Predicates and Quantifiers

Introduce Predicat Logic

Predicate logic uses the following new features:

- Variables: x, y, z
- Predicates: Properties for variables
- Quantifiers

Propositional functions, denoted $P(x)$, $Q(x)$, ..., are a generalization of propositions.

- They contain variables and a predicate.
- Variables can be replaced by elements from their domain.

Propositional functions become propositions when their variables replaced by a value from the domain.

Example: Let $P(x)$ denote "$x > 0$". Then it's variable is x and it's predicate is "> 0". Let the domain be the integers. Then: $P(-9)$ is false; $P(0)$ is false; $P(3)$ is true.

$P(x, y, z)$ denote „$x + y = z$"

$R(2, -1, 5)$ is false; $R(3, 4, 7)$ is true; $R(x, 3, 2)$ is false.

Compound Expressions

Connectives from propositional logic carry over to predicate logic.

If $P(x)$ denotes „$x>0$", find these truth values:

$P(3) \vee P(-1)$: T

$P(3) \wedge P(-1)$: F

$P(3) \rightarrow P(-1)$: F

Quantifiers

We need quantifiers to express the meaning of English words including all and some: „All men are Mortal." „Some cats do not have fur."

The two most important qualifiers are:

Universal Quantifier, „For all," symbol: $\forall$

Existential Quantifier, „There exist," symbol: $\exists$

We write as in $\forall x\, P(x)$ and $\exists x\, P(x)$.

$\forall x\, P(x)$ asserts $P(x)$ is true for <u>every</u> x in the domain.

$\exists x\, P(x)$ asserts $P(x)$ is true for <u>some</u> x in the domain.

Universal Quantifier

$\forall x\, P(x)$ is read as „For all x, $P(x)$" or „For every x, $P(x)$"

Existential Quantifier

$\exists x\, P(x)$ is read as „For some x, $P x$" or as „There is an x such that $P(x)$," or „For at least one x, $P(x)$."

The truth value of $\exists x\, P(x)$ and $\forall x\, P(x)$ depend on both the propositional function $P(x)$ and on the domain.

$\forall x\, (S(x) \rightarrow J(x))$

$\forall x\, (S(x) \wedge J(x)) \longrightarrow$ For all x student in this class AND x has taken a course.

"Some Student in this class has taken a course in Java."

Depends on the domain U.

$\exists x\,(S(x) \land J(x))$

$\exists x\,(S(x) \rightarrow J(x))$ = For some students