Problem N1

12 = 3+3+3+3

13 = 3+3+7

14 = 7+7

15 = 3+3+3+3+3

Problem N2

$2+4+\cdots+2k=k^2+k$

We must prove it for n=k+1:

$2+4+\ldots+2k + 2(k+1)$.

By the inductive hypothesis, the sum up to 2k is $k^2 + k$ so

$2 + 3 + \ldots + 2k +2 (k+1) = (k^2 + k) + 2(k+1) = (k^2 + k) + 2(k+1) = k^2+k+2k+2= k^2+3k+2=$

$(k+1)^2+(k+1)$

This matches the required form with n replaced by k+1
Thus by induction, the formula holds for all $n\geq1$

Problem N31

$n^2+n = n(n+1)$

since n and n+1 are consecutive integers, one of them must be even. Hence their product is divisible by 2

Problem N32

$n = 1:1^3 + 2(1) = 1 + 2 =3$, divisible by 3

$(k+1)^3 + 2(k+1) = (k^3 + 3k^2 + 3k + 1) + (2k+2) = (k^3 +2k) + 3k^2 + 3k + 2k +3 =$

$(k^3 + 2k) + 3(k^2+k)+3$

By the inductive hypothesis $k^3 + 2k$ is multiple of 3

Problem 33

n = 0: $n^5 - n = 0 - 0 = 0$ divisible by 5.

$(k^5 - k)$

For k + 1

$(k+1)^5 - (k+1)$

$(k+1)^5 = k^5 + 5k^4 + 10k^3 + 10k^2 + 5k + 1$

$(k+1)^5 - (k+1) = (k^5 - k) + (5k^4 + 10k^3 + 10k^2 + 5k) + (1-1)$

First bracket is divisible by 5 bu hypothesis. ; each term in the second bracket clearly has a factor of 5

Problem 34

$n^3 - n = n(n^2\ 1) = n(n-1)(n+1)$. This is the product of three consecutive integers (n-1),n and (n+1). Among any three consecutive integers, one is multiple of 3 and at least one is even, so the product is a multiple of 2*3 =6.