Foundations for DS.

Week 2

Arithmetic Series

a.

$a_1, a_1+d, a_1+2d, a_1+3d \ldots$

$\sum_{k=0} 17k+19 = (17\cdot 0+19)+(17\cdot 1+19)\ldots$

$\boxed{\sum 17k+19}$ - sigma notation

$\sum_{k=1}^{100} 17(k-1)+19$

Sum notation

$(-2006) \overset{+2}{+} (-1989) \overset{+2}{+} (-1992) + \ldots + 1984$

Recursive notation

$a_1 = 9$

$n \geq 2,\ a_n = a_{n-1} + 17$

a_4 - ?

$a_4 = 60$

$a_4 = a_3 + 17 = 60$

$a_3 = a_2 + 17 \in 43$

$a_2 = a_1 + 17 = 26$

b. N-the term formula

$a_n = a_1 + (n-1)d \qquad d = 17$

$a_1 = 9$

$a_4 = a_1 9 + (3)\cdot 17$

$a_4 = 60$

c. Mean formula

$a_x \quad a_y \quad x \leq y \quad y = x+1$ is odd

$a_2 \rightarrow - = a_6$

$a_4 = \frac{a_2 + a_2}{2}$

$a_2 = a_1 + 1d$

$+\ a_6 = a_1 + 5d$

$\frac{2\cdot a_1 + 6d}{2}$

$a_4 = a_1 + 3d$

d. d - ?

$\frac{a_6 - a_2}{6-2} = d$

$a_x \quad a_y \quad x \leq y$

$\boxed{d = \frac{a_y - a_x}{y-x}}$

2. Sum of the N terms

i. a_1, d

$a_1 + a_2 + \dots + a_{n-1} + a_n$

$$S_n = \frac{a_1 + a_n}{2} \cdot n = \frac{a_1 + (a_1 + (n-1)d)}{2} \cdot n =$$

$$= \frac{2a_1 + (n-1)d}{2} \cdot n$$

$1 + 2 + 3 + \dots + n$

$a_1 = 1 \quad d = 1$

$$S_n = \frac{1 + n}{2} \cdot n = \frac{n(n+1)}{2}$$

$$2S_n = n \cdot (n+1)$$

$$S_n = \frac{n \cdot (n+1)}{2}$$

Basic Step: $n = 1 \quad S_n = 1 = \frac{1 \cdot (1+1)}{2} = 1$ ✓

Inductive Step: $S_n = \frac{n(n+1)}{2} \Rightarrow S_{n+1} = \frac{(n+1)}{2}$

$1 + 2$

$$S_{n+1} = 1 + 2 + 3 \dots n + (n+1)$$

$$\frac{n(n+1)}{2} + (n+1)$$

$$= \frac{n(n+1)}{2} + \frac{n(n+2)}{2}$$

$$= \frac{(n+1)(n+2)}{2}$$

✓ Proof of arithmetic

$$S_n = \frac{a_1 + a_n}{2} \cdot n$$

$$S_n = a_1 + a_2 + a_3 \dots + a_{n-2} + a_{n-1} + a_n$$

$$S_n = a_n + a_{n-1} + a_{n-2} \dots + a_2 + a_1$$

$$(a_1 + a_n) + (a_1 + a_n) +$$

$a_2 + a_{n-1}$

$a_1 + d \quad a_1 + (n-2)d$

$2a_1 + (n-1)d$

$a_1 + (a_1 + (n-1)d)$

$a_1 + a_n$

$$S_n = \frac{a_1 + a_n}{2} \cdot n$$