Problem N1

12 = 3+3+3+3

13 = 3+3+7

14 = 7+7

15 = 3+3+3+3+3

Problem N2

2+4+⋯+2k=k2+k

We must prove it for n=k+1:

2+4+…+2k  +  2(k+1).

By the inductive hypothesis, the sum up to 2k is k2 + k so

2 + 3 + … + 2k +2 (k+1) = (k2 + k) + 2(k+1) = (k2 + k) + 2(k+1) = k2+k+2k+2= k2+3k+2=

(k+1)2+(k+1)

This matches the required form with n replaced by k+1  
Thus by induction, the formula holds for all n≥1

Problem N31

n2+n = n(n+1)

since n and n+1 are consecutive integers, one of them must be even. Hence their product is divisible by 2

Problem N32

n = 1:13 + 2(1) = 1 + 2 =3, divisible by 3

(k+1)3 + 2(k+1) = (k3 + 3k2 + 3k + 1) + (2k+2) = (k3 +2k) + 3k2 + 3k + 2k +3 =

(k3 + 2k) + 3(k2+k)+3

By the inductive hypothesis k3 + 2k is multiple of 3

Problem 33

n = 0: n5 – n = 0 – 0 = 0 divisible by 5.

(k5 – k)

For k + 1

(k+1)5 – (k+1)

(k+1)5 = k5 + 5k4 + 10k3 + 10k2 + 5k + 1

(k+1)5 – (k+1) = (k5 –k) + (5k4 + 10k3 + 10k2 + 5k) + (1-1)

First bracket is divisible by 5 bu hypothesis. ; each term in the second bracket clearly has a factor of 5

Problem 34

n3 –n = n(n2 1) = n(n-1)(n+1). This is the product of three consecutive integers (n-1),n and (n+1). Among any three consecutive integers, one is multiple of 3 and at least one is even, so the product is a multiple of 2\*3 =6.