Tautologies, Contradictions and Contingencies

A tautology, denoted T, is a proposition which is always true.
A contradiction, denoted F, is a proposition which is always false.
A contingency is a proposition which is neither a tautology nor a contradiction, such as p.

De Morgan's Law

$$\neg(p \wedge q) \equiv \neg p \vee \neg q$$
$$\neg(p \vee q) \equiv \neg p \wedge \neg q$$

Equivalence Proofs

$\neg(p \vee (\neg p \wedge q))$ is equivalent to $\neg p \wedge \neg q$.

$\neg p \wedge \neg q$

$\neg p \wedge q \to$ True

$p \vee$ True $\to$ True

$\neg$ True $\to$ False

$$\neg(p \vee (\neg p \wedge q)) = \neg p \wedge \neg(\neg p \wedge q)$$
$$= \neg p \wedge [\neg(\neg p) \vee \neg q]$$
$$= \neg p \wedge (p \vee \neg q)$$
$$= (\neg p \wedge p) \vee (\neg p \wedge \neg q)$$
$$= F \vee (\neg p \wedge \neg q)$$
$$= \underbrace{(\neg p \wedge \neg q) \cancel{\vee} \cancel{F}}$$
$$= (\neg p \wedge \neg q)$$

Show that $(p \wedge q) \rightarrow (p \vee q)$ is a tautology.

$$\left.\begin{array}{l} p \wedge q \rightarrow \text{True} \\ p \vee q \rightarrow \text{True} \end{array}\right\} \text{ is a tautology.}$$

$p \rightarrow q \Leftrightarrow \neg p \vee q$

$$(p \wedge q) \rightarrow (p \vee q) \equiv \neg(p \wedge q) \vee (p \vee q)$$
$$= (\neg p \vee \neg q) \vee (p \vee q)$$
$$= (\neg p \vee p) \vee (q \vee \neg q)$$
$$= T \vee T$$
$$= \text{True}$$

If we have all OR we can change position of variables

Propositional Satisfiability

- A compound proposition is satisfiable if there is an assignment of truth values to its variables that makes it true.
- When no such assignment exist, the compound proposition is unsatisfiable.
- A compound proposition is unsatisfiable if and only if its negation is tautology.

Notation

$\bigvee_{j=1}^{n} p_j$ is used for $p_1 \vee p_2 \vee \dots p_n$

$\bigwedge_{j=1}^{n} p_j$ is used for $p_1 \wedge p_2 \wedge \dots p_n$