Find the domain and range of the following function

$h(x) = -2x^2 + 4x - 9$

$(-2x^2 + 4x - 4) + 4) - 9$

$(-2x^2 + 4x - 4) - 5$

$-2x \qquad 2$

$x \qquad -2$

$\frac{-b^2 \pm \sqrt{4ac}}{2a}$

$\frac{4 \pm \sqrt{4(+2)\cdot(+9)}}{2\cdot(-2)}$

4 ± -8

$\frac{4 \pm 6\sqrt{2}}{4}$

$\frac{2 \pm 3\sqrt{2}}{2}$

Vertex : $-\frac{4}{2\cdot 2}$; $f\left(-\frac{4}{2\cdot 2}\right)$

$(-1, f(-1))$ which is $(-1, -2\cdot(-1)^2 - 9$

R: $[-11, \infty)$

Find the domain of the following : $f(x) = \frac{x -}{x^2 -}$

$(x-5)(x+3) = 0$

$x - 5 = 0$; $x = 5$

$x + 3 = 0$, $x = -3$

$(-\infty, -3) \cup (3, -5) \cup (-5, \infty)$

$$f(x) = \begin{cases} -2x+1 & -1 \le x < 0 \\ x^2+2 & 0 \le x \le 2 \end{cases}$$

x	$f(x)$
-1	3
0	1

x^2+2

x	$f(x)$
0	2
1	3
2	6

Slope of the line $(-1,2)$ and $(3,-4)$

Slope: $m = \frac{y_2-y_1}{x_2-x_1} = \frac{-4-2}{3-(-1)} = \frac{-6}{4} = -\frac{3}{2}$

Graph point $(1,-1)$ where slope: $m = \frac{3}{4}$

$\frac{rise}{run} = \frac{3}{4}$; rise $= 3$; run $= 4$

-3; -4

Avergge rate of $f(x) = x^2 - \frac{1}{x}$; interval $[2,4]$

$f(2) = 2^2 - \frac{1}{2}$ · $f(4) =$

$= 4 - \frac{1}{2} = 7/2$

Avergge rate of change:

$\frac{f(4)-f(2)}{4-2} = \frac{63}{4} - \frac{7}{2} =$

$= \frac{49}{2} = \frac{49}{8}$

Given $f(t) = t^2 - t$ and $h(x) = 3x+2$, $f(h(1))$

$h(1) = 3 \cdot 1 + 2$

$h(1) = 5$

$f(5) = 5^2 - 5 = 20$

Find the domain of $(f \cdot g)(x) = \frac{5}{x-1}$ and $g(x) = \frac{4}{3x-2}$; $g(x) = 1$

$x \ne \frac{2}{3}$; $x \ne 2$

$x \cdot \frac{4}{3x-2} = 1 \cdot 4$

$3x-2 = 4$

$3x = 6$

$x = 2$

$(-\infty; \frac{2}{3}) \cup (\frac{2}{3}; 2) \cup (2; \infty)$

Find and simplify the functions $(g-f)$ and $(\frac{g}{f})$ given $f(x) = x-1$ and $g(x) = x^2-1$. Are they same function?

$(g-f)(x) = g(x) - f(x)$

$(g-f)(x) = x^2 - 1 - (x-1) =$

$= x^2 - x = x(x-1)$

$(\frac{g}{f})(x) = \frac{x^2-1}{x-1} = \frac{(x+1)\cancel{(x-1)}}{\cancel{(x-1)}} = x+1$

Write a formula for transformation of the function $f(x) = \frac{1}{x}$ that shifts the function's graph one unit to the right and one unit up.

$g(x) = \frac{1}{x-1} + 1$

Is the function $f(x) = x^3 + 2x$ even, odd, or neither?

even functions: $f(-x) = (-x)^3 + 2(-x) =$

$= -x^3 - 2x$;- not even

odd functions:

$-f(-x) = -(-x^3 - 2x) = x^3 + 2x$

it is odd function

Is the function $f(s) = s^4 + 3s^2 + 7$ even, odd or neither?

even : $f(-s) = (-s)^4 + 3 \cdot (-s)^2 + 7 = -s^4 - 3s^2 + 7$

odd : $-f(-s) = -(-s)^4 + 3 \cdot (-s)^2 + 7 =$

$= -s^4 + 3s^2 + 7$

Point-Slope Form of a Linear Function:

The point-slope form of linear equation takes the form: $\boxed{y - y_1 = m(x - x_1}$, where m is the slope x_1 and y_1 are the x- and y-coordinates of a specific point through which the line passes.

Write the point-slope form of an equation a line that passes through the point $(5,1)$ and $(8,7)$

Then write it in the slope-intercept form.

slope-intercept form is $y = mx + b$

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{7-1}{8-5} = \frac{6}{3} = \frac{2}{1} = 2$

$y - y_1 = m(x - x_1)$

$y - 1 = \frac{2}{1}(x - 5)$

$y - 1 = 2x - 10$

$y = 2x - 9$

If $f(x)$ is a linear function, and $(3,-2)$ and $(8,1)$ are points on the line, find the slope. Is this functions increasing or decreasing?

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1-(-2)}{8-3} = \frac{-3}{5} = -\frac{3}{5} = \frac{3}{5}$

Absolute Maxima and Minima

- The absolute Maxima of f at $x = c$ is $f(c)$ where $f(c) \geq f(x)$ for all x in the domain of f.
- The absolute minimum of f at $x = d$ is $f(d)$ where $f(d) \leq f(x)$ for all x in the domain of f.

For the function f shown in Figure; find all absolute maxima and minima.

Scanned with CamScanner

Maxima : $x = -2$; $x = 2$; $y = 16$

Minima : $x = 3$; $y = -10$

Local Minima and local Maxima

A function f is an increasing function on an open interval if $f(b) > f(a)$ for every a, b interval where $b > a$.

A function f is an decreasing function on an open interval if $f(b) < f(a)$ for every a, b intervah where $b < a$.

A function f has a local maximum at a point b in an open interval (a, c) if f(b) is greater than or equal to f(x) for every point x (x doesn't equal b) in the interval. Likewise, f has a local minimum at a point b in (a, c) if ~~f~~ f(b) is less than or equal to f(x) for every x (x doesn't equal b) in the interval.

Local Maxima; $x = 1$; $y = 2$

Local Minima; $x = -1$; $y = -2$

Given the function below, identify the functions whose graphs are a pair of parallel lines and a pair of perpendicular lines.

$f(x) = 2x + 3$ $\quad h(x) = -2x + 2$

$g(x) = \frac{1}{2}x - 4$ $\quad i(x) = 2x - 6$

Slope int: ~~f(x)~~ $y = 2x + 3$

f(x) and i(x) ; same slope $m = 2$ it means parallel lines.

g(x) and h(x) has negative slope it means perpendicullar lines.

$$\begin{cases} 2x + y = 7 \\ x - 2y = 6 \end{cases}$$

$$\begin{cases} y = 2x - 7 \\ x - 2y = 6 \end{cases} \quad \begin{cases} x - 2(2x - 7) = 6 \\ x - \cancel{4x}\,14 + 4x = 6 \end{cases} \quad \begin{cases} x + 4x = \\ 5x = 20 \end{cases}$$

$$\begin{cases} x = 4 \\ y = 7 - 2x \end{cases} \quad \begin{cases} \cancel{2}\ 5 - 2(\tfrac{5}{2} \cdot 5) - 7) = 6 \\ \cancel{5 - 20 - 8}\ 5 + 6 = 6 \end{cases}$$

$$\begin{cases} \cancel{4 - 2}\ 2 \cdot 4 + y = 7 \\ 8 + y = 7 ;\ y = -1 \end{cases} \qquad (4, -1)$$

$$\begin{cases} 2(4) + (-1) = 7 \\ 7 = 7 \end{cases} \quad \begin{cases} 4 - 2(-1) = 6 \\ 6 = 6 \end{cases}$$

Scanned with CamScanner

$$\begin{cases} 4x + 2y = 4 \\ 6x - y = 8 \end{cases} \qquad \begin{cases} 2y = \frac{4 - 4x}{2} \\ y = 2 - 2x \end{cases} \qquad \begin{cases} 2y = -4x + \\ y = -2x + 2 \end{cases}$$

$6x - (-2x + 2) = 8$

$6x + 2x - 2 = 8$

$$\begin{cases} 8x = 10 \\ x = \frac{10}{8} = \frac{5}{4} \end{cases}$$

$$\begin{cases} 4\left(\frac{5}{4}\right) + 2y = 4 \\ 5 + 2y = 4 \;;\; y = -\frac{1}{2} \end{cases} \qquad \begin{cases} x = \frac{5}{4} \\ y = -\frac{1}{2} \end{cases}$$

$$\begin{cases} 4\left(\frac{5}{4}\right) + 2 \cdot \left(-\frac{1}{2}\right) = 4 \\ 5 + -1 = 4 \end{cases} \qquad \begin{cases} 6\left(\frac{5}{4}\right) - \left(-\frac{1}{2}\right) = 8 \\ \frac{15}{2} - \left(-\frac{1}{2}\right) = \frac{16}{2} = 8 \end{cases}$$

Definitions: Forms of Quadratic Functions

A quadratic functions is a function of degree two. The graph of a quadratic function is parabola.

The general form of a quadratic function is $f(x) = ax^2 + bx + c$ where $a, b,$ and c are real number and $a \neq 0$.

The standard form of a quadratic function is $f(x) = (x - h)^2 + k$.

The vertex (h, k) is located at.

$$h = -\frac{b}{2a},\; k = f(h) = f\left(\frac{-b}{2a}\right).$$

Find the vertex of the quadratic function $f(x) = 2x^2 - 6x + 7$

$a = 2 \;;\; b = -6 \;;\; c = 7$

$\text{Vertex} = -\frac{b}{2a} = \frac{6}{2 \cdot 2} = \frac{6}{4} = \frac{3}{2}$

$k = f(h) = f\left(\frac{3}{2}\right) = 2\left(\frac{3}{2}\right)^2 - 6\left(\frac{3}{2}\right) + 7 = \frac{5}{2}$

Standard form: $2\left(\frac{3}{2}x - \frac{3}{2}\right)^2 + \frac{5}{2}$

Find the domain and range of $f(x) = -5x^2 + 9x - 1$

$a = -5 \;;\; b = 9 \;;\; c - 1$

$$D = \frac{-b \pm \sqrt{4ac}}{2a} = \frac{-9 \pm \sqrt{4 \cdot (-5) \cdot (-1)}}{2 \cdot (-5)} = \frac{-9 \pm \sqrt{20}}{-10}$$

$$x = -\frac{b}{2a} = \frac{-9}{2 \cdot (-5)} = \frac{9}{10}$$

$$y = f\left(\frac{9}{10}\right) = -5\left(\frac{9}{10}\right)^2 + 9\left(\frac{9}{10}\right) - 1 = \frac{61}{20}$$

The range : $\left(-\infty ; \frac{61}{20}\right]$.

Find the y- and x- intercepts of the quadra

$f(x) = 3x^2 + 5x - 2$

$y : f(0) = 3(0)^2 + 5 \cdot 0 - 2 = -2$

$x : 3x^2 + 5x - 2 = 0$

$a = 3 \;;\; b = 5 \;;\; c = -2$

$x = -\frac{b}{2a} \;;\; \frac{-5}{2 \cdot 3} = -\frac{5}{6}$

Scanned with CamScanner

$3x^2+5x-2=0$

$3x^2+6x-x-2=0$

$3x(x+2)-(x-2)=0$

$(x+2)(3x-1)=0$

$x+2=0$; $3x-1=0$

$x_1=-2$; $3x-1=\frac{1}{3}=x_2$

a. $-1\le 2x-5<7$

$-1+5\le 2x<7+5$

$\frac{4}{2}\le\frac{2x}{2}<\frac{12}{2}$

$2\le x<6$

$[2,6)$

b. $x^2+7x+10<0$

$(x+5)(x+2)<0$

$(x+5)>0 \cup (x+2)<0$

$x>-5 \cup x<-2$

c. $-6<x-2<4$

$-4<x<6$

$(-4,6)$

Solve:

$10-(2y+1)\le-4(3y+2)-3$

$10-2y-1\le-12y-8-3$

$-2y+9\le-12y-11+12y$

$-2y+12y+9\le-11$

$10y+9\le-11-9$

$10y\le-20$

$y\le-\frac{20}{10}=-2$

~~Solve $x(x+3)^2(x-4)<0$~~

Solve: $2x^4>3x^3+9x^2$

$2x^4-3x^3-9x^2>0$

$x^2(2x^2-3x-9)=0$

$x^2(2x+3)(x-3)=0$

$(-\infty;-\frac{3}{2})\cup(3,\infty)$

Function

$f(x)=-\frac{1}{2}|4x-5|+3\le0$

$|4x-5|\ge6$

$4x-5=0$

Scanned with CamScanner