. (3.5.5),

$$H[\delta(x - \alpha, y - \beta)] = h(x - \alpha, y - \beta) \quad ...(3.5.12)$$

Equation (3.5.11) reduces to

$$g(x, y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(\alpha, \beta)\, h(x - \alpha, y - \beta)\, d\alpha\, d\beta \quad ...(3.5.13)$$

Que 3.6. **Describe briefly the concept of adaptive filter.**

Answer

1. An adaptive filter is a system with a linear filter that has a transfer function controlled by variable parameters and a means to adjust those parameters according to an optimization algorithm.
2. All adaptive filters are digital filters because of the complexity of the optimization algorithm.
3. Adaptive filters are required for some applications because some parameters of the desired processing operation are not known in advance or are changing.
4. The closed loop adaptive filter uses feedback in the form of an error signal to refine its transfer function.

5. The closed loop adaptive process involves the use of a cost function, which is a criterion for optimum performance of the filter, to feed an algorithm, which determines how to modify filter transfer function to minimize the cost on the next iteration.
6. The most common cost function is the mean square of the error signal.
7. As the power of digital signal processors has increased, adaptive filters have become much more common and are now routinely used in devices such as mobile phones and other communication devices, camcorders and digital cameras, and medical monitoring equipment.

Que 3.7. **Given below is a 3 × 3 image. What will be the value of the centre pixel change to when this image is passed through :**

a. Arithmetic mean filter
b. Geometric mean filter
c. Harmonic mean filter
d. Max filter
e. Min filter

5	1	7
6	2	3
4	2	1

Fig. 3.7.1.

Answer

a. Arithmetic mean filter :

$$\hat{f}(x,y) = \frac{1}{MN}\sum_{(s,t)\in S_{xy}} g(s,t)$$

$$= \frac{1}{3\times 3}\Sigma(5+1+7+6+2+3+4+2+1)$$

$$= \frac{1}{9}\times 31 = 3.44 \approx 3$$

b. Geometric mean filter :

$$\hat{f}(x,y) = \left[\prod_{(s,t)\in S_{xy}} g(s,t)\right]^{\frac{1}{MN}}$$

$$= [5\times1\times7\times6\times2\times3\times4\times2\times1]^{\frac{1}{3\times3}}$$

$$= [10080]^{\frac{1}{9}} = 2.78 \approx 3$$

c. **Harmonic mean filter :**

$$\hat{f}(x,y) = \frac{MN}{\sum_{(s,t)\in S_{xy}} \frac{1}{g(s,t)}}$$

$$= \frac{3\times 3}{\frac{1}{5}+\frac{1}{1}+\frac{1}{7}+\frac{1}{6}+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{2}+\frac{1}{1}}$$

$$= \frac{9}{4.0928} = 2.19 \approx 2$$

d. **Max filter :**

$$\hat{f}(x,y) = \max_{(s,t)\in S_{xy}} \{g(s,t)\}$$
$$= \max\{5, 1, 7, 6, 2, 3, 4, 2, 1\} = 7$$

e. **Min filter :**

$$\hat{f}(x,y) = \min_{(s,t)\in S_{xy}} \{g(s,t)\}$$
$$= \min\{5, 1, 7, 6, 2, 3, 4, 2, 1\} = 1$$

Que 3.8. **Consider the following image. What will be the new value of the pixel (2, 2) if smoothing is done using a 3 × 3 :**

0	1	0	2	7
2	7	7	4	0
5	6	4	3	3
1	1	0	7	5
5	4	2	2	5

Fig. 3.8.2.

i. **Mean filter**
ii. **Weighted average filter**
iii. **Median filter**
iv. **Min filter**
v. **Max filter**

AKTU 2015-16, Marks 10

Answer

Pixel (2, 2) is (4)

0	1	0	2	7
2	7	7	4	0
5	6	4	3	3
1	1	0	7	5
5	4	2	2	5

Fig. 3.8.2.

3 × 3 neighbourhood is :

7	7	4
6	4	3
1	0	7

Fig. 3.8.3.

i. **Mean filter :**

$$= \frac{7+7+4+6+4-3+1+0+7}{9} = \frac{39}{9} = 4.33 = 4$$

Mean filter = 4

ii. **Weighted average filter :**

1	1	1
1	2	1
1	1	1

Fig. 3.8.4.

Accordingly the response of weighted average filter is

$$= \frac{7\times1+7\times1+4\times1+6\times1+2\times4+3\times1+1\times1+0\times1+7\times1}{9}$$

$$= \frac{7+7+4+6+8+3+1+7}{9} = 4.77 = 5$$

iii. **Median filter :**

0, 1, 3, 4, 4 , 6, 7, 7, 7

median = 4

iv. **Min filter :**

0 , 1, 3, 4, 4, 6, 7, 7, 7

min = 0

v. **Max filter :**

0, 1, 3, 4, 4, 6, 7, 7, 7

max = 7

Que 3.9. **Explain periodic noise reduction using band reject filter.**

AKTU 2015-16, Marks 10

Answer

1. Band reject filter is used for noise removal in the application where the general location of the noise components in the frequency domain is approximately known.
2. For example, an image corrupted by additive periodic noise that can be approximated as two-dimensional sinusoidal functions.

3. It is not difficult to show that the Fourier transform of a sine consists of two impulses that are mirror images of each other about the origin of the transform.
4. The impulses are both imaginary (the real part of the Fourier transform of a sine is zero) and are complex conjugates of each other.
5. Unfortunately, when the distances of the periodic noise components about the origin of the 2D Fourier transform are different, there is necessary to use either a wide band-reject filter, or several narrow band-reject filters.
6. In both cases, the restored image may lose some important image information.

Que 3.10. **Explain band reject filters and band pass filters in detail.**

AKTU 2015-16, Marks 10

Answer

Band reject filters : Refer Q. 3.9, Page 3–12A, Unit-3.

Band pass filters :

1. A band pass filter performs the opposite operation of a band reject filter.
2. The transfer function $H_{BP}(u, v)$ of a band pass filter is obtained from a corresponding band reject filter with transfer function $H_{BR}(u, v)$ by using the equation :

$$H_{BP}(u, v) = 1 - H_{BR}(u, v)$$

Derivation :

a. **Ideal band pass filter :**

$$H_{IBP}(u, v) = \begin{cases} 0 & \text{if } D(u, v) < D_0 - \dfrac{W}{2} \\ 1 & \text{if } D_0 - \dfrac{W}{2} \le D(u, v) \le D_0 + \dfrac{W}{2} \\ 0 & D(u, v) > D_0 + \dfrac{W}{2} \end{cases}$$

b. **Butterworth band pass filter :**

$$H_{BBP}(u, v) = 1 - \frac{1}{1 + \left[\dfrac{D(u, v)W}{D^2(u, v) - D_0^2}\right]^{2n}} = \frac{\left[\dfrac{D(u, v)W}{D^2(u, v) - D_0^2}\right]^{2n}}{1 + \left[\dfrac{D(u, v)W}{D^2(u, v) - D_0^2}\right]^{2n}}$$

c. **Gaussian band pass filter :**

$$H_{GBP}(u, v) = 1 - \left[1 - e^{-\frac{1}{2}\left[\frac{D^2(u, v) - D_0^2}{D(u, v)W}\right]^2}\right]$$

$$= e^{-\frac{1}{2}\left[\frac{D^2(u, v) - D_0^2}{D(u, v)W}\right]^2}$$

PART-2

Optimum Notch Filtering, Inverse Filtering, Wiener Filtering.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 3.11. **Explain notch filters and optimum notch filtering.**

Answer

Notch filters :

1. A notch filter is also known as a band stop filter or band reject filter.
2. These filters reject/attenuate signals in a specific frequency band called the stop band frequency range and pass the signals above and below this band.
3. When selecting a band stop filter or notch filter the following parameters are used :
   a. **Stop band frequency :** Any signal in this frequency range will be attenuated by the notch filter.
   b. **Attenuation/rejection in the stop band :** This is the level of attenuation that the filter will provide in the stop band. This is usually represented in dB.
   c. **Power handling capability of the filter :** This is the maximum power that the notch filter can handle.
   d. **Package type :** There are a number of form factors and package types in which these notch filters are available.

Optimum notch filtering :

1. Optimum notch filtering minimizes local variances of the restored image.
2. First step is to extract the principal frequency components of the interference pattern.

3. This can be fulfilled by placing a notch pass filter $H_{NP}(u,v)$ at the location of each spike.
4. If the filter is constructed which passes only components associated with the interference pattern then the Fourier transform of the interference noise pattern can be expressed by :

$$N(u, v) = H_{NP}(u, v)\, G(u, v).$$

where H_{NP} is superimposed response of all necessary notch-pass filter and $G(u, v)$ is the 2-D Fourier transform of the contaminated image.

Que 3.12. Write a note on noise models in image restoration. Describe Wiener filter and inverse filtering.

Answer

Noise model : Refer Q. 3.3, Page 3–4A, Unit-3.

Inverse filtering :

1. We know that the input image $f(x, y)$ gets convolved by a blurring function $h(x, y)$ and changes to $g(x, y)$.
2. Hence to retrieve $f(x, y)$ from $g(x, y)$ we take the inverse of $h(x, y)$.
3. Assuming the noise term to be zero, equation can be written as,

$$g(x, y) = \sum_k \sum_l h(x-k, y-l) f(k,l) \quad ...(3.12.1)$$

4. We now take the Fourier transform :

$$G(u, v) = H(u, v) \times F(u, v)$$

$$\therefore \quad F(u, v) = \frac{G(u, v)}{H(u, v)} \quad ...(3.12.2)$$

5. Let, $$H_I(u, v) = \frac{1}{H(u, v)}$$

$\therefore$ Inverse filtering is given by,

$$F(u, v) = H_I(u, v) \times G(u, v)$$

6. Taking Inverse Fourier transform, we get

$$f(x, y) = h_I(x, y) * g(x, y)$$

i.e., if we convolve the blurred image with the inverse of the blurring function, we get back the original image.

7. The main limitation of inverse filtering is that this is sensitive to noise.

Wiener filter :

1. The Wiener filter is the most popular filter used for restoration.
2. The Wiener filter exploits the statistical properties of the image and can be used to restore images in the presence of blur as well as noise.
3. Let $f(x, y)$, $g(x, y)$, and $\tilde{f}(x, y)$ be zero mean random sequences.

Zero mean random sequences imply $E[f(x, y)] = 0$, $E[g(x, y)] = 0$ and $E[\tilde{f}(x, y)] = 0$.

4. Similarly, stationary sequences can be defined in terms of correlation.

$$\left.\begin{array}{l} E[f(x,y)\cdot f(i,j)] = r_{ff}(x-i, y-j) \\ E[g(x,y)\cdot g(i,j)] = r_{gg}(x-i, y-j) \end{array}\right\} \rightarrow \text{Auto correlation} \quad ...(3.12.3)$$

$$E[f(x, y) \cdot g(i, j)] = r_{fg}(x-i, y-j) \rightarrow \text{cross correlation}$$

5. The zero mean image model is given by the equation,

$$g(x, y) = \sum_i \sum_j h_D(i, j) \cdot f(x-i, y-j) \quad ...(3.12.4)$$

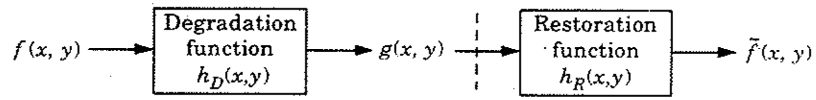


Fig. 3.12.1.

6. Image restoration involves finding the restored image $\tilde{f}(x, y)$ from the degraded image $g(x, y)$ such that the mean squared error between $f(x, y)$ and $\tilde{f}(x, y)$ is minimum.
7. The mean squared error (mse) is given by the formula,

$$\text{mse} = E[\{f(x, y) - \tilde{f}(x, y)\}^2] \quad ...(3.12.5)$$

$$\text{mse} = \frac{1}{N^2} \sum_{x=0}^{N-1} \sum_{y=0}^{N-1} \{f(x,y) - \tilde{f}(x,y)\}^2$$

8. The mse is minimized if and only if the orthogonal condition is satisfied *i.e.*,

$$E[\{f(x, y) - \tilde{f}(x, y)\}^2] \perp g(x, y) \quad ...(3.12.6)$$

9. In second part of Fig. 3.12.2, $h_R(x, y)$ is the restoration filter.

$$\tilde{f}(x, y) = \sum_{i=0}^{N-1} \sum_{j=0}^{N-1} g(i, j) \cdot h_R(x-i, y-j) \quad ...(3.12.7)$$

10. $\tilde{f}(x, y)$ should be such that the mean squared error (mse) between $f(x, y)$ and $\tilde{f}(x, y)$ is minimum.
11. The mse is minimized if and only if the orthogonality property is satisfied (Dot product should be zero for orthogonality).

$$E[f(x, y) - \tilde{f}(x, y)] \cdot g(i, j) = 0 \quad ...(3.12.8)$$

We now solve this equation,

$$E[f(x,y)\cdot g(i,j)] - \hat{f}(x,y)\cdot g(i,j)] = 0 \quad ...(3.12.9)$$

$$E[f(x,y)\cdot g(i,j)] - E[\hat{f}(x,y)\cdot g(i,j)] = 0 \quad ...(3.12.10)$$

Substituting eq. (3.12.9) in eq. (3.12.10)

We get

$$E[f(x,y)\cdot g(i,j)] - E\left[\sum_{i=0}^{N-1}\sum_{j=0}^{N-1} g(i,j)\cdot h_R(x-i,y-j)\cdot g(i,j)\right] = 0$$

$$E[f(x,y)\cdot g(i,j)] - E\left[\sum_{i=0}^{N-1}\sum_{j=0}^{N-1} h_R(x-i,y-j)\cdot g(i,j)g(i,j)\right] = 0$$

i.e.,

$$E[f(x,y)\cdot g(i,j)] - \sum_{i=0}^{N-1}\sum_{j=0}^{N-1} h_R(x-i,y-j)\cdot E[g(i,j)\cdot g(i,j)] = 0$$

$$\therefore\ E[f(x,y)\cdot g(i,j)] = \sum_{i=0}^{N-1}\sum_{j=0}^{N-1} h_R(x-i,y-j)\cdot E[g(i,j)\cdot g(i,j)]$$

12. Using eq. (3.12.5) we get,

$$r_{fg}(x,y) = \sum_{i=0}^{N-1}\sum_{j=0}^{N-1} h_R(x-i,y-j)\cdot r_{gg}(i,j)$$

We now take the Fourier transform of the above equation.

$$S_{fg}(u,v) = H_R(u,v)\cdot S_{gg}(u,v) \quad ...(3.12.11)$$

Where $S_{gg}(u,v)$ is the power spectral density and $S_{fg}(u,v)$ is the cross spectral density. Using eq. (3.12.13), we get

$$H_R(u,v) = \frac{S_{fg}(u,v)}{S_{gg}(u,v)} \quad ...(3.12.12)$$

Here, eq. (3.12.13) and eq. (3.12.14) are called the Wiener filter equations. Eq. (3.12.14) gives us the Wiener filter frequency response.

Que 3.13. **Explain SNR using frequency domain.**

Answer

1. Signal-to-Noise (SNR) ratio, approximated using frequency domain quantities and it is defined as :

$$\text{SNR} = \frac{\sum_{u=0}^{M-1}\sum_{v=0}^{N-1}|F(u,v)|^2}{\sum_{u=0}^{M-1}\sum_{v=0}^{N-1}|N(u,v)|^2}$$

2. This ratio gives a measure of the level of information bearing signal power (*i.e.*, of the original, undegraded image) to the level of noise power.
3. Image with low noise tends to have a high SNR and, conversely, the same image with a higher level of noise has a lower SNR.
4. This ratio by itself is of limited value, but it is an important metric used in characterizing the performance of restoration algorithms.

VERY IMPORTANT QUESTIONS

Following questions are very important. These questions may be asked in your SESSIONALS as well as UNIVERSITY EXAMINATION.

Q. 1. What is image restoration ? Explain in detail the image restoration in presence of noise only.

Ans. Refer Q. 3.1.

Q. 2. What is the difference between image enhancement and image restoration ? Mention some important causes of image degradation.

Ans. Refer Q. 3.2.

Q. 3. What is noise ? Define any two noise models in detail.

Ans. Refer Q. 3.3.

Q. 4. Discuss order statistics filters with suitable example.

Ans. Refer Q. 3.4.

Q. 5. Draw the diagram for image restoration/degradation process. Explain the linear position invariant property of degradation function.

Ans. Refer Q. 3.5.

Q. 6. Explain periodic noise reduction using band reject filter.

Ans. Refer Q. 3.9.

Q. 7. Explain band reject filters and band pass filters in detail.

Ans. Refer Q. 3.10.

☺☺☺

4 UNIT

Image Segmentation

CONTENTS

PART-1

Edge Detection, Edge Linking Via Hough Transform, Thresholding, Region based Segmentation, Region Growing, Region Splitting and Merging.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 4.1. **Write short note on image segmentation.**

AKTU 2016-17, Marks 05

OR

Discuss image segmentation. What are the characteristics of segmentation process ?

Answer

1. Segmentation subdivides (segments) an image into its constituent regions or objects.
2. In image segmentation, we address some aspects of analyzing the content of an image, *i.e.*, what is in the picture.
3. Segmentation forms a section of computer vision *i.e.*, we use segmentation, when we want the computer to make decisions.
4. Segmentation is not required when the images have to be shown to a human being.
5. This is because a human visual system has an inherent quality to segment the image shown to it. Examples where image segmentation is used :
   a. Automated blood cell counting
   b. Finger print matching in forensic
6. Image segmentation algorithms are generally based on one of the two basic properties *i.e.*, image segmentation can be achieved in any one of the two ways viz.
   a. Segmentation based on discontinuities in intensity.
   b. Segmentation based on similarities in intensity.
7. In the first method, the approach is to partition an image based on abrupt changes in intensity, such as edges, while in the second method, we partition an image into regions that are similar according to a set of predefined criteria.

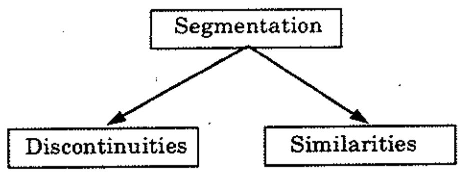


Fig. 4.1.1.

Characteristics of the segmentation process are :

1. If the subregions are combined, the original region can be obtained. Mathematically, it can be stated that $\cup R_i = R$ for $i = 1, 2, ..., n$.
2. The subregions R_i should be connected. In other words, the region cannot be open-ended during the tracing process.
3. The regions $R_1, R_2, ... , R_n$ do not share any common property. Mathematically, it can be stated as $R_i \cup R_j = \phi$ for all i and j where $i \neq j$. Otherwise, there is no justification for the region to exist separately.
4. Each region satisfies a predicate or a set of predicates such as intensity or other image statistics, that is the predicate (P) can be colour, gray scale value, texture, or any other image statistic. Mathematically, this is stated as $P(R_i)$ = True.

Que 4.2. **Explain different ways of classifying segmentation algorithm.**

Answer

Different ways of classifying the segmentation algorithm are :

1. **Based on user interaction :**

a. **Manual method :**

i. A manual method of extraction is time consuming, highly subjective, prone to human error, and has poor intra-observer reproducibility.

ii. However, manual methods are still used commonly by experts to verify and validate the results of automatic segmentation algorithms.

b. **Automatic segmentation algorithm :**

i. Automatic segmentation algorithms segment the structures of the objects without any human intervention.

ii. They are preferred if the tasks need to be carried out for a large number of images.

c. **Semi-automatic algorithm :**

i. Semi-automatic algorithms are a combination of automatic and manual algorithms.

ii. In semi-automatic algorithms, human intervention is required in the initial stages.

2. **On the basis of pixel relationship :**

a. **Contextual algorithm :**

i. Contextual algorithms group pixels together based on common properties by exploiting the relationships that exist among the pixels.

ii. These are also known as region-based or global algorithms.

iii. In region-based algorithms, the pixels are ground based on some sort of similarity that exists between them.

b. **Non-contextual algorithm :**

i. Non-contextual algorithms are also known as pixel-based or local algorithms.

ii. These algorithms ignore the relationship that exists between the pixels of features.

iii. Instead, the idea is to identify the discontinuities that are present in the image such as isolated lines and edges.

iv. These are then simply grouped into a region based on some global level property.

Que 4.3. **Explain region based segmentation with an example.**

AKTU 2017-18, Marks 10

Answer

1. Region based segmentation is a technique in which segmentation is carried out based on the similarities in the given image.
2. The region based approach to segmentation seeks to create regions directly by grouping together pixels which share common features into areas or regions of uniformity.
3. Let R represent the entire image :

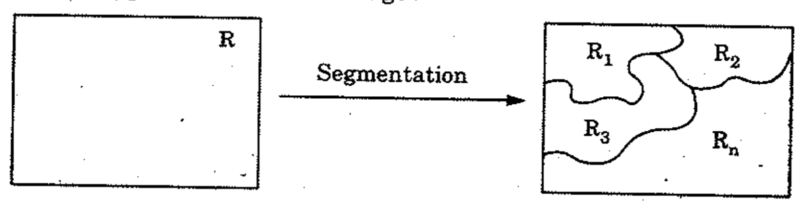


Fig. 4.3.1.

4. We can view segmentation as a process that partitions R into sub-regions $R_1, R_2, ... R_n$ such that

a. $\bigcup_{i=0}^{n} R_i = R$

b. R_i is a connected region, $i = 1, 2, \ldots, n$

c. $R_i \cap R_j = \phi$ for all i and j; $i \neq j$

d. $P(R_i)$ = True for $i = 1, 2, \ldots, n$

e. $P(R_i \cup R_j)$ = False for $i \neq j$

where $P(R_i)$ is the logical predicate over the points in set R and ϕ is the null set.

Que 4.4. Explain intensity transformations in details. What would happen to the dynamic range of an image if all the slopes in the contrast stretched algorithm (l, m, n) are less than 1 ? Answer using illustration.

AKTU 2017-18, Marks 10

Answer

Intensity transformation :

a. In intensity transformation technique, we find out the contribution made by each bit to the final image.

b. An image is defined as say a 256 × 256 × 8 image.

c. In this, 256 × 256 is the number of pixels present in the image and 8 is the number of bits required to represent each pixel. 8-bits simply mean 2^8 or 256 gray levels.

d. Now each pixel will be represented by 8-bits. For example, black is represented as 00000000 and white is represented as 11111111 and between them, 254 gray levels are accommodated.

Dynamic range of an image :

1. The idea behind contrast stretching is to increase the contrast of the images by making the dark portions darker and the bright portions brighter.
2. Fig. 4.4.1 shows the transformation used to achieve contrast stretching.
3. In Fig. 4.4.1, the dotted line indicates the identity transformation and the solid line is the contrast stretching transformation.
4. The formulation of the contrast stretching algorithm is given as :

$$s = \begin{cases} l.r & 0 \le r < a \\ m.(r-a)+v & a \le r < b \\ n.(r-b)+w & b \le r < L-1 \end{cases}$$

where l, m and n are the slopes. It is clear from the Fig. 4.4.1 that l and n are less than one while m is greater than one.

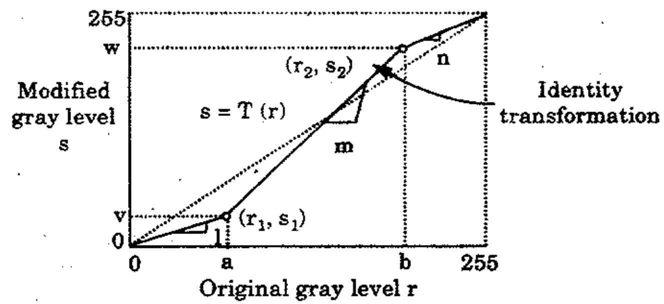


Fig. 4.4.1.

5. As slope < 1, it will result in measurement inconsistencies as well as overly optimistic measurements. No meaningful image will be visible.
6. The contrast stretching transformation increases the dynamic range of the modified image.

Que 4.5. Explain the Hough Transforms to join the points. And also explain the problem of HT with their solutions. Given the four points in the x-y plane with the following coordinates (1,1), (2,2), (3,3), (4,4). Use Hough Transform to join these points.

AKTU 2017-18, Marks 10

Answer

Hough Transform :

1. The Hough transform is a technique which can be used to isolate features of a particular shape within an image.
2. It requires that the desired features should be specified in some parametric form, the classical Hough transform is used for the detection of regular curves such as lines, circles, ellipses, etc.
3. A generalized Hough transform can be employed in applications where a simple analytic description of a feature is not possible.
4. Despite its domain restrictions, the classical Hough transform retains many applications, as most manufactured parts (and many anatomical parts investigated in medical imagery) contain feature boundaries which can be described by regular curves.
5. The main advantage of the Hough transform technique is that it is tolerant of gaps in feature boundary descriptions and is relatively unaffected by image noise.
6. The Hough technique is particularly useful for computing a global description of a feature(s) (where the number of solution classes need not be known a priori), given (possibly noisy) local measurements.

Problem of Hough Transform (HT) with solution :

1. The Hough transform fails when image space is transformed in a parameter space.
2. Generalized Hough Transform is an efficient implementation of match template within the presence of Gaussian.
3. For its robustness, this approach has widely accepted owing to high cost of computation and high storage space requirements.
4. One of the key drawbacks of template matching techniques is their high computational cost.
5. To overcome this problem, active contour models is proposed.
6. Contour model emphases on initialization invariant edge based method that brings huge freedom in contour initialization.

Numerical :

1. Four points form a straight line. But we come to this conclusion only because our visual system is very complex and it helps us to visualize a line joining these points.

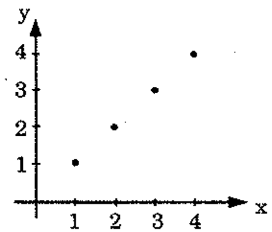


Fig. 4.5.1.

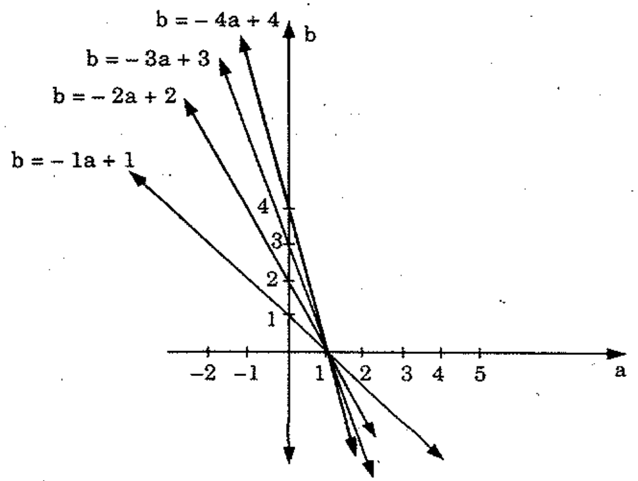


Fig. 4.5.2.

2. This example can be solved on a graph paper to get accurate results.
3. We use the equation $y = ax + b$
4. Changing the coordinates we get

$$b = -ax + y$$

x	y		
1	1	→ b = – a + 1	...(1)
2	2	→ b = – 2a + 2	...(2)
3	3	→ b = – 3a + 3	...(3)
4	4	→ b = – 4a + 4	...(4)

5. Using eq. (1), (2), (3) and (4) we draw four lines in the parameter space ab, each representing a point from the xy-plane.
6. From the graph, we get a single intersection point which has coordinates (1, 0) *i.e.*, $a = 1$, $b = 0$.
7. Taking these values of a and b, we draw a line using the equation :

$$y = ax + b$$

i.e., $$y = 1 \times x + b$$

i.e., $$y = x$$

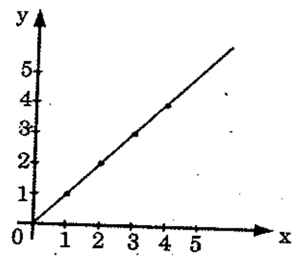


Fig. 4.5.3.

This line passes through points (1, 1) (2, 2), (3, 3), (4, 4).

Que 4.6. **Write short note on edge detection and line detection algorithm.**

AKTU 2017-18, Marks 10

Answer

Edge detection algorithm :

1. Edge detection is an image processing technique used for finding the boundaries of objects within images.
2. It works by detecting discontinuities in brightness.
3. Edge detection is used for image segmentation and data extraction in areas such as image processing, computer vision, and machine vision.

4. Common edge detection algorithms include Sobel, Canny, Prewitt, Roberts, and fuzzy logic methods.
5. Edge detection algorithm is used for image segmentation and data extraction in image processing, computer vision.

Line detection algorithm :

1. In image processing, line detection is an algorithm that takes a collection of n edge points and finds all the lines on which these edge points lies.
2. The most popular line detectors are the Hough transform and convolution based techniques.
3. The Hough transform can be used to detect lines and the output is a parametric description of the lines in an image.
4. For example $\rho = r \cos(\theta) + c \sin(\theta)$. If there is a line in a row and column based image space, it can be defined ρ, the distance from the origin to the line along a perpendicular to the line, and θ, the angle of the perpendicular projection from the origin to the line measured in degrees clockwise from the positive row axis.
5. In a convolution based technique, the line detector operator consists of a convolution masks tuned to detect the presence of lines of a particular width n and θ orientation.

Que 4.7. **Explain edge detection and edge linking. Also write the difference between edge detection and edge linking.**

AKTU 2018-19, Marks 10

Answer

Edge detection : Refer Q. 4.6, Page 4–8A, Unit-4.

Edge linking :

1. Edge detection is always followed by edge linking.
2. Edge detectors yield pixels in an image that lie on edges.
3. The next step is to try to collect these pixels together into a set of edges.
4. Thus, our aim is to replace many points on edges with a few edges themselves.
5. The practical problem may be much more difficult than the idealised case :
   a. Small pieces of edges may be missing.
   b. Small edge segments may appear to be present due to noise where there is no real edge, etc.
6. Edge linking methods can be classified into two categories :
   a. **Local edge linkers :** In local edge linkers, edge points are grouped to form edges by considering each point's relationship to any neighbouring edge points.

   b. **Global edge linkers :** In global edge linkers, where all edge points in the image plane are considered at the same time and sets of edge points are sought according to some similarity constraint, such as points which share the same edge equation.

Difference :

Edge detection : Edge detection is the process of identifying points in a digital image at which the image brightness changes sharply or has discontinuities.

Edge linking : Edge linking is the process of forming an ordered list of edges from an unordered list.

Que 4.8. **Explain :**

i. **Stereo imaging**
ii. **Multi-level thresholding**
iii. **Image registration**

AKTU 2018-19, Marks 10

OR

Write short note on image registration.

AKTU 2017-18, Marks 05

Answer

i. **Stereo imaging :**

1. Stereo imaging refers to the aspect of sound recording and reproduction concerning the perceived spatial locations of the sound source(s), both laterally and in depth.
2. An image is considered to be good if the location of the performers can be clearly located, the image is considered to be poor if the location of the performers is difficult to locate.
3. We saw in perspective transformation that when we map a 3D scene onto a 2D image plane, we lose the z-axis information.
4. This z-axis represents depth. This depth information can be obtained using stereo imaging.
5. A stereo system has two speakers as opposed to a mono system which has only one speaker.
6. The stereo system gives us the location from where sound is coming (provided it is recorded properly).
7. When this concept is applied to imaging, we get stereo imaging.
8. Because we possess a pair of eyes, two images of the same scene are captured.
9. These two images are processed by the brain and the depth information is extracted.

10. Hence, to compute the depth information, we need two sensors (cameras) placed at a particular distance capturing the same scene.

ii. Multi-level thresholding :

1. Multi-level thresholding is a process that segments a gray level image into several distinct regions.
2. This technique determines more than one threshold for the given image and segments the image into certain brightness regions, which correspond to one background and several objects.

iii. Image registration :

1. Image registration is a process of overlaying two or more images of the same scene taken at different times or at the same time by different sensors.
2. Image registration is a important step in all image analysis tasks where the final information is gained from combining various data.
3. Example of image registration is image fusion and remote sensing imaging.
4. Images of the earth are taken at different instances of time via satellites.
5. These images are used to study various effects like deforestation, rise in sea water levels etc.
6. Image registration basically involves finding a correspondence *i.e.*, matching of identical shapes in the related image pair.
7. This matching between image pair is achieved using geometric transformations.

Que 4.9. **Let A be an image and B a structuring element, given as follows. Find $A \ominus B$ and $A \oplus B$. Note : X denotes the origin, which is not part of the structuring element.**

	1	1	1		
	1		1		
	1		1	1	
				1	1
X					

1	X	1

AKTU 2014-15, Marks 05

Answer

Given,

A =

	1	1	1		
	1		1		
	1		1	1	
				1	1
X					

B =

1	X	1

A ⊕ B =

1	1	1	1	1	
1	1	1	1	1	
1	1	1	1	1	1
			1	1	1

A ⊖ B =

		1			
		1			
		1			

Fig. 4.9.1.

Que 4.10. **Thin the following image. Show the image after each step.**

AKTU 2014-15, Marks 05

Answer

1	1	0	0	0	0	0	1	1	1
1	1	1	1	1	1	1	1	0	0
1	1	1	1	1	1	1	1	0	0
1	1	1	1	1	1	1	1	0	0
1	1	1	0	0	1	1	1	0	0

Fig. 4.10.1.

$$A \otimes B = A \cup (A \otimes B) = A \cap (A \otimes B)^C$$

$$\{B\} = \{B^1, B^2, B^3, \ldots B^n\}$$

$$A \otimes \{B\} = (\ldots(A \otimes B^1) \otimes B^2) \otimes B^3)\ldots)\otimes B^n)$$

X	1	X
1	1	1

B^1

X		
1	1	
1	1	X

B^2

1	X	
1	1	
1	X	

B^3

1	1	X
1	1	
X		

B^4

1	1	1
X	1	X

B^5

X	1	1
	1	1
		X

B^6

	X	1
	1	1
	X	1

B^7

		X
	1	1
X	1	1

B^8

1	0	0	0	0	0	0	1	1	1
1	1	1	1	1	1	1	1	0	0
1	1	1	1	1	1	1	1	0	0
1	1	1	1	1	1	1	1	0	0
1	1	1	0	0	1	1	1	0	0

$(A \otimes B^2)$

1	0	0	0	0	0	0	1	1	1
1	1	1	1	1	1	1	1	0	0
1	1	1	1	1	1	1	0	0	0
1	1	1	1	1	1	1	1	0	0
1	1	1	0	0	1	1	1	0	0

$(A \otimes B^3)$

1	0	0	0	0	0	0	1	1	1
1	1	1	1	1	1	1	1	0	0
1	1	1	1	1	1	1	1	0	0
1	1	1	1	1	1	1	1	0	0
1	1	1	0	0	1	1	1	0	0

$(A \otimes B^2)$

1	0	0	0	0	0	0	1	1	1
1	1	1	1	1	1	1	1	0	0
1	1	1	1	1	1	1	0	0	0
1	1	1	1	1	1	1	1	0	0
1	1	1	0	0	1	1	1	0	0

$(A \otimes B^3)$

1	0	0	0	0	0	0	1	1	1
1	1	1	1	1	1	1	0	0	0
1	1	1	1	1	1	0	0	0	0
1	1	1	0	0	1	0	0	0	0
1	0	0	0	0	1	1	0	0	0

$(A \otimes B^6)$

1	0	0	0	0	0	0	1	1	1
1	1	1	1	1	1	1	1	0	0
0	1	1	1	1	1	1	0	0	0
1	1	1	0	0	1	0	0	0	0
1	0	0	0	0	1	1	0	0	0

$(A \otimes B^7)$

1	0	0	0	0	0	0	1	1	1
1	1	0	0	0	0	1	1	0	0
0	1	1	1	1	1	1	0	0	0
1	1	1	0	0	0	1	0	0	0
1	0	0	0	0	0	1	1	0	0

$(A \otimes B^{1,2,3})$

1	0	0	0	0	0	?	1	1	1
1	1	0	0	0	0	1	1	0	0
0	1	1	1	1	1	1	0	0	0
1	1	0	0	0	0	1	0	0	0
1	0	0	0	0	0	1	1	?	0

$(A \otimes B^{1,2,3,4,5,6,7,8})$

Fig. 4.10.2.

Que 4.11. **Extract the connected component from the following image :**

0	0	0	0	0	0	0	0	0
0	0	1	1	1	0	0	0	0
0	0	1	1	0	1	1	0	0
0	1	1	1	1	1	1	1	0
0	1	1	1	0	1	1	0	0
0	0	0	0	0	0	0	0	0

AKTU 2014-15, Marks 05

Answer

$A \ominus B =$

0	0	0	0	0	0	0	0	0
0	0	1	1	1	0	0	0	0
0	0	1	1	0	1	1	0	0
0	1	1	1	1	1	1	1	0
0	1	1	1	0	1	1	0	0
0	0	0	0	0	0	0	0	0

Fig. 4.11.1.

$$X_k = (X_{k-1} \oplus B) \cap A$$
$$k = 1, 2, 3, \ldots$$
$$X_0 = p$$

if $X_k = X_{k-1}$ algorithm has converged $Y = X_k$

1	1	1
1	1	1
1	1	1

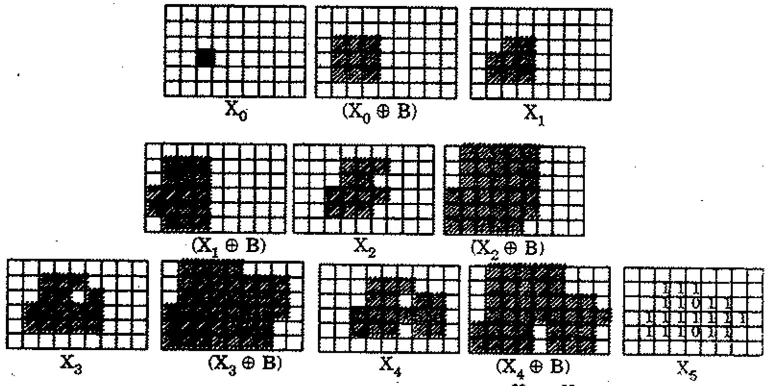


Fig. 4.11.2.

Que 4.12. **Explain the procedure of region filling with an example.**

AKTU 2014-15, Marks 05

Answer

Region filling is a morphological algorithm in image processing, which deals with filling the region in the image with some colors. The image region can be selected in two ways :

1. **Interior region :**
   a. Interior regions are defined by assigning the same value to all the pixels inside that region.
   b. The algorithms used to change the values of all pixels in the interior regions to new values are flood-fill algorithms.
2. **Boundary region :**
   a. Boundary regions are defined by assigning the same value to all the pixels on the boundary of the region.
   b. Boundary region pixels and the interior region pixels should not have the same values.
   c. The algorithms used to change the value of all pixels in boundary regions to new value are boundary-fill algorithms.

Example : Refer Q. 4.11, Page 4–13A, Unit-4.

Que 4.13. **Explain the process of image segmentation using region growing.**

AKTU 2014-15, Marks 05

Answer

1. The region-based segmentation is partitioning of an image into similar/homogenous areas of connected pixels through the application of homogeneity/similarity criteria among candidate sets of pixels. Each of the pixels in a region is similar with respect to some characteristics or computed property such as colour, intensity and/or texture.
2. Region growing is a simple region-based image segmentation method. It is also classified as pixel-based image segmentation method since it involve the selection of initial seed points.
3. This approach to segmentation examines neighbouring pixels of initial seed points and determines whether the pixel neighbours should be added to the region.
4. The process is iterated on, in the same manner as general data clustering algorithms.

Que 4.14. **Describe the technique of thresholding for image segmentation.** **AKTU 2014-15, Marks 05**

Answer

Technique of thresholding for image segmentation :

a. **Global thresholding :**
   - i. In global thresholding, we partition the image histogram by using a single thresholding function T.
   - ii. In this case, $A(x, y)$ is the entire image.
   - iii. Since, we have a single threshold T for the entire image, T is called the global threshold.
   - iv. Segmentation is achieved by scanning the entire image pixel by pixel and labelling each pixel as an object or a background pixel depending on the value of that pixel.
   - v. Global thresholding works only if the histogram of the image is partitioned properly.
   - vi. Steps involved in performing global thresholding :
     1. Read the given image.
     2. Plot the histogram of the given image.
     3. Based on the histogram, choose the threshold T.
     4. Using this value of T, segment the image into objects and background.

b. **Local thresholding :**
   - i. There is no way we can segment an image based on global thresholding and separate the object from its background.

   - ii. In such case, we work with local thresholds :

$$T = T\{A(x, y), f(x, y)\}$$

   - iii. In this, T is dependent upon a neighbourhood property of the pixel as well as its gray level.
   - iv. We divide the entire image into sub-images of size A and then use a separate threshold to segment each sub-image A.

PART-2

Morphological Processing, Erosion and Dilation, Segmentation by Morphological Watersheds, Basic Concepts, Dam Construction, Watershed Segmentation Algorithm.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 4.15. **Write short note on morphology processing.**

Answer

1. Image morphology is the study of shapes of the objects present in the image and extraction of image features.
2. Image features are necessary for object recognition.
3. The theory of mathematical morphology is based on set theory. We can visualize the binary image as a set.
4. Then set theory can be applied to the sample set extracted from images.
5. Morphological operators take binary image and a mask known as a structuring element as inputs.
6. Then the set operators such as intersection, union, inclusion and complement can be applied to the images.
7. In a binary image, the set is a collection of coordinates where the pixel may be black or white.

Que 4.16. **Prove that opening and closing are dual transformations.** **AKTU 2014-15, Marks 10**

Answer

Opening and closing are duals of each other with respect to set complementation and reflection that is,

$(A \bullet B)^c = (A^c \circ \hat{B})$ and $(A \circ B)^c = (A^c \bullet \hat{B})$

$$A = \begin{array}{|c|c|c|c|c|c|}\hline 0&0&0&0&0&0\\\hline 0&1&1&1&0&0\\\hline 0&1&1&1&0&0\\\hline 0&0&1&1&1&0\\\hline 0&0&1&1&1&0\\\hline 0&0&0&0&0&0\\\hline\end{array} \quad B = \begin{array}{|c|c|c|}\hline 0&1&0\\\hline 1&1&1\\\hline 0&1&0\\\hline\end{array}$$

$$X_1 = (A \ominus B) \begin{array}{|c|c|c|c|c|c|}\hline 0&0&0&0&0&0\\\hline 0&0&0&0&0&0\\\hline 0&0&0&0&0&0\\\hline 0&0&0&0&0&0\\\hline 0&0&0&0&0&0\\\hline 0&0&0&0&0&0\\\hline\end{array}$$

$$(X_1 \oplus B) = \begin{array}{|c|c|c|c|c|c|}\hline 0&0&0&0&0&0\\\hline 0&0&0&0&0&0\\\hline 0&0&0&0&0&0\\\hline 0&0&0&0&0&0\\\hline 0&0&0&0&0&0\\\hline 0&0&0&0&0&0\\\hline\end{array}$$

$$(A \oplus B)^c = \begin{array}{|c|c|c|c|c|c|}\hline 1&1&1&1&1&1\\\hline 1&1&1&1&1&1\\\hline 1&1&1&1&1&1\\\hline 1&1&1&1&1&1\\\hline 1&1&1&1&1&1\\\hline 1&1&1&1&1&1\\\hline\end{array}$$

$$A^c = \begin{array}{|c|c|c|c|c|c|}\hline 1&1&1&1&1&1\\\hline 1&0&0&0&1&1\\\hline 1&0&0&0&1&1\\\hline 1&1&0&0&0&1\\\hline 1&1&0&0&0&1\\\hline 1&1&1&1&1&1\\\hline\end{array} \quad \hat{B} = \begin{array}{|c|c|c|}\hline 0&1&0\\\hline 1&1&1\\\hline 0&1&0\\\hline\end{array}$$

$(A^c \circ \hat{B}) = (A^c \oplus \hat{B}) \ominus A\hat{B}$

$$A^c \oplus \hat{B} = \begin{array}{|c|c|c|c|c|c|}\hline 1&1&1&1&1&1\\\hline 1&0&1&1&1&1\\\hline 1&1&1&1&1&1\\\hline 1&1&1&1&1&1\\\hline 1&1&1&1&1&1\\\hline 1&1&1&1&1&1\\\hline\end{array}$$

$$(A^c \bullet \hat{B}) = \begin{array}{|c|c|c|c|c|c|}\hline 1&1&1&1&1&1\\\hline 1&0&1&1&1&1\\\hline 1&1&1&1&1&1\\\hline 1&1&1&1&1&1\\\hline 1&1&1&1&1&1\\\hline 1&1&1&1&1&1\\\hline\end{array}$$

Que 4.17. **Prove that rotation and translation are not commutative operations.**

AKTU 2014-15, Marks 10

Answer

1. We translate a point in the xy-plane to new place by adding a vector (h, k).
2. It is not difficult to see that between a point (x, y) and its new place (x', y'), we have $x' = x + h$ and $y' = y + k$.
3. Let us use a form similar to the homogeneous coordinates.
4. That is, a point becomes a column vector whose third component is 1.
5. Thus, point (x, y) becomes :

$$\begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

6. Then, the relationship between (x, y) and (x', y') can be put into a matrix form like :

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & h \\ 0 & 1 & k \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -h \\ 0 & 1 & -k \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix}$$

7. Therefore, if a line has an equation $Ax + By + C = 0$, after plugging the formula for x and y, the line has a new equation $Ax' + By' + (-Ah - Bk + C) = 0$.
8. If a point (x, y) is rotated an angle a about the coordinate origin to become a new point (x', y'), the relationships can be described as follows :

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos a & -\sin a & 0 \\ \sin a & \cos a & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} \cos a & \sin a & 0 \\ -\sin a & \cos a & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix}$$

9. Thus, rotating a line $Ax + By + C = 0$ about the origin with an angle a brings it to a new equation :

$(A \cos a - B \sin a) x' + (A \sin a + B \cos a) y' + C = 0$

10. Translations and rotations can be combined into a single equation as :

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos a & -\sin a & h \\ \sin a & \cos a & k \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} \cos a & \sin a & -h \cos a - k \sin a \\ -\sin a & \cos a & h \sin a - k \cos a \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix}$$

11. This means that rotates the point (x, y) at an angle a about the origin and translates the rotated result in the direction of (h, k).
12. However, if translation (h, k) is applied first followed by a rotation of angle a (about the origin), we will have the following :

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos a & -\sin a & h \cos a - k \sin a \\ \sin a & \cos a & h \sin a + k \cos a \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} \cos a & \sin a & -h \\ -\sin a & \cos a & -k \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix}$$

13. Therefore, rotation and translation are not commutative.

Que 4.18. What is shearing ? Give the transformation matrix and its inverse to carry out shearing in both x-and y-directions with shearing factors 10 and 30. **AKTU 2014-15, Marks 10**

Answer

Shearing :

1. In plane geometry, a shear mapping is a linear map that displaces each point in fixed direction, by an amount proportional to its signed distance from the line that is parallel to that direction and goes through the origin.
2. This type of mapping is called shear transformation, or shearing.

x-direction :

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & s_1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \quad ...(4.18.1)$$

$$\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & -s_1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix}$$

Using equation (4.18.1) we get,

$$x' = x + s_1 y$$

y-direction :

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ s_2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \quad ...(4.18.2)$$

$$\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -s_2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix}$$

Using equation (4.18.2) we get,

$$y' = s_2 x + y$$

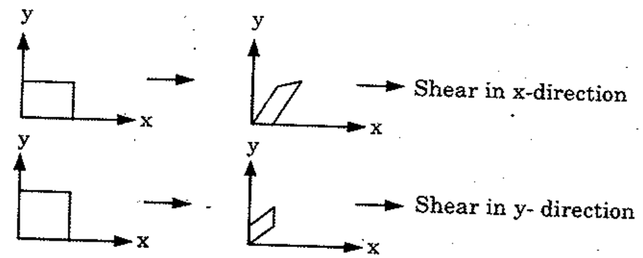


Fig. 4.18.2.

Numerical : Transformation matrix in the x-direction :

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 10 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

And its inverse is :

$$\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & -10 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix}$$

Similarly, transformation matrix in the y-direction is :

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 30 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

And its inverse is :

$$\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -30 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix}$$

Que 4.19. **Find a matrix to perform the following transformations to an object :**

i. Scale in the x-direction using a scale factor 10.

ii. Followed by a rotation about z-axis 30 degree.

AKTU 2014-15, Marks 10

Answer

i. Let the original point be (x, y) and transformed point be (x', y')

The scaling matrix can be :

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 10 & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

ii. Rotation about z-axis 30 degree :

$$X = x;$$
$$Y = y^* \cos(30) - z^* \sin(30);$$
$$Z = y^* \sin(30) + z^* \cos(30);$$

Que 4.20. **Given the image A :**

0	0	0	0	0	0
0	0	1	1	0	0
0	1	1	1	1	0
0	0	1	1	0	0
0	0	0	0	0	0

And structuring element B :

1
1
1

Compute

i. A dilated by B

ii. A^c eroded by B

AKTU 2015-16, Marks 10

Answer

i. **A dilated by B :**

$A \oplus B =$

0	0	0	0	0	0
1	1	1	1	1	1
1	1	1	1	1	1
1	1	1	1	1	1
0	0	0	0	0	0

ii. **A^c eroded by B :**

$A^c =$

1	1	1	1	1	1
1	1	0	0	1	1
1	0	0	0	0	1
1	1	0	0	1	1
1	1	1	1	1	1

$A^c \ominus B =$

1	1	1	1	1	1
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
1	1	1	1	1	1

Que 4.21. **Explain convex hull with the help of an example.**

AKTU 2015-16, Marks 10

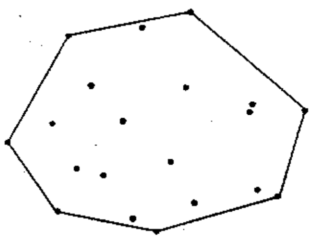
OR

Explain the following in details :
i. **Stereo imaging**
ii. **Region filling**
iii. **Convex hull**

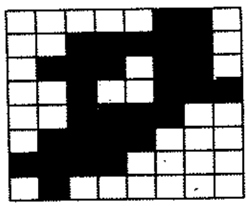
AKTU 2016-17, Marks 10

Answer

i. **Stereo imaging :** Refer Q. 4.8, Page 4–10A, Unit-4.
ii. **Region filling :** Refer Q. 4.12, Page 4–14A, Unit-4.
iii. **Convex hull :**
1. Convex hull of a set Q of points, denoted by CH(Q) is the smallest convex polygon P for which each points in Q is either on the boundary of P or in its interior.



2. The convex hull method uses the Hit and Miss transformation.
3. For example, let the input image and the structuring element is shown as :



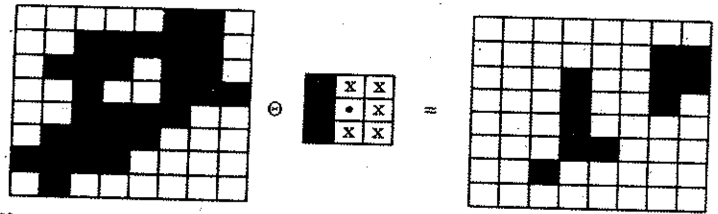
The step-by-step approach to the determination of the convex hull of the input image is given as :

Step 1 : The value of Y_1^1 is determined using $Y_1^1 = HM(Y_0^1, B^1) \cup X$.

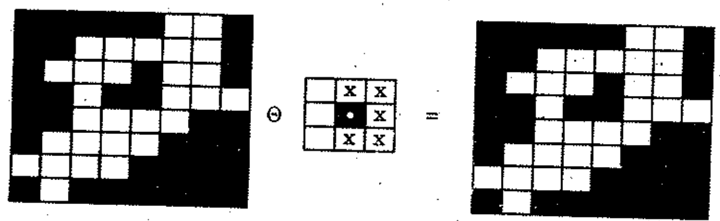
Step 2 : To find $HM(Y_0^1, B^1)$

$$HM(Y_0^1, B^1) = (Y_0^1 \ominus B^1) \cap ((Y_0^1)^c \ominus (W - B^1))$$

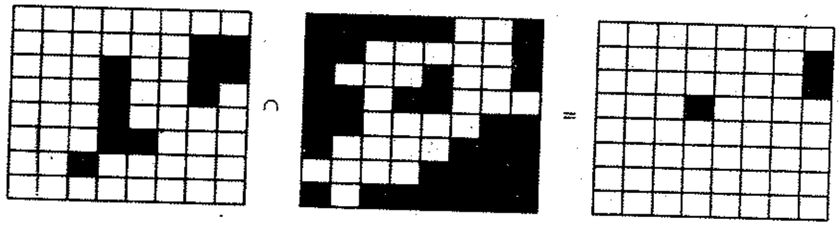
We find $Y_0^1 \ominus B^1$ as :



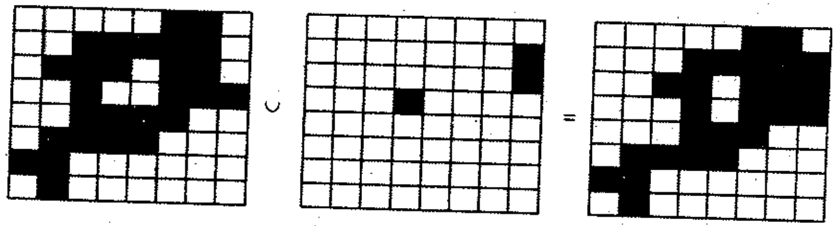
The next step is to find $(Y_0^1)^c \ominus (W - B^1)$. The result of $(Y_0^1)^c \ominus (W - B^1)$ is shown as :



Then we find the intersection of above two results as :



Step 3 : The union of the input image with the result obtained in step 2 will give the convex hull of the input image which is :



Que 4.22. **State and explain various approaches used for edge detection.**

AKTU 2015-16, Marks 10

Answer

Various approaches used for edge detection are :

a. Pattern fit algorithm :

1. The approach of pattern fit algorithm towards edge detection is to reconstruct the continuous domain from the discrete domain.
2. This ensures the detection of edges with more precision at the pixel level.
3. However, construction of continuous domain from the discrete domain is a difficult process and so the reconstruction is attempted in a smaller neighbourhood.
4. The logic is to model the image as analytical functions such as biquadratic or bicubic equations.
5. The individual functions are called facets.
6. The aim is to fit a pattern over a neighbourhood of pixels where the edge strength is to be calculated.
7. The properties of the edge points are calculated based on the parameters of the analytical functions.
8. A very simple technique is given by Hueckel to model a step edge.
9. A step edge model represents two surfaces with intensities α and β.
10. The step edge model is described as,

$$h(x, y) = \begin{cases} \alpha & \text{if } x \sin\theta \geq y \cos\theta \\ \beta & \text{otherwise} \end{cases}$$

θ is the edge orientation.

b. Morphological edge detection :

1. In morphology, gray-scale edge detection is obtained by taking the difference between an image and its erosion/dilation image by a structuring element.
2. This may be preceded by preprocessing or followed by postprocessing or both.
3. The difference image is an image of edge strength.
4. Most popularly used structuring element for edge detection is rod shaped with flat top.
5. To define the gray-scale rod structuring element having flat top and rod shaped domain, let (0, 0) denotes the centre of local neighbourhood and a point by (r, c) at an offset of r along row direction and c along column direction.
6. Then the domain of rod structuring element, say, of radius 1 (using city-block distance) is denoted by D_{rod1} and is defined as

$$D_{rod1} = \{(0, -1), (0, 1), (0, 0) (-1, 0), (1, 0)\}$$

and its value is a mapping $b : D_{rod1} \rightarrow \{0...255\}$. Since rod is flat on the top, the gray-scale value of $b(r, c) = 0 \;\forall\; (r, c) \in D_{rod1}$.

c. First order derivatives :

1. The first order derivative of an image containing gray value pixels must fulfill the following conditions :
   a. It must be zero in flat segments (area of constant gray level values).
   b. It must be non-zero at the beginning of a gray level or ramp.
   c. It must be non-zero along the ramp (constant change in grey values).
2. The first order derivative of a one-dimensional function $f(x)$ can be obtained by using :

$$df / dx = f(x + 1) - f(x)$$

3. The first order derivative is given by estimating the finite difference :

$$\frac{\partial f}{\partial x} = \lim_{h \to 0} \frac{f(x + h, y) - f(x, y)}{h}$$

$$\frac{\partial f}{\partial y} = \lim_{h \to 0} \frac{f(x, y + h) - f(x, y)}{h}$$

$$\frac{\partial f}{\partial x} = \frac{f(x + h, y) - f(x, y)}{h_x} f(x + 1, y) - f(x, y), (h_x = 1)$$

$$\frac{\partial f}{\partial y} = \frac{f(x, y + h) - f(x, y)}{h_y} f(x, y + 1) - f(x, y), (h_y = 1)$$

4. Most edge-detecting operators can be thought of as gradient calculators.
5. Because the gradient is a continuous function concept and the input signal is a finite signal (image), the gradient computations have to be approximated.

d. Second order derivatives :

1. It was shown that first order derivative does enhance the edges of the image.

We know,

$$\nabla F = \frac{\partial f}{\partial x} + \frac{\partial f}{\partial y}$$

$$\therefore \quad \frac{\partial f}{\partial x} = f(x + 1, y) - f(x, y)$$

and $$\frac{\partial f}{\partial y} = f(x, y + 1) - f(x, y)$$

2. The second order derivative is given by,

$$\nabla^2 F = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

where $\frac{\partial^2 f}{\partial x^2} = f(x+1, y) + f(x-1, y) - 2f(x, y)$

and $\frac{\partial^2 f}{\partial y^2} = f(x, y+1) + f(x, y-1) - 2f(x, y)$

$$\nabla^2 F = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

$$\therefore \quad |\nabla^2 F| = [f(x+1, y) + f(x-1, y) + f(x, y+1) + f(x, y-1) - 4f(x, y)]$$

3. Considering the 3 × 3 neighbourhood

	$y-1$	y	$y+1$
$x-1$	Z_1	Z_2	Z_3
x	Z_4	Z_5	Z_6
$x+1$	Z_7	Z_8	Z_9

4. This equation in the discrete form reduces to
$$|\nabla^2 F| = [Z_8 + Z_2 + Z_6 + Z_4 - 4Z_5]$$
5. This equation can be implemented using a mask shown as :

0	1	0
1	–4	1
0	1	0

6. This is known as the Laplacian operator.

e. Canny edge detection :

1. Canny operator is a first derivative edge detector coupled with noise cleaning. Like in the LoG, a Gaussian function is used to smoothen the noise.
2. In the Canny edge detector, we first smoothen the image using a Gaussian low pass filter and then take the first derivative.
3. Compare Fig. 4.22.1(c) and Fig. 4.22.1(e) and note the similarity between the shape of the first derivative of the Gaussian and the second derivative of the ramp edge.

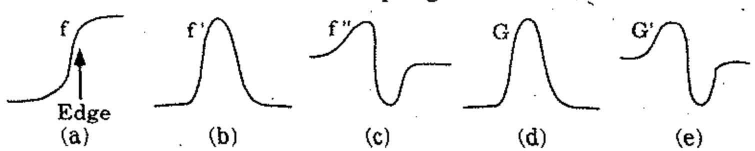


Fig. 4.22.1. (a) A ramp edge, (b) First derivative of ramp edge, (c) Second derivative of ramp edge, (d) Gaussian function, (e) First derivative of Gaussian function.

4. Hence, the derivative of the bell shape of the Gaussian function approximates the second derivative or zero-crossing operator.

Que 4.23. **Write the procedures for boundary extraction and region filling. Mention at least one real life application of both. What is the result of applying successive opening on the same set with the same structuring element ?**

AKTU 2015-16, Marks 10

OR

Write short note on region extraction. **AKTU 2017-18, Marks 05**

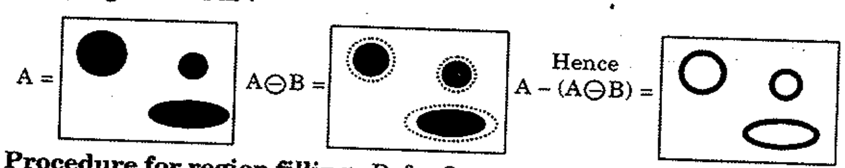
Answer

Procedures for boundary extraction :

1. Given a binary image it is fairly simple to extract the boundary of the image.
2. If A is the image and B is the structuring element, then boundary extraction can be achieved using the formula,
$$\text{Boundary } (A) = A - (A \ominus B) \quad ...(4.23.1)$$
3. Subtracting the erode image of A from the original image.
4. This statement makes sense as erosion reduces the size of the image.

If $B = \begin{array}{|c|c|c|}\hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline \end{array}$

5. Then $A \ominus B$ would be the same as A except one pixel less from all the sides.
6. Hence, $A - (A \ominus B)$ would give us this one pixel difference. Consider the image shown as :



Procedure for region filling : Refer Q. 4.12, Page, Unit-4.

Application for boundary extraction : Boundary extraction is used for isolating the boundary pixels in a real image.

Application for region filling : Region filling is used to increase the sharpness of image.

Result : After the opening has been carried out, the all new boundaries of foreground regions will be such that the structuring element fits inside them, and so further openings with the same element have no effect. The property is known as idempotence.

Que 4.24. **Write short notes on :**

a. Chain code
b. Skeletons/MAT
c. Hough transform for boundary shape detection

AKTU 2015-16, Marks 10

Answer

a. Chain code :

1. Chain code is used to represent a boundary by a connected sequence of straight-line segments.
2. This representation is based on 4-connectivity or 8-connectivity of the segments.
3. The chain code works best with binary images and is a concise way of representing a shape contour.
4. The chain code direction convention is shown in Fig. 4.24.1.

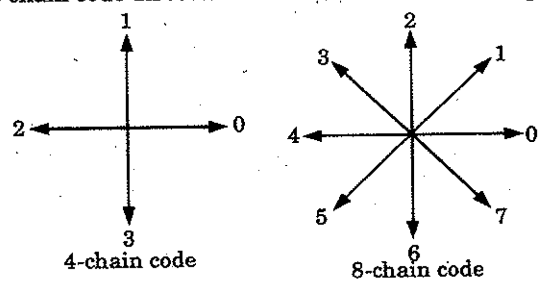


Fig. 4.24.1.

5. As an edge is traced from its beginning point to the end point, the direction that must be taken to move from one pixel to the next is given by the number present in either the 4-chain code or the 8-chain code.
6. An edge can be completely described in terms of its starting coordinate and its sequence of chain code descriptors.
7. Of the two chain codes, the 4-chain code is easier requiring only four different code values.

b. Skeletonization (skeleton extraction) :

1. It is a way to reduce binary objects to thin strokes that retain important structural information about the shapes of the original objects.
2. The skeleton of A can be expressed in terms of erosions and opening as follows :

$$S(A) = \bigcup_{k=0}^{K} S_k(A)$$

with $\quad S_k(A) = (A \ominus kB) - (A \ominus kB) \circ B$

where B is a structuring element, and $(A \ominus kB)$ indicates k successive erosions of A :

$$(A \ominus kB) = (\ldots (A \ominus B) \ominus B) \ominus \ldots) \ominus B$$

c. Hough transform for boundary shape detection : Refer Q. 4.5, Page 4–6A, Unit-4.

Que 4.25. Explain opening and closing operation for gray-scale image processing.

AKTU 2017-18, Marks 10

OR

Explain the following morphological operations :
i. Opening
ii. Closing
iii. Region filling

AKTU 2018-19, Marks 10

Answer

i. Opening operation :

1. Morphological opening of an image is basically erosion followed by dilation, using the same structuring element.
2. If A is the image and B is the structuring element then, opening of A by B is given as,

   OPEN$(A, B) = D(E(A)) \rightarrow D \rightarrow$ Dilation $E \rightarrow$ Erosion

   It is also written as,

$$A \circ B = (A \ominus B) \oplus B$$

3. Opening generally smoothes the contours of the image, breaks down narrow bridges and eliminates thin protrusion.
4. Thus, opening isolates objects which may be just touching one another.

ii. Closing operation :

1. Morphological closing of an image is basically dilation followed by erosion, using the structuring element.

   CLOSE $(A, B) = E(D(A))$

2. It is also written as,

$$A \bullet B = (A \oplus B) \ominus B$$

3. Closing generally tends to fuse narrow breaks and eliminates small holes. This simplifies the process of assessing the separation of particles.

iii. Region filling : Refer Q. 4.12, Page 4–14A, Unit-4.

Que 4.26. **Describe fundamental operations of morphological image processing.**

AKTU 2017-18, Marks 10

Answer

Fundamental operations of morphological image processing are :

i. Dilation operation :

1. With A and B as two sets in Z^2 (2D integer space), the dilation of A by B is defined as,

$$A \oplus B = \{Z \mid (\hat{B})_z \cap A \neq \phi\} \qquad ...(4.26.1)$$

2. In the eq. (4.26.1), $(\hat{B})_z$ simply means taking the reflection of B about its origin and shifting it by Z.
3. Hence dilation of A with B is a set of all displacements, Z, such that $(\hat{B})$ and A overlap by at least one element.
4. Flipping of B about the origin and then moving to it past image A is analogous to the convolution process.
5. In practice, flipping of B is not always done.
6. Dilation adds pixels to the boundaries of objects in an image.
7. The number of pixels added depends on the size and shape of the structuring element.
8. Based on this definition, dilation can also be defined as,

$$A \oplus B = \{Z \mid [(\hat{B})_z \cap A] \in \phi\} \qquad ...(4.26.2)$$

ii. Erosion operation :

1. For image A and structuring element B in the Z^2 (2D integer space), erosion is defined as,

$$A \ominus B = \{Z \mid [(\hat{B})_z \in A\} \qquad ...(4.26.3)$$

2. This equation indicates that erosion of A by B is the set of all points Z such that B, translated (shifted by Z), is a subset of A *i.e.*, B is entirely contained within A.
3. Erosion reduces the number of pixels from the object boundary.
4. The number of pixels removed depends on the size of the structuring element.

iii. & iv. Opening and closing operation : Refer Q. 4.25, Page 4–30A, Unit-4.

Que 4.27. **Define boundary extraction. Perform boundary extraction on image A with the help of structuring element B.**

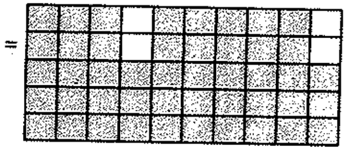
A = 

Fig. 4.27.1.

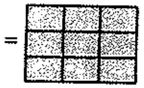
B = 

Fig. 4.27.2.

AKTU 2016-17, Marks 10

Answer

Boundary extraction : Refer Q. 4.23, Page 4–28A, Unit-4.

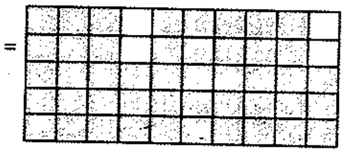
Numerical :

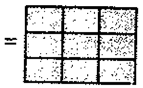
1. The boundary of a set A, denoted by $\beta(A)$, can be obtained by first eroding A by B and then performing the set difference between A and its erosion. That is,

$$\beta(A) = A - (A \ominus B) \qquad ...(4.27.1)$$

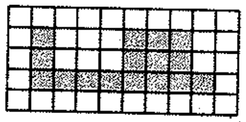
where B is a suitable structuring element.

2. Let us perform the erosion of A by B.

A = 

B = 

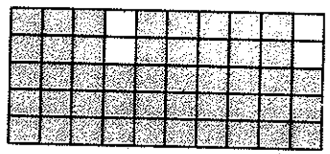
After eroding A by B we get,



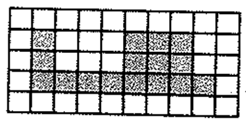
$A \ominus B$

Fig. 4.27.1.

3. Let us now perform the set difference between A and its erosion.

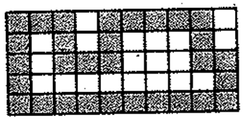


A



$A \ominus B$

We get,



β(A)

Fig. 4.27.2.

Hence, this is the boundary extracted image of given image *A*.

Que 4.28. **What is geometric transformation ? Also discuss Euclidean transformation.**

AKTU 2016-17, Marks 10

Answer

Geometric transformation :

1. A geometric transformation is any bijection of a set having some geometric structure to itself or another such set. Specifically, a geometric transformation is a function whose domain and range are sets of points.
2. Most often the domain and range of a geometric transformation are both R^2 or both R^3. Often geometric transformations are required to be 1-1 functions, so that they have inverses.

Euclidean transformations : The Euclidean transformation is either a translation, a rotation.

1. **Translation :**
   i. Suppose a point (x, y) in the xy-plane gets translated (shifted) to a new point $x'\,y'$ where $x' = x + h$, and $y' = y + k$.
   ii. This relationship between (x, y) and (x', y') can be written in a matrix form as shown :

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & h \\ 0 & 1 & k \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \qquad ...(4.28.1)$$

Similarly,
$$\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -h \\ 0 & 1 & -k \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix}$$

2. **Rotation :**
   i. If a point (x, y) is rotated by an angle θ about the origin to become a new point (x', y'), the relationship in matrix form can be written as :

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \qquad ...(4.28.2)$$

Similarly,
$$\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix}$$

   ii. Translations and rotations can be implemented using a single equation *i.e.*,

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & h \\ -\sin\theta & \cos\theta & k \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

and
$$\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta & -h\cos\theta - k\sin\theta \\ -\sin\theta & \cos\theta & h\sin\theta - \cos\theta \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix}$$

   iii. This transformation means we rotate (x, y) by an angle θ and then shift it by (h, k) *i.e.*, rotation followed by translation.
   iv. If we translate (x, y) by (h, k) first and then rotate it, the transformation obtained will be,

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & h\cos\theta - k\sin\theta \\ \sin\theta & \cos\theta & h\sin\theta + k\cos\theta \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

and
$$\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta & -h \\ -\sin\theta & \cos\theta & -k \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix}$$

   v. From eq. (4.28.1) and eq. (4.28.2) rotation and translation are not commutative.
   vi. In general, the Euclidean transformation can be written as,

$$T = \begin{bmatrix} r_{11} & r_{12} & t_1 \\ r_{21} & r_{22} & t_2 \\ 0 & 0 & 1 \end{bmatrix}$$

where the top 2×2 sub-matrix is the rotation matrix while (t_1, t_2) is the translational matrix.

Que 4.29. **How dilation and erosion is used in morphological operations ? How it is used in opening and closing operations ?**

AKTU 2016-17, Marks 10

Answer

Dilation and erosion in morphological operation :

1. Dilation operation is used to add pixels to the boundaries of objects in an image, while erosion operation is used to remove pixels from object boundaries.
2. The number of pixels added or removed from the objects in an image depends on the size and shape of the structuring element used to process the image.
3. In the morphological dilation and erosion operations, the state of any given pixel in the output image is determined by applying a rule to the corresponding pixel and its neighbours in the input image.
4. The rule used to process the pixels defines the operation as dilation or erosion.

Opening and closing in morphological operation :

1. Opening operation is used to removes small objects from the foreground (usually taken as the bright pixels) of an image, placing them in the background, while closing operation is used to remove small holes in the foreground, changing small islands of background into foreground.
2. These techniques can also be used to find specific shapes in an image.
3. Opening operation can be used to find things into which a specific structuring element can fit (edges, corners).

Que 4.30. **Explain thinning and thickening operation with suitable example.** **AKTU 2018-19, Marks 10**

Answer

Thinning operation :

1. The thinning of a set A by a structuring element B, denoted $A \otimes B$, can be defined in terms of the Hit and Miss transform :

$$A \otimes B = A - (A \circledast B)$$

$$= A \cap (A \circledast B)^c \qquad ...(4.30.1)$$

2. A useful expression for thinning A symmetrically is based on a sequence of structuring elements :

$$\{B\} = \{B^1, B^2, B^3, \ldots, B^n\} \qquad ...(4.30.2)$$

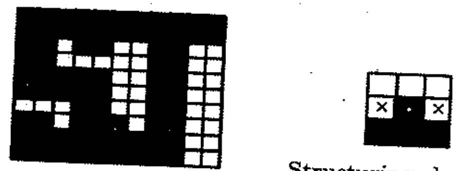
where B^i is a rotated version of B^{i-1}.

3. Using this concept, we now define thinning by a sequence of structuring elements as,

$$A \otimes \{B\} = ((\ldots((A \otimes B^1) \otimes B^2) \ldots) \otimes B^n) \qquad ...(4.30.3)$$

4. The process is to thin A by one pass with B^1, then thin the result with one pass of B^2, and so on, until A is thinned with one pass of B^n.
5. The entire process is repeated until no further changes occur.
6. Each individual thinning pass is performed using eq. 4.30.1.

For example : Apply the thinning process to the image using the structuring elements shown as :



Input image A Structuring element B^1

Fig. 4.30.1.

As we know, $A \otimes B = A - (A \otimes B)$

and $(A \otimes B^1) = (A \otimes B^1) \cap [A^c \otimes (W - B^1)]$ by Hit or Miss transformation

Step 1 :

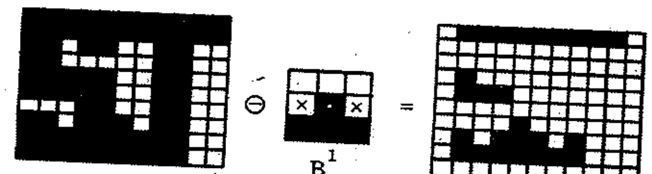


Fig. 4.30.2.

Step 2 :

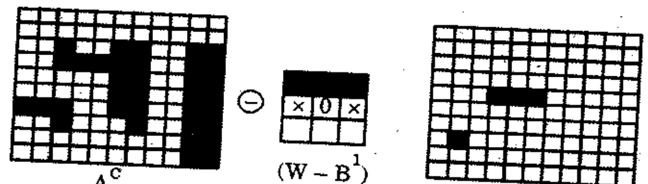


Fig. 4.30.3.

Step 3 :

1. The Hit or Miss transformation is the intersection of the above results.

2. This resultant image is subtracted from the original input image and we get the thinned original image. This is illustrated in Fig. 4.30.4.

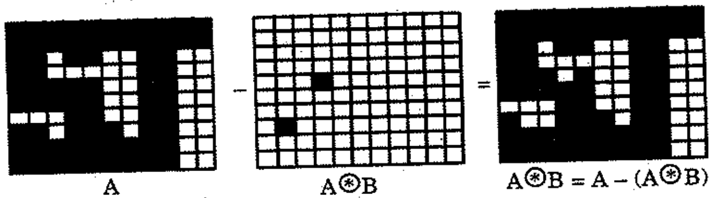


Fig. 4.30.4.

Thickening operation :

1. Thickening is the morphological dual of thinning and is defined by the expression :

$$A \odot B = A \cup (A \otimes B)$$

where B is a structuring element suitable for thickening.

2. As in thinning, thickening can be defined as a sequential operation :

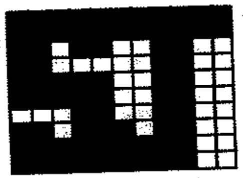
$$A \odot \{B\} = (1 \ldots ((A \odot B^1) \odot B^2) \ldots) \odot B^n)$$

For example : Consider the following image as a input image A and use B^1 as a structuring element and find the thickening process of the given image.

As we know the thickening is,

$$A \odot B^1 = A \cup (A \otimes B^1)$$

and $(A \otimes B^1) = (A \otimes B^1) \cap [A^c \otimes (W - B^1)]$ by Hit or Miss transformation



Input Image A

Fig. 4.30.5.

Step 1 :

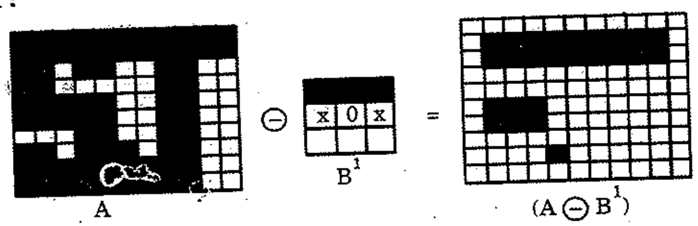


Fig. 4.30.6.

Step 2 :

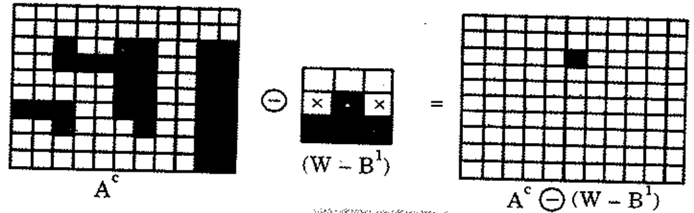
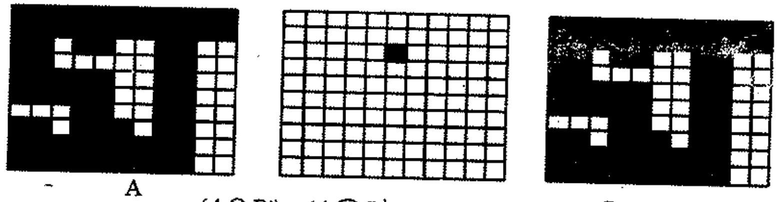


Fig. 4.30.7.

Step 3 : Now, find the intersection of the output of steps 1 and 2. This result is combined (union) with the original image which is shown in Fig. 4.30.8.



$(A \otimes B') = (A \ominus B') \cap [A^c - (W - B')]\ A \odot B = A \cup (A \otimes B')$

Fig. 4.30.8.

1. Thinning and thickening are derived from the Hit or Miss transform. For a image A and a composite structuring element $B = (B_1, B_2)$, thinning can be defined as,

$$A \oslash B = A - (A \otimes B)$$

where "–" is the set difference. This equation can also be written as,

$$A \oslash B = A \cap (A \otimes B)^c$$

2. Thickening is defined as,

$$A \odot B = A \cup (A \otimes B)$$

3. In thinning, a part of the boundary of the object is subtracted from the object and in thickening, a part of the boundary of the background is added to the object.
4. Thinning and thickening are dual operations. *i.e.*,

$$(A \odot B)^c = A^c \oslash B$$

Que 4.31. Explain the process of minimum mean square error restoration.

AKTU 2018-19, Marks 10

Answer

1. To design a filter that can estimate original images from the observed ones optimally in a Minimum Mean-Square Error (MMSE) sense for a large number of images.
2. So the i-th recorded image g_i can be expressed as

$$g_i = Hf_i + \eta_i \qquad ...(4.31.1)$$

3. Let the estimated image be $\hat{f}_i$ corresponding to g_i.
4. Since our image formation model is linear and space-invariant, we consider similar model for the restoration filter Φ which may be represented by the pre-multiplying g_i by the matrix P. Hence,

$$\hat{f}_i = Pg_i \qquad ...(4.31.2)$$

5. Thus the corresponding error vector e_i is obtained as

$$e_i = f_i - \hat{f}_i \qquad ...(4.31.3)$$

   Where the elements of the error vector e_i may be positive or negative.
6. We consider a non-negative quantity $e_i^T e_i$ error in estimating i-th image.
7. The MMSE criterion requires the mean error over the entire ensemble of the images under consideration be minimum. Thus, we pose the optimization problem as

   minimize $\varepsilon\{(e^Te)\}$ = minimize $[\varepsilon\{Tr(ee^T)\}]$ ...(4.31.4)

   ε and Tr represent expectation and trace operator respectively.

   Substituting eq. (4.31.1), (4.31.2), and (4.31.3), into the optimization criterion given we obtain the objective function $J(P)$. Thus

$$\begin{aligned} J(P) &= \varepsilon[Tr\{[f - P(Hf + \eta)][f - P(Hf + \eta)]^T\}] \\ &= \varepsilon[Tr\{(f - PHf - P\eta)(f - PHf - P\eta)^T\}] \\ &= \varepsilon[Tr\{(f - PHf - P\eta)(f^T - f^TH^TP^T - \eta^TP^T)\}] \\ &= \varepsilon[Tr\{(ff^T - ff^TH^TP^T - f\eta^TP^T - PHff^T + PHff^TH^TP^T \\ &\quad + PHf\eta^TP^T - P\eta f^T + P\eta f^TH^TP^T + P\eta\eta^TP^T)\}] \end{aligned} \qquad ...(4.31.5)$$

8. Since both Tr and ε are linear operators, they can be interchanged. We have assumed signal-independent noise, *i.e.*, $\varepsilon\{\eta f^T\} = \varepsilon\{\eta f^T\} = 0$. Also $Tr\{A\} = Tr\{A^T\}$. Hence, eq. (4.31.5) becomes.

$$J(P) = Tr[R_f - 2PHR_f + PHR_fH^TP^T + PR_\eta P^T] \qquad ...(4.31.6)$$

   Where R_f and R_η are auto-correlation matrices defined as $R_f = \varepsilon\{ff^T\}$ and $R_\eta = \varepsilon\{\eta\eta^T\}$, respectively.

9. So, for $J(P)$ be minimum, we differentiate $J(P)$ with respect to P and equate the derivative to zero as :

$$-2R_fH^T + 2PHR_fH^T + 2PR_\eta = 0$$

10. Hence, the filter matrix P is given by

$$P = R_fH^T(HR_fH^T + R_\eta)^{-1}$$

   The correlation matrices R_f and R_h are in block-Toeplitz form.

Que 4.32. **Prove the validity of the duality expressions $(A \bullet B)^c = (A^c o\hat{B})$ and $(AoB)^c = (A^c o\hat{B})$.**

AKTU 2018-19, Marks 10

Answer

Starting with the definition of closing,

$$\begin{aligned} (A \bullet B)^c &= [(A \oplus B) \ominus B]^c \\ &= (A \oplus B)^c \oplus \hat{B} \\ &= (A^c \ominus \hat{B}) \oplus \hat{B} \\ &= (A^c \circ \hat{B}) \end{aligned}$$

Starting with the definition of opening,

$$\begin{aligned} (A \circ B)^c &= [(A \ominus B) \oplus B]^c \\ &= (A \ominus B)^c \ominus \hat{B} \\ &= (A^c \oplus \hat{B}) \ominus \hat{B} \\ &= (A^c \circ \hat{B}) \end{aligned}$$

Que 4.33. **The so-called compass gradient operators of size 3 × 3 are designed to measure gradients of edges oriented in eight directions : E, NE, N, NW, W, SW, S, and SE. Give the form of these eight operators using coefficients valued 0, 1, or -1.**

AKTU 2018-19, Marks 10

Answer

To measure gradients of edges oriented in eight directions: E, NE, N, NW, W, SW, S and SE, the compass gradient operators of size 3 × 3 can be obtained from four Prewitt masks. The compass gradient operators for each direction are shown in Fig. 4.33.1.

$$\underset{W}{\begin{bmatrix} -1 & -1 & -1 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}} \quad \underset{S}{\begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix}} \quad \underset{E}{\begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ -1 & -1 & -1 \end{bmatrix}} \quad \underset{N}{\begin{bmatrix} 1 & 0 & -1 \\ 1 & 0 & -1 \\ 1 & 0 & -1 \end{bmatrix}}$$

$$\underset{SE}{\begin{bmatrix} 0 & 1 & 1 \\ -1 & 0 & 1 \\ -1 & -1 & 0 \end{bmatrix}} \quad \underset{SW}{\begin{bmatrix} -1 & -1 & 0 \\ -1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}} \quad \underset{NW}{\begin{bmatrix} 0 & -1 & -1 \\ 1 & 0 & -1 \\ 1 & 1 & 0 \end{bmatrix}} \quad \underset{NE}{\begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & -1 \\ 0 & -1 & -1 \end{bmatrix}}$$

Fig. 4.33.1.

Que 4.34. Explain watershed segmentation process.

Answer

1. A grey scale image can be viewed as a topological surface. In a topological surface, the value of a pixel is its height.

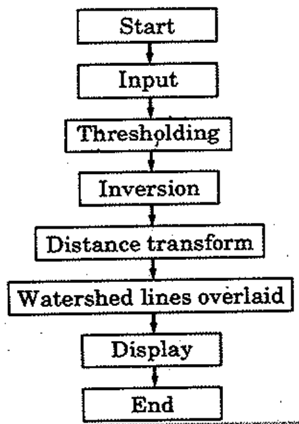


Fig. 4.34.1. Watershed segmentation process.

2. The complement image is the image where the peak becomes the valley.
3. It can be imagined that these are slowly filled with water.
4. The water starts filling the distinct catchment basins. Dams are built wherever it is necessary to prevent the merging of two adjacent basins.
5. Once the surface is immersed in water, the dams outline the watershed lines. Thus, watershed lines mark the boundaries of the catchment basins.

6. These segment the image into the desired regions. If it is the morphological gradient, then the object will now be low-level depressions bounded by the peaks.
7. These are the original edges. If these images are immersed in water, then water meets at the peaks. These are the watershed lines.
8. The problem of this method is that this algorithm leads to over segmentation.
9. This is also because of noise. Hence it is better if the images are smoothened before the segmentation process.
10. The post segmentation algorithms can be involved to connect the smaller regions to prevent over segmentation.

VERY IMPORTANT QUESTIONS

Following questions are very important. These questions may be asked in your SESSIONALS as well as UNIVERSITY EXAMINATION.

Q. 1. Write short note on image segmentation.
Ans. Refer Q. 4.1.

Q. 2. Explain region based segmentation with an example.
Ans. Refer Q. 4.3.

Q. 3. Explain intensity transformations in details. What would happen to the dynamic range of an image if all the slopes in the contrast stretched algorithm (l, m, n) are less than 1 ? Answer using illustration.
Ans. Refer Q. 4.4.

Q. 4. Explain the Hough Transforms to join the points. And also explain the problem of HT with their solutions. Given the four points in the x-y plane with the following coordinates (1,1), (2,2), (3,3), (4,4). Use Hough Transform to join these points.
Ans. Refer Q. 4.5.

Q. 5. Write short note on edge detection and line detection algorithm.
Ans. Refer Q. 4.6.

Q. 6. Explain edge detection and edge linking. Also write the difference between edge detection and edge linking.

Ans. Refer Q. 4.7.

Q. 7. Write short note on morphology processing.

Ans. Refer Q. 4.15.

Q. 8. Explain the following in details :

i. Stereo imaging

ii. Region filling

iii. Convex hull

Ans. Refer Q. 4.21.

Q. 9. Write short note on region extraction.

Ans. Refer Q. 4.23.

Q. 10. Explain the following morphological operations :

i. Opening

ii. Closing

iii. Region filling

Ans. Refer Q. 4.25.

☺☺☺

5 UNIT

Image Compression and Recognition

CONTENTS

PART-1

Need For Data Compression, Huffman, Run Length Encoding, Shift Codes, Arithmetic Coding, JPEG Standard, MPEG.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 5.1. Describe briefly about image compression.

Answer

1. Image compression is the process of minimizing the size in bytes of a graphics file without degrading the quality of the image to an unacceptable level.
2. The reduction in file size allows more images to be stored in a given amount of disk or memory space.
3. It also reduces the time required for images to be sent over the Internet or downloaded from Web pages.
4. There are several different ways in which image files can be compressed. For Internet use, the two most common compressed graphic image formats are the JPEG format and the GIF format.
5. The JPEG method is used for photographs, while the GIF method is used for line art and other images in which geometric shapes are relatively simple.
6. Other techniques for image compression include the use of fractals and wavelets. These methods have not gained widespread acceptance for use on the Internet.

Que 5.2. What is data compression and why we need it ? Explain compression and reconstruction with the help of block diagram.

Answer

1. Data compression is the process of encoding information using fewer bits and specific encoding schemes.
2. It is the art or science of representing information in a compact form. This compaction of information is done by identifying the structure that exists in the data.
3. Compressed data communication only works when both the sender and the receiver of the information understand the encoding scheme.

4. For example, any text makes sense only if the receiver understands that it is intended to be interpreted as characters representing the English language.
5. Similarly, the compressed data can only be understood if the decoding method is known by the receiver.

Need of data compression :

1. Compression is needed because it helps to reduce the consumption of expensive resources such as a hard disk space or transmission bandwidth.
2. As an uncompressed text or multimedia (speech, image or video) data requires a huge amount of bits to represent them and thus require large bandwidth, this storage space and bandwidth requirement can be decreased by applying proper encoding scheme for compression.
3. The design of data compression schemes involves trade off among various factors including the degree of compression, the amount of distortion introduced and the computational resources required to compress and decompress the data.

Compression and reconstruction :

1. A compression technique or compression algorithm includes two algorithms *i.e.*, compression algorithm and reconstruction algorithm.
2. The compression algorithm takes an input X and generates a representation X_c that requires fewer bits, and the reconstruction algorithm operates on the compressed representation X_c to generate the reconstruction Y. These operations are shown in Fig. 5.2.1.

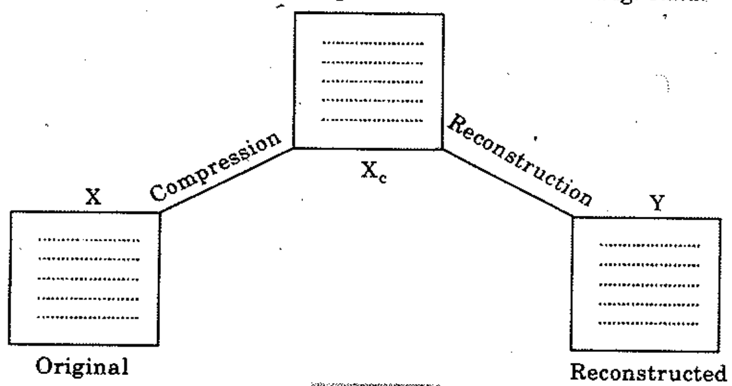


Fig. 5.2.1.

Que 5.3. What do you mean by data compression ? Explain its application areas.

Answer

Data compression : Refer Q. 5.2, Page 5–2A, Unit-5.

Applications of data compression :

1. **Audio :**
   a. Audio data compression reduces the transmission bandwidth and storage requirements of audio data.
   b. Audio compression algorithms are implemented in software as audio codecs.
   c. Lossy audio compression algorithms provide higher compression at the cost of allegiance and are used in numerous audio applications.
   d. These algorithms rely on psychoacoustics to eliminate or reduce allegiance of less audible sounds, thereby reducing the space required to store or transmit them.
2. **Video :**
   a. Video compression uses modern coding techniques to reduce redundancy in video data.
   b. Most video compression algorithms and codecs combine spatial image compression and temporal motion compensation.
   c. Video compression is a practical implementation of source coding in information theory.
3. **Genetics :** Genetics compression algorithms are the latest generation of lossless algorithms that compress data using both conventional compression algorithms and genetic algorithms adapted to the specific datatype.
4. **Emulation :**
   a. In order to emulate CD-based consoles such as the Playstation 2, data compression is desirable to reduce huge amounts of disk space used by ISO (International Organization for Standardization).
   b. For example, Final Fantasy XII (Computer Game) is normally 2.9 gigabytes. With proper compression, it is reduced to around 90% of that size.

Que 5.4. **Explain image compression algorithm.**

Answer

Following are different image compression algorithm :

1. **Entropy coding :**
   a. The logic behind entropy coding is that if pixels are not uniformly distributed, then an appropriate coding scheme can be selected that can encode the information so that the average number of bits is less than the entropy.
   b. Entropy specifies the minimum number of bits used to encode information.

   c. Hence, the coding is based on the entropy of the source and on the possibility of occurrence of the symbols.
   d. This leads to the idea of variable length coding.
   e. Some examples of this type of coding are Huffman coding, arithmetic coding, and dictionary-based coding.
2. **Predictive coding :**
   a. The idea behind predictive coding is to remove the mutual dependency between the successive pixels and then perform the encoding.
   b. Normally the samples would be very large, but the differences would be small.
   c. Examples of this category include Differential Pulse Code Modulation (DPCM) and delta modulation techniques.
3. **Transform coding :**
   a. The idea behind transform coding is to exploit the information packing capability of the transform.
   b. The energy is packed into fewer components and only these components are encoded and transmitted.
   c. The human eye is more sensitive to the lower spatial frequencies than the higher spatial frequencies.
   d. The idea is to remove the redundant high frequency components to create compression.
   e. The removal of these frequency components leads to loss of information.
   f. However, this loss of information, if tolerable, can be used for imaging and video applications.
   g. Thus the basis of transform coding is frequency selection, information packing, and the concept of basis images.
4. **Layered coding :**
   a. Layered coding is useful in the case of layered images.
   b. Sometimes the image is represented in the form of layers.
   c. Data structures such as pyramids are useful to represent an image in this multi-resolution form.
   d. The layers of a pyramid would be sent depending on the application.
   e. At times, these images are segmented as foreground and background and based on the needs of the application, encoding is performed.
   f. This is also in the form of selected frequency coefficients or selected bits of pixels of an image.

Que 5.5. **Write short note on Huffman coding.**

Answer

Huffman coding is a type of variable length coding. In Huffman coding, the coding redundancy can be eliminated by choosing a better way of assigning the codes. The Huffman coding algorithm is given as follows :

1. List the symbols and sort them.
2. Pick two symbols having the least probabilities.
3. Create a new node. Add the probabilities of the symbols selected in step 2 and label the new node with it.
4. Repeat steps 2 and 3 till only one node remains.
5. Start assigning code 0 for the left tree and code 1 for the other branch.
6. Trace the code from the root to the leaf that represents each label.
7. The running time of the algorithm is $O(n \log n)$.

Que 5.6. **Calculate the Huffman coding for the set of symbols shown in Table 5.6.1.**

Table 5.6.1. Symbols and their probabilities.

Symbol	A	B	C	D
Probability	0.4	0.3	0.2	0.1

Answer

1. The Huffman tree is constructed as shown in Fig. 5.6.1

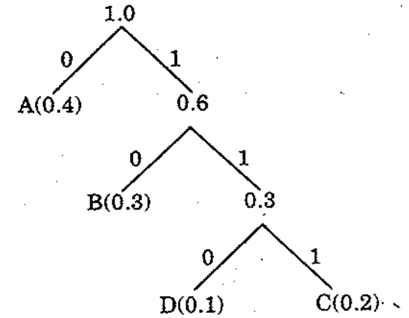


Fig. 5.6.1. Huffman code tree.

This yields the code shown in Table 5.6.2.

Table 5.6.2. Symbols and their Huffman codes—version 1.

Symbol	A	B	C	D
Code	0	10	111	110

The problem with Huffman codes is that they are not unique. The data given can be differently combined to yield the result shown in Fig. 5.6.2.

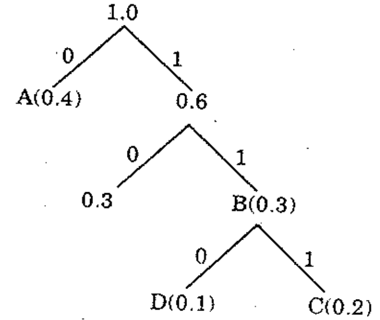


Fig. 5.6.2. Huffman code tree—alternate version.

This yields the code shown in Table 5.6.3.

Table 5.6.3. Symbols and their Huffman codes—version 2.

Symbol	A	B	C	D
Code	0	11	101	100

Que 5.7. **Differentiate between lossless and lossy compression.**

Answer

S. No.	Lossless compression	Lossy compression
1.	This is a reversible process and no information is lost.	This is a non-reversible process and information is lost.
2.	Compression ratio is usually less.	Compression ratio is very high.
3.	It is used for data that humans can handle directly such as text data.	It is useful for diffused data that humans cannot understand or interpret directly.
4.	Compression is independent of the psychovisual system.	Compression is dependent on the psychovisual characteristics.
5.	It is required in domains where reliability is very crucial such as executable files and medical data.	It is useful in domains where loss of data is acceptable.

Que 5.8. **What is run-length encoding ?**

Answer

1. Run-Length Encoding (RLE) is a simple form of lossless data compression in which runs of data (sequences in which the same data value occurs in many consecutive data elements) are stored as a single data value and count, rather than as the original run.
2. Run-Length Coding (RLC) exploits the repetitive nature of the image.
3. It tries to identify the length of the pixel values and encodes the image in the form of a run.
4. Each row of the image is written as a sequence.
5. Then the length is represented as a run of black or white pixels. This is called run-length coding.
6. Run-length coding is a CCITT (Consultative Committee of the International Telegraph and Telephone), standard that is used to encode binary and grey-level image.
7. The technique scans the image row by row and identifies the run.
8. The output run-length vector specifies the pixel value and the length of the run.
9. Given an input string, write a function that returns the Run Length Encoded string for the input string :
   a. Pick the first character from source string.
   b. Append the picked character to the destination string.
   c. Count the number of subsequent occurrences of the picked character and append the count to destination string.
   d. Pick the next character and repeat steps (b) (c) and (d) if end of string is NOT reached.

Que 5.9. **Explain arithmetic coding. What are the applications of arithmetic coding ?**

Answer

1. Arithmetic coding is a data compression technique that encodes data by creating a code string which represents a fractional value on the number line between 0 and 1.
2. The coding algorithm is symbolwise recursive *i.e.*, it operates upon and encodes (decodes) one data symbol per iteration or recursion.
3. On each recursion, the algorithm successively partitions an interval of the number line between 0 and 1, and retains one of the partitions as the new interval.
4. Thus, the algorithm successively deals with smaller intervals, and the code string viewed as a magnitude, lies in each of the nested intervals.

5. The data string is recovered by using magnitude comparisons on the code string to recreate how the encoder must have successively partitioned and retained each nested sub-interval.
6. Arithmetic coding differs considerably from the more familiar compression coding techniques such as prefix (Huffman) code.
7. It should not be confused with error control coding, whose object is to detect and correct errors in computer operations.

Applications of arithmetic coding :

1. Arithmetic coding is used in a variety of lossless and lossy compression applications.
2. It is a part of many international standards. In the area of multimedia, there are few principle organizations that develop standards.
3. Arithmetic coding is used in image compression, audio compression, and video compression standards.

Que 5.10. **Differentiate between arithmetic coding and Huffman coding.**

Answer

S. No.	Arithmetic coding	Huffman coding
1.	This is complex technique for coding short messages.	This is a simple technique for coding characters.
2.	It is always optimal.	It is optimal only if the probabilities of the symbols are negative powers of two.
3.	Precision is a big issue.	Precision is not an important factor.
4.	There is no slow reconstruction.	There is slow reconstruction when the number of symbols is very large and changing rapidly.

Que 5.11. **Explain JPEG and MPEG briefly.**

Answer

JPEG :

1. JPEG stands for Joint Photographic Experts Group.
2. It uses lossy compression technique.
3. It supports 24-bit colors.
4. It is supported by all browsers.
5. It is suitable for photographs.

6. It has smaller file size as compared to GIF.
7. It uses extension .jpg or .jpeg.

MPEG :

1. It stands for Moving Picture Experts Group.
2. MPEG files employ loss data compression.
3. MPEG files use the .mp3 filename extension.
4. Digital music, pod casts and audio books are often saved as MPEG3 files.
5. Older MPEG files are usually designated with an extension of .mp2.

PART-2

Boundary Representation, Boundary Description, Fourier Descriptor, Regional Description : Topological Feature, Texture, Patterns and Pattern Classes, Recognition Based on Matching.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 5.12. Explain the different approaches used in boundary representation. What are the approaches used to describe the boundary of a region ?

Answer

Different approaches used in boundary representation are :

1. **Minimum perimeter polygons :**
   a. We visualize minimum perimeter polygons enclosure as two walls corresponding to the outside and inside boundaries of the strip of cells, and think of the object boundary as a rubber band contained within the wall.
   b. If the rubber band is allowed to shrink, it takes the shape shown in Fig. 5.12.1.

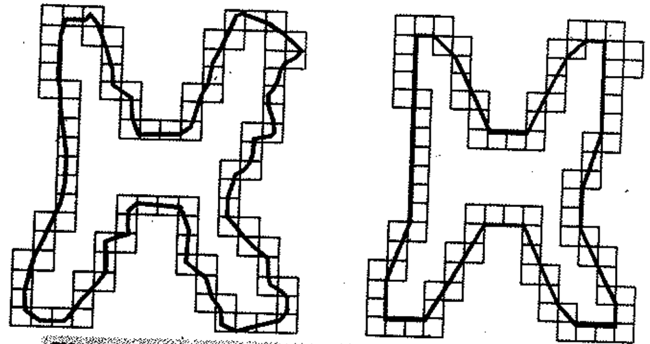


Fig. 5.12.1. Object boundary enclosed by cells, and Minimum perimeter polygon.

2. **Merging techniques :**
   a. Merging techniques based on average error or other criteria have been applied to the problem of polygonal approximation.
   b. The approach is to merge points along a boundary until the least square error line fit of the points merged so far exceeds a preset threshold.

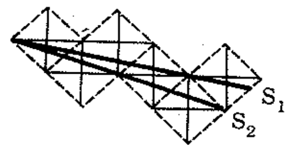


Fig. 5.12.2.

3. **Splitting techniques :**
   a. An approach of the splitting techniques is to subdivide a segment successively into two parts until a criterion is satisfied.
   b. For instance, a requirement might be that the maximum perpendicular distance from a boundary segment to the line joining its two end point not exceed a preset threshold.
4. **Signature :**
   a. Signature is an approach that translates 2D function to 1D function.
   b. One of the simplest example is to plot the distance from the center to the boundary as a function of angle, as illustrated in Fig. 5.12.3.

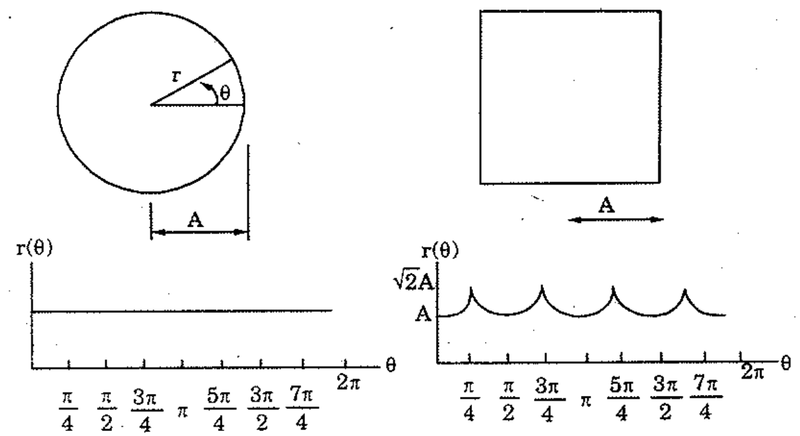


Fig. 5.12.3 (a), (b): Distance signature of circle and rectangular shapes.

5. **Boundary segments :**
   a. In boundary segments, a boundary is decomposed into segment which reduces the boundary's complexity.
   b. This approach is useful when the boundary contains one or more concavities.
   c. In this case use of the convex hull of the region enclosed by the boundary is a powerful tool for robust decomposition of the boundary.
6. **Skeleton :** Refer Q. 4.24, Page 4–28A, Unit-4.

Descriptor used to describe the boundaries of a region are :

1. **Eccentricity :** Eccentricity is the ratio of major axis and minor axis.
2. **Curvature :** Curvature is defined as the rate of the boundary that use convex and concave to describe the boundary.

Que 5.13. **Write short note on Fourier descriptor.**

Answer

1. The Fourier descriptors are starting at an arbitrary point (x, y). Each coordinate pair can be treated as a complex number so that
$$sk = x(k) + jy(k) \quad ...(5.13.1)$$
2. This representation reduces a 2D to a 1D problem. The Discrete Fourier Transform (DFT) of $s(k)$ is
$$a(u) = \frac{1}{K}\sum_{k=0}^{K-1} s(k)e^{-j2\pi uk/K} \quad ...(5.13.2)$$
for $u = 0, 1, 2, ..., K-1$

3. The complex coefficients $a(u)$ are called the Fourier descriptors of the boundary. The inverse Fourier transform of these coefficients restore $s(k)$. That is,
$$s(k) = \sum_{u=0}^{K-1} a(u)e^{j2\pi uk/K}$$
for $k = 0, 1, 2, ..., K-1$
4. Suppose, that instead of all the Fourier coefficients, only the first P coefficient is used. This is equivalent to setting $a(u)$ for $u > P-1$. The result is the following approximation to $\hat{s}(k)$:
$$\hat{s}(k) = \sum_{u=0}^{P-1} a(u)e^{j2\pi uk/K}$$
5. When P is small, more details are lost on the boundary. When P is large, there is less loss of detail on boundary.

Que 5.14. **Explain the term topological. What are topological features ?**

Answer

1. Topological is the study of properties of a figure that are unaffected by any deformation, as long as there is no tearing.
2. It can be defined by the number of holes, and connected component. It is defined by the Euler formula :
$$E = V - Q + F = C - H$$
V: the number of vertices
Q: the number of edges
F: the number of faces
C: the number of connected component
H: the number of holes

Following are the topological features :

1. **Connected component :** The connected component is defined as the regions that are populated by pixels that share common characteristics.
2. **Euler number :** This is also known as genus. This is one of the most important topological properties. It is given as
$$E = C - H$$
Here C is the connected component and H is the number of holes. For a polygonal network, it is given as
$$\text{Euler number} = V - E + F$$
Here V is the number of vertices, E is the number of edges, and F is the number of faces.

3. **Number of holes present :** This is a count of the number of holes that are present in the object.
4. **Total hole area :** This is the total pixel area of the interior holes present in the object.
5. **Total hole area / object area :** This parameter is a measure of object proliferation. The value of this parameter can range from zero to one. If the value is one, then the entire object itself is a hole.

Que 5.15. Describe briefly the approaches used for region description.

Answer

Approaches used for region description are :

1. **Statistical approaches :**
   i. Statistical approaches yield characterizations of textures as smooth, coarse, and grainy, and so on.
   ii. The nth moment of z about the mean is

$$u_n(z) = \sum_{i=0}^{L-1} (z_i - m)^n p(z_i)$$

Where $\quad m = \sum_{i=0}^{L-1} z_i p(z_i)$

   iii. The second moment is a measure of gray- level contrast that can be used to establish descriptors of relative smoothness.
   iv. For example, the measure of constant intensity

$$R = 1 - \frac{1}{1 + \sigma^2(z)}$$

   v. The third moment is a measure of the skewness of the histogram.
   vi. The fourth moment is a measure of its relative flatness.
   vii. The fifth and higher moments are not so easily related to histogram shape. Some useful approaches established in the histogram is measuring of uniformity

$$U = \sum_{i=0}^{L-1} p^2(z_i)$$

and measuring of an average entropy

$$e - \sum_{i=0}^{L-1} p(z_i) \log_2 p(z_i)$$

2. **Structural approaches :**
   i. Structural techniques deal with the arrangement of image primitives.
   ii. Suppose that we have a rule of the form $s \to as$, which indicates that the symbols may be rewritten *as*.
   iii. If a represents a circle [Fig. 5.15.1(a)] and the meaning of "circles to the right" is assigned to a string of the form aaa...., the rule $s \to as$ allows generation of the texture pattern shown in Fig. 5.15.1(b).

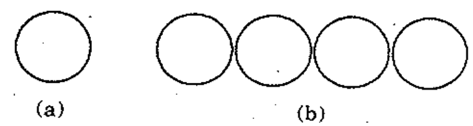


Fig. 5.15.1.

3. **Spectral approaches :**
   i. Spectral techniques are based on properties of the Fourier spectrum and are used primarily to detect global periodicity in an image by identifying high energy, narrow peaks in the spectrum.
   ii. Interpretation of spectrum features are simplified by expressing the spectrum in polar coordinates to yield a function, $S(r, \theta)$ where S is the spectrum function and r and θ are the variables in this coordinate system.
   iii. Analyzing $S_\theta(r)$ for a fixed value of θ yield the behaviour of the spectrum along a radial direction from the origin, whereas analyzing $S_r(\theta)$ for a fixed value of r yields the behaviour along a circle centered on the origin.

$$S(r) = \sum_{\theta=0}^{\pi} S_\theta(r)$$

$$S(\theta) = \sum_{r=1}^{R_0} S_r(\theta)$$

Discriminating the different texture patterns by analyzing their corresponding $S(\theta)$ and $S(r)$ waveforms would be straightforward.

Que 5.16. Explain relational descriptor.

Answer

1. Relational descriptor is applied to boundaries or regions, and their main purpose is to capture repetitive patterns in a boundary or region.
2. By defining the two primitive elements a and b, we use it to establish the code, and use rewriting rules to transform it.
   a. $S \to aA$
   b. $A \to bS$
   c. $A \to b$

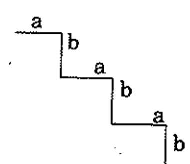


Fig. 5.16.1.

3. Because strings are 1D structures, their application to image description requires establishing an appropriate method for reducing 2D positional relations to 1D form.
4. An approach is to follow the contour of an object and code the result with segments of specified direction and/or length.
5. Another approach is to describe sections of an image by directed line segments. String descriptions are best suited for applications in which connectivity of primitives can be expressed in a head-tail or other continuous manner.
6. Regions that are similar in terms of texture or other descriptor may not be contiguous, then tree descriptors are required for describing such situations.

Que 5.17. What do you mean by binary code ? How a sequence is coded ?

Answer

Binary code :

1. A binary code is a way of representing text or computer instructions by the use of the binary number system 0 and 1.
2. This is accomplished by assigning a bit string to each particular symbol or instruction.
3. For example, a binary string of eight binary digits (bits) can represent any of 256 symbols, letters or instructions.
4. In computing, binary codes are used for many methods of encoding data, such as character strings into bit strings.
5. These methods may be fixed-width or variable-width. In a fixed-width binary code, each letter, digit, or other interpreted as a binary number, is usually displayed in code tables in octal, decimal or hexadecimal notation.
6. There are many character sets and character encoding for them.
7. A bit string, interpreted as a binary number, can be translated into a decimal number.

Coding a sequence :

1. In order to distinguish a sequence of symbols from another sequence of symbols, we need to tag it with a unique identifier.
2. One possible set of tags for representing sequences of symbols are the numbers in the unit interval [0, 1).
3. Because the number in the unit interval is infinite, it should be possible to assign a unique tag to each distinct sequence of symbols.

4. In order to do this, we need a function that will map sequences of symbols into the unit interval.
5. A function that maps random variables and sequences of random variables into the unit interval is the cumulative distribution function (*cdf*) of the random variable associated with the source.
6. A random variable maps the outcomes, or sets of outcomes, of an experiment to values on the real number line.
7. For example, in a coin-tossing experiment, the random variable could map a head to zero and a tail to one (or it could map a head to 2367.5 and a tail to –192).
8. To use this technique, we need to map the source symbols or letters to numbers.
9. For convenience, we will use the mapping :

$$X(a_i) = i \qquad a_i \in A$$

where $A = \{a_1, a_2, ... a_m\}$ is the alphabet for a discrete source and X is a random variable.

10. This mapping means that given a probability model P for the source, we also have a probability density function for the random variable.

$$P(X = i) = P(a_i)$$

and the cumulative density function can be defined as,

$$F_X(i) = \sum_{k=1}^{i} P(X = k)$$

Que 5.18. How a tag is generated in arithmetic coding ?

Answer

1. The procedure for generating the tag works by reducing the size of the interval in which the tag resides as more and more elements of the sequence are received.
2. Start by first dividing the unit interval into sub-intervals of the form $[F_x(i-1), F_x(i)], i = 1, ..., m$.
3. Because the minimum value of the *cdf* is zero and the maximum value is one, this exactly partitions the unit interval.
4. The sub-interval $[F_x(i-1), F_x(i)]$ with the symbol a_i.
5. The appearance of the first symbol in the sequence restricts the interval containing the tag to one of these sub-intervals.
6. Suppose the first symbol was a_k.
7. Then the interval containing the tag value will be the sub-interval $[F_x(k-1), F_x(k)]$.

8. This sub-interval is now portioned in exactly the same proportions as the original interval.
9. The j^{th} interval corresponding to the symbol a_j is given by,
   $[F_x(k-1) + F_x(j-1)/F_x(k) - F_x(k-1)),\ F_x(k-1) + F_x(j)/F_x(k) - F_x(k-1)))$.
10. Thus, the second symbol in the sequence is a_j, then the interval contains the tag value becomes $[F_x(k-1) + F_x(j-1)/F_x(k) - F_x(k-1) + F_x(j)/(F_x(k) - F_x(k-1)))$.
11. Each succeeding symbol causes the tag to be restricted to a sub-interval that's further partitioned in the same proportions.

VERY IMPORTANT QUESTIONS

Following questions are very important. These questions may be asked in your SESSIONALS as well as UNIVERSITY EXAMINATION.

Q. 1. Describe briefly about image compression.

Ans. Refer Q. 5.1.

Q. 1. What is data compression and why we need it ? Explain compression and reconstruction with the help of block diagram.

Ans. Refer Q. 5.2.

Q. 1. Differentiate between lossless and lossy compression.

Ans. Refer Q. 5.7.

Q. 1. What is run-length encoding ?

Ans. Refer Q. 5.8.

Q. 1. What do you mean by binary code ? How a sequence is coded ?

Ans. Refer Q. 5.17.

Q. 1. How a tag is generated in arithmetic coding ?

Ans. Refer Q. 5.18.

☺☺☺

1 UNIT

Digital Image Fundamental (2 Marks Questions)

1.1. Define image. What is dynamic range ?

AKTU 2016-17, Marks 02

OR

Define image. What is range ? **AKTU 2017-18, Marks 02**

Ans. **Image :** An image is defined as a two-dimensional function, $F(x, y)$, where x and y are spatial coordinates, and the amplitude of F at any pair of coordinates (x, y) is called the intensity of that image at that point. When x, y and amplitude values of F are finite, we call it a digital image.

Dynamic range : Dynamic range is defined as the ratio between the maximum and minimum measurable light intensities (white and black, respectively).

1.2. What is meant by binary image, colour image, gray-scale image ? **AKTU 2017-18, Marks 02**

Ans. **Binary image :** It is the image obtained by converting gray-scale image to a black and white image.

Colour image : It is the image represented by three visible bands *i.e.*, red, green and blue.

Gray-scale image : It is image represented by range of shades between white and black or vice-versa.

1.3. Define components of image processing system.

AKTU 2018-19, Marks 02

Ans. **Components of image processing system are :**
1. Image sensors
2. Specialized image processing hardware
3. Computer
4. Image processing software

1.4. Name some applications of digital image processing.

AKTU 2018-19, Marks 02

Ans. **Applications of digital image processing are :**
1. Gamma-ray imaging
2. X-ray imaging
3. Imaging in the ultraviolet band
4. Imaging in the visible and infrared band
5. Imaging in the microwave band
6. Imaging in the radio band

1.5. What do you understand by Weber ratio ? What does a low value for Weber ratio indicate ? AKTU 2015-16, Marks 02

Ans. Weber ratio is defined as the ratio of increment of illumination *i.e.*, ΔI_c to the background illumination *i.e.*, I. A low value of $\Delta I_c/I$ mean that a small percentage change in intensity is discriminable. This represents good brightness discrimination.

1.6. What would happen to the dynamic range of an image if all the slopes in the contrast stretched algorithm (l, m, n) are less than one ? AKTU 2015-16, Marks 02

Ans. If all the slopes in the contrast stretched algorithm (l, m, n) are less than one, then for the input range (r), the output range decreases. When dynamic range is reduced, the image would become darker.

1.7. What steps are related with high level processing in digital image processing ? AKTU 2015-16, Marks 02

Ans. **Steps related with high level processing in digital image processing are :**
1. Construction of the model of the real world object or scene.
2. Construction of the model from the image.
3. A matching process, initiated between the real world model and the model created from the image, which results in partial or complete matching.
4. A feedback mechanism that invokes additional routines to update the models if necessary.

1.8. Define digital image processing.

Ans. Digital image processing refers to the processing of digital images by means of digital computer. Digital image processing requires a computer to process images and two special input/output equipments *i.e.*, an image digitizer and an image display device.

1.9. Name three elements that govern visual perception.

Ans. **Three elements that govern visual perception are :**
1. Structure of eyes
2. Brightness adaptation and discrimination
3. Image formation

1.10. Define mach bands effects.

Ans. Mach bands are an optical illusion where a band of gradients will appear in places to be lighter or darker than they actually are. Mach band provide a valuable way to investigate how the eye and brain process visual information.

1.11. What are the types of sampling in image processing ?

Ans. **Three types of sampling in image processing are :**
1. Ideal sampling 2. Natural sampling 3. Flat top sampling

1.12. Give merits and demerits of digital images.

Ans. **Merits of digital images :**
1. The processing of images is faster and cost-effective.
2. Copying a digital image is easy.

Demerits of digital images :
1. A digital file cannot be enlarged beyond a certain size without compromising on quality.
2. Misuse of copyright has become easier because images can be copied from the internet.

1.13. Derive, why we multiply with $(-1)^{x+y}$ in case of frequency domain filtering ? AKTU 2015-16, Marks 02

Ans. Multiplying $f(x)$ by the exponential term shown shifts the data so that the origin, $f(0)$ is located at u_0. If we let $u_0 = M/2$, the exponential term becomes $e^{j\pi x}$ which is equal to $(-1)^x$ because x is an integer. In this case,

$$f(x)(-1)^x \leftrightarrow F(u - M/2)$$

i.e., multiplying $f(x)$ by $(-1)^x$ shifts the data so that $F(0)$ is at the center of the interval $[0, M-1]$. In 2D, the principle is same. A shifted DFT is obtained by multiplying $f(x, y)$ by $(-1)^{x+y}$ before computing $F(u, v)$.

1.14. Discuss the key approach in homomorphic filtering.

Ans. The key approach in homomorphic filtering is to separate the illumination and reflectance components.

1.15. What do you mean by aliasing in the context of image sampling ?

Ans. Aliasing is a phenomenon that occurs when an image is under-sampled. This phenomenon would not occur if the image is sampled at a rate of at least twice the highest frequency component in the image. Additional frequency components are present in an undersampled image that results in a change in the way the image looks.

☺☺☺

2 UNIT

Image Enhancement (2 Marks Questions)

2.1. Consider the following two 8-bit images :

Fig. 1.

Each of these images has dimensions 20 × 20. Show the histograms of these images. Please note that the borders of the images shown in black are just to highlight the boundaries. The border is not a part of the image.

AKTU 2015-16, Marks 02

Ans. Histogram of first image is :

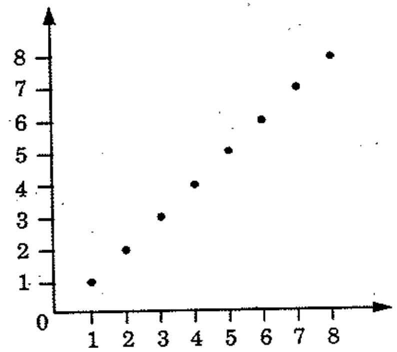


Fig. 2.

Histogram of second image is :

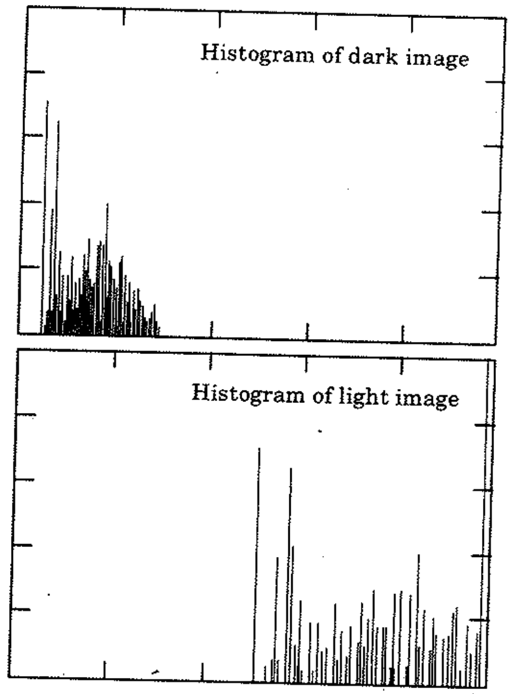


Fig. 3.

2.2. Write down the filter mask for Sobel and Prewitt filters.

AKTU 2015-16, Marks 02

AKTU 2018-19, Marks 02

Ans. **Filter mask for Sobel filters are :**

–1	–2	–1
0	0	0
1	2	1

–1	0	1
–2	0	2
–1	0	1

Filter mask for Prewitt filters are :

–1	–1	–1
0	0	0
1	1	1

–1	0	1
–1	0	1
–1	0	1

2.3. Draw the graph for power law (Gamma) transformation (for gamma > 1). AKTU 2015-16, Marks 02

Ans.

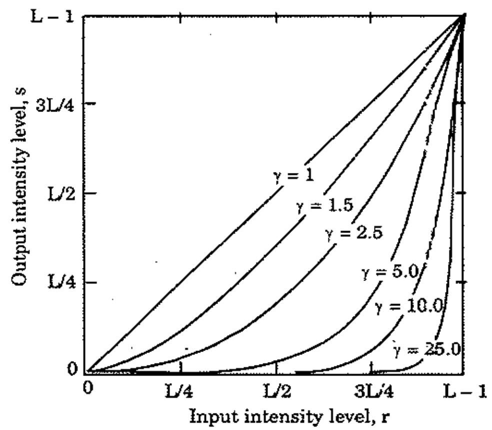


Fig. 4.

2.4. Explain Laplacian filter. AKTU 2018-19, Marks 02

Ans.

1. First derivative does enhance the edges of the image.
   We know,

$$\nabla F = \frac{\partial f}{\partial x} + \frac{\partial f}{\partial y}$$

$$\therefore \quad \frac{\partial f}{\partial x} = f(x+1, y) - f(x, y)$$

and $$\frac{\partial f}{\partial y} = f(x, y+1) - f(x, y)$$

2. The second derivative is given by,

$$\nabla^2 F = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

where

$$\frac{\partial^2 f}{\partial x^2} = f(x+1, y) + f(x-1, y) - 2f(x, y)$$

and $$\frac{\partial^2 f}{\partial y^2} = f(x, y+1) + f(x, y-1) - 2f(x, y)$$

$$\nabla^2 F = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

$$\therefore \quad |\nabla^2 F| = [f(x+1, y) + f(x-1, y) + f(x, y+1) + f(x, y-1) - 4f(x, y)]$$

3. Considering the 3 × 3 neighbourhood

	$y-1$	y	$y+1$
$x-1$	Z_1	Z_2	Z_3
x	Z_4	Z_5	Z_6
$x+1$	Z_7	Z_8	Z_9

4. This equation in the discrete form reduces to

$$|\nabla^2 F| = [Z_8 + Z_2 + Z_6 + Z_4 - 4Z_5]$$

5. This equation can be implemented using a mask shown as :

0	1	0
1	–4	1
0	1	0

6. This is known as the Laplacian filter.

2.5. What is geometric transformation ?

AKTU 2018-19, Marks 02

AKTU 2016-17, Marks 02

Ans. A geometric transformation is any bijection of a set having some geometric structure to itself or another such set. Specifically, a geometric transformation is a function whose domain and range are set of points.

2.6. What are first order derivative filters ?

AKTU 2018-19, Marks 02

Ans. **First order derivative filters are :**

1. Gradient filters
2. Prewitt gradient filters
3. Sobel gradient filters
4. Gaussian gradient filters

2.7. Explain harmonic mean filter. AKTU 2017-18, Marks 02

Ans. In the harmonic mean filter, the colour value of each pixel is replaced with the harmonic mean of colour values of the pixels in a surrounding region.

The harmonic mean is defined as :

$$H = \frac{n}{\frac{1}{x_1} + \frac{1}{x_2} + \ldots + \frac{1}{x_n}}$$

2.8. Explain Affine transform. AKTU 2016-17, Marks 02

Ans. Affine transformation is the combined effect of translation, rotation, scaling and shear. Through affine transformation, lines transform to lines but circles become ellipses.

2.9. Give classification of spatial domain technique.

Ans. Spatial domain technique can be classified as :

1. Point operation 2. Mask operation 3. Global operation

2.10. What is image filtering ?

Ans. Image filtering is the process of modifying the pixels in an image based on function of a local neighbourhood of the pixels.

2.11. Why does histogram equalization (discrete histogram equalization) not produce a perfectly flat histogram ?

Ans. Discrete histogram equalization basically performs the remapping of histogram components on the intensity scale. To obtain a flat histogram, the pixel intensities should be redistributed in such a way that there are L groups of n/L pixels with the same intensity, where L is the number of allowed discrete intensity levels and n is the total number of pixels in the input image. The histogram equalisation method has no provisions for this type of redistribution process hence discrete histogram equalization does not yield a flat histogram.

2.12. What does the standard deviation of a histogram tell us about the image ?

Ans. The standard deviation (square root of the variance) tells us about the contrast. It describes the spread in the data, so a high-contrast image will have a high variance and a low-contrast image will have a low variance.

2.13. Write procedure to perform histogram equalization.

Ans. Histogram equalization is done by performing the following steps :

i. Find the running sum of the histogram values.
ii. Normalise the values from step (i) by dividing by the total number of pixels.
iii. Multiply the values from step (ii) by the maximum gray-level value and round.
iv. Map the gray-level values to the results from step (iii) using a one-to-one correspondence.

2.14. What is the value of the marked pixel after a 5 × 5 median filter ?

$$\begin{bmatrix} 2 & 1 & 3 & 4 & 5 \\ 1 & 1 & 0 & 2 & 3 \\ 2 & 0 & \textcircled{0} & 1 & 2 \\ 5 & 1 & 2 & 3 & 1 \\ 4 & 3 & 1 & 2 & 0 \end{bmatrix}$$

Ans. In order to perform a median filter, the pixels have to be sorted either in the ascending or descending order.

Step : 1 Arranging the pixels in ascending order :

0000111111122222233334455

Step : 2 To determine the median value :

0000111111122222233334455

The median value is found to be 2. Hence, the marked pixel 0 will be replaced by 2.

2.15. What is the value of the central pixel (marked by a round) if it is smoothened by a 3 × 3 box filter ?

$$\underset{\textbf{Input image}}{\begin{bmatrix} 1 & 2 & 3 \\ 2 & \textcircled{4} & 5 \\ 3 & 4 & 3 \end{bmatrix}} \xrightarrow{\textbf{3 × 3 box filter}} \underset{\textbf{Output image}}{\begin{bmatrix} & & \\ & ? & \\ & & \end{bmatrix}}$$

Ans. The 3 × 3 box filter is given by $\frac{1}{9} \times \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$. If this box filter is applied to the input image, the central value '4' will be replaced by the average of the neighbourhood defined by the mask. This is given by averaging operation = $\frac{1}{9} \times (1+2+3+2+4+5+3+4+3)$

$= \frac{27}{9} = 3$. The pixel value '4' will be replaced by the average of the neighbourhood which results in '3'.

$$\underset{\text{Input image}}{\begin{bmatrix} 1 & 2 & 3 \\ 2 & \textcircled{4} & 5 \\ 3 & 4 & 3 \end{bmatrix}} \xrightarrow{\text{3 × 3 box filter}} \underset{\text{Output image}}{\begin{bmatrix} & & \\ & 3 & \\ & & \end{bmatrix}}$$

2.16. Define alpha blending.

Ans. Alpha blending refers to addition of two images, each with 0 to 1 fractional masking weights. Alpha blending is useful for transparency and compositing.

2.17. Write the procedure of unsharp masking.

Ans. The procedure to perform unsharp masking is :

1. Filter the blur image.
2. Subtract the result obtained from step 1 from the original image.
3. Multiply the result obtained in step 2 by some weighting fraction.
4. Add the result obtained in step 3 to the original image.

2.18. Suggest a suitable filter that will reduce the impact of salt-and-pepper noise at the same time it preserves the edges in the image.

Ans. The filter that will reduce the impact of salt-and-pepper noise with minimum blurring is a median filter.

2.19. Suggest a suitable filter that could minimise the impact of 'Gaussian noise' ?

Ans. Gaussian filter is an optimum filter that could reduce the impact of Gaussian noise.

2.20. What is contrast stretching ? **AKTU 2016-17, Marks 02** **AKTU 2017-18, Marks 02**

Ans. Contrast stretching is an image enhancement technique that attempts to improve the contrast in an image by stretching the range of intensity values it contains to span a desired range of value.

2.21. List edge detection operators. **AKTU 2016-17, Marks 02**

Ans. **Edge detection operators are :**

1. Roberts operator
2. 4-Neighbour operator
3. Prewitt operator
4. Sobel operator

2.22. Explain the types of connectivity. **AKTU 2016-17, Marks 02**

Ans. **Types of connectivity :**

4-connectivity : Two pixels p and q with gray levels from the set V are said to be 4-connected if q is in the set $N_4(p)$.

8-connectivity : Two pixels p and q with gray levels from the set V are said to be 8-connected if q is in the set $N_8(p)$.

***m*-connectivity :** Two pixels p and q with gray levels values from the set V are 8-connected if q is in $N_8(p)$ or q is in $N_D(p)$ and the set $N_4(p) \cap N_4(q)$ is a null set.

m-connectivity is used to eliminate the multiple path connections that may arise when 8-connectivity is used.

☺☺☺

3 UNIT

Image Restoration (2 Marks Questions)

3.1. Define the terms image restoration and image degradation.

Ans. **Image restoration :** It is defined as the process of removal or reduction of degradation in an image through linear or non-linear filtering.

Image degradation : The process by which the original image is blurred is usually very complex and often unknown. To simplify the calculations, the degradation is often modeled as a linear function which is often referred as point-spread function.

3.2. Diagrammatically show image degradation / restoration model.

Ans.

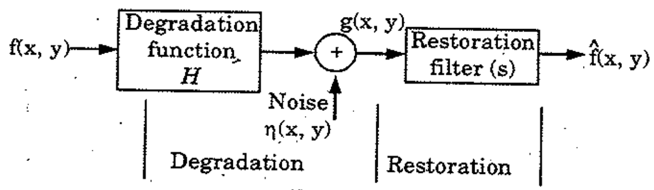


Fig. 1.

3.3. What are the types of noise ?

Ans. **Noise in an image are of two types :**

1. **Correlated noise :** These noise are periodic in nature, chances of existence of these types of noises are rare and can be easily filtered out to restore.
2. **Uncorrelated noise :** These noise are random in nature, chances of existence of these types of noises are common and they cannot be filtered out to restore completely.

3.4. List three main properties of a median filter.

Ans. **The three main properties of median filters are :**

1. A median filter smoothen additive noise.
2. A median filter does not degrade edges.
3. A median filter is effective in removing impulses.

3.5. List types of blur image.

Ans. Types of blur image are :
1. Gauss blur
2. Out-of-focus blur
3. Motion blur

3.6. With the help of flow chart show the classification of image restoration techniques.

Ans.

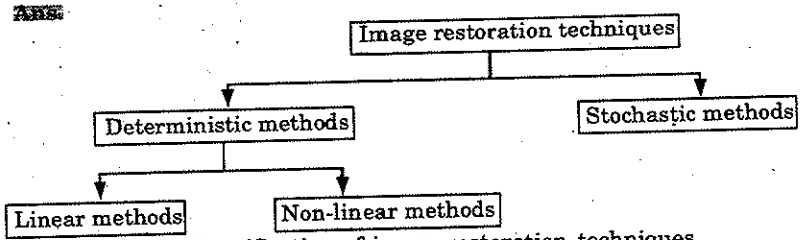


Fig. 2. Classification of image restoration techniques.

3.7. Give some applications of digital image restoration.

Ans. Applications of digital image restoration :
1. Astronomical imaging
2. Medical imaging
3. Printing industry

3.8. How degradation function is estimated ?

Ans. Degradation function is estimated using following techniques :
1. Estimation by image observation
2. Estimation by experimentation
3. Estimation by modeling

3.9. When will Wiener filter reduce to an inverse filter ?

Ans. The frequency domain representation of a Wiener filter is given by

$$G(\omega_1, \omega_2) = \frac{H^*(\omega_1, \omega_2) \times S_{ff}(\omega_1, \omega_2)}{|H(\omega_1, \omega_2)|^2 \times S_{ff}(\omega_1, \omega_2) + S_{\eta\eta}(\omega_1, \omega_2)}$$

In the absence of noise, $S_{\eta\eta}(\omega_1, \omega_2) = 0$, the expression for a Wiener filter is given by $G(\omega_1, \omega_2) = \dfrac{1}{H(\omega_1, \omega_2)}$.

Thus, a Wiener filter reduces to an inverse filter in the absence of noise.

3.10. What are the advantages of a Wiener filter over an inverse filter ?

Ans. A Wiener filter is better than the inverse filter in the presence of noise because a Wiener filter uses a prior statistical knowledge of the noise field. The transfer function of the Wiener filter is chosen to minimise the mean square error using statistical information on both image and noise fields.

3.11. Comment on the choice of regularization parameter used in constrained least square restoration technique ?

Ans. When regularization parameter is large, it leads to more regularization, and the restored image tends to have more ringing. With smaller values of the regularization parameter, the restored image tends to have more amplified noise effects.

3.12. Differentiate between image enhancement and restoration.

AKTU 2018-19, Marks 02

Ans.

S. No.	Image enhancement	Image restoration
1.	It gives better visual representation.	It removes effects of sensing environment.
2.	No model required.	Mathematical model of degradation is needed.

3.13. What is the advantage and drawback of Wiener filter ?

Ans. The main advantage of a Wiener filter is that it is stable in the presence of nulls in $H(\omega_1, \omega_2)$. It also reduces the effect of additive noise and produces best reconstruction in terms of the mean least square error. The main drawback of Wiener filter is that to get an optimum solution, one should know the statistics of the image and of the noise.

3.14. Explain noise model. **AKTU 2016-17, Marks 02**

Ans. Noise models are those models which help in building good restoration system. Following are the noise model :
1. Gaussian noise model
2. Rayleigh noise model
3. Gamma noise model
4. Exponential noise model
5. Uniform noise model
6. Salt and pepper noise model

☺☺☺

4 UNIT

Image Segmentation (2 Marks Questions)

4.1. Define structuring elements.
OR
Define morphological image processing.

AKTU 2018-19, Marks 02

Ans. Morphological image processing is a processing of non-linear processes which can be applied to an image to remove details smaller than a certain reference shape, and that reference shape is called structuring element.

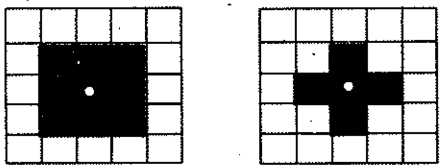


Fig. 1. Examples of structuring elements.

4.2. What do you mean by binary image processing ?

Ans. Binary image processing is the process of converting a gray scale image to a black and white image. Binary image processing is done by using global thresholding.

4.3. Binary image given below : find $\bar{A}, A \cup B, A \cap B, A\ XOR\ B$.

$$A = \begin{matrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{matrix} \qquad B = \begin{matrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{matrix}$$

Ans.

$$\bar{A}\ \textbf{(Complement)} = \begin{matrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{matrix}$$

$$A \cup B\ \textbf{(Union)} = \begin{matrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{matrix}$$

$$A \cap B\ \textbf{(Intersection)} = \begin{matrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{matrix}$$

$$A\ XOR\ B\ \textbf{(Exclusive-OR)} = \begin{matrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{matrix}$$

4.4. What are morphological filters ? Explain using term "Increasing" operation and "Idempotent" operation.

Ans. Any transformations that are increasing and idempotent are called the morphological filters.

Increasing operation : For any morphological operation [$\oplus$, $\ominus$, $\circ$, $\bullet$], if following condition satisfies then operator is increasing :

$$A \subseteq B \Rightarrow A \oplus C \subseteq B \oplus C$$

Idempotent operation : Any operator when repetitively applied for same object and it does not change the resultant object, the operator is called as idempotent operator and operation is called as idempotent operation, such as;

$$(A \circ B) \circ B = A \circ B$$

$$(A \bullet B) \bullet B = A \bullet B$$
$$(((A \circ B) \bullet C) \circ B) \bullet C = (A \circ B) \bullet C$$
$$(((A \bullet B) \circ C) \bullet B) \circ C = (A \bullet B) \circ C$$

4.5. What are the effects of the dilation process ?

Ans. The effects of the dilation process are :
1. Dilation expands the input image.
2. This process affects both the inside and outside borders of the input image.
3. Dilation fills the holes in the image.
4. The dilation process smoothens out the image contour.
5. The dilation process depends on the choice of the structuring element.

4.6. What are the major effects in the erosion process ?

Ans. The effects of the erosion process are :
1. The erosion process shrinks the given input image.
2. Pixels and holes inside the image are removed by the erosion process.
3. This process removes small objects like noise.

4.7. What is Beucher gradient ?

Ans. The Beucher gradient is defined as :

$$\rho = (X \oplus B) - (X \ominus B),$$

where X is the input image and B is the structuring element. This Beucher gradient is also called the gradient operator.

4.8. What is geodesic dilation ?

Ans. Geodesic dilation is the result obtained after dilating the image X is masked using a mask image g. This can be defined as $X \oplus_g B = (X \oplus B) \cap g$.

4.9. What is geodesic erosion ?

Ans. Geodesic erosion is the result after erosion X is masked with a mask image g. It is defined as $X \ominus_g B = (X \ominus B) \cup g$.

4.10. Mention the properties of opening and closing operators.

Ans. Properties of opening and closing operators :
1. Opening and closing are dual operators.
2. Closing is an extensive transform.
3. Opening is an anti-extensive transform.
4. Opening and closing keep the ordering relation between two images.
5. Opening and closing are idempotent transforms.

4.11. What is the use of the closing operation ?

Ans. The closing operation is used to smoothen the contour of the input image. It can be used to connect small gaps in the image and also used to remove small holes in the input image.

4.12. List some properties of morphological operations.

Ans. Properties of morphological operations :
1. Increasing
2. Expansivity
3. Duality
4. Chain rule
5. Idempotency

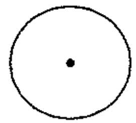
4.13. Draw the skeleton of :

a. Circle

b. Triangle

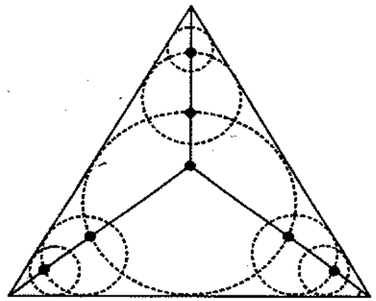
Ans.

a. The skeleton of a circle is the same circle.



(a)

b. The skeleton of a triangle is shown in Fig. 2(b)



(b)

Fig. 2.

4.14. Find the shape number and order of the given boundary. Use 4-chain code.

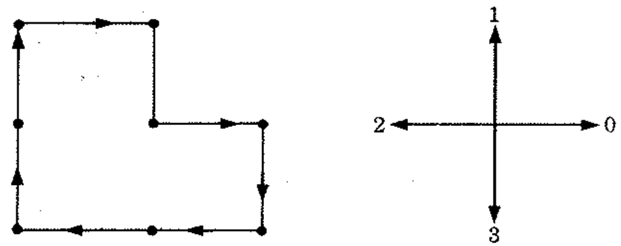


Fig. 3.

Ans.

4-direction chain code	: 0 3 0 3 2 2 1 1
First difference	: 3 1 3 3 0 3 0
Circular first difference	: 3 3 1 3 3 0 3 0
Shape number (Smallest magnitude of circular first difference)	: 0 3 0 3 3 1 3 3
Order	: 8

4.15. Define the term skeletons.

Ans. Skeletons are the minimal representation of objects in an image while retaining the Euler number of the image. The Euler number is the number of objects in an image minus the number of holes in those objects. The skeletons of objects in an image can be found by successive thinning until stability is reached.

4.16. What is a hit-or-miss transformation ?

Ans. The hit-or-miss transformation is the morphological operator used for finding local patterns of pixels. Here, local refers to the size of the structuring element. A hit-or-miss transform is a variant of template matching that finds collection of pixels with certain shape properties.

4.17. Give a few uses of morphological operations in the field of image processing.

Ans. **Uses of morphological operations are :**

1. The morphological operators can be used to remove noise in the image which is widely used in image preprocessing.
2. The morphological operators can be used to enhance the object structure which includes region thinning, thickening, convex hull etc.
3. Morphological operators can be used to segment objects from the background.
4. Morphological operators are effective in the quantitative description of objects.

4.18. Find the number of bits required to store a 256 × 256 image with 32 gray levels. **AKTU 2016-17, Marks 02**

Ans. For binary image, one bit is sufficient for representing the pixel value. So, the number of bits required will be 256 × 256 × 1 bits

= 65536 bits

= 8192 byte

= 8.192 kB

4.19. What do you mean by dilation and erosion ?

AKTU 2016-17, Marks 02

AKTU 2017-18, Marks 02

OR

Define dilation process. **AKTU 2018-19, Marks 02**

Ans. **Dilation :** Dilation is a morphological operation that adds pixels to the boundaries of objects in an image.

With A and B as two sets in Z^2 (2D integer space), the dilation of A by B is defined as,

$$A \oplus B = \{Z \mid (\hat{B})_z \cap A \neq \phi\}$$

Erosion : Erosion is a morphological operation that reduces the number of pixels from the object boundary. The number of pixels removed depends on the size of the structuring element.

For image A and structuring element B in the Z^2 (2D integer space), erosion is defined as,

$$A \ominus B = \{Z \mid [(\hat{B})_z \in A\}$$

4.20. Explain the concept of thresholding.

AKTU 2016-17, Marks 02

Ans. Thresholding is segmentation techniques used to produce regions of uniformity within the given image based on some threshold criteria T.

4.21. What are the different approaches for segmentation ?

AKTU 2015-16, Marks 02

Ans. **Different approaches for segmentation are :**

1. Region approach
2. Boundary approach
3. Edge approach

4.22. In which situation we use region merging and region splitting ? **AKTU 2015-16, Marks 02**

Ans. Region merging technique is used when homogeneous regions are small. Region splitting technique is used when the regions in an image are homogeneous.

4.23. What are the issues involved for stereo imaging problem ?

AKTU 2015-16, Marks 02

Ans. **Issues involved for stereo imaging problem are :**

1. Phase cancellation :

a. Phase cancellation happens when the two microphones are placed so that they each receive the sound at slightly different times.

b. When this occurs, we do not hear the bass as well because the low frequencies drop off.

c. Improper mic placement or two mics that are out of phase which one another can cause phase cancellation.

2. Poor stereo imaging :

a. Poor stereo imaging occurs when we cannot tell where things fall from left to right (or right to left, if that's the way we think), or when we can not hear a clear center point in the sound.

b. Poor stereo imaging is more difficult to correct than phase cancellation, but we can fix it.

4.24. What is the use of boundary extraction?

AKTU 2018-19, Marks 02

Ans. Boundary extraction is used for isolating the boundary pixels in a real image.

4.25. What is meant by illumination and reflectance ?

AKTU 2016-17, Marks 02

OR

What is meant by reflectance ? **AKTU 2017-18, Marks 02**

Ans. **Illumination :** Illumination is the amount of light incident on the object/image.

Reflectance : Reflectance is the amount of light reflecting from the object/image.

☺☺☺

5 UNIT

Image Compression and Recognition (2 Marks Questions)

5.1. Define image compression.

Ans. Image compression is the process of minimizing the size in bytes of a graphics file without degrading the quality of the image to an unacceptable level. The reduction in file size allows more images to be stored in a given amount of disk or memory space.

5.2. Why we need data compression.

Ans. Data compression is needed because it helps to reduce the consumption of expensive resources such as a hard disk space or transmission bandwidth.

5.3. What are the applications of data compression ?

Ans. **Applications of data compression are :**

1. Audio
2. Video
3. Genetics
4. Emulation

5.4. Define Huffman coding.

Ans. Huffman coding is a lossless statistical method that always finds a variable-length code with minimum redundancy. It uses a binary encoding tree for representing commonly occurring values in few bits and less common values in more bits.

5.5. What is run-length encoding ?

Ans. Run-Length Encoding (RLE) is a simple form of lossless data compression in which runs of data (sequences in which the same data value occurs in many consecutive data elements) are stored as a single data value and count, rather than as the original run.

5.6. Define arithmetic coding.

Ans. Arithmetic coding is a data compression technique that encodes data by creating a code string which represents a fractional value on the number line between 0 and 1.

5.7. Define JPEG and MPEG.

Ans. JPEG :

1. It stands for Joint Photographic Experts Group.
2. It uses lossy compression technique.

MPEG :

1. It stands for Moving Picture Experts Group.
2. MPEG file uses the .mp3 file name extension.

5.8. What are different approaches used in boundary representation ?

Ans. Different approaches used in boundary representation are :

1. Minimum perimeter polygons
2. Merging techniques
3. Splitting techniques
4. Signature
5. Boundary segments
6. Skeleton

5.9. Define the term topology.

Ans. Topology is defined as the study of properties of a figure that are unaffected by any deformation, as long as there is no tearing or joining of figure

5.10. What are the approaches used in region description ?

Ans. Approaches used in region description are :

1. Statistical approaches
2. Structural approaches
3. Spectral approaches

5.11. Explain counter predictive coding.

AKTU 2017-18, Marks 02

Ans. Counter predictive coding is image compression algorithm which is used to remove mutual dependency between the successive pixels and then perform encoding.

☺☺☺

B. Tech.
(SEM. VII) EVEN SEMESTER THEORY EXAMINATION, 2014-15
DIGITAL IMAGE PROCESSING

Time : 3 Hours **Max. Marks : 100**

Note : Attempt **all** questions.

1. Attempt any **four** parts of the following : (5 × 4 = 20)

a. What is digital image processing ? Discuss some of its major applications.

Ans. Refer Q. 1.1, Page 1–2A, Unit-1.

b. Consider two image subsets S_1 and S_2 as shown in the Fig. 2. For V = [0] determine whether the regions are :

i. 4-Adjacent ii. 8-Adjacent iii. m-Adjacent

Give reasons for your answer.

S_1				S_2			
1	1	1	1	1	1	0	0
1	1	0	1	1	0	1	1
1	1	0	1	0	0	1	1
1	0	0	0	1	1	1	1

Fig. 1.

Ans. Refer Q. 2.8, Page 2–11A, Unit-2.

c. Write short notes on :

i. Sampling & quantization

ii. Homomorphic filtering

Ans.

i. Sampling and quantization : Refer Q. 1.8, Page 1–11A, Unit-1.

ii. Homomorphic filtering : Refer Q. 2.27, Page 2–37A, Unit-2.

d. Given $H(u, v)$ as follows. Discuss its frequency response.

	$\frac{1}{6}$	
$\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{6}$
	$\frac{1}{6}$	

Fig. 2.

Ans. Refer Q. 2.25, Page 2–34A, Unit-2.

e. Find the DFT of $f(x)$ = {0, 1, 2, 1}.

Ans. Refer Q. 1.19, Page 1–19A, Unit-1.

2. Attempt any **four** parts of the following : (5 × 4 = 20)

a. What is bit-plane slicing ? Given the following 3 × 3 image, find its bit-planes.

1	2	3
4	5	0
7	2	1

Fig. 3.

Ans. Refer Q. 2.4, Page 2–7A, Unit-2.

b. Write short notes on the following :

i. Gamma correction

ii. Piecewise-linear transformation

Ans. Refer Q. 2.9, Page 2–12A, Unit-2.

c. Consider the following image. What will be the new value of the pixel (2, 2) if smoothing is done using a 3 × 3 :

0	1	0	2	7
2	7	7	4	0
5	6	4	3	3
1	1	0	7	5
5	4	2	2	5

Fig. 4.

i. Mean filter **ii. Weighted average filter**

iii. Median filter **iv. Min filter**

v. Max filter

Ans. Refer Q. 3.8, Page 3–11A, Unit-3.

d. Briefly explain the working of a Laplacian mask. What will be the effect of applying the filter (a) on the image (b) ?

1	1	1
1	–8	1
1	1	1

(a)

50	50	50	50	50	50
50	50	50	50	50	50
50	50	50	50	50	50
100	100	100	100	100	100
100	100	100	100	100	100
100	100	100	100	100	100

(b)

Fig. 5.

Ans. Refer Q. 2.23, Page 2–31A, Unit-2.

e. Perform histogram equalization on the following 8 × 8 image. The gray level distribution of the image is given below :

Gray levels (r_k)	0	1	2	3	4	5	6	7
Number of pixels (p_k)	8	10	10	2	12	16	4	2

Ans. Refer Q. 2.16, Page 2–22A, Unit-2.

3. Attempt any **four** parts of the following : (5 × 4 = 20)

a. In an image the gray scale spans from black to near white in only three increments. A certain noise has corrupted the image. The image and its histogram are as follows. What type of mean filters can you use to eliminate the noise ? Explain.

Fig. 6.

Ans. Refer Q. 2.19, Page 2–25A, Unit-2.

b. Give a model of image degradation/restoration process.

Ans. Refer Q. 3.1, Page 3–2A, Unit-3.

c. What is the difference between image enhancement and image restoration ? Mention some important causes of image degradation.

Ans. Refer Q. 3.2, Page 3–3A, Unit-3.

d. Explain any two noise models in detail.

Ans. Refer Q. 3.3, Page 3–4A, Unit-3.

e. What are order-statistic filters ?

Ans. Refer Q. 3.4, Page 3–7A, Unit-3.

4. Attempt any **four** parts of the following : (5 × 4 = 20)

a. Let *A* be an image and *B* a structuring element, given as follows. Find $A \ominus B$ and $A \oplus B$. Note : *X* denotes the origin, which is not part of the structuring element.

	1	1	1		
	1		1		
	1		1	1	
				1	1
X					

1	X	1

Fig. 7.

Ans. Refer Q. 4.9, Page 4–11A, Unit-4.

b. Thin the following image. Show the image after each step.

Fig. 8.

Ans. Refer Q. 4.10, Page 4–12A, Unit-4.

c. **Extract the connected component from the following image :**

0	0	0	0	0	0	0	0	0
0	0	1	1	1	0	0	0	0
0	0	1	1	0	1	1	0	0
0	1	1	1	1	1	1	1	0
0	1	1	1	0	1	1	0	0
0	0	0	0	0	0	0	0	0

F*ig. 9.

Ans. Refer Q. 4.11, Page 4–13A, Unit-4.

d. **Explain the procedure of region filling with an example.**

Ans. Refer Q. 4.12, Page 4–14A, Unit-4.

e. **Prove that opening and closing are dual transformations.**

Ans. Refer Q. 4.16, Page 4–16A, Unit-4.

5. Attempt any **four** parts of the following : **(5 × 4 = 20)**

a. **Prove that rotation and translation are not commutative operations.**

Ans. Refer Q. 4.17, Page 4–18A, Unit-4.

b. **What is shearing ? Give the transformation matrix and its inverse to carry out shearing in both x-and y-directions with shearing factors 10 and 30.**

Ans. Refer Q. 4.18, Page 4–19A, Unit-4.

c. **Find a matrix to perform the following transformations to an object :**

i. **Scale in the x-direction using a scale factor 10.**

ii. **Followed by a rotation about z-axis 30 degree.**

Ans. Refer Q. 4.19, Page 4–21A, Unit-4.

d. **Explain the process of image segmentation using region growing.**

Ans. Refer Q. 4.13, Page 4–14A, Unit-4.

e. **Describe the technique of thresholding for image segmentation.**

Ans. Refer Q. 4.14, Page 4–15A, Unit-4.

☺☺☺

B.Tech.

(SEM. VII) EVEN SEMESTER THEORY EXAMINATION, 2015-16

DIGITAL IMAGE PROCESSING

Time : 3 Hours **Max. Marks : 100**

Section – A

1. Attempt **all** parts. All parts carry equal marks. Write answer of each part in short : **(2 × 10 = 20)**

a. **What do you understand by Weber ratio ? What does a low value for Weber ratio indicate ?**

Ans. Refer Q. 1.5, Page SQ–2A, Unit-1, Two Marks Questions.

b. **Consider the following two 8-bit images :**

Fig. 1.

Each of these images has dimensions 20 × 20. Show the histograms of these images. Please note that the borders of the images shown in black are just to highlight the boundaries. The border is not a part of the image.

Ans. Refer Q. 2.1, Page SQ–4A, Unit-2, Two Marks Questions.

c. **What would happen to the dynamic range of an image if all the slopes in the contrast stretched algorithm (l, m, n) are less than one ?**

Ans. Refer Q. 1.6, Page SQ–2A, Unit-1, Two Marks Questions.

d. **Write down the filter mask for Sobel and Prewitt filters.**

Ans. Refer Q. 2.2, Page SQ–5A, Unit-2, Two Marks Questions.

e. **What are the different approaches for segmentation ?**

Ans. Refer Q. 4.21, Page SQ–19A, Unit-4, Two Marks Questions.

f. **In which situation we use region merging and region splitting ?**

Ans. Refer Q. 4.22, Page SQ–19A, Unit-4, Two Marks Questions.

g. **Derive, why we multiply with $(-1)^{x+y}$ in case of frequency domain filtering ?**

Ans. Refer Q. 1.13, Page SQ–3A, Unit-1, Two Marks Questions.

h. **What steps are related with high level processing in digital image processing ?**

Ans. Refer Q. 1.7, Page SQ–2A, Unit-1, Two Marks Questions.

i. **Draw the graph for power law (Gamma) transformation (for gamma > 1).**

Ans. Refer Q. 2.3, Page SQ–6A, Unit-2, Two Marks Questions.

j. What are the issues involved for stereo imaging problem ?

Ans. Refer Q. 4.23, Page SQ–19A, Unit-4, Two Marks Questions.

Section – B

Note : Attempt any **five** questions from this section : **(10 × 5 = 50)**

2. What do you understand by digital image processing ? Explain the components of an image processing system.

Ans. Refer Q. 1.2, Page 1–3A, Unit-1.

3. Given the image A :

0	0	0	0	0	0
0	0	1	1	0	0
0	1	1	1	1	0
0	0	1	1	0	0
0	0	0	0	0	0

And structuring element B :

1
1
1

Compute **i. A dilated by B**
ii. A^c eroded by B

Ans. Refer Q. 4.20, Page 4–22A, Unit-4.

4. Consider the image segment :

3	4	1	2	0
0	1	0	4	2(*q*)
2	2	3	1	4
3(*p*)	0	4	2	1
1	2	0	3	4

Let $V = \{2, 3, 4\}$. Compute the lengths of the shortest-8 and path between p and q. If a particular path does not exist between these two points, explain why ? What is the significance of 'm' path ?

Ans. Refer Q. 2.11, Page 2–15A, Unit-2.

5. Derive the frequency domain transformation function $H(u, v)$ for the following spatial domain filter $h(x, y)$.

0	–1	0
–1	8	–1
0	–1	0

How homomorphic filtering is implemented ?

Ans. Refer Q. 2.27, Page 2–37A, Unit-2.

6. Draw the diagram for image restoration/degradation process. Explain the linear position invariant property of degradation function.

Ans. Refer Q. 3.5, Page 3–8A, Unit-3.

7. Explain periodic noise reduction using band reject filter.

Ans. Refer Q. 3.9, Page 3–12A, Unit-3.

8. Explain convex hull with the help of an example.

Ans. Refer Q. 4.21, Page 4–23A, Unit-4.

9. State and explain various approaches used for edge detection.

Ans. Refer Q. 4.22, Page 4–24A, Unit-4.

Section – C

Note : Attempt any **two** questions from this section : **(15 × 2 = 30)**

10. What are the linear and non-linear smoothing filters in spatial domain ? Compute the new pixel values after applying the 3*3 box filter on the following 5*5 matrix of an 8-bit image.

139	128	237	126	129
145	129	123	89	132
146	122	128	87	135
141	125	134	131	139
112	127	138	133	142

1/9*

1	1	1
1	1	1
1	1	1

Ans. Refer Q. 2.28, Page 2–38A, Unit-2.

11. Write the procedures for boundary extraction and region filling. Mention at least one real life application of both. What is the result of applying successive opening on the same set with the same structuring element ?

Ans. Refer Q. 4.23, Page 4–28A, Unit-4.

12. Write short notes on :

a. Chain code

b. Skeletons/MAT

c. Hough transform for boundary shape detection

Ans. Refer Q. 4.24, Page 4–28A, Unit-4.

☺☺☺

B.Tech.
(SEM. VIII) EVEN SEMESTER THEORY EXAMINATION, 2016-17
DIGITAL IMAGE PROCESSING

Time : 3 Hours **Max. Marks : 100**

Note : Be precise in your answer. In case of numerical problem assume data wherever not provided.

SECTION-A

1. Attempt **all** parts of the following questions : **(2 × 10 = 20)**

a. **Define image. What is dynamic range ?**

Ans. Refer Q. 1.1, Page SQ–1A, Unit-1, Two Marks Questions.

b. **What is meant by illumination and reflectance ?**

Ans. Refer Q. 4.25, Page SQ–20A, Unit-4, Two Marks Questions.

c. **Find the number of bits required to store a 256 × 256 image with 32 gray levels.**

Ans. Refer Q. 4.18, Page SQ–18A, Unit-4, Two Marks Questions.

d. **Explain the types of connectivity.**

Ans. Refer Q. 2.22, Page SQ–10A, Unit-2, Two Marks Questions.

e. **What is contrast stretching ?**

Ans. Refer Q. 2.20, Page SQ–10A, Unit-2, Two Marks Questions.

f. **What do you mean by dilation and erosion ?**

Ans. Refer Q. 4.19, Page SQ–18A, Unit-4, Two Marks Questions.

g. **Explain noise model.**

Ans. Refer Q. 3.15, Page SQ–13A, Unit-3, Two Marks Questions.

h. **List edge detection operators.**

Ans. Refer Q. 2.21, Page SQ–10A, Unit-2, Two Marks Questions.

i. **Explain Affine transform.**

Ans. Refer Q. 2.8, Page SQ–7A, Unit-2, Two Marks Questions.

j. **Explain the concept of thresholding.**

Ans. Refer Q. 4.20, Page SQ–19A, Unit-4, Two Marks Questions.

SECTION-B

2. Attempt any **five** parts of the following questions : **(10 × 5 = 50)**

a. **What is digital image processing ? Draw a block diagram. And discuss some of its major applications.**

Ans. Refer Q. 1.1, Page 1–2A, Unit-1.

b. **Write a short note on :**

i. **Sampling and quantization**

ii. **Homomorphic filtering**

Ans.

i. Refer Q. 1.8, Page 1–11A, Unit-1.

ii. Refer Q. 2.27, Page 2–37A, Unit-2.

c. **Explain histogram equalization. And equalize the given histogram.**

Gray level	0	1	2	3	4	5	6	7
Number of Pixel	790	1023	850	656	329	245	122	81

Ans. Refer Q. 2.15, Page 2–19A, Unit-2.

d. **Define boundary extraction. Perform boundary extraction on image *A* with the help of structuring element *B*.**

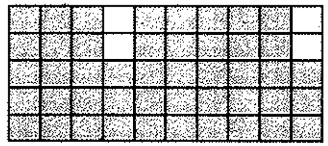
A = 

Fig. 3.

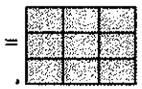
B = 

Fig. 4.

Ans. Refer Q. 4.27, Page 4–31A, Unit-4.

e. **What is noise ? Define any two noise models in detail.**

Ans. Refer Q. 3.3, Page 3–4A, Unit-3.

f. **What is geometric transformation ? Also discuss Euclidean transformation.**

Ans. Refer Q. 4.28, Page 4–33A, Unit-4.

g. **How dilation and erosion is used in morphological operations ? How it is used in opening and closing operations ?**

Ans. Refer Q. 4.29, Page 4–34A, Unit-4.

h. Write a short note on :
i. Image segmentation
ii. Sampling and quantization
iii. Illumination and reflectance

Ans. i. Refer Q. 4.1, Page 4–2A, Unit-4.
ii. Refer Q. 1.8, Page 1–11A, Unit-1.
iii. Refer Q. 4.25, Page SQ–20A, Unit-4, Two Marks Questions.

SECTION-C

Attempt any **two** parts of the following question. **(15 × 2 = 30)**

3. What are the different stages of digital image processing ? Explain each stage in detail.

Ans. Refer Q. 1.7, Page 1–8A, Unit-1.

4. Explain the following in details :
i. Stereo imaging
ii. Region filling
iii. Convex hull

Ans. Refer Q. 4.21, Page 4–23A, Unit-4.

5. What is image restoration ? Draw and explain the basic block diagram of the restoration process. Give two areas where restoration process can be applied.

Ans. Refer Q. 3.1, Page 3–2A, Unit-3.

☺☺☺

B. Tech.
(SEM. VIII) EVEN SEMESTER THEORY EXAMINATION, 2017-18
DIGITAL IMAGE PROCESSING

Time : 3 Hours **Max. Marks : 100**

Note : Attempt **all** Section. If require any missing data; then choose suitable.

SECTION-A

1. Attempt **all** questions in brief. **(2 × 10 = 20)**

a. Define Image. What is range ?

Ans. Refer Q. 1.1, Page SQ–1A, Unit-1, Two Marks Questions.

b. What is meant by reflectance ?

Ans. Refer Q. 4.25, Page SQ–20A, Unit-4, Two Marks Questions.

c. What is meant by binary image, colour image, gray-scale image ?

Ans. Refer Q. 1.2, Page SQ–1A, Unit-1, Two Marks Questions.

d. Explain harmonic mean filter.

Ans. Refer Q. 2.7, Page SQ–7A, Unit-2, Two Marks Questions.

e. What is contrast stretching ?

Ans. Refer Q. 2.20, Page SQ–10A, Unit-2, Two Marks Questions.

f. What do you mean by dilation and erosion ?

Ans. Refer Q. 4.19, Page SQ–18A, Unit-4, Two Marks Questions.

g. Explain counter predictive coding.

Ans. Refer Q. 5.11, Page SQ–22A, Unit-5, Two Marks Questions.

h. List edge detection operators.

Ans. Refer Q. 2.21, Page SQ–10A, Unit-2, Two Marks Questions.

i. Explain Affine transform.

Ans. Refer Q. 2.8, Page SQ–7A, Unit-2, Two Marks Questions.

j. Explain the concept of thresholding.

Ans. Refer Q. 4.20, Page SQ–19A, Unit-4, Two Marks Questions.

SECTION-B

2. Attempt any **three** of the following : **(10 × 3 = 30)**

a. What do you mean by digital image representation ?

Ans. Refer Q. 1.3, Page 1–4A, Unit-1.

b. Compare and contrast between linear spatial filtering and non-linear spatial filtering.

Ans. Refer Q. 2.30, Page 2–44A, Unit-2.

c. What is image restoration ? Draw and explain the basic block diagram of the restoration process. Give two areas where restoration process can be applied.

Ans. Refer Q. 3.1, Page 3–2A, Unit-3.

d. What do you understand by hit-miss transform and why they are used explain in brief ?

Ans. Refer Q. 2.12, Page 2–15A, Unit-2.

e. Prove that Prewitt and Sobel operator act as a low pass and high pass filter.

Ans. Refer Q. 2.31, Page 2–45A, Unit-2.

SECTION-C

3. Attempt any **one** part of the following : **(10 × 1 = 10)**

a. Explain region based segmentation with an example.

Ans. Refer Q. 4.3, Page 4–4A, Unit-4.

b. Explain intensity transformations in details. What would happen to the dynamic range of an image if all the slopes in the contrast stretched algorithm (l, m, n) are less than 1 ? Answer using illustration.

Ans. Refer Q. 4.4, Page 4–5A, Unit-4.

4. Attempt any **one** part of the following : **(10 × 1 = 10)**

a. Explain intensity transformations in details. What would happen to the dynamic range of an image if all the slopes in the contrast stretched algorithm (l, m, n) are less than 1 ? Answer using illustration.

Ans. Refer Q. 4.4, Page 4–5A, Unit-4.

b. Explain the Hough Transforms to join the points. And also explain the problem of HT with their solutions. Given the four points in the x-y plane with the following coordinates (1,1), (2,2), (3,3), (4,4). Use Hough Transform to join these points.

Ans. Refer Q. 4.5, Page 4–6A, Unit-4.

5. Attempt any **one** part of the following : **(10 × 1 = 10)**

a. What is histogram equalization ?

Ans. Refer Q. 2.15, Page 2–19A, Unit-2.

b. Explain Laplacian filter.

Ans. Refer Q. 2.25, Page 2–34A, Unit-2.

6. Attempt any **one** part of the following : **(10 × 1 = 10)**

a. Explain opening and closing operation for gray-scale image processing.

Ans. Refer Q. 4.25, Page 4–30A, Unit-4.

b. Describe fundamental operations of morphological image processing.

Ans. Refer Q. 4.26, Page 4–31A, Unit-4.

7. Attempt any **one** part of the following : **(10 × 1 = 10)**

a. Write short note on following :

i. Region extraction

Ans. Refer Q. 4.23, Page 4–28A, Unit-4.

ii. Image registration

Ans. Refer Q. 4.8, Page 4–10A, Unit-4.

b. Write short note on following :

i. Edge detection algorithm

Ans. Refer Q. 4.6, Page 4–8A, Unit-4.

ii. Line detection algorithm

Ans. Refer Q. 4.6, Page 4–8A, Unit-4.

☺☺☺

B. Tech.
(SEM. VIII) EVEN SEMESTER THEORY EXAMINATION, 2018-19
DIGITAL IMAGE PROCESSING

Time : 3 Hours **Max. Marks : 100**

Note : Attempt **all** Section. If require any missing data; then choose suitable.

SECTION-A

1. Attempt **all** questions in brief. (2 × 10 = 20)

a. Define components of image processing system.

Ans. Refer Q. 1.3, Page SQ–1A, Unit-1, Two Marks Questions.

b. Name some applications of digital image processing.

Ans. Refer Q. 1.4, Page SQ–1A, Unit-1, Two Marks Questions.

c. Write down mask of Sobel filter.

Ans. Refer Q. 2.2, Page SQ–5A, Unit-2, Two Marks Questions.

d. Differentiate between image enhancement and restoration.

Ans. Refer Q. 3.12, Page SQ–13A, Unit-3, Two Marks Questions.

e. Explain Laplacian filter.

Ans. Refer Q. 2.4, Page SQ–6A, Unit-2, Two Marks Questions.

f. Define dilation process.

Ans. Refer Q. 4.19, Page SQ–18A, Unit-4, Two Marks Questions.

g. What is the use of boundary extraction?

Ans. Refer Q. 4.24, Page SQ–20A, Unit-4, Two Marks Questions.

h. Define morphological image processing.

Ans. Refer Q. 4.1, Page SQ–14A, Unit-5, Two Marks Questions.

i. What is geometric transformation ?

Ans. Refer Q. 2.5, Page SQ–7A, Unit-2, Two Marks Questions.

j. What are first order derivative filters ?

Ans. Refer Q. 2.6, Page SQ–7A, Unit-2, Two Marks Questions.

SECTION-B

2. Attempt any **three** of the following : (10 × 3 = 30)

3	1	2	1 (q)
2	2	0	2
1	2	1	1
(p) 1	0	1	2

i. Take $V = \{0, 1\}$ and compute the lengths of shortest 4, 8, and m-path between p and q. If a particular path does not exist between these two points. Explain why ?

ii. Repeat for $V = \{1, 2\}$

Ans. Refer Q. 2.10, Page 2–14A, Unit-2.

b. What are the linear and non-linear smoothing filters in spatial domain ? Compute the new pixel values after applying the 3 × 3 box filter on the following 5 × 5 matrix of a 3-bit image.

2	3	7	0	7
6	4	4	1	5
1	3	2	2	3
4	5	0	6	4
6	1	7	6	4

Ans. Refer Q. 2.29, Page 2–41A, Unit-2.

c. Explain edge detection and edge linking. Also write the difference between edge detection and edge linking.

Ans. Refer Q. 4.7, Page 4–9A, Unit-4.

d. What is image restoration ? Explain in detail the image restoration in presence of noise only.

Ans. Refer Q. 3.1, Page 3–2A, Unit-3.

e. Explain the following morphological operations :

i. Opening ii. Closing iii. Region filling

Ans. Refer Q. 4.25, Page 4–30A, Unit-4.

SECTION-C

3. Attempt any **one** part of the following : (10 × 1 = 10)

a. Explain low level, mid level and high level image processing. Also explain sampling and quantization process.

Ans. Refer Q. 1.10, Page 1–12A, Unit-1.

b. Explain the process of filtering in frequency domain. Discuss low pass and high pass frequency domain filters.

Ans. Refer Q. 2.24, Page 2–34A, Unit-2.

4. Attempt any **one** part of the following : **(10 × 1 = 10)**

a. Explain piecewise-linear transformations of image enhancement with suitable example.

Ans. Refer Q. 2.9, Page 2–12A, Unit-2.

b. Write notes on :

i. Bit-plane slicing **ii. Homomorphic filter**

iii. Image histogram

Ans.

i. **Bit-plane slicing :** Refer Q. 2.3, Page 2–3A, Unit-3.

ii. **Homomorphic filter :** Refer Q. 2.27, Page 2–37A, Unit-3.

iii. **Image histogram :** Refer Q. 2.13, Page 2–17A, Unit-2.

5. Attempt any **one** part of the following : **(10 × 1 = 10)**

a. Discuss order-statistics filters with suitable example.

Ans. Refer Q. 3.4, Page 3–7A, Unit-3.

b. Explain the process of minimum mean square error restoration.

Ans. Refer Q. 4.31, Page 4–38A, Unit-4.

6. Attempt any **one** part of the following : **(10 × 1 = 10)**

a. Explain thinning and thickening operation with suitable example.

Ans. Refer Q. 4.30, Page 4–35A, Unit-4.

b. Prove the validity of the duality expressions $(A \bullet B)^C = \left(A^C o \hat{B}\right)$ and $(AoB)^C = \left(A^C o \hat{B}\right)$.

Ans. Refer Q. 4.32, Page 4–40A, Unit-4.

7. Attempt any **one** part of the following : **(10 × 1 = 10)**

a. Explain :

i. Stereo imaging

ii. Multi-level thresholding

iii. Image registration

Ans. Refer Q. 4.8, Page 4–10A, Unit-4.

b. The so-called compass gradient operators of size 3 × 3 are designed to measure gradients of edges oriented in eight directions : E, NE, N, NW, W, SW, S, and SE. Give the form of these eight operators using coefficients valued 0, 1, or – 1.

Ans. Refer Q. 4.33, Page 4–40A, Unit-4.

☺☺☺