Data Structure

Sorting

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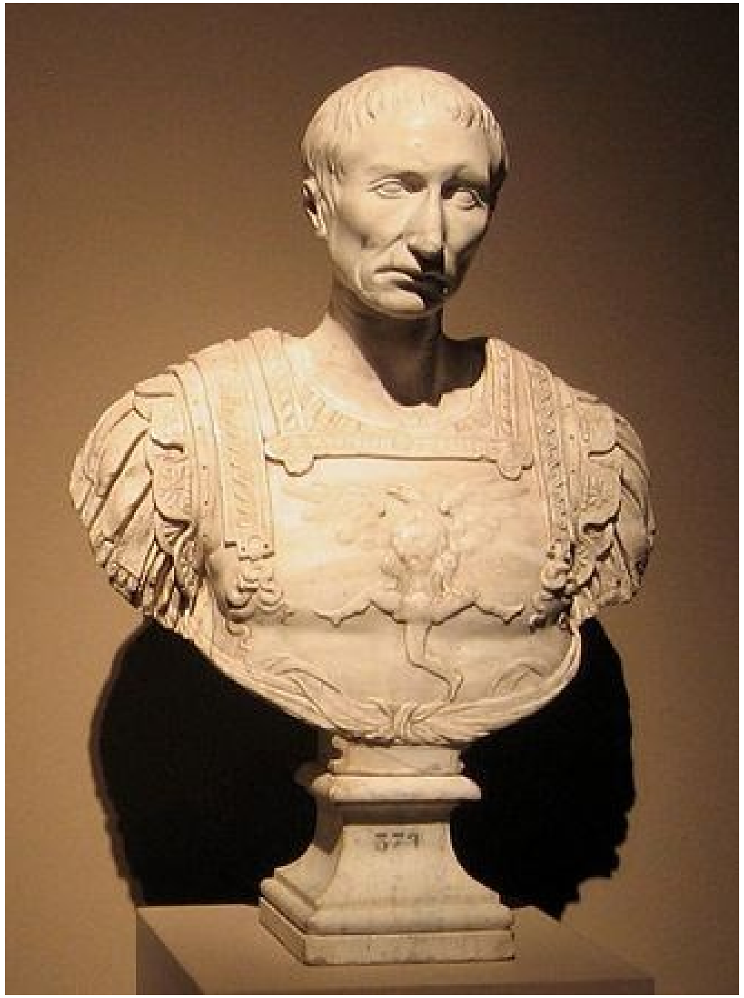
25 May 2023



DS&A. Chapter 11.1 Merge-sort
DS&A. Chapter 11.2 Quick-sort

Merge-sort (1/2)

- a sorting problem is to produce an ordered representation of a given sequence of objects stored in a linked list or an array
  - according to a given comparator that defines a total order on the given objects

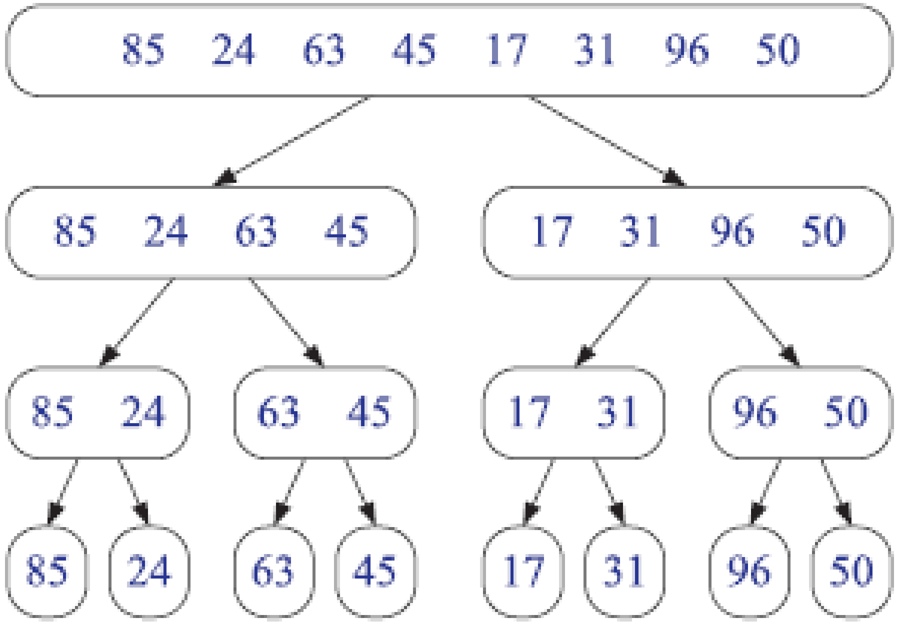
- merge-sort is based on an algorithmic design pattern called divide-and-conquer which typically consists of the followings:
  - divide the input data into two or more disjoint subsets, and recursively solve the subproblems with the subsets
  - take the solutions of the subproblems and merge them into a solution to the original program

Merge-sort (2/2)

- To sort a sequence S with n elements:

  1. Return S immediately if n is zero or one.
     Otherwise, create S_1 and S_2 by dividing S evenly
  2. Sort S_1 and S_2, recursively
  3. Put back the elements into S by merging S_1 and S_2

Example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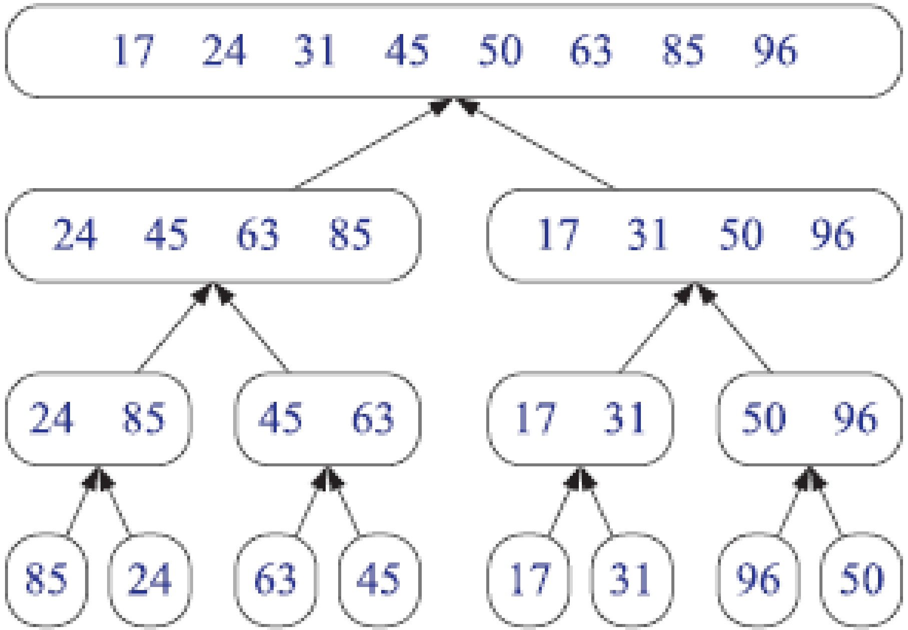
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Running Time of Merge-sort

- suppose that n is a power of 2 (i.e., $n = 2^m$)

- merge-sort of 2^k elements is invoked for 2^{m-k} times
  - at merge-sort execution, it takes linear time to divide the given elements, and merge the two sorting results into one
  - the maximum depth of recursive call is m

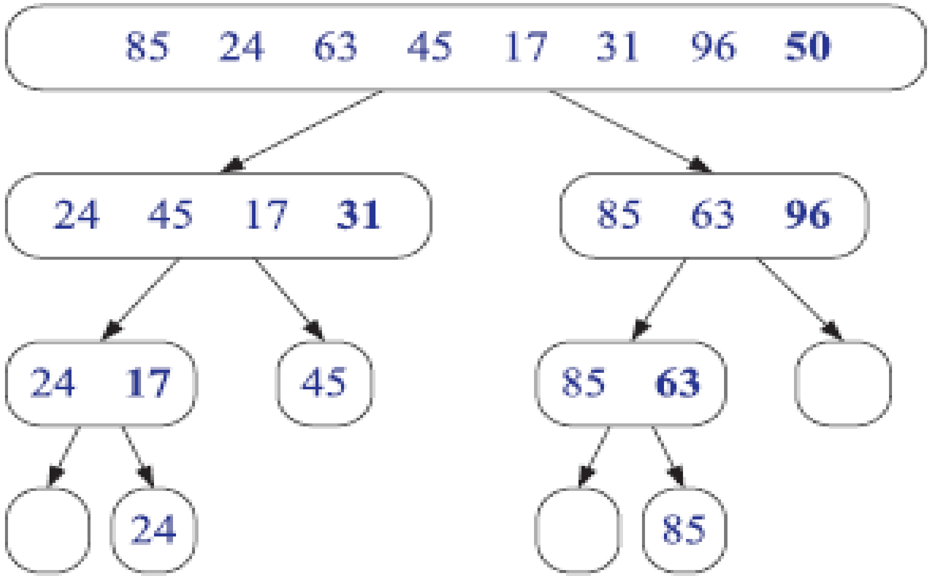
- the time complexity is $O(n \log n)$

Quick-sort

- To sort a sequence *S* with *n* elements:

  1. if *S* has at least two elements, select a specific element *x* from *S* as the pivot
  2. Remove all elements from *S* and put them into three sequences:
     - *L*, storing the elements less than *x*
     - *E*, storing the element equal to *x*
     - *G*, storing the element greater *x*
  3. Recursively sort L and G
  4. Put back the elements into S in order of *L*, *E*, and *G*

Example



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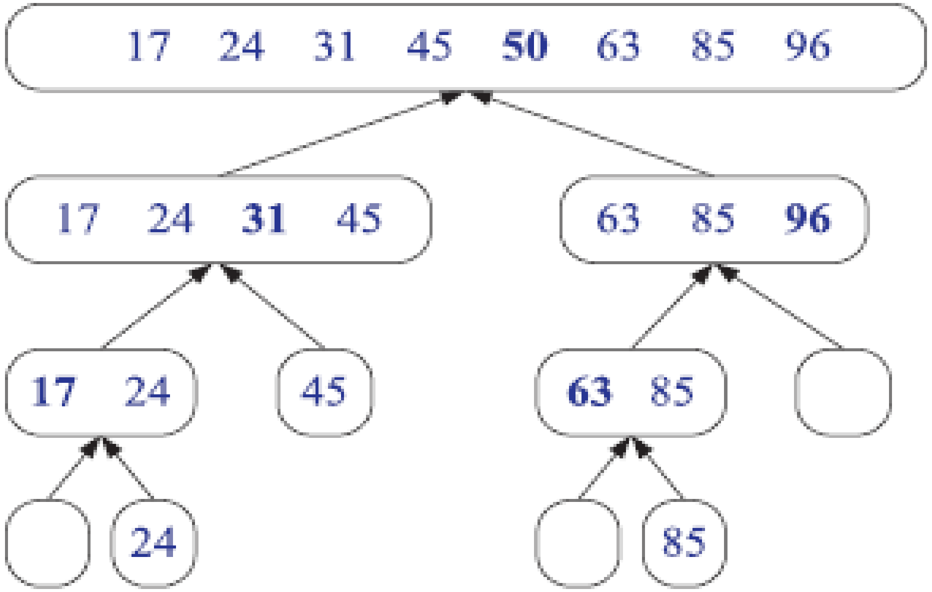

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Running Time of Quick-sort

- Time spent at each quick-sort is linear of the number of elements

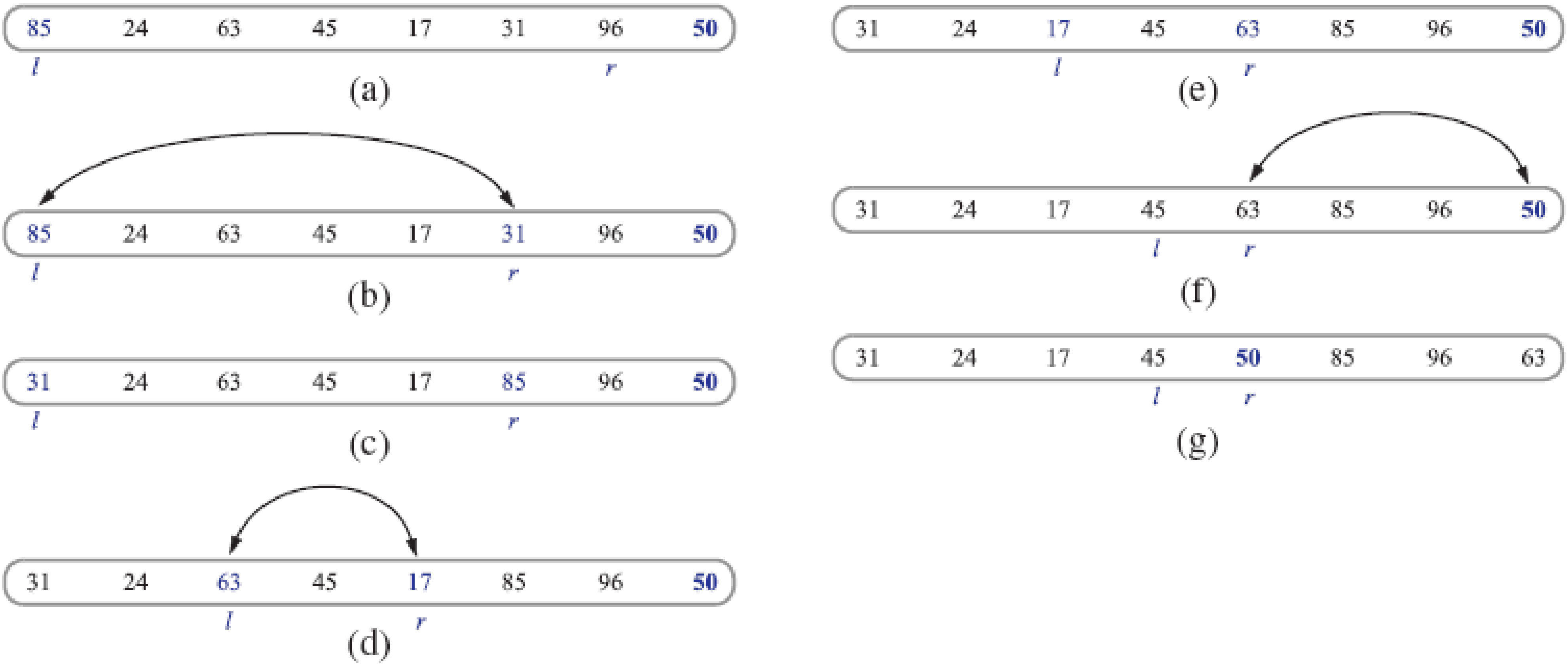
- In worst case, the number of element is reduced by one at each recursion and the recursion continues up to n-1 times, thus it takes $O(n^2)$
  - pivot does not divide given elements evenly

- The best case is where at each quick-sort invocation, the pivot divides the given elements into two subsequences of an equal size, thus it takes $O(n\log n)$

Randomized Quick-sort

- The worst case happens if a pivot does not divide the given input sequence every time
    - e.g., initially, given elements are arranged in decreasing order
- Instead of picking a pivot as the last element, select an element at a random index as the pivot
    - then, the expected running time is O(n log n)

In-place Quick-sort



Bucket-sort

- Sort n entries whose keys are integers in the range $[0, N - 1]$ in $O(n + N)$
  - assume that $N << n$
  - it is not based on comparison
- Allocate a bucket array B that has cells indexed from 0 to $N – 1$, place each entry with key k in $B[k]$, and then enumerate entries in B in order

Stability of Sorting

- Let S = ((k_0, x_0), ..., ($k_{n\text{-}1}$, $x_{n\text{-}1}$)) be a sequence of entries
- We say that a sorting algorithm is stable if, for any two entries (k_i, x_i) and (k_j, x_j) of S such that $k_\mathrm{i} = k_j$ and i < j, (k_i, x_i) precedes (k_j, x_j) after sorting

Radix-sort

- Suppose that a key is d digits each of which is an integer in the range $[0, N\text{-}1]$

- Iteratively run bucket-sort with i-th digit as key, for $i = d, d-1, \ldots, 1$
  - exploit the stability of bucket-sort
  - last digit is least-significant digit
  - e.g., 233, 46, 82, 2, 958, 33, 143, 67, 146, 92

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- Radix-sort takes $O(d(n+N))$