# Week 2
## Geometric Series

a. Definition

$a_1 = 3 \quad a_2 = a_1 \cdot r \quad a_3 = a_1 \cdot r^2$

$a_4 = a_1 \cdot r^3$

$\sum_{n=0}^{99} 3 \cdot 2^n = \underbrace{3 \cdot 2^0}_{3} + \underbrace{3 \cdot 2^1}_{6} + \underbrace{3 \cdot 2^2}_{12} + \ldots$ Shift $\sum_{n=1}^{100} 3 \cdot 2^{(n-1)}$

$S_n$

b. N-th term formula

$a_n = a_1 \cdot r^{n-1}$

$a_x \quad a_y \quad y-x+1$ is odd

$a_2 \quad a_6$

$\frac{a_{x+y}}{2} = \sqrt{a_x a_y}$

$a_4 = \sqrt{a_2 \cdot a_6}$

$a_1 \cdot r^1 \quad a_1 \cdot r^5$

$\sqrt{a_1^2 \cdot r^6}$

$a_4 = a_1 \cdot r^3$

$r - ? \qquad x \leq y$

$a_x \quad a_y \qquad \frac{a_y}{a_x} = \frac{a_1 r^{y-1}}{a_1 \cdot r^{x-1}} = r^{y-1-(x-1)}$

$= r^{y-1-x+1} = r^{y-x} = \sqrt[y-x]{\phantom{a}}$

Sum of the N-terms

$\boxed{S_n = \frac{a_1(r^n - 1)}{r-1}}$

$S_n = \frac{1(2^n - 1)}{2-1} = 2^n - 1$

$r S_n$

$S_n - r \cdot S_n = a_1 - a_1 r^n \;;\; S_n(1-r) = a_1(1-r^n)$

$S_n = \frac{a_1(1-r^n)}{1-r} = \frac{a_1(r^n - 1)}{r-1}$

Sum of the infinite decreasing geometric sequence

$a_1 = 1 \quad a_n = a_{n-1} \cdot \frac{1}{2} \qquad 0 < r < 1$

$\underbrace{1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \ldots}$ $\qquad 0 < |r| < 1$

$S_\infty = \frac{a_1(1-r^n)}{1-r} = \frac{1}{1-\frac{1}{2}} = \frac{\frac{1}{1}}{\frac{1}{2}} = 2$

$S_n = \frac{a_1(1-r^n)}{1-r}$

$0 < r < 1$

$\lim_{n \to \infty} \left( \frac{a_1(1-r^n)}{1-r} \right) = \frac{a_1(1-0)}{1-r} = \frac{a_1}{1-r}$