CV 能量泛函：

$$
\begin{aligned}
F^{CV}\left(c_1, c_2, \phi\right) = {} & \lambda_1 \int_\Omega \left|u_0(x,y) - c_1\right|^2 H_\mathcal{E}(\phi(x,y)) dxdy \\
& + \lambda_2 \int_\Omega \left|u_0(x,y) - c_2\right|^2 \left[1 - H_\mathcal{E}(\phi(x,y))\right] dxdy \\
& + \nu \int_\Omega \left|\nabla H_\mathcal{E}(\phi(x,y))\right| dxdy
\end{aligned}
$$

固定 ϕ 对 c_1 和 c_2 分别求导并使得偏导数为零,

$$
\frac{\partial \mathcal{F}}{\partial c_1} = 0, \frac{\partial \mathcal{F}}{\partial c_2} = 0
$$

$$
\begin{aligned}
\int_\Omega \left(u_0(x,y) - c_1\right) H_\mathcal{E}(\phi(x,y)) dxdy = 0 \\
\int_\Omega \left(u_0(x,y) - c_2\right) \left[1 - H_\mathcal{E}(\phi(x,y))\right] dxdy = 0
\end{aligned}
$$

化简：

$$
\begin{aligned}
\int_\Omega u_0(x,y) H_\mathcal{E}(\phi(x,y)) dxdy - c_1 \int_\Omega H_\mathcal{E}(\phi(x,y)) dxdy = 0 \\
\int_\Omega u_0(x,y) H_\mathcal{E}(\phi(x,y)) dxdy - c_2 \int_\Omega \left[1 - H_\mathcal{E}(\phi(x,y))\right] dxdy = 0
\end{aligned}
$$

可得：

$$
c_1 = \frac{\displaystyle\int_\Omega u_0(x,y) H_\mathcal{E}(\phi(x,y)) dxdy}{\displaystyle\int_\Omega H_\mathcal{E}(\phi(x,y)) dxdy}
$$

$$
c_2 = \frac{\displaystyle\int_\Omega u_0(x,y) H_\mathcal{E}(\phi(x,y)) dxdy}{\displaystyle\int_\Omega \left[1 - H_\mathcal{E}(\phi(x,y))\right] dxdy}
$$

我们有 $Euler - Lagrange$ 方程 (推导过程见 RSF 模型推导)：

$$
\frac{\partial F}{\partial u} - \frac{d}{dx}\left(\frac{\partial F}{\partial u_x}\right) - \frac{d}{dy}\left(\frac{\partial F}{\partial u_y}\right) = 0
$$

对应的梯度下降流方程为：

$$
\frac{\partial u}{\partial t} = -\frac{\partial F}{\partial u} + \frac{d}{dx}\left(\frac{\partial F}{\partial u_x}\right) + \frac{d}{dy}\left(\frac{\partial F}{\partial u_y}\right)
$$

在 F^{CV} 中：

$$F=\lambda_1\left|u_0(x,y)-c_1\right|^2 H_{\mathcal{E}}(\phi(x,y))+\lambda_2\left|u_0(x,y)-c_2\right|^2\left[1-H_{\mathcal{E}}(\phi(x,y))\right]+\nu\left|\nabla H_{\mathcal{E}}(\phi(x,y))\right|$$

$$u=\phi$$

F^{CV} 对应的梯度下降流方程为：

$$\frac{\partial\phi}{\partial t}=-\frac{\partial F}{\partial\phi}+\frac{d}{dx}\left(\frac{\partial F}{\partial\phi_x}\right)+\frac{d}{dy}\left(\frac{\partial F}{\partial\phi_y}\right)$$

$$\frac{\partial F}{\partial\phi}=\lambda_1\left|u_0-c_1\right|^2\delta_{\mathcal{E}}(\phi)-\lambda_2\left|u_0-c_2\right|^2\delta_{\mathcal{E}}(\phi)$$

先求：

$$\begin{aligned}
\frac{\partial F}{\partial\phi_x}&=\nu\frac{\partial\left|\nabla H_{\mathcal{E}}(\phi)\right|}{\partial\phi_x}=\nu\frac{\partial\sqrt{\left(\frac{\partial H_{\mathcal{E}}(\phi)}{\partial x}\right)^2+\left(\frac{\partial H_{\mathcal{E}}(\phi)}{\partial y}\right)^2}}{\partial\phi_x}\\
&=\nu\frac{\partial\sqrt{\left(\frac{\partial H_{\mathcal{E}}(\phi)}{\partial\phi}\frac{\partial\phi}{\partial x}\right)^2+\left(\frac{\partial H_{\mathcal{E}}(\phi)}{\partial\phi}\frac{\partial\phi}{\partial y}\right)^2}}{\partial\phi_x}\\
&=\nu\frac{\partial\sqrt{\left(\delta_{\mathcal{E}}(\phi)\phi_x\right)^2+\left(\delta_{\mathcal{E}}(\phi)\phi_y\right)^2}}{\partial\phi_x}\\
&=\nu\frac{1}{2}\frac{2\delta_{\mathcal{E}}(\phi)\phi_x\delta_{\mathcal{E}}(\phi)}{\sqrt{\left(\delta_{\mathcal{E}}(\phi)\phi_x\right)^2+\left(\delta_{\mathcal{E}}(\phi)\phi_y\right)^2}}\\
&=\nu\frac{\delta_{\mathcal{E}}(\phi)\phi_x}{\sqrt{\left(\phi_x\right)^2+\left(\phi_y\right)^2}}\\
&=\nu\delta_{\mathcal{E}}(\phi)\frac{\phi_x}{|\nabla\phi|}
\end{aligned}$$

那么：

$$\frac{d}{dx}\left(\frac{\partial F}{\partial\phi_x}\right)=\nu\delta_{\mathcal{E}}(\phi)\frac{d}{dx}\left(\frac{\phi_x}{|\nabla\phi|}\right)$$

同理：

$$\frac{d}{dy}\left(\frac{\partial F}{\partial\phi_y}\right)=\nu\delta_{\mathcal{E}}(\phi)\frac{d}{dy}\left(\frac{\phi_y}{|\nabla\phi|}\right)$$

所以：

$$
\begin{aligned}
\frac{d}{dx}\left(\frac{\partial F}{\partial \phi_x}\right)+\frac{d}{dy}\left(\frac{\partial F}{\partial \phi_y}\right) &= \nu\delta_{\mathcal{E}}(\phi)\frac{d}{dx}\left(\frac{\phi_x}{|\nabla\phi|}\right)+\nu\delta_{\mathcal{E}}(\phi)\frac{d}{dy}\left(\frac{\phi_y}{|\nabla\phi|}\right)\\
&= \nu\delta_{\mathcal{E}}(\phi)\left(\frac{d}{dx}\left(\frac{\phi_x}{|\nabla\phi|}\right)+\frac{d}{dy}\left(\frac{\phi_y}{|\nabla\phi|}\right)\right)\\
&= \nu\delta_{\mathcal{E}}(\phi)div\left(\frac{\nabla\phi}{|\nabla\phi|}\right)
\end{aligned}
$$

将

$$
\frac{\partial F}{\partial \phi}=\lambda_1\left|u_0-c_1\right|^2\delta_{\mathcal{E}}(\phi)-\lambda_2\left|u_0-c_2\right|^2\delta_{\mathcal{E}}(\phi)
$$

$$
\frac{d}{dx}\left(\frac{\partial F}{\partial \phi_x}\right)+\frac{d}{dy}\left(\frac{\partial F}{\partial \phi_y}\right)=\nu\delta_{\mathcal{E}}(\phi)div\left(\frac{\nabla\phi}{|\nabla\phi|}\right)
$$

代入

$$
\frac{\partial \phi}{\partial t}=-\frac{\partial F}{\partial \phi}+\frac{d}{dx}\left(\frac{\partial F}{\partial \phi_x}\right)+\frac{d}{dy}\left(\frac{\partial F}{\partial \phi_y}\right)
$$

可得 CV 能量泛函对应的梯度下降流方程为：

$$
\begin{aligned}
\frac{\partial \phi}{\partial t} &= \nu\delta_{\mathcal{E}}(\phi)div\left(\frac{\nabla\phi}{|\nabla\phi|}\right)-\lambda_1\left|u_0-c_1\right|^2\delta_{\mathcal{E}}(\phi)+\lambda_2\left|u_0-c_2\right|^2\delta_{\mathcal{E}}(\phi)\\
&= \delta_{\mathcal{E}}(\phi)\left(\nu div\left(\frac{\nabla\phi}{|\nabla\phi|}\right)-\lambda_1\left|u_0-c_1\right|^2+\lambda_2\left|u_0-c_2\right|^2\right)
\end{aligned}
$$