C++ Real-Time Audio Programming with Bela

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Founder and Director, Bela

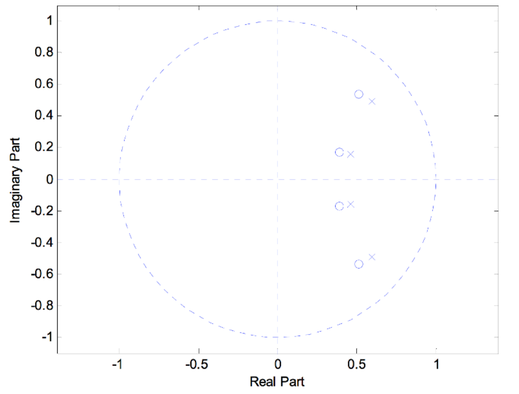
Course topics

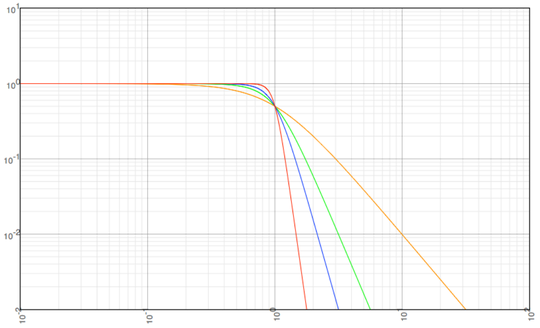
Programming topics	Music/audio topics
Working in real time	Oscillators
Buffers and arrays	Samples
Parameter control	Wavetables
Classes and objects	Control voltages
Analog and digital I/O	Gates and triggers
Filtering	Filters
Circular buffers	Delays and delay-based effects
Timing in real time	Metronomes and clocks
State machines	Envelopes
MIDI	ADSR
Block-based processing	MIDI
Threads	Additive synthesis
Fixed point arithmetic	Phase vocoders
ARM assembly language	Impulse reverb

Today

The course in a nutshell

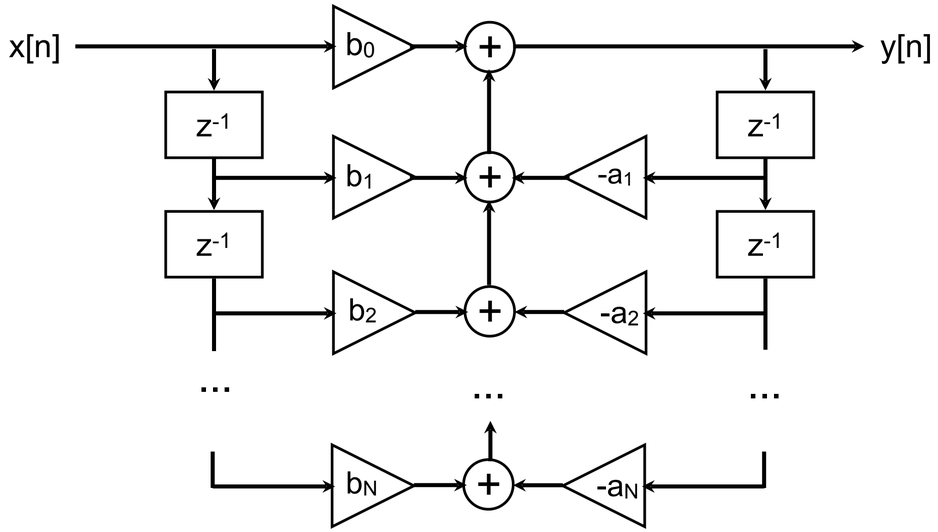
Specifications





Design

$$y[n] = \sum_{k=1}^{N} a_k y[n-k] + \sum_{k=0}^{M} b_k x[n-k]$$



Implementation

```
void render(BelaContext *context, void *
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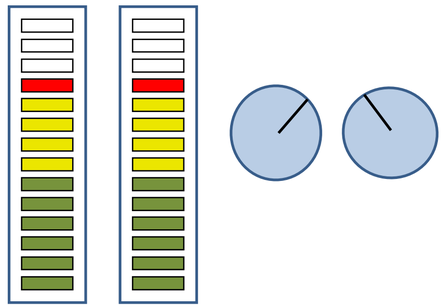
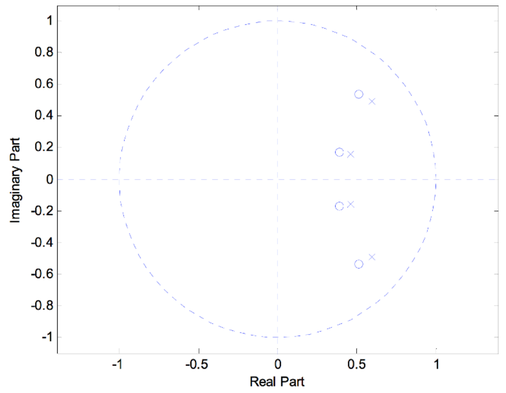
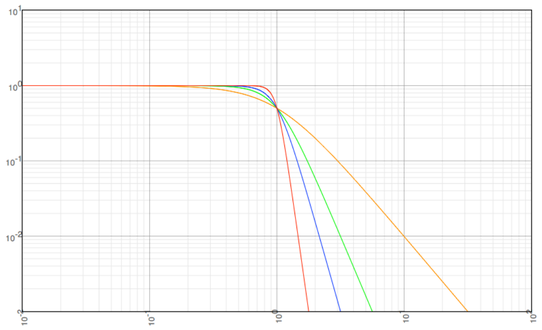


Typical digital signal processing course

The course in a nutshell

Specifications





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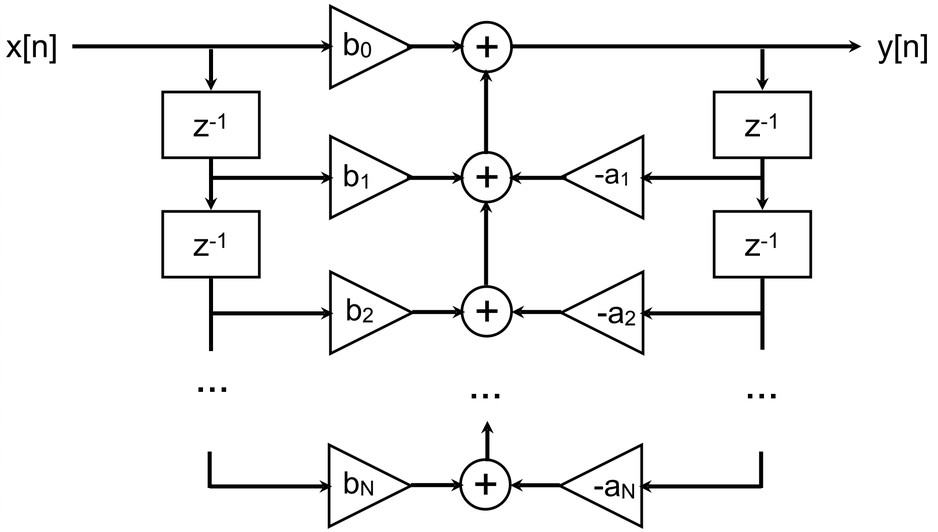

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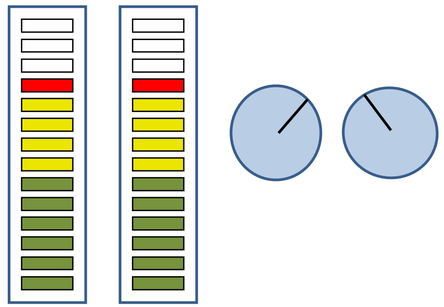

This course

The course in a nutshell

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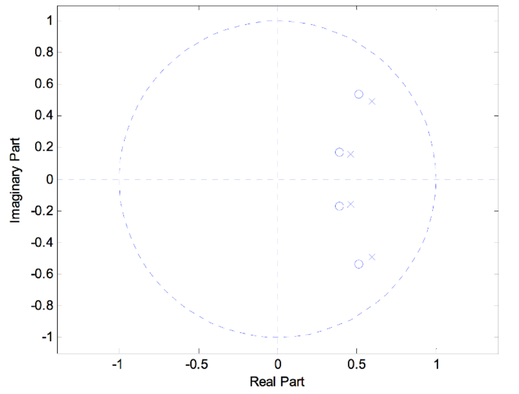

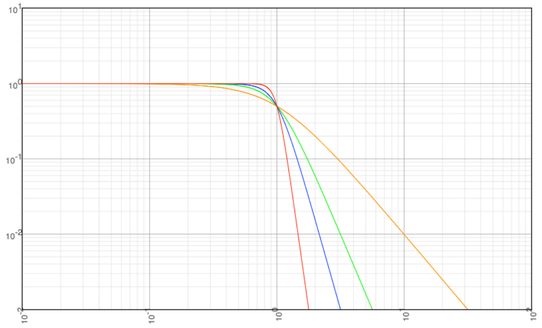

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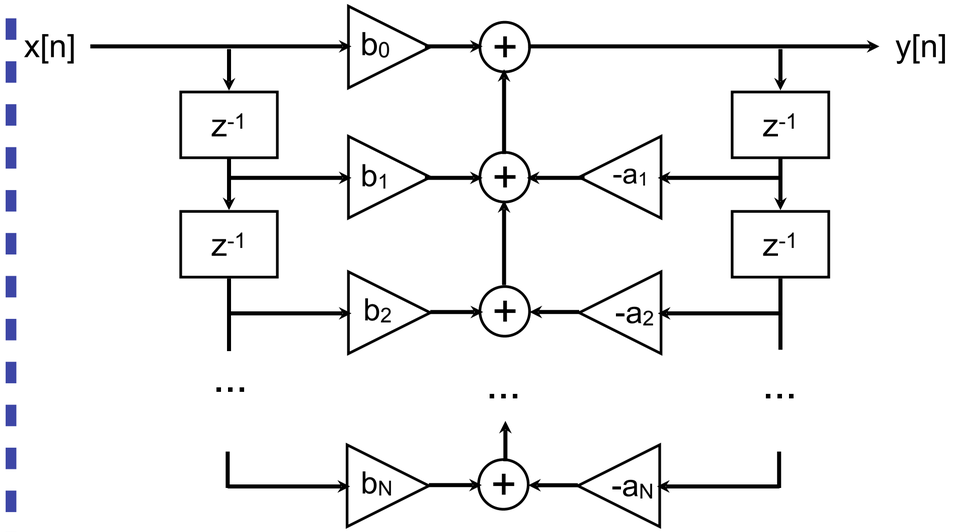

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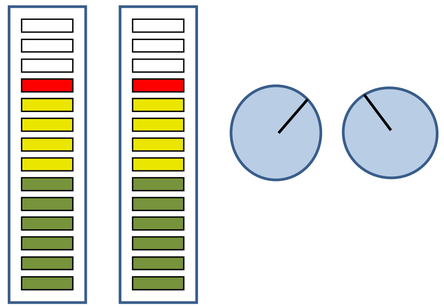


- We will turn signal processing theory into efficient C++ code
- We'll use Bela to process audio and sensor signals in real time

Lecture 8: Filters

What you'll learn today:

Basic theory of digital filters
Filter terminology: cutoff, bandwidth, Q, order
Turning filter equations into code

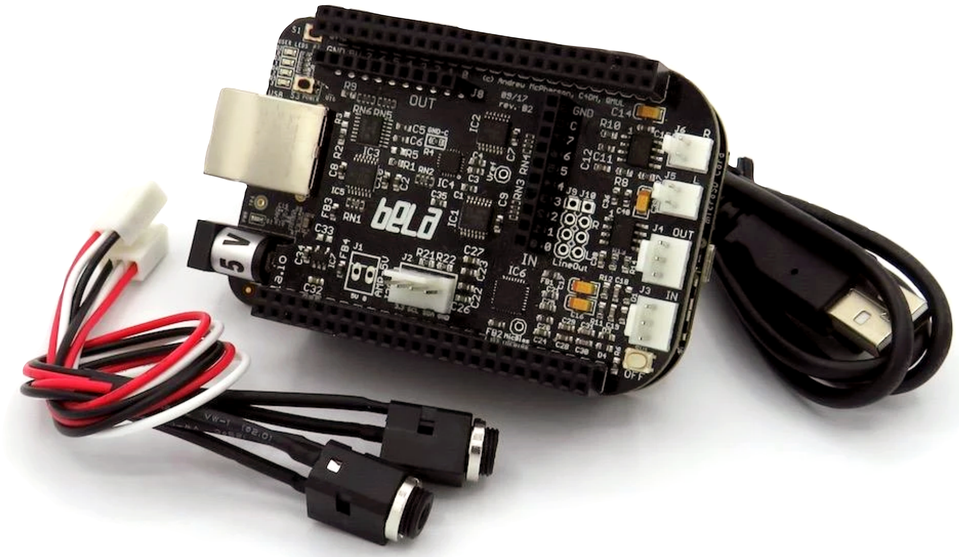
What you'll make today:

An adjustable resonant lowpass filter

Companion materials:

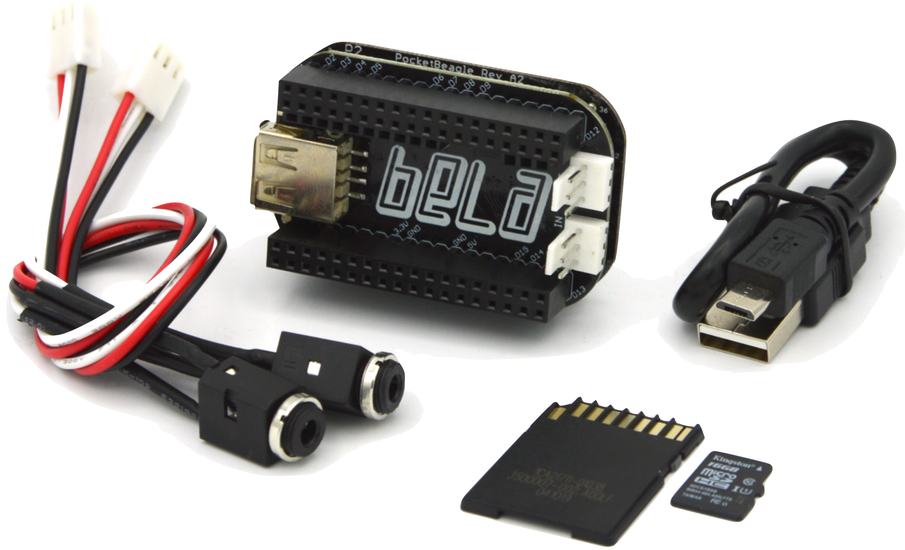
github.com/BelaPlatform/bela-online-course

What you'll need



Bela Starter Kit

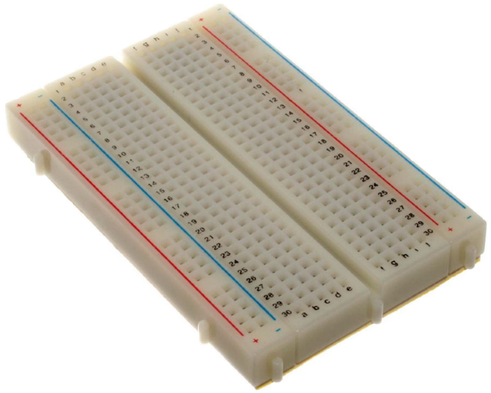
or

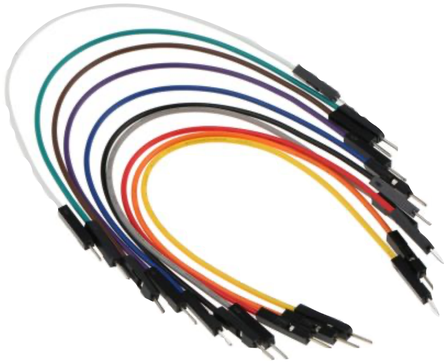


Bela Mini Starter Kit

[shop.bela.io]

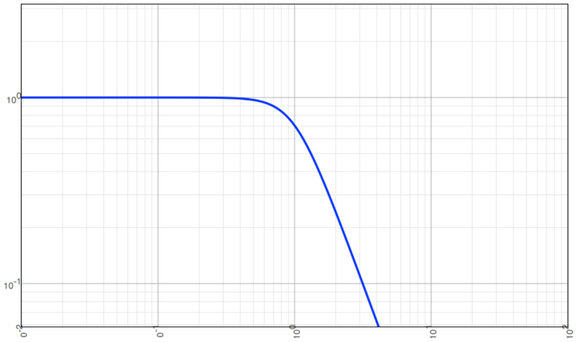
Recommended for some lectures:



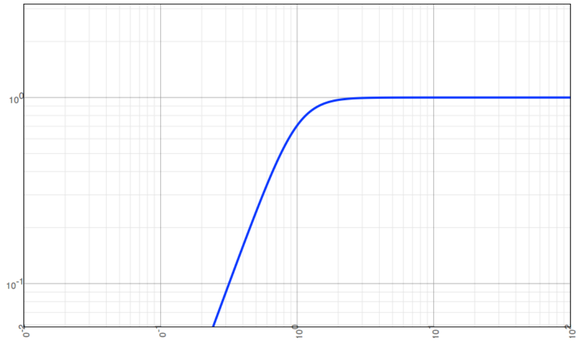


Filters

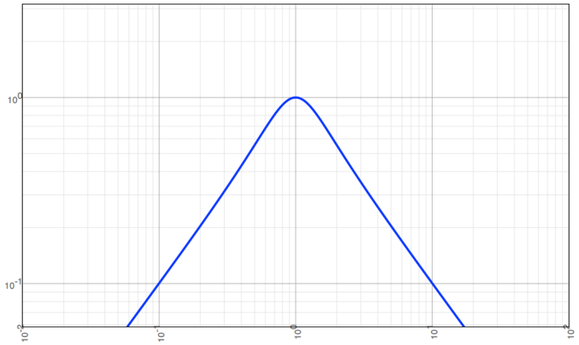
- Filters are a fundamental building block of DSP systems
  - They alter the frequency and phase content (and therefore the waveforms) of signals
- There are a huge number of filter types and applications
  - Some common types in audio:



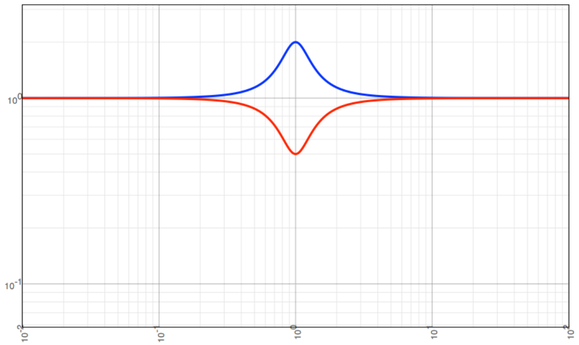
Lowpass



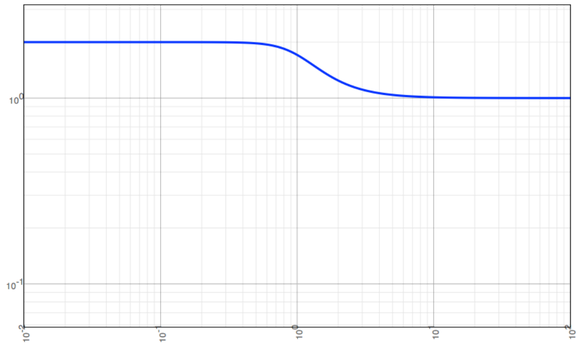
Highpass



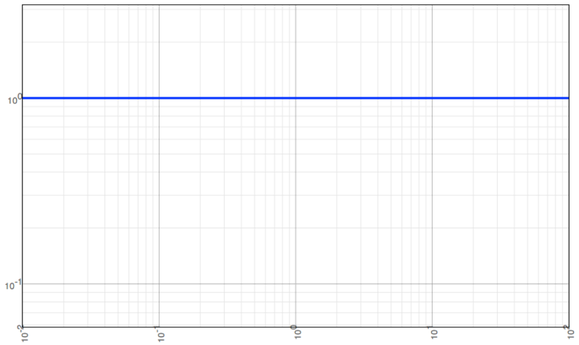
Bandpass



Peak / Notch



Shelving



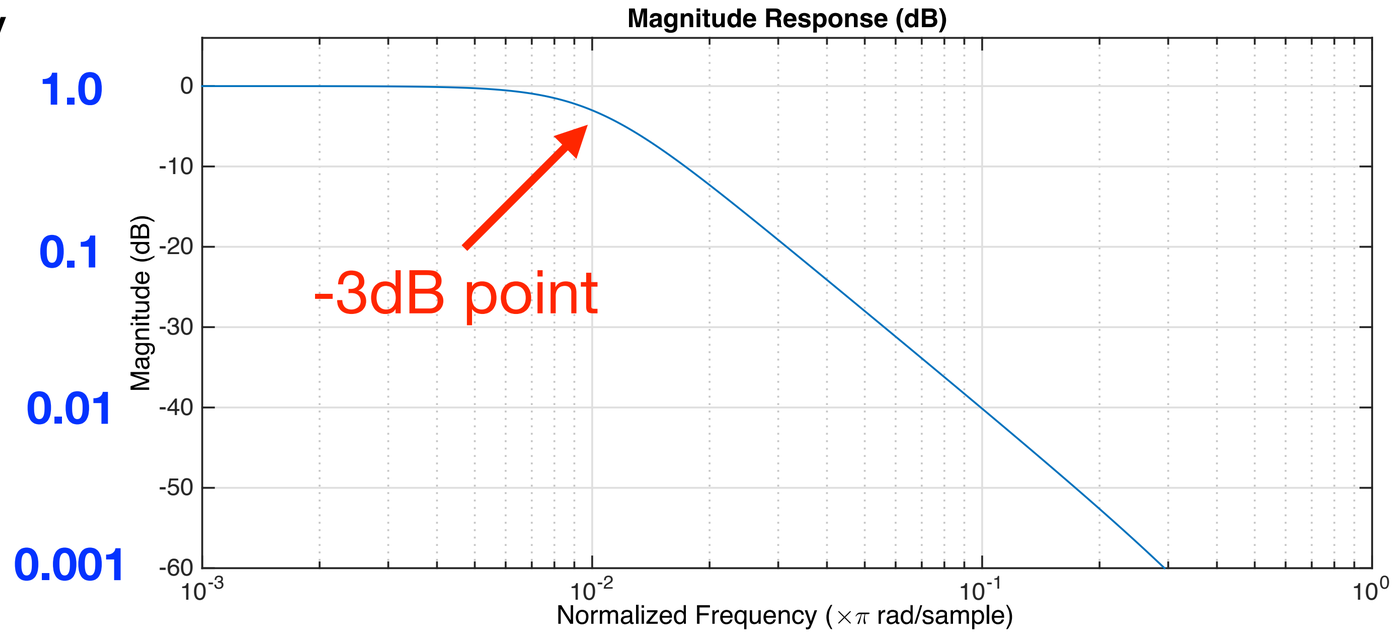
Allpass

Decibel amplitude scale

- The decibel scale is a logarithmic scale used across science and engineering to express ratios between values
  - For a given ratio *A*, we could express the same thing in linear or decibel terms:

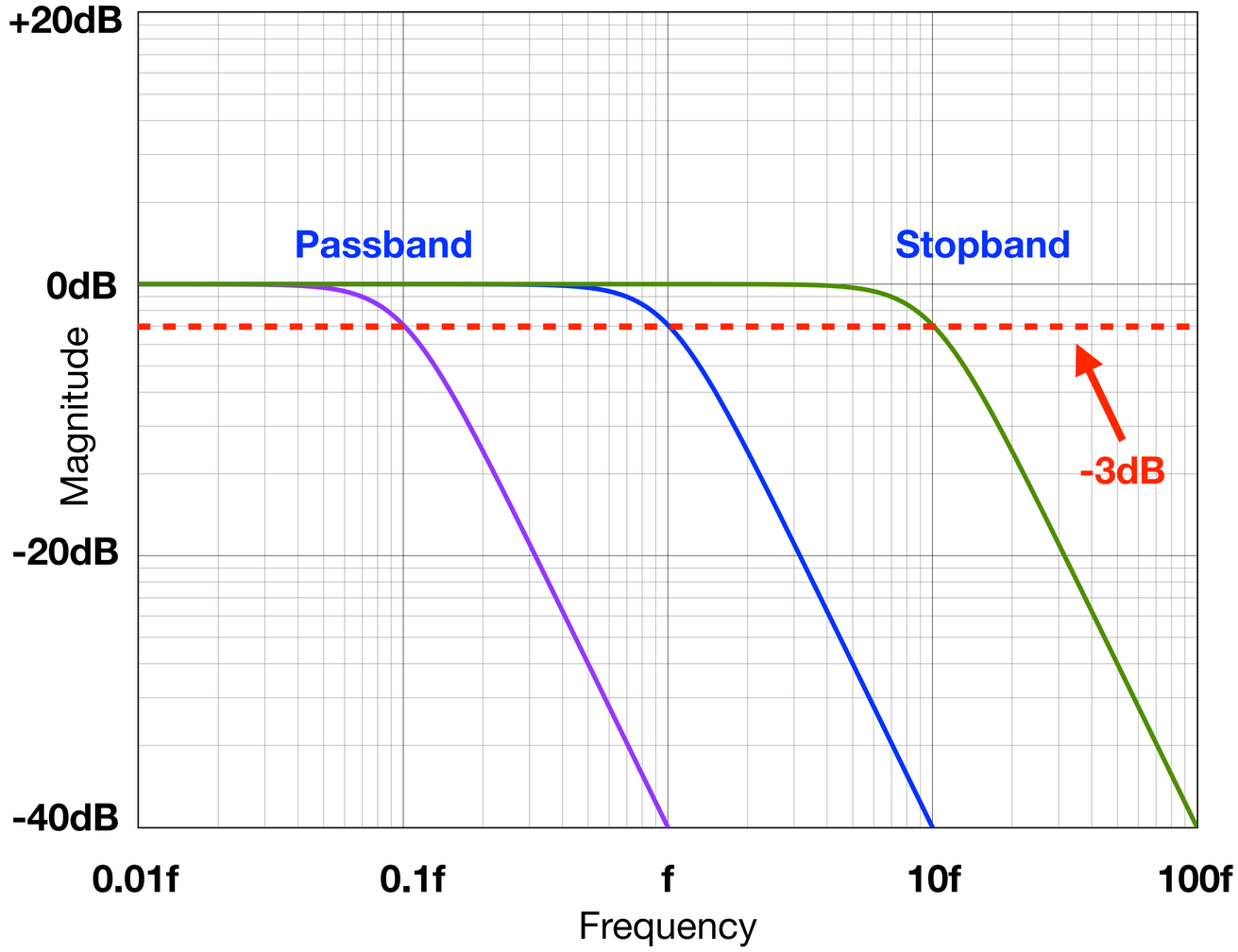
$$A_{dB} = 20\log_{10}(A_{linear}) \quad \longleftrightarrow \quad A_{linear} = 10^{\frac{A_{dB}}{20}}$$

- Decibel scales are often used in specifying filter magnitude response
  - By convention, the critical frequency of many standard filter types is the -3dB point
  - What is -3dB as a linear ratio?
    - $10^{\frac{-3}{20}} \approx .707$
    - More precisely, we are often interested in where the response drops by a factor of $\sqrt{2}$



Filter terminology

- Cutoff frequency
  - Transition between the passband and stopband
  - Often specified as the -3dB point
    - The point where the magnitude response is down 3dB compared to the passband level
    - Not necessarily a raw gain of -3dB!
  - For a bandpass, peak or notch filter, we might talk about centre frequency instead of cutoff frequency

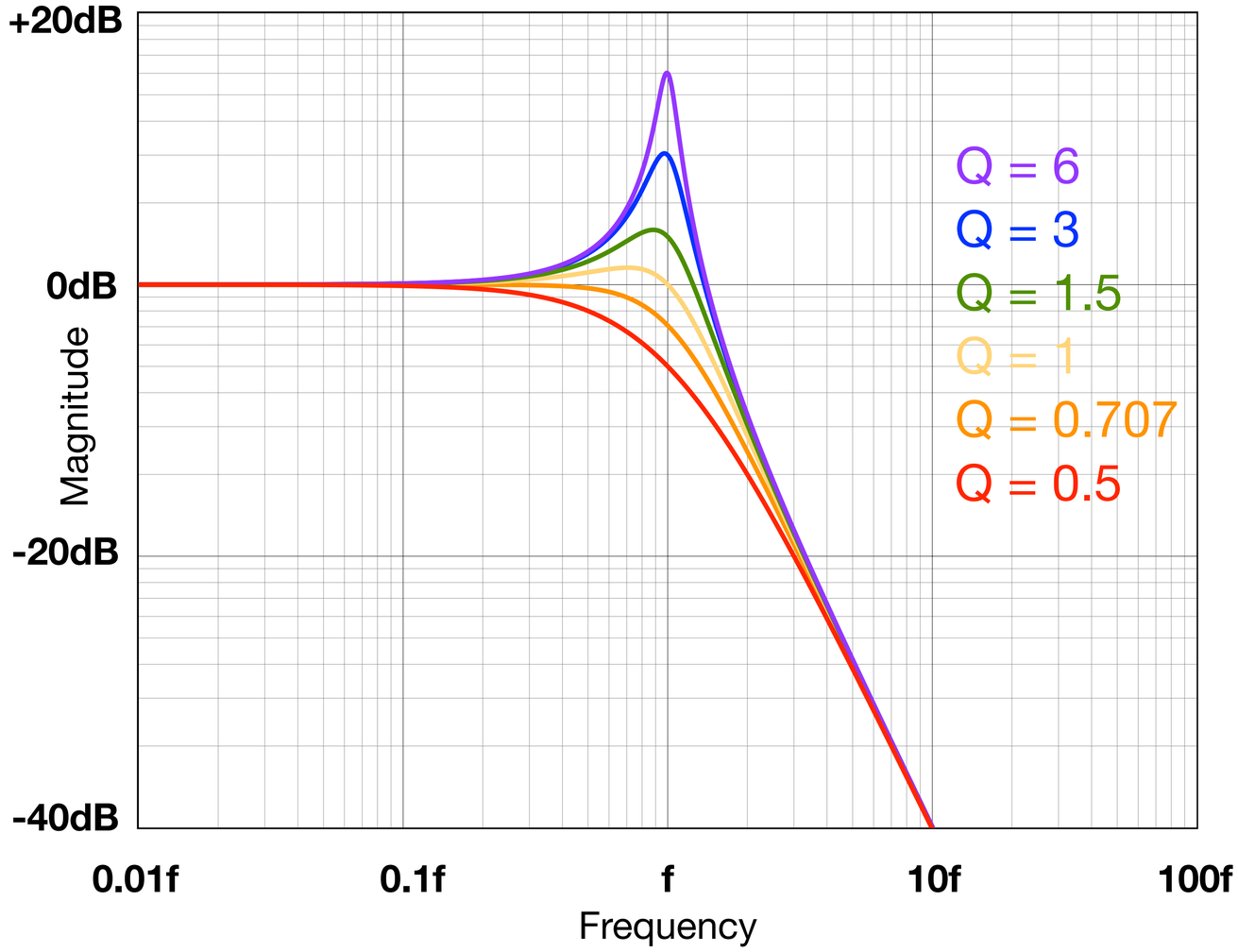


Filter terminology

- Bandwidth
  - For bandpass filters or filters with a resonant peak: bandwidth is the width of the peak
  - In bandpass filters, bandwidth measured between frequencies where gain is -3dB less than peak
- Q factor
  - Defined as the centre frequency divided by the bandwidth:

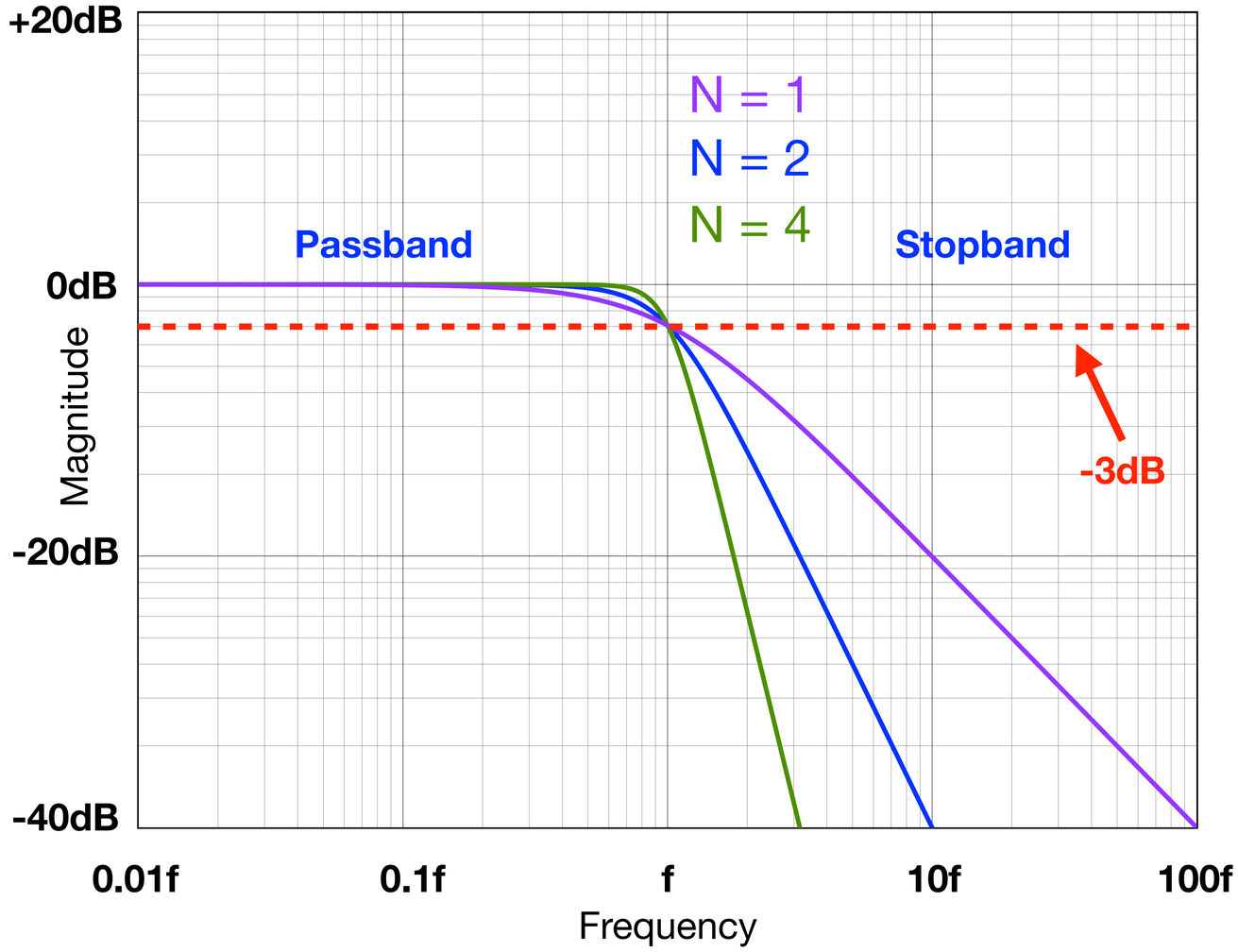
$$Q = \frac{f_c}{\Delta f}$$

  - Low/highpass filters have a Q related to the sharpness of the resonant peak



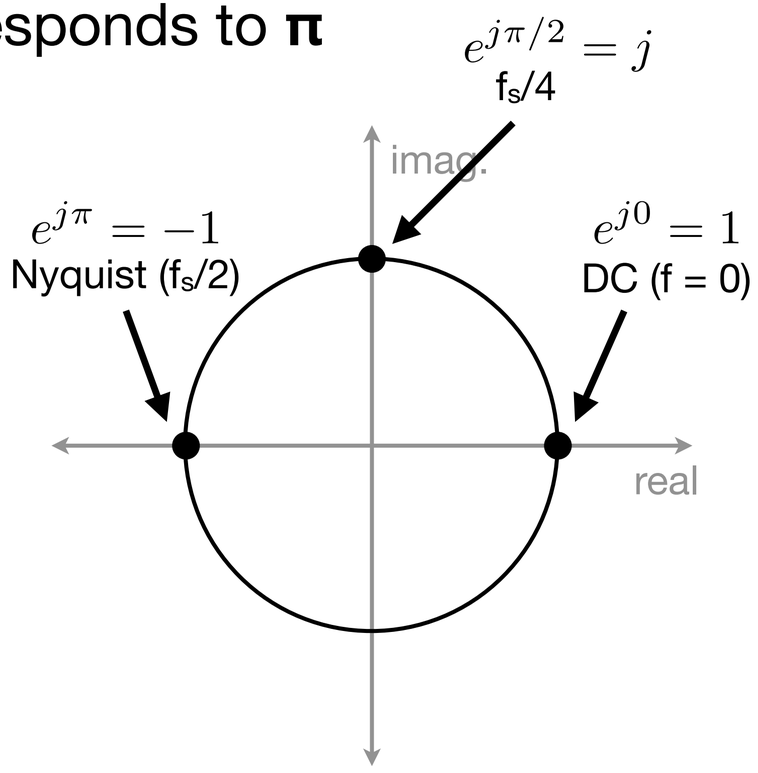
Filter terminology

- Order
  - Determines the steepness of the filter rolloff in the stopband
  - For order *N*, the rolloff is:
    - 6**N* dB/octave, or
    - 20**N* dB/decade
  - In discrete time, the number of taps (previous inputs/outputs) determines the order
    - For example, a 2nd-order IIR filter will use the previous 2 inputs and 2 outputs
  - Any higher-order filter can be built out of second-order sections



Frequency in discrete time

- In continuous time, we represent frequency in Hertz (cycles per second)
- In discrete time, we have a different mathematical formulation for frequency
  - Frequency normalised between **0** and **2π** (2π corresponds to the sample rate)
  - The Nyquist rate, the highest representable frequency, corresponds to **π**
- We will often see frequency-domain representations of discrete-time filters written in terms of ***z***
  - This is based on the *z*-transform (see references in the companion materials)
  - Frequency in the *z*-domain is mapped to the unit circle
  - $z = e^{j\omega}$ where ω (range 0-2π) is the discrete-time frequency
  - If you're given a filter *H*(*z*) and you want to find out its response at a given frequency:
    - Plug in $z = e^{j\omega}$ and calculate the magnitude and phase



Filter equation

- In signal processing, we typically deal with linear time-invariant (LTI) filters
- These filters are specified by their coefficients:

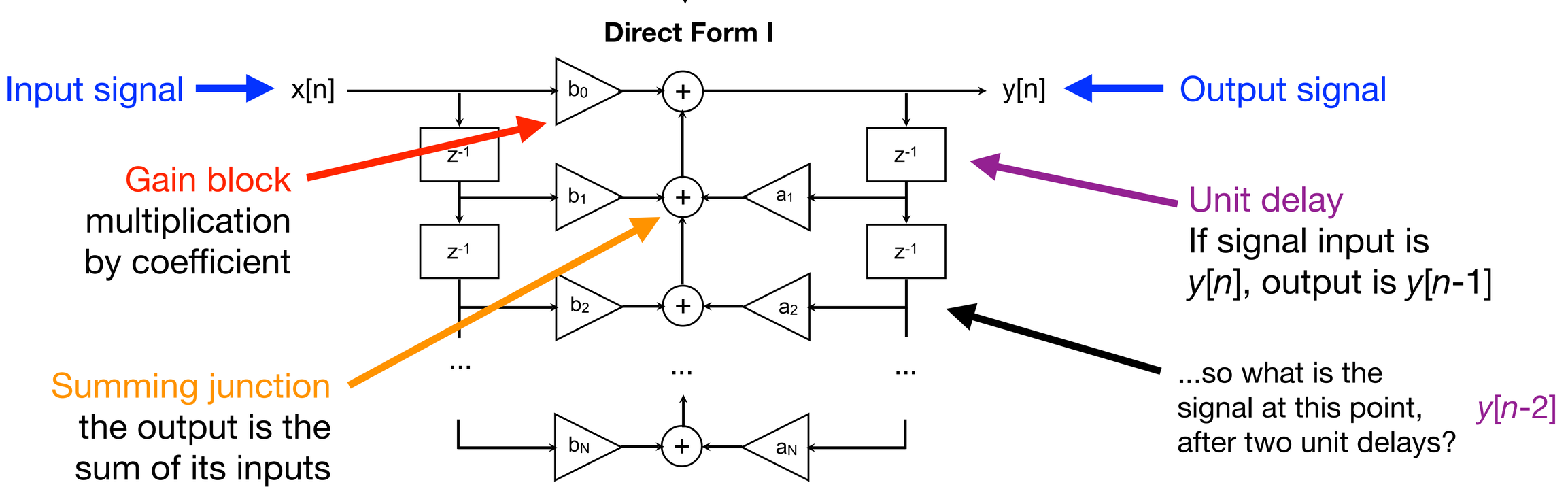
$$y[n] = \sum_{k=1}^{N} a_k y[n-k] + \sum_{k=0}^{M} b_k x[n-k]$$

  - The current output sample is the weighted sum of previous inputs and outputs
  - The coefficients b_k specify the weights of previous inputs
  - The coefficients a_k specify the weights of previous outputs
  - Going from specifications to coefficients is a topic for a DSP class
- Given a set of coefficients, how do we implement a filter in code?
  - Assume the coefficients are calculated for us, and stay the same over time
  - For each new sample, we need to calculate the equation above

Filter block diagram

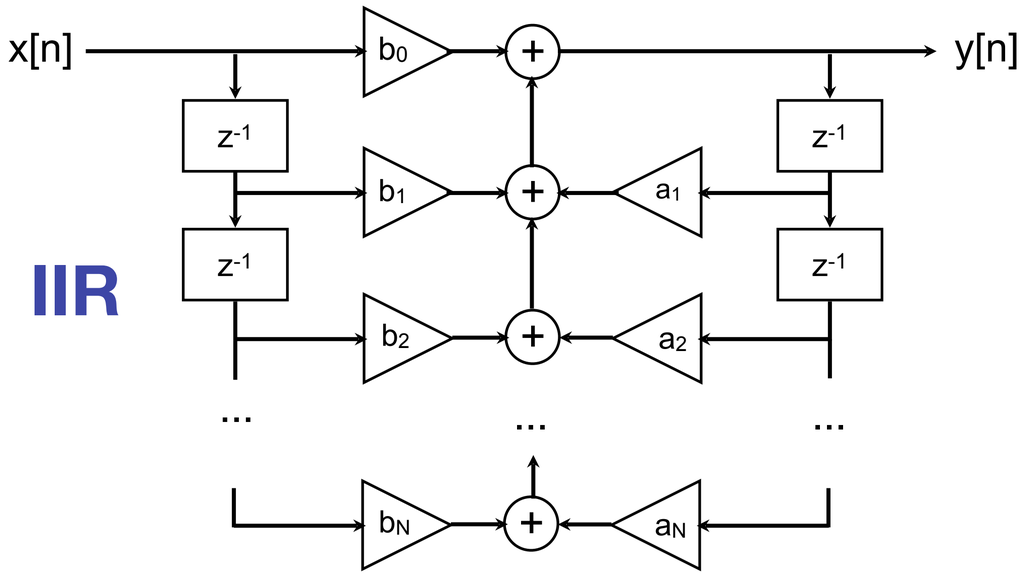
- Filter equations can also be written as block diagrams:

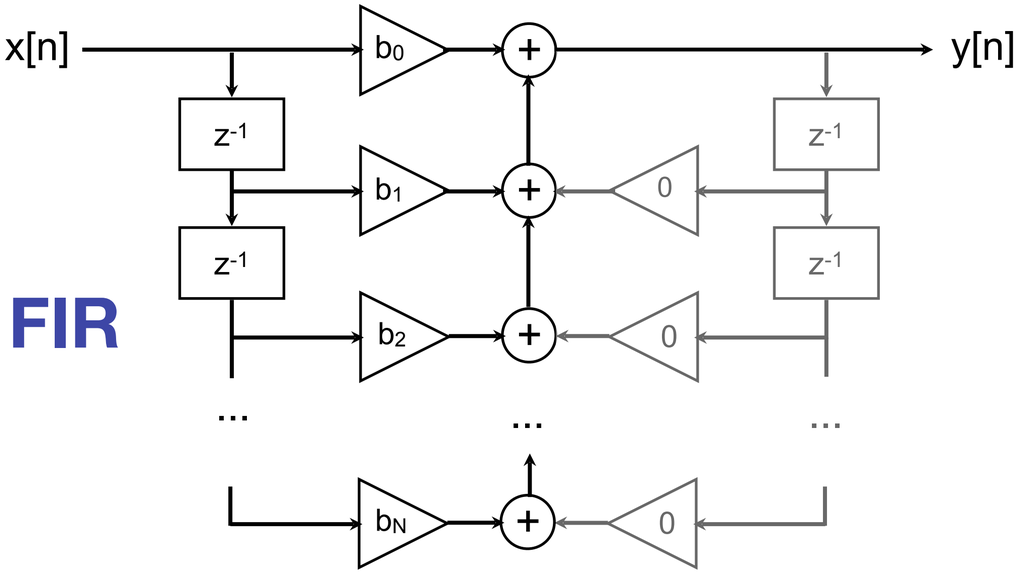
$$y[n] = \sum_{k=1}^{N} a_k y[n-k] + \sum_{k=0}^{M} b_k x[n-k]$$



IIR and FIR filters

- If a filter depends on its previous outputs as well as previous inputs, it is an Infinite Impulse Response (IIR) filter
  - Feedback means that after an impulse comes in, the output will never return exactly to 0





- If a filter depends only on its previous inputs (all $a_k = 0$), it is a Finite Impulse Response (FIR) filter
  - With an impulse input, the output will return to 0 after a finite number of samples

Filter example

- Let's implement a simple first-order difference filter: $y[n] = x[n] - x[n-1]$
  - Current output equals the current input minus the previous input
  - Is this FIR or IIR?
- DSP theory review: is this a high-pass or low-pass filter?
  - Intuitively: what happens if *x*[*n*] is a constant value?
  - Let's look at the z-transform:

$$Y(z) = X(z) - z^{-1}X(z) \longrightarrow H(z) = Y(z)/X(z) = 1 - z^{-1} = \frac{z-1}{z}$$

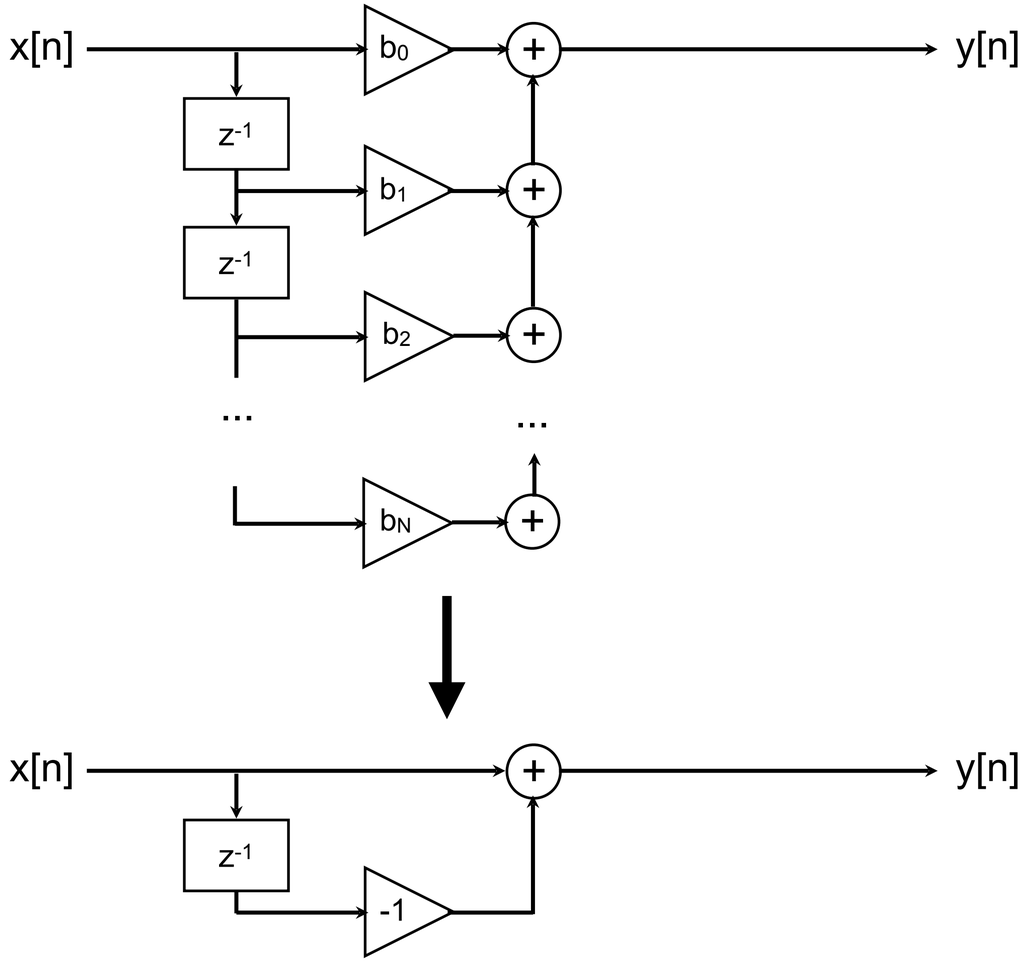
  - There's a zero at $z = 1$. What frequency is that?

$$e^{j\omega} = 1 \implies \omega = 0$$

  - So it's a very simple high-pass filter

Filter example

- We want to implement this filter: $y[n] = x[n] - x[n-1]$
- Block diagram for an FIR filter:
- What are the coefficients b_n?
  - $b_0 = 1$
  - $b_1 = -1$
- Task: in the sample-player-filter project, implement this filter in code
  - What information do you need to keep track of?
    - Previous value of $x[n]$
    - Use a global variable



Filter code

```
float gLastSample = 0;
```

global variable to hold *x*[*n* - 1]

```
for(unsigned int n = 0; n < context->audioFrames; n++) {
    // Read the next sample from the buffer
    float in = gPlayer.process();

    // y[n] = x[n] - x[n-1]
    float out = in - gLastSample;
    gLastSample = in;

    for(unsigned int channel = 0; channel < context->audioOutChannels; channel++) {
       // Write the sample to every audio output channel
       audioWrite(context, n, channel, out);
    }
}
```

so that the **next** time it holds x[n-1]

set `gLastSample` to x[n]...

Filter code

```
for(unsigned int n = 0; n < context->audioFrames; n++) {
    // Read the next sample from the buffer
    float in = gPlayer.process();

    // y[n] = x[n] - x[n-1]
    float out = in - gLastSample;
    gLastSample = in;

    // ....
}
```

n	in	gLastSample	out
0 (begin)	x[0]	0	x[0] - 0
0 (end)	x[0]	x[0]	x[0] - 0
1 (begin)	x[1]	x[0]	x[1] - x[0]
1 (end)	x[1]	x[1]	x[1] - x[0]
2 (begin)	x[2]	x[1]	x[2] - x[1]
2 (end)	x[2]	x[2]	x[2] - x[1]

Filter example 2

- This time, let's start from a frequency-domain function: $\frac{Y(z)}{X(z)} = \frac{1-\alpha}{1-\alpha z^{-1}}$
  - Rewrite as: $Y(z) - \alpha z^{-1} Y(z) = (1-\alpha)X$
  - Convert to time domain (z^{-N} turns into an N sample delay)

$$y[n] - \alpha y[n-1] = (1-\alpha)x[n]$$

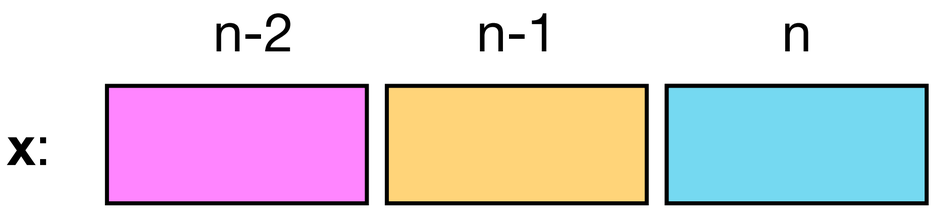
$$\downarrow$$

$$y[n] = \alpha y[n-1] + (1-\alpha)x[n]$$

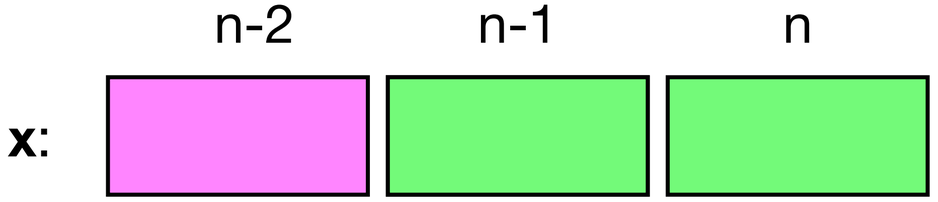
  - This is a first-order IIR lowpass filter
  - What values should we choose for α to get a low cutoff frequency?
    - Close to (but less than) 1.0 - try 0.99, for example
- Task: implement this using the sample-player-filter example
  - What previous value(s) do we need to keep track of this time?

Second-order filters

- Now let's implement a general second-order IIR filter
  - We can use 2nd-order ("biquad") sections to implement any type and order of IIR filter
- We won't focus in these lectures on how to calculate filter coefficients
  - See the companion materials for references
  - Instead, given a set of coefficients, how do we implement the equations as code?
- How many previous values of *x* and *y* do we need for a 2nd-order IIR filter?
  - 2 each. So how many global variables? 4
  - How do we keep track of *x*[*n*-2] and *y*[*n*-2]?
    - Analogously to what we did before: copy *x*[*n*-1] and *y*[*n*-1] at the end of the loop
    - Be careful about the order!



NO:



Second-order filter task

- Task: in the resonant-lowpass example
  - Implement the filter calculation in `render()`
  - Declare any global variables you need to remember the previous state
  - Coefficients for a resonant lowpass are calculated for you in `calculate_coefficients()`
  - Once it works, add GUI controls or analog inputs for frequency and Q
- Possible point of confusion: the sign of the *a* coefficients
  - Sometimes you will see the IIR filter equation written like this:

$$y[n] = \sum_{k=0}^{N} b_k x[n-k] + \sum_{k=1}^{N} a_k y[n-k]$$

  - Sometimes (including in this example), it's like this:

$$y[n] = \sum_{k=0}^{N} b_k x[n-k] - \sum_{k=1}^{N} a_k y[n-k]$$

Filters: from analog to digital

- The IIR filters we most commonly use in DSP are based on analog prototypes
  - Start with a continuous-time model (expressed in terms of its Laplace transform)
  - Transform it to the discrete-time domain, making a digital approximation of the analog behaviour
- Useful tool: the bilinear transformation
  - Start with a continuous-time filter $H_a(s)$
  - Perform this substitution:
    - where *T* is sampling period $s = \frac{2}{T}\frac{1 - z^{-1}}{1 + z^{-1}}$
  - Now implement the digital filter: $H_d(z) = H_a\left(\frac{2}{T}\frac{1 - z^{-1}}{1 + z^{-1}}\right)$
  - More info (Julius O. Smith, *Physical Audio Signal Processing*)
    - https://ccrma.stanford.edu/~jos/pasp/Bilinear_Transformation.html
- Bela provides the Biquad library for implementing common 2nd-order filters
  - Based on open-source code by Nigel Redmon (earlevel.com)

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