Efficient Distributed Estimation using Adaptive Value of Information based Self-censoring

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Outline

Distributed Sensing Under Uncertainties and Communication Constraints

- Variety of Multi-agent Distributed Systems
  - Military: formation controls
  - Robotics: team cooperation
  - Agriculture: soil condition monitoring

Multi-agent Distributed Missions

- **Problem**: estimate global parameters to maintain situation awareness and consistency
- **Challenge**
  - dynamic system, uncertain environments, constrained resources

Literature Review

- **Full-Relay**[?]: all measurements are broadcast or relayed
  - Comparable to centralized estimation
  - Inefficient: communication cost very high

- **Consensus** [? ? ? ? ?]: agents average parameters with neighbors
  - Example: Consensus; Gossip
  - Works for arbitrary connected network
  - Comm. cost lower than FR, but agents still communicate at all times
  - Purposefully censoring agents will lead to bias [?]

  - **Random Relay** [?]: randomly censor sensors
    - Communication cost reduced and no bias
    - Randomness in performance and longer convergence time

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Literature Review

- **Graphical Models** [? ? ? ?]: agent correlate local distributions
  - Example: channel filters, Bayesian network, Markov random field

  - **Censoring** [? ? ? ? ?]: compute *Value of Information* (VoI) and censor uninformative measurements/agents
    - Comm. cost significantly reduced without much performance loss

  - Not easily scalable to cyclic graphs
    - Multiple paths between two agents $\Rightarrow$ duplicate messages
    - Approximate algorithms leads to bias
    - Exact methods have high overhead computation and communication

Main Result (Mu et al. [? ?])

- Developed Value of Information based Distributed Sensing (VoIDS)
  - Differentiate highly informative agents from less informative ones
  - Agents self-censor when measurements have low-value information
  - Works for arbitrary connected network topologies
  - Trade-off between comm. cost and performance
- Developed **Adaptive VoIDS** (A-VoIDS)
  - Strikes **balance between comm. cost and estimation error**
- Gave theoretical bounds on performance of VoIDS and A-VoIDS

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Bayesian Inference and Conjugacy

- **Bayesian update** (θ: parameters; ω: hyperparameters)

$$p(\theta|z,\omega) = \frac{p(z|\theta)p(\theta|\omega)}{\int p(z|\theta)p(\theta|\omega)\,d\theta}$$

  - **closed from solution** exists for exponential family distributions
    $\Rightarrow$ efficient Bayesian inference

- **Conjugacy**
  - **Exponential Family likelihood**: $p(\mathbf{x}|\theta) = \exp\left\{\theta^T T(\mathbf{x}) - A(\theta)\right\}$
    - $T(\mathbf{x})$: sufficient statistics; $A(\theta)$: log partition
  - **Conjugate prior**: $p(\theta|\omega,\nu) = \exp\left\{[\omega,\nu]^T[\theta,-A(\theta)] - \Lambda(\omega,\nu)\right\}$
    - ω, ν: hyperparameters; Λ: log partition of conjugate prior
  - **Posterior**: same form as prior; with additive update to hyperparameters

$$\omega \leftarrow \omega + T(\mathbf{z}), \quad \nu \leftarrow \nu + n$$

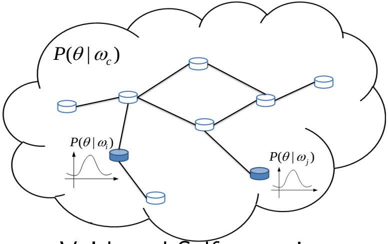
Two Compared Method

- **Hyperparameter Consensus (HPC)** [?]: Linear consensus on hyperparameters:
  - Guaranteed to asymptotically converge to centralized Bayesian estimate
  - Agents continuously communicate with neighbors

- **Random Relay** [?]:
  - **Censoring + Broadcasting**
    - Agents randomly censor themselves
    - Uncensored agents form **Active Set** C, while self-censored agents act as relays (**Relay Set**)
  - Works for dynamic, cyclic network topologies, comm. cost reduced
  - Wide variance in performance, longer convergence time
  - Approach ignores **Value of Information (VoI)**

Key idea: VoI based Self-censoring

- Agents compute VoI of local measurements, compare with **threshold** V^*
- **Informative Set** C
  - An agent declares itself as informative when VoI $> V^*$
  - Informative agents broadcast their messages to neighbors
- **Relay Set**
  - An agent becomes a relay if VoI $\leq V^*$
  - Uninformative agents censor themselves but relay messages for others



VoI based Self-censoring

- Agents estimate global parameters $P(\theta|\omega_c)$
- Dark ones are informative agents with higher VoI
- White ones with lower VoI censor themselves

Value of Information Metric

- **Divergence**: dissimilarity between two distributions
  - Many possible measures [**? ? ?**], usually hard to compute
  - **Kullback-Leibler (KL) divergence** [**? ?**]

$$\mathrm{D}_{KL}(P\|Q) = \int \ln \frac{P(x)}{Q(x)}\, dQ(x)$$

    - Closed-form solution for exponential family distributions
    - Can get exact value with little computation

- **Value of Information**: KL divergence between prior and posterior

$$\mathrm{VoI}(\omega, \nu, \mathbf{z}) = \mathrm{D}_{KL}(\ p(\theta|\omega, \nu)\ \|\ p(\theta|\mathbf{z}, \omega, \nu)\ )$$

  - Closed-form for exponential family distributions [**?**]

VoI Realized Distributed Sensing (VoIDS)

Algorithm 1 VoIDS

initiate hyperparameters $\omega[0], \nu[0]$
for t **do**
  for each agent i **do**
    take measurement, compute local VoI
    if $V_i[t] > V^*$ **then**
      agent i is informative, broadcasts messages
    end if
  end for
  Relay message for informative agents
  for each broadcast message **do**
    update the global posterior
  end for
end for

Performance Guarantees on VoIDS

- Communication Cost

Theorem

Communication frequency of agents → 0 a.s. when $t \to \infty$.

  - Incremental communication cost → 0 a.s when $t \to \infty$.

- Error $e[t]$: KLD between global estimate by agents and centralized Bayes estimate

$$e[t] = D_{\mathrm{KL}}\left(\text{global estimate}\|\text{centralized estimate}\right)$$

Theorem

VoIDS estimation error *is bounded above by* $f(N)\mathrm{V}^*$

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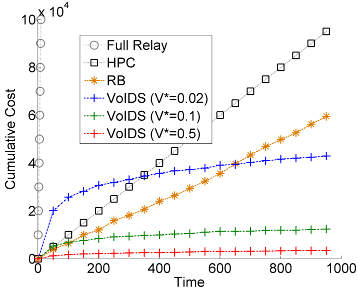
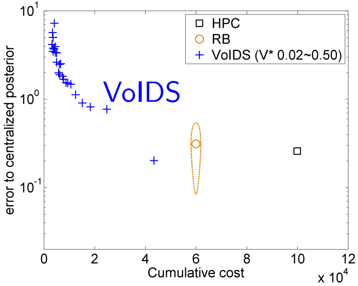
Example: Poisson Distribution

▶ Simulation settings

- Likelihood: Poisson distribution $\mathrm{Poi}(\lambda)$
- Conjugate prior: gamma distribution $\Gamma(\lambda|\alpha,\beta)$
  - Update law: $\alpha \leftarrow \alpha + \mathbf{z}, \quad \beta \leftarrow \beta + n$
- $N = 100$ agents measure corrupted λ: $\lambda_i \sim U(4,6)$



Total Communication Cost



Cost-Error Summary

▶ Communication cost gradually levels off, but error persists

$\Rightarrow$ **trade-off between cost and accuracy**

Adaptive VoI Realized Distributed Sensing (A-VoIDS)

- Control frequency of communication by **adjusting $V^\star$ in response to communication load**

Adaptive VoI

Given c as the desired communication cost,

$$\begin{aligned} &\text{communication cost} < c: && \text{decrease } V^\star \\ &\text{communication cost} \geq c: && \text{increase } V^\star \end{aligned}$$

  - c: targeted communication cost, can be tuned to reflect available communication bandwidth

- Intuition:
  - If many nodes are informative, increase $V^\star$ to reduce communication load
  - If communication load is low, decrease $V^\star$ to increase accuracy

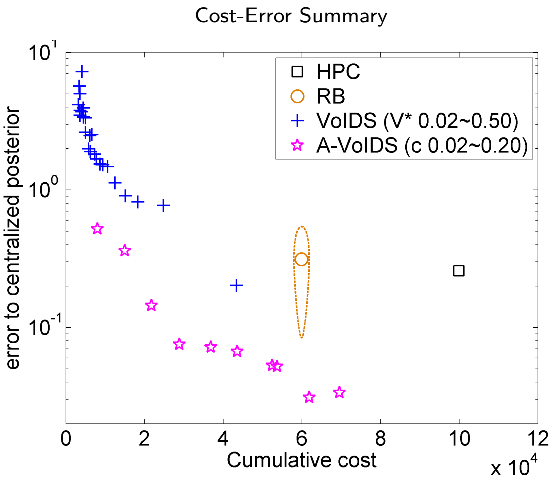
Performance Guarantees of A-VoIDS

Theorem

Estimation error $e[t]$ *satisfies:* $\lim_{t\to\infty} e[t] = 0$ *a.s.*

- Error is asymptotically decreasing $\Rightarrow$ algorithm asymptotically converges to true parameters
- Communication cost in each step is tunable $\Rightarrow$ communication bandwidth can be fully utilized
- Balance between comm. cost and inference error

Simulation Result of Adaptive VoI



- A-VoIDS's performance curve dominates those of other algorithms considered

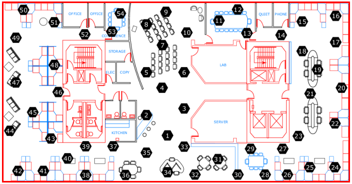
Performance Comparison

Algorithm	**Comm. cost per step**	**KLD Error**
Full Relay	**fixed, high**	**0**
HPC	**fixed, high**	**converge to 0**
Random Broadcast	tunable	converge to 0, randomness
VoIDS	**converge to 0**	**bounded**
A-VoIDS	tunable	**converge to 0**

Intel Lab Dataset [? ? ?]

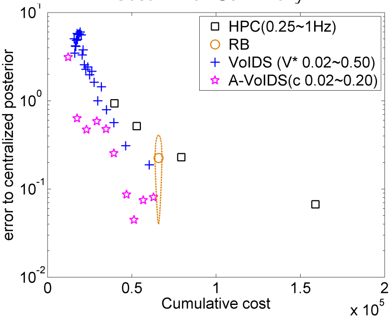
- **Goal**: estimate room temperature distribution
  - Likelihood function: $z \sim \mathcal{N}(\theta, 1)$; conjugate prior: $\theta \sim \mathcal{N}(\mu, \sigma^2)$
  - Update law: $\mu \leftarrow (\mu + \sum_{i=1}^{n} z_i)/(\frac{1}{\sigma^2} + n)$, $\frac{1}{\sigma^2} \leftarrow \frac{1}{\sigma^2} + n$
- Sensors collect data every 30s, update global posterior every 1s.

Sensor Layout



54 sensors in Intel Berkeley Lab.
collect time stamped information
such as temperature, humidity, light

Cost-Error Summary



- A-VoIDS's performance curve closer to bottom-left of plot
  $\Rightarrow$ better balance between accuracy and comm. cost

Conclusions

- Presented Value of Information (VoI) based Distributed Sensing (VoIDS) algorithm
  - Overcome known shortcomings (excessive communication cost and slow convergence speed) of traditional consensus based algorithms
  - Does not require knowledge of network topology
  - Not limited to acyclic networks
  - However, dynamic trade-off exists between estimation accuracy and communication cost
- Presented Adaptive-VoIDS (A-VoIDS) algorithm
  - Adaptively change VoI threshold to better exploit available communication bandwidth
  - Better balance comm. cost and error
- Initial results suggest VoIDS and A-VoIDS work well with real data
- Theoretical bounds on VoIDS and A-VoIDS provided

Future Work

- Consider more complicated situations
  - e.g. multi-variance, dynamic estimation problems
- Other metrics on Value of Information
  - Approximation algorithms and metrics on VoI in no conjugate, no closed form cases
- Non-exponential family distributions
  - e.g. multi-model distributions, non-analytical pdfs
- Information exploitation
  - Mobile agents
  - VoI based planning

References I

VoI Realized Distributed Sensing (VoIDS)

- **Informative Set C** and **Relay Set**
  - Agents locally compute VoI of buffered measurements
  - Broadcast if $\mathrm{VoI} \geq V^\star$; otherwise self-censor local measurements

- **Algorithm**:
  1. Initialize: $\omega_i[0] = \omega, \quad \nu_i[0] = \nu$
  2. Take measurements:

$$S_i[t] = S_i[t-1] + T(z_i[t]), \quad n_i[t] = n_i[t-1] + 1$$

  3. Agents locally check VoI $\boxed{i \in C[t] \quad \text{if} \quad V_i > V^\star}$
  4. Broadcast and relay informative updates $S_{C[t]}[t], \quad n_{C[t]}[t]$
  5. Every node computes posterior

$$\forall i, \, \omega_i[t] = \omega_i[t-1] + \sum_{j \in \nu[t]} S_j[t], \quad \nu_i[t] = \nu_i[t-1] + \sum_{j \in \nu[t]} n_j[t]$$

- Overall cost depends on size of informative set, or V^*

Hyperparameter Consensus Algorithm (HPC)

- Fusion can be done by running linear consensus on hyperparameters
  1. Initialization

$$\omega_i[0] = \omega^- + \beta_i(\omega_i - \omega^-), \quad \nu_i[0] = \nu^- + \beta_i(\nu_i - \nu^-)$$

  2. Measurement update

$$\omega_i[k] \leftarrow \omega_i[k] + \beta_i T(\mathbf{z}_i), \quad \nu_i[k] \leftarrow \nu_i[k] + \beta_i n_i$$

  3. Consensus protocol

$$\omega_i[k+1] = \omega_i[k] + \epsilon \sum_{j \in \mathcal{N}_i} (\omega_j[k] - \omega_i[k]), \quad \nu_i[k+1] = \nu_i[k] + \epsilon \sum_{j \in \mathcal{N}_i} (\nu_j[k] - \nu_i[k])$$

- Variables used
  - $\mathcal{N}_i$: neighborhood of agent i
  - β_i: pre-computed weight of agent i, guarantee convergence on sum. Related to network topology
  - $\epsilon \in (0, 1/\max_i |\mathcal{N}_i|)$ is a weighting constant
- **Theorem** [?]: Algorithm guaranteed to asymptotically converge to Bayesian fused estimate over a strongly connected known network

Censoring

- **Censoring**
  - Censoring output of nodes studied for centralized estimation [**? ? ? ? ? ?**]
  - Cetin et al. [**?**] used a VoI metric to censor measurements on a graphical model in the context of a data association problem
  - Censoring is hard to do on consensus based algorithms ⇒ dynamic network topology causes bias [**?**]
- Censoring + Broadcasting
  - Upon getting a new measurement, agent stores them in a local buffer
  - Agent generates a random number, if bigger than threshold, becomes active (**Active Set** C) and broadcast its updates
  - Inactive agents act as relays (**Relay Set**)
- Highly scalable, can function in dynamic, unknown network topologies

Random Broadcast Algorithm

- **Random Broadcast** (inspired by [**?**])
  1. Initialization: $\omega_i[0] = \omega$, $\nu_i[0] = \nu$
  2. Take measurements: $S[t] = S[t-1] + T(\mathbf{z_i})$, $\nu_i[t] = \nu_i[t-1] + n$
  3. If locally generated random number bigger than a threshold: $i \in C[t]$
  4. Broadcast and relay updates: $S_{C[t]}[t]$, $\nu_{C[t]}[t]$
  5. Every node computes the posterior: $\omega_i[t] = \omega_i[t-1] + \sum_{j \in \nu[t]} S_j[t]$, $\nu_i[t] = \nu_i[t-1] + \sum_{j \in C[t]} n_j[t]$
- Communication cost reduced compared to HPC, but reduced frequency of communication leads to slower convergence
- Wide dispersion in performance (censoring based on a random process)
- Approach ignores **Value of Information (VoI)**

Value of Information Metric

- Alternatives: agents determine VoI for broadcast
  - **Divergence** measure of difference between two distributions
  - Many possible divergence measures [? ? ?]:

Metric	Formula
Kullback-Leibler	$D_{\text{KL}}(p\|\|q) = \int \log(\frac{p}{q})dp(x)$
Renyi	$D_{\alpha}(p\|\|q) = \frac{1}{\alpha-1}\log\int p^{\alpha}q^{1-\alpha}dx$
Chernoff	$D_{c}(p\|\|q) = \log\int p^{\alpha}q^{1-\alpha}dx$
f-divergence	$D_{f}(p\|\|q) = \int f(\frac{p}{q})dq(x)$
Variational	$V(p\|\|q) = \int \|p-q\|dx$
Generalized Matusita	$D_{M}(p\|\|q) = \left[\int \|p^{1/r}-q^{1/r}\|^{r}dx\right]^{1/r},\ r>0$

- Typically no closed-form solutions (special case: Renyi divergence for exponential family)