Week 2 — 3/5

Polynomials

a. $\underbrace{\text{Poly}}_{\text{many}}$ $\underbrace{\text{nomial}}_{\text{parts}}$

$\overbrace{5x,\ 3,\ 3x^2}^{\text{monomial}}$

$\underbrace{3x^2-5,\ x-1,\ 5x^5+3x^2}_{\text{Bionomial}}$

$(3x^2-5)+(5x^5+3x^2)$

$5x^5+6x^2-5 \rightarrow$ polynomial

$a_n x^n + a_{n-1}x^{n-1} \dots$ — n-th degree polynomial

$a_n \neq 0$

b. $(3x^2-5)(5x^5+6x^2-5) = 15x^7 + 18x^4 - 15x^2 - 25x^5 + \cancel{15x} + 30x^2 + 25 = \cancel{15x^7 + 18x^4}\ 15x^7 - 25x^5 + 18x^4 - 45x^2 + 25$

c. Long division of polynomial

$$\begin{array}{l|l} x^2+5x+6 & x+2 \\ -(x^2+2x) & x+3 \\ \hline 3x+6 & \\ -(3x+6) & \\ \hline 0 & \end{array}$$

$$\begin{array}{l|l} 2x^3+8x^2-6x+10 & x-2 \\ -(2x^3-4x^2) & 2x^2+12x+18 \\ \hline 12x^2-6x+10 & \\ -(12x^2-24x) & \\ \hline 18x+10 & \\ -(18x-36) & \\ \hline \end{array}$$

remainder of division (46)

$(2x^2+12x+18)(x-2)+46 = 2x^3+8x^2-6x+10$

d. Spein bionomial products

$a^2-b^2 = (a+b)(a-b)$

$= a^2 + \cancel{a\cdot(-b)} + \cancel{b\cdot a} + b\cdot(-b)$

$= a^2-b^2$ — Proved

1) $(ax+b)(ax-b) = (ax)^2-b^2$

$(a+b)^2 = a^2+2ab+b^2$

e. Factors and divisibility

$20x^2y^3 \,/\, x$

$2x^2+6x = 2x(x+3)$

$24x^4 \,/\, 8x^3 = 3x$

$32x^3 \,/\, 4x^6 = \frac{8}{x^3} = 8\cdot x^{-3}$ — no polynomial

$6x \,/\, 12x^2 = \frac{1}{2x}$ ✗

$\underline{15x^2y^6}$

$5x$ ✓

$3x^2y^5$ ✓

$10x^4y^3$ ✗

x^2+5x+4

$x \quad 4 \quad 4x$

$x \quad 1 \quad 1x$

$(x+4)(x+1) = x^2+5x+4$

$x_1 \quad x_2$

GCD/LCM of monomials

$15x^2y^6$ $\quad$ $10x^4y^3$

$3\cdot 5\cdot x\cdot x\cdot y\,y\,y\,y\,y\,y$ $\quad$ $2\cdot 5\cdot x\cdot x\cdot x\cdot x\cdot y\,y\,y$

$GCD = 5x^2y^3$

$LCM = 2\cdot 3\cdot 5\cdot x^4\cdot y^6 = 30x^4y^6$

9. Factoring quadratic

$2x^2 + 7x + 3$ $\qquad$ $6x^2 + 49x + 8$

$2x \quad 1 \quad 2x$ $\qquad$ $6x \quad 8$

$x \quad 3 \quad 3x$ $\qquad$ $x \quad 1$

$5x \neq 7x$

$6x + x = 7x$

$(2x+1)(x+3)$

10. Binomial Theorem & Pascal's Triangle

$F(x) = a_n x^n + a_{n-1} x^{n-1} \ldots$ $\quad$ $\boxed{a \neq 0}$

R $\quad$ C

7 $\quad$ 0

$ax^2 + bx + c$

$D = b^2 - 4ac$

$x_{1,2} = \frac{-b \pm \sqrt{D}}{2a}$

$3x^2 - 6x^2 + \ldots$

Pascal's triangle & Binomial Theorem

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1

$\boxed{C_n^k = \frac{n!}{k!(n-k)!}}$

$(x+y)^2 =$

$x^2 + xy + xy + y^2$

$= x^2 + 2xy + y^2$

$n = 3$

$(x+y)(x+y)(x+y) =$

$= x^3 + 3x^2y + 3xy^2 + y^3$