Algebra Part 3 Lecture Part 1/3

1. Logarithmic Formulas

a. Formulas and Exercise

$\log_b x = y$

$b^y = x$

$\log_2 2^8 = 8$

$\log_3 27 = 3$

$\log_2 1 = 0$

$2^0 = 1$

1. $\log_a 1 = 0 \Leftrightarrow a>0,\ a\neq1$
2. $\log_a a = 1 \Leftrightarrow a>0,\ a\neq1$
3. $\log_a(b\cdot c) = \log_a b + \log_a c \Leftrightarrow a>0,\ b>0,\ c>0, a\neq1$
4. $\log_a \frac{b}{c} = \log_a b - \log_a c \Leftrightarrow a>0,\ b>0,\ c>0, a\neq1$
5. $\log_a b^n = n\cdot\log_a b \Leftrightarrow a>0,\ b>0,\ a\neq1$
6. $\log_a b = \frac{\log_c b}{\log_c a}$
7. $\log_a b = \frac{1}{\log_b a} \Leftrightarrow a>0,\ b>0,\ a\neq1,\ b\neq1$
8. $\log_{a^n} b = \frac{1}{n}\log_a b \Leftrightarrow a>0,\ b>0,\ a\neq1,\ n\neq0$
9. $\log_{a^{\frac{n}{m}}} b = \frac{m}{n}\cdot\log_a b \Leftrightarrow a>0,\ b>0,\ a\neq1$
10. $a^{\log_c b} = b^{\log_c a} \Leftrightarrow a>0,\ b>0,\ c>0,\ a\neq1,\ b\neq1,\ c\neq1$
11. $a^{\log_a b} = b \Leftrightarrow a>0,\ b>0,\ a\neq1$

$a^{\log_a b} = b$

$\log_a \frac{1}{a} = -1$

$\log_a a^{-1}$

$\log_{a^k} a = \frac{1}{k}$

$(a^k)^{\frac{1}{k}} = a$

$\log_{a^k} a^m = \frac{m}{k} \rightarrow (a^k)^{\frac{m}{k}} = a^{k\cdot\frac{m}{k}} = a^m$

$\log_a a^m = m$

$\log_a bc = \log_a b + \log_a c$

$\log_2(32) = \log_2(8\cdot4) = \log_2(\underset{2^3}{8}) + \log_2(\underset{2^2}{4})$

$\underbrace{5 = 3 + 2}$

$\log_a \frac{b}{c} = \log_a b - \log_a c$

$\log_2 \frac{32}{8} = \log_2 32 - \log_2 8 = 5 - 3 = 2$

$\log_{a^k} b = \frac{1}{k} \cdot \log_a b$

$(a^k)^{\frac{1}{k}\cdot \log_a b} = a^{k \cdot \frac{1}{k} \cdot \log_a b} = a^{\log_a b} = b$

$\log_a b^m = m \cdot \log_a b$

$a^{m \cdot \log_a b}$

$$\underbrace{\log_a b = \frac{\log_c b}{\log_c a}}$$

$$a^{\log_c b} = b^{\log_c a}$$

$$\log_a b = \frac{1}{\log_b a}$$

Exercise:

1. $\log(x^2 \cdot y) = \log x^2 + \log y = 2 \cdot \log x + \log y$

2. $\log\left(\frac{\sqrt{x} \cdot \sqrt[3]{y^2}}{z^4}\right) = (\log \sqrt{x} \cdot \sqrt[3]{y^2}) - \log z^4 =$

$= (\log \sqrt{x} + \log \sqrt[3]{y^2}) - 4 \log z =$

$= (\log x^{\frac{1}{2}} + 2 \cdot \log y^{\frac{1}{3}}) - 4 \log z =$

$= \underline{\frac{1}{2} \log x + \frac{2}{3} \log y - 4 \log z}$

3. $\log x \sqrt{\frac{\sqrt{x}}{z}} = \log x + \log \sqrt{\log \sqrt{x} - \log z} =$

$= \log x + \log \sqrt{\frac{1}{2} \log x - \log z} = \log x + \frac{1}{2} \log + (\frac{1}{2} \log x - \log z$

$= \log x + \frac{1}{2}(\log x - \log z) = \frac{5}{4} \log x + \frac{1}{2} \log z$

$\ln(x) = \log_e$; $\boxed{e = 2.71828\ldots}$

$$\log_4(-x) + \log_4(6-x) = 2$$

$$\log_4(-x(6-x)) = 2$$

$$-x(6-x) = 4^2 = 16$$

$$(-x)(6) + (-x)(-x) = 16$$

$$-6x + x^2 = 16$$

$$x^2 - 6x - 16 = 0$$

$$\begin{array}{lll} x & -8 & -8x \\ x & 2 & 2x \\ & & -6x \end{array}$$

$$(x+2)(x-8) = 0$$

$$x = -2, \quad x = 8; \quad = 0$$

Logarithms are only defined for positive number

$$\log_4(-x) + \log_4(6-x) = 2$$

$$\log_4(-(-2)) + \log_4(6-(-2)) = 2$$

$$2 \qquad\qquad 8$$

$$\log_{2^2}(2) + \log_{2^2}(2^3)$$

$$\frac{1}{2} + \frac{3}{2} = \frac{4}{2} = \underline{2}$$

$$\log_2(x+1) - \log_2(2-x) = 3$$

$$\log_2 \frac{x+1}{2-x} = 3 \quad ; \quad \frac{x+1}{\cancel{2-x}} = 8 \cdot (2-x)$$

$$x+1 = 8(2-x)$$

$$x+1 = 16 - 8x$$

$$9x = 15$$

$$x = \frac{15}{9} = \frac{5}{3}$$

$$\log_2\left(\frac{5}{3}+1\right) - \log_2\left(\frac{1}{3}\right) = 3$$

1000 6.5% 30

$$f(t) = 1000 \cdot (1+0.065)^t$$

$$f(30) = 1000 \cdot 1.065^{30} = 6614.36$$

100 80Y 20

$$f(t) = 10 \cdot \left(\frac{1}{2}\right)^{20}$$

$$f(t_{min}) < 2$$

$$10\left(\frac{1}{2}\right)^{\frac{t}{80}} < 2$$

$$\left(\frac{1}{2}\right)^{\frac{t}{80}} < \frac{1}{5}$$

$$\log_{\frac{1}{2}}\left(\frac{1}{2}\right)^{\frac{t}{80}} \quad \log_{\frac{1}{2}}\left(\frac{1}{5}\right)$$

$\log_a$

$0 < a < 1$

$\log_b \; ; \; b > 1$

$$\frac{t}{80} > \log_{\frac{1}{2}}\left(\frac{1}{5}\right)$$

$$t_{min} > 80 \cdot \log_{\frac{1}{2}}\left(\frac{1}{5}\right)$$

t_{min}