Boolean Logic
The Foundations: Logic and Proofs.
Part 1. Propositional Logic

A propositions is a declarative sentence that is either True or False.

Compound Propositions:

- Negation
- Conjuction
- Disconjuction
- Implication
- Biocenditional

Negation – Отрицание : Unary (single variable)

The negation of proposition p is denoted by $\neg p$.

Conjunction – Соединение (And operator)

The conjunction of propositions p and q is denoted by $p \land q$ – read p and q.

Table

p	q	p∧q
T	T	T
T	F	F
F	T	T
F	F	F

Conjunction not unary they are binary. They take 2 variables.

```
int x = 5;   // 000....101 → 5
int y = 3;   // 000....011 → 3
int res = x & y;   // 000....001 → 1
```

$\underbrace{\text{int}}_{32\text{-bit}}\ x = \underline{4};\ \sim \underbrace{000000\ldots 100}_{32\text{-bit}}$ → $\overbrace{111111 \ldots 011}$ so negation is

Disjunction (OR)

The Disjunction of propositions p and q is denoted by $p \vee q$ - p or q.

Table

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

Disjunction is True, when one of the variables are True.

`z = x | y;`

Exclusive OR (XOR)

The exclusive or of propositions is denoted by $p \oplus q$.

Table

p	q	$p \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

Exclusive OR is when p or q should be True. But not in the same time.

`z = p ^ q;`

Implication

If p and q are propositions, then $p \to q$ is a conditional statement or implication. p is the hypothesis and q is the conclusion.

Table

p	q	$p \to q$	$\neg p$	$\neg p \vee q$
T	T	T	F	F
T	F	F	F	F
F	T	T	T	F
F	F	T	T	T

Converse, Contrapositive and Inverse.

have the same truth table

From $\boxed{p \to q}$ we can form new conditional statements.

- $q \to p$ is the converse of $p \to q$.
- $\boxed{\neg q \to \neg p}$ is the contrapositive of $p \to q$.
- $\neg p \to \neg q$ is the inverse of $p \to q$.

Biocondiitional

If p and q are propositions, then we can form the bio-conditional proposition $p \leftrightarrow q$, read as "p if and only if q."

Table

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

Biocondiitional is true, when both variable are at the same time True or False, otherwise False.