1 Calc 1 Exam 1

1. (a) Given that $g(3) = 3/2$ and $g'(3) = \pi$ construct the tangent line of $g(x)$ at $x = 3$
   (b) Using the fact that $g''(x) < 0$ for all x, what are the possible values for $g(6)$? Why
2. Find the derivative of the following functions using the limits definition of derivative. (Note: you will not receive any credit for simply applying a law of differentiation. You must use the limit definition)
   (a) $g(x) = \sqrt{x+2}$
   (b) $h(x) = (x-3)^{-1}$

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3. Let $g(x) = \frac{6-x}{2x+1}$ then
   (a) Find the derivative at the value $x = 1$.
   (b) Construct the equation of the tangent line at the value $x = 1$.
   (c) Construct the equation of the line perpendicular to the tangent line at $x = 1$.
4. Given the following values

x	-2	0	2	4	6
$f(x)$	-3	-1	2	0	-2
$g(x)$	5	2	-3	2	0
$f'(x)$	-2	2	4	-1	0
$g'(x)$	-3	-1	2	3	4

   (a) If $h_1(x) = f \circ g(x)$ find $h_1'(0)$.
   (b) If $h_2(x) = f(x)g(x)$ find $h_2'(2)$.
   (c) If $h_3(x) = \frac{f(x)}{g(x)}$ find $h_3'(-2)$.
   (d) If $h_4(x) = 2 \cdot f^{-1}(x)$ find $h_4'(0)$.
5. Find the derivatives to these implicit functions.
   (a) $x\ln(y) + y^2 = \ln(x)$
   (b) $e^{xy} = 2$

   (a) Find the local linear approximation of $y = \sqrt[3]{x}$ at $x = 8$.
   (b) Use your result to approximate $\sqrt[3]{9}$.
   (c) Is this an over or under estimate? (Show work here)
6. Let $f(x) = x\ln(x) + x$, on the interval $(0, \infty)$.
   (a) Find the interval(s) where $f(x)$ is increasing.
   (b) Find the interval(s) where $f(x)$ is concave up.

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7. Decide if the statement is true or false and give an explaination for your answer: The Racetrack Principle can be used to justify the statement that if two horses start a race at the same time, the horse that wins must have been moving faster than the other throughout the race.
8. Let $p(x)$ be a seventh degree polynomial with 7 distinct zeros. How many zeros does $p'(x)$ have? Justify your answer.

9. Sketch a possible graph of $y = f(x)$ using information about the derivatives. Assume that the function is defined and continuous for all real x.

10. Find the global maximum and minimum for the function on the closed interval.

$$f(x) = x^3 - 3x^2 + 20, \quad -1 \leq x \leq 3$$

11. Find the global maximum and minimum for the function on the closed interval.

$$f(x) = x - 2\ln(x+1), \quad 0 \leq x \leq 2$$

12. Find a formula for the family of cubic polynomials with an inflection point at the origin. How many parameters are there?

13. Find the x-value maximizing the shaded area. One vertex is on the graph of $f(x) = x^2/3 - 50x + 1000$.

14. A rectangle has one side on the x-axis and two vertices on the curve

$$y = \frac{1}{(1+x^2)}.$$

Find the vertices of the rectangle with maximum area.

15. When production is 2000, marginal revenue is \$4 per unit and marginal cost is \$3.25 per unit. Do you expect maximum profit to occur at a production level above or below 2000? Explain.

16. Let $C(q)$ be the total cost of producing a quantity q of a certain product. See Figure 4.76.
(a) What is the meaning of $C(0)$?
(b) Explain the shape of the curve in terms of economics.

17. Gasoline is pouring into a cylindrical tank of radius 3 ft. When the depth of the gasoline is 4 ft, the depth is increasing at 0.2 ft/sec. How fast is the volume of gasoline changing at that instant?

18. The radius of a spherical balloon is increasing by 2 cm/sec. At what rate is air being blown into the balloon at the moment when the radius is 10 cm? Give units in your answer.

19. Evaluate the following limit:

$$\lim_{x \to 0} \frac{1 - \cos x}{x^2}$$

20. Evaluate the following limit:

$$\lim_{x \to 0^+} x^x$$

21. (a) State precisely the Mean Value Theorem
(b) Does the function $f(x) = \ln(x+1)$ satisfy the hypothesis of the Mean Value Theorem on the interval $[0, e-1]$? Explain.
(c) Show that the function $f(x) = \ln(x+1)$ satisfies the conclusion of the Mean Value Theorem on $[0, e-1]$ by finding the value c.

22. Use the Racetrack Principle to show that $\ln x \leq x - 1$ for all $x \geq 1$.

23. Find and classify all critical points for the following functions:

(a) $f(x) = x^3 - x^2 - 5x + 7$

(b) $f(x) = e^{x^2 - 4x}$

24. Find and classify all critical points of the function $f(x) = x^2 e^{-3x}$

25. Find the global maximum and global minimum of the function $f(x) = 6xe^{-x/2}$ on the interval $[0, \infty)$

26. Find a formula for the function described: a cubic polynomial with a local maximum at $x = -2$, a local minimum at $x = 1$, a y-intercept of -2, and an x^3-term whose coefficient is 1.

27. A rectangle has one side on the x-axis and two vertices on the curve $e^{-x^2/6}$. What x-value gives the maximum area? What is the maximum area?

28. A Landscape architect plans to enclose a 125 square foot rectangular region in a botanical garden. She will use wire fencing that costs \$8 per foot along three sides and wooden fencing that costs \$12 along the fourth side. Find the minimum total cost.

29. Suppose you are selling q units of a product with a cost function of $C(q)$ and a revenue function of $R(q)$. If $C'(300) = 120$ and $R'(300) = 90$, should the quantity produced be increased or decreased from $q = 300$ in order to increase profits? Explain.

30. The total cost $C(q)$ and total revenue $R(q)$ of producing q goods are given by the equations:

$$C(q) = 20 + 10q \qquad R(q) = 50q - q^2$$

Write a function that gives total profit earned and find the quantity that maximizes profit.

31. A 25 ft ladder is leaning against a wall. The floor is slightly slippery and the foot of the ladder slips away from the wall at a rate of 3 in/s. How fast is the top of the ladder sliding down the wall when the base is 20 ft from the wall?

32. A spherical snowball is melting at a rate of 6 cm^3/min. How fast is the radius of the snowball decreasing when its radius is 18 cm?

33. For the amusement of guests, some hotels have elevators on the outside of the building. One such hotel is 300 feet high. You are standing by a window 100 feet above the ground and 150 feet away from the hotel, and the elevator descends at a constant speed of 25 ft/sec, starting at time $t = 0$, where t is in seconds. Let θ be the angle between the line of your horizon and your line of sight to the elevator. How fast is the angle θ changing after 6 seconds?

34. Calculate the following limits:
    - $\lim_{x\to\infty} \frac{e^x}{x^2}$
    - $\lim_{x\to 0} \frac{1}{\sin x} - \frac{1}{\cos x - 1}$
    - $\lim_{x\to\infty} e^{-x} \ln x$
    - $\lim_{x\to 0} (1 + 7x)^{\frac{1}{x}}$

35. Suppose that the position of a particle at time t is given by $x(t) = -3t + e^t$ and $y(t) = 4t - 4$.
(a) Find the position of the particle at time $t = 0$.
(b) Find $\frac{dy}{dx}$ when $t = 0$.

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36. At time, t, in seconds, your velocity, v, in meters/second, is given by

$$v(t) = 1 + t^2 \quad \text{for} \quad 0 \le t \le 6.$$

Use $\Delta t = 2$ to estimate the distance traveled during this time. Find right hand and left hand estimates, and then average the two.

37. At time, t, in seconds, your velocity, v, in meters/second, is given by

$$v(t) = t^2 - 3t + 5 \quad \text{for} \quad 0 \le t \le 8.$$

Use Δ four partitions ($n = 4$) to estimate the distance traveled during this time using both right hand and left hand estimates.

38. Find the difference between the right hand and left hand estimates for the area under $f(t)$ on the interval $a \le x \le b$ for n subdivisions.

$$f(t) = \sin t, \quad a = 0, \quad b = \pi/2, \quad n = 100$$

39. Use Figure 5.31 to find the values of

    (a) $\int_a^b f(x)\,dx$ (b) $\int_b^c f(x)\,dx$
    (c) $\int_a^c f(x)\,dx$ (d) $\int_a^c |f(x)|\,dx$

40. Find the average value of the function $f(x) = 4\sin(x)$ on the interval $[0, \pi]$

41. In 2005, the population of Mexico was growing at 1% a year. Assuming that this growth rate continues into the future and that t is in years since 2005, the Mexican population, P, in millions, will be given by

$$P = 103(a)^t.$$

    Where $a = 1.01$. Predict the average population of Mexico between 2005 and 2055. (Consider a as a constant and keep your answers in terms of a, $\ln a$, etc.)

42. Find the area of the region between $y = x^{1/2}$ and $y = x^{1/3}$ for $0 \le x \le 1$.

43. Find the absolute area between the curves $f(x) = \cos(x)$ and $g(x) = \sin(x)$ on the interval $[0, \pi/2]$.

44. Suppose $\int_0^b f(x)\,dx = 12$. Calculate the integral $\int_{-b}^b f(x)\,dx$ if:
    (a) $f(x)$ is an odd function (b) $f(x)$ is an even function

45. Use Figure 6.14 and the fact that $F(2) = 3$ to sketch the graph of $F(x)$. Label the values of at least four points.

46. Calculate the following definite integrals and provide a family of antiderivatives for any indefinite integrals.

    (a) $\int x^9 - 3x^5 + 12x^3 - 2\,dx$
    (b) $\int e^t - 7\,dt$
    (c) $\int y(y^2 - \frac{3}{y^2})\,dy$
    (d) $\int x\sqrt{x} + \frac{1}{x\sqrt{x}}\,dx$
    (e) $\int_0^3 \frac{1}{w} + w\,dw$
    (f) $\int_1^2 3^t \ln(3)\,dt$
    (g) $\int_0^{\pi/4} \cos(x) - \sin(x)\,dx$
    (h) $\int_{-4}^4 (\sqrt{y} + 1)^2\,dy$
    (i) $\int t^2 + \frac{1}{t^2}\,dt$ $\int_1^2 \frac{1+y^2}{y}\,dy$

47. Find the area enclosed by the curve $y = x^2(1-x)^2$ and the x-axis.

48. A car is travelling at 40ft/s and decelerates at a constant rate of 5ft/s. How far does the car travel before coming to a complete stop?

49. A rock is thrown with a vertical velocity of 20 m/s from a height of 60m. What is the maximum height the rock reaches? How long does it take the rock to hit the ground? (You may assume that the acceleration of gravity is -10 m/s^2)

50. Calculate the following:

    (a) $\frac{d}{dx}\int_4^x \cos(x)\,dx$
    (b) $\frac{d}{dx}\int_{-3}^{x^2} 5^x\,dx$
    (c) $\frac{d}{dx}\int_{3x^2}^{\ln(x)} \tan(x)\,dx$
    (d) $\frac{d}{dx}\int_{-15}^{x^2-2x} \frac{1}{x}\,dx$