# Universal Set and Empty Set

The universal set $U$ is the set containing everything currently under consideration.

- Sometimes implicit.
- Sometimes explicitly stated.
- Contents depend on the context.

The empty set, denoted $\emptyset$ or $\{\}$, is the set with no element.

# Set-Builder Notation

A predicate can be used to define a set: for a universal set $U$ and a propositional function $P(x)$, there exists

$$S = \{x \in U : P(x)\}$$

Example: Set of positive rational numbers:

$$Q^+ = \{x \in R \mid x = p/q, \text{ for some positive integers } p, q\}.$$

Set below such

# Three definition of Sets: Equal, Subset and Proper Subset.

Equal — when set A has the same elements in B. $A = B$

Subset - A is a subset if and only if every element of A is also an element of B. $A \subseteq B$

Proper subset — when set B has element in A and more.

# Russell's Paradox

Let $S$ be the set of all sets which are not numbers of themselves. Namely,

$$S = \{x \mid x \notin x\}.$$

A paradox results from trying to answer the question „Is $S$ a number of itself?"

# Set Cardinality

If there are exactly $n$ distinct elements in $S$ where $n$ is a non-negative integer, we say that $S$ is finite. Otherwise it is infinite.

The cardinality of a finite set $A$, denoted by $|A|$, is the number of elements of $A$.

Examples:
1. $|\emptyset| = 0$
2. $|\{1, 2, 3\}| = 3$
3. $|\{\emptyset\}| = 1$
4. the set of integers is infinite.

$\{a\} = 1 \qquad \{\{a\}\} = 1$

$\{a, \{a\}\} = 2 \qquad \{a, \{a\}, \{a, \{a\}\}\} = 3$

# Power Sets

The set of all subset of a set $A$, denoted $P(A)$, is called the Power set of $A$.

Ex: If $A = \{a, b\}$ then

$$P(A) = \{\emptyset, \{a\}, \{b\}, \{a, b\}\}$$

If a set has $n$ elements, then the cardinality of its power set is $2^n$.

$\{a\} = \{\emptyset, \{a\}\}$

$\{a, b\} = \{\emptyset, \{a\}, \{b\}, \{a, b\}\}$

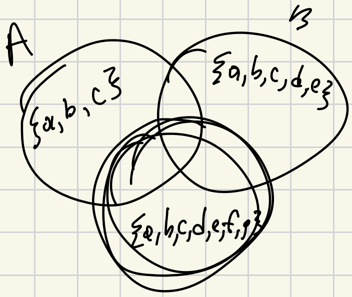
$\{\emptyset, \{\emptyset\}\} = \{\emptyset, \{\emptyset\}, \{\{\emptyset\}\}, \{\emptyset, \{\emptyset\}\}\}$

Find $A^2$ if.

a) $A = \{0, 1, 3\}$. b) $A = \{1, 2, a, b\}$

a) $A \times A = \{0, 1, 3\} \times \{0, 1, 3\} = \{0, 0\}; \{0, 1\}; \{0, 3\}; \{1, 0\}; \{1, 1\}; \{1, 3\}; \{3, 0\}; \{3, 1\}; \{3, 3\}.$

b) $A \times A = \{1, 2, a, b\} \times \{1, 2, a, b\} = \{1, 1\}; \{1, 2\}; \{1, a\}; \{1, b\};$
$\{2, 1\}; \{2, 2\}; \{2, a\}; \{2, b\}; \{a, 1\}; \{a, 2\}; \{a, a\}; \{a, b\};$
$\{b, 1\}; \{b, 2\}; \{b, a\}; \{b, b\}$




$A \subset B$ and $B \subset C$

If $A = \{a, b, c\}$ so $B = \{a, b, c, d, e\}$