Week 2 H/W

Problem 1.

~~A~~rithmetic Series $S = 5 + 9 + 13 \ldots 89$

$a_1 = 5$

$d = 4$

$a_n = 89$

$89 = 5 + (n-1)d$

$84 = 4(n-1)$

$n - 1 = 21$

$n = 22$

$= \sum_{k=1}^{22} = [5 + (k-1) \cdot 4]$

Problem 2.

$\sum_{k=3}^{15} (2k+1)$ start at $k=1$

$j = k - 2$ $k=3;\ j=1$

$k=15;\ j=13$

$\sum_{k=1}^{13} = [2(j+2)+1) = [2j+5]$

Problem 3.

Recursive notation

$a_1 = 12$ $a_{10} = 57$ $a_{25} - ?$

$d - ?$

$a_n = a_{n-1} + d$

$d = \cancel{\frac{a_{10} - a_2}{10 - 2}} = \cancel{\frac{57 - 12}{8}} =$

$a_n = a_1 + (n-1)d$

$a_{10} = 12 + (10-1)d$

$57 = 12 + (10-1)d$

$\frac{45}{9} = d = 5$

$a_{25} = 12 + (25-1)5$

$a_{25} = 132$

Problem 4

Find the sum all multiples of 7 between 100 and 2000.

$a_1 = 7 \cdot 15 = 105$

$a_n = 7 \cdot 142 = 994$

$n = \frac{a_n - a_1}{d} + 1 = \frac{994 - 105}{7} + 1 = 128$

$S_n = \frac{n}{2}(a_1 + a_n) = \frac{128}{2}(994 + 105) = 64 \cdot 1099 = 20336$

Problem 5

$S = \sum_{k=1}^{n} (3k+2),\ n - ?$ $S = 2{,}650$

$S_n = \frac{n}{2}(a_1 + a_n) = \frac{n}{2}(5 + (3n+2)) = \frac{n}{2}(3n+7)$

$a_1 = 3(1) + 2 = 5$

$a_n = 3n + 2$

$2 \cdot 2650 = \frac{n}{2}(3n+7)$; $5300 = n(3n+7)$

$5300 = n(3n + 7)$

$5300 = 3n^2 + 7n - 5$

$3n^2 + 7n - 5300 = 0$

$n = \frac{-7 \pm \sqrt{7^2 - 4 \cdot 3 \cdot (-5300)}}{2 \cdot 3} = \frac{-7 + 252{,}29}{6} \approx \frac{245{,}29}{6} =$

40.88

$n = 41$

$S_{41} = \frac{41}{2}\left([5 + 3 \cdot 41 + 2]\right) = \frac{41}{2}(5 + 125) = 41 \cdot 65$

2665

There is no integer value of n such as that $S = 2680$

Problem 6

$a_5 = 20$; $a_{15} = 60$

a_{10} is arithmetic mean of a_5 and a_{15}

$a_{10} = 20 + 9d$ $\qquad a_{10} = a_1 + 9d$

$a_{10} =$

$\cancel{a_5 = 20 + 4d}$ $\qquad 20 = a_1 + 4d$

$20 = a_1 + 4 \cdot 4$ $\qquad 60 = a_1 + 14d$

$a_1 = 4$ $\qquad 40 = 10d$

$a_{10} = 4 + 9 \cdot 4$ $\qquad d = 4$

$a_{10} = 40$

Mean Formula $= \frac{a_5 + a_{15}}{2} = \frac{20 + 60}{2} = 40$

Answers: a_{10} is arithmetic mean of a_5 and a_{15}

Problem 7

$n = 20$ steps.

$a_1 = 5$

$d = 0.5$

$S_{20} = \frac{20}{2}(a_1 + a_n) = \frac{20}{2}(5 + (5 + (20-1)0.5)) =$

$= 195$ cm

Problem 8

$a_1 = 11$

$d = 3$

$n - ?$

$\boxed{\text{Sum formula} = S_n = \frac{n}{2}[2a_1 + (n-1)d]}$

$S_n = \frac{n}{2}[22 + 3(n-1)]$

$S_n = \frac{n}{2}[22 + 3n - 3] = \frac{n}{2}[19 + 3n]$

$= \frac{n(3n + 19)}{2} > 1000$

$S_n > 1000$. $\qquad n(3n + 19) > 2000$

$3n^2 + 19n - 2000 = 0$

$n = \frac{-19 \pm \sqrt{19^2 - 4 \cdot 3 (-2000)}}{2 \cdot 3} \approx 22.84$

$S_{23} = \frac{23}{2}(2 \cdot 11 + (23-1)3)$

$n = 23$ is smallest

Problem 9

Rewrite $\sum_{k=3}^{12} 4\left(\frac{1}{2}\right)^k$ as a sum starting from

$k = 0$

$j = k - 3$; $k = 3$; $j = 0$; $k = 12$; $j = 9$

$$\sum_{k=3}^{12} 4\left(\frac{1}{2}\right)^k = \sum_{j=0}^{9} 4\left(\frac{1}{2}\right)^{j+3} = \sum_{j=0}^{9} \left(\frac{1}{8}\right)\left(\frac{1}{2}\right)^j =$$

$$\sum_{j=0}^{9} \left(\frac{1}{2}\right)^j \frac{1}{2} = \sum_{j=0}^{9} \left(\frac{1}{2}\right)^{j+1}$$

Problem 10

N-th formula

$a_2 = -6$ and $a_5 = 48$ $\quad a_n = a_1 \cdot r^{n-1}$

$a_2 = a_1 r^{2-1}$; $-6 = a_1 r^1$

a_5 : $48 = a_1 r^4$

$\frac{48}{-6} = r^3$

$-8 = r^3$

$r = -2$

let's find a_1:

$= a_1 \cdot r^{n-1}$; $-6 = a_1 \cdot (-2)^1$; $a_1 = 3$

a_{10} - ?

$a_{10} = a_1 \cdot r^9$

$a_{10} = 3 \cdot (-2)^9$

$a_{10} = -1536$

Problem 11

$a_4 = 54$ and $a_7 = 1458$

r - ?

$a_4 = 54 = a_1 \cdot r^3$

$a_7 = 1458 = a_1 \cdot r^6$

$\frac{1458}{54} = r^3$

$27 = r^3$

$r = 3$

Problem 12

Sum of the first N terms

S_{15} - ? , where $a_1 = 8$ and $r = \frac{3}{4}$

$$\boxed{S_n = \frac{a_1(r^n - 1)}{r - 1}} \qquad S_{15} = \frac{8\left(\frac{3}{4}^{15} - 1\right)}{\frac{3}{4} - 1} = 8\frac{\left(\frac{3}{4}^{15} - 1\right)}{\left(\frac{3}{4} - 1\right)}$$

$= 8 \cancel{\frac{1}{4}}^{14} \quad 8 \frac{1 - r^{15}}{\frac{1}{4}} = 32(1 - r^{15})$

$S_{15} = 32 \cdot 1 \approx 32$

Problem 13

$P(x) = x^5 + x^3 + x^2 - 7$

degree - 5

Number of quadratic - quadratic

Problem 14

Simplify $(2x^4 - 3x^3 + x - 5) + (x^3 - 2x^2 + 4x + 7)$

$2x^4 - 3x^3 + x - 5 + x^3 - 2x^2 + 4x + 7 = 2x^4 - 2x^3 + 5x + 2$

Problem 15

$(x^2 - x + 2)(x^2 + x + 1) = x^4 + \cancel{x^3} + \cancel{x^2} - \cancel{x^3} - \cancel{x^2} - x + 2x^2 + 2x + 2 = x^4 + 2x^2 + x + 2$

Problem 16

Find GCD and LCM for $24x^3y^2z^5$ and $36x^5y^3z^2$

~~GCD = $6x^2yz^3$~~

~~LCM~~ $= 4 \cdot 6 \cdot 6\ x^5y^3z^5 = 144x^5yz^5$

GCD $= 12x^3y^2z^2$

LCM $= \cancel{2} \cdot 72 = 72x^5y^3z^5$

Problem 17

Factor $x^4 - 13x^2 + 36$

$x^2(x^2 - 13x + 36)$

$x \quad 9$

$x \quad 4$

$(x-9)(x-4) = 0$

$(x^2-9)(x-4) = (x-3)(x+3)(x-2)(x+2)$

Problem 18

Expand $(2x+3y)^5$

Using $(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^{n-k} b^k$

$k=0: \binom{5}{0}(2x)^5(3y)^0 = 32\cancel{x^5 + 3y} = 32x^5$

$k=1: \binom{5}{1}(2x)^4(3y)^1 = 5 \cdot 16x^4 \cdot 3y = 240x^4y$

$k=2: \binom{5}{2}(2x)^3(3y)^2 = 10 \cdot 8x^3 \cdot 9y^2 = 720x^3y^2$

$k=3: \binom{5}{3}(2x)^2(3y)^3 = 10 \cdot 4x^2 \cdot 27y^3 = 1080x^2y^3$

$k=4: \binom{5}{4}(2x)^1(3y)^4 = 5 \cdot 2x \cdot 81y^4 = 810xy^4$

$k=5: \binom{5}{5}(2x)^0(2)^0(3y)^5 = 1 \cdot 1 \cdot 243y^5 = 243y^5$

$(2x+3y)^5 = 32x^5 + 240x^4y + 720x^3y^2 + 1080x^2y^3 + 810xy^4 + 243y^5$

Problem 19

$6x^3 + 11x^2 - 31x + 15 \mid 3x - 2$

$-6x^3 - 4x^2 \qquad 2x^2 + 5x - 7$

$-15x^2$

$-15x^2 - 10x$

$-21x$

$-21x + 15$