C++ Real-Time Audio Programming with Bela

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Course topics

Programming topics

Working in real time
Buffers and arrays
Parameter control
Classes and objects
Analog and digital I/O
Filtering
Timing in real time
Circular buffers
State machines
MIDI
Block-based processing
Threads
Fixed point arithmetic
ARM assembly language

Music/audio topics

Oscillators
Samples
Wavetables
Control voltages
Gates and triggers
Filters
Metronomes and clocks
Delays and delay-based effects
Envelopes
ADSR
MIDI
Additive synthesis
Phase vocoders
Impulse reverb

Today

Lecture 19: Phase vocoder, part 2

What you'll learn today:

Working in the frequency domain
Frequency detection from phase
Window functions

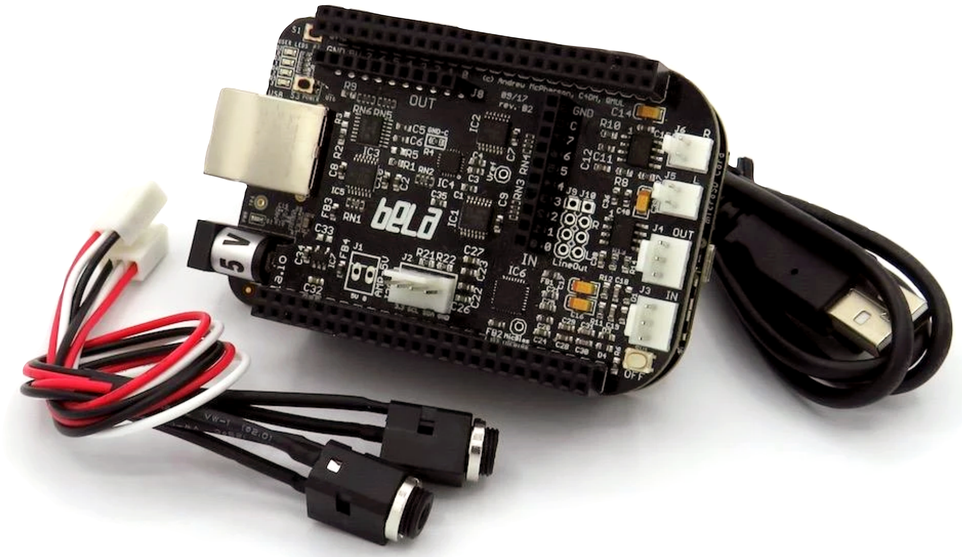
What you'll make today:

An FFT-based frequency detector

Companion materials:

github.com/BelaPlatform/bela-online-course

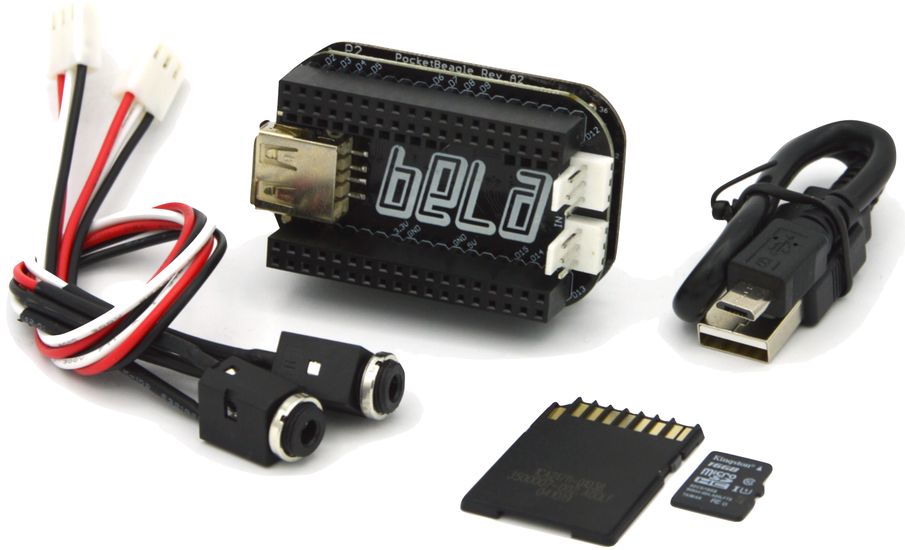
What you'll need



Bela Starter Kit

or

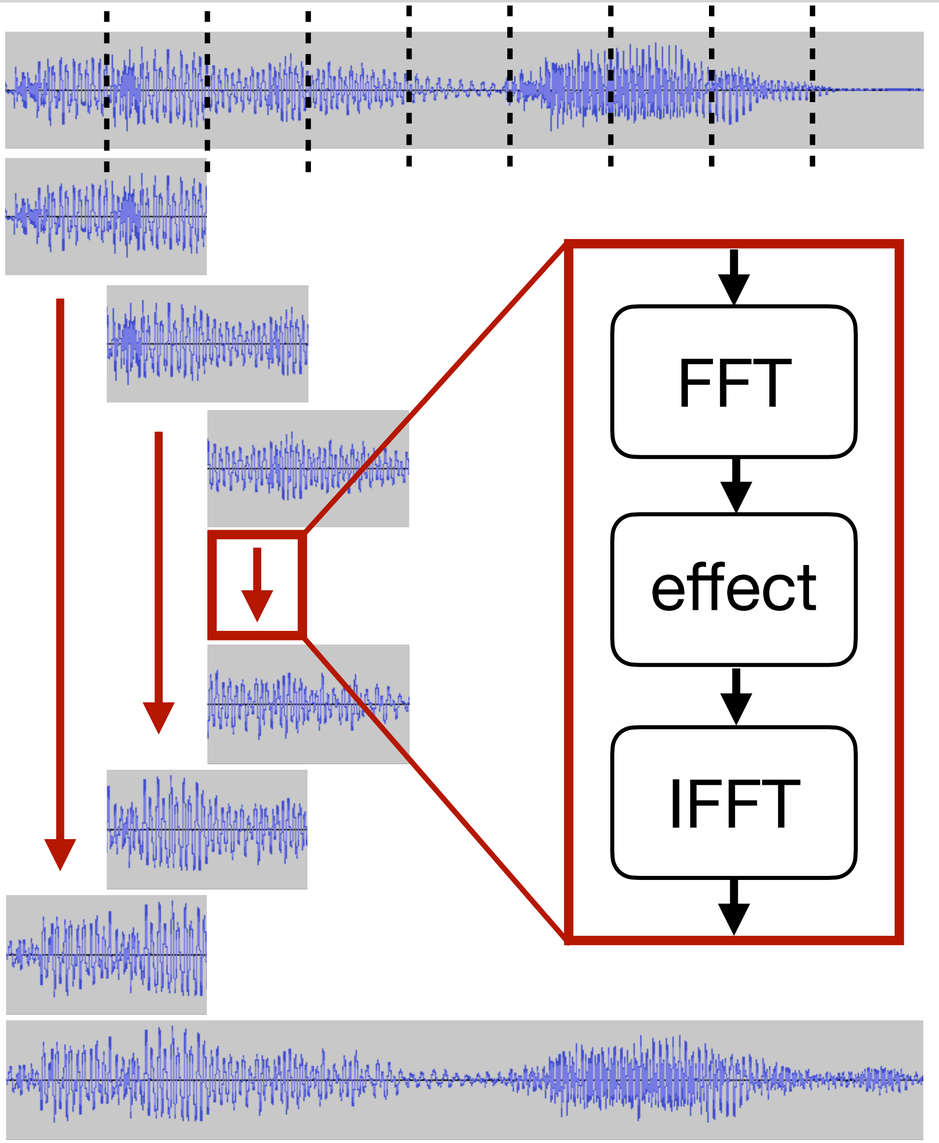
[shop.bela.io]



Bela Mini Starter Kit

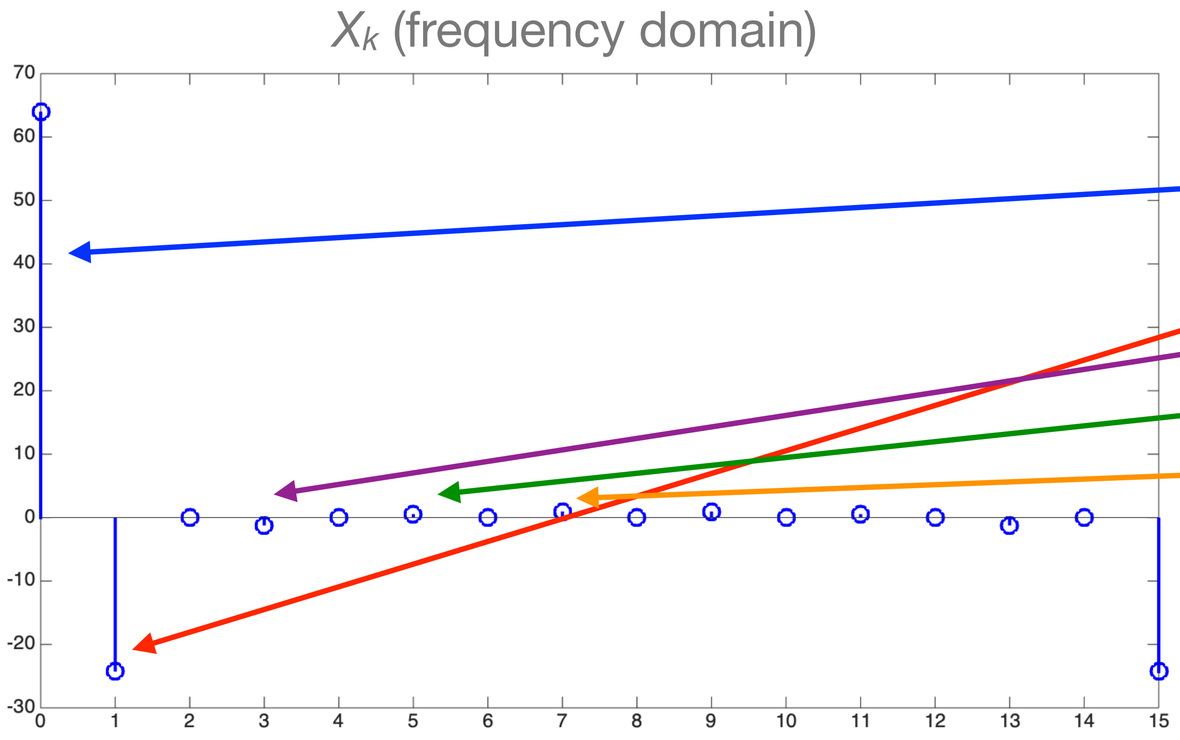
Overlap-Add

- A standard approach for block-based processing with overlapping blocks is called overlap-add:
  1. Isolate a block of length M using a windowing function
  2. Take an FFT of length N ≥ M of the segment
     - If N > M, zero-pad the block (add zeros to end of the window)
  3. Do something interesting to frequency-domain data
  4. Take an IFFT to get a new time domain segment
  5. Add the segment to an output buffer which also contains the preceding segments
     - As we'll see, we can't write the segment directly to the audio output
  6. Advance by the hop size (H) to the next frame and repeat
     - In real time, count samples until H more have arrived
     - Hop size H is less than window size M, hence the **overlap**



The Fast Fourier Transform

- Most common use of block-based processing: the Fast Fourier Transform (FFT)
  - Efficient computational algorithm for calculating the Discrete Fourier Transform (DFT)
  - The DFT is a mathematical formula for calculating the frequency content of a signal
- Any discrete-time signal of N points can be expressed as the sum of N sinusoids:



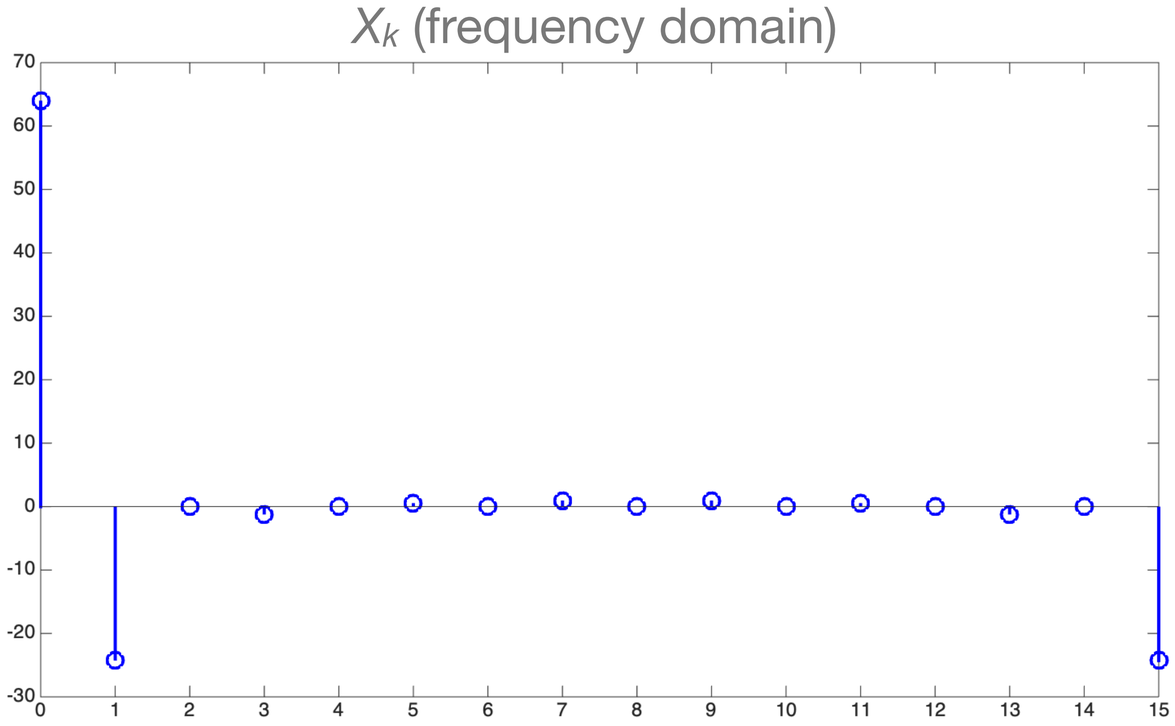
Frequency	Amplitude	Phase
0	4.00	0
π/8	1.64	-.875*π
3π/8	0.20	-.625*π
5π/8	0.09	-.375*π
7π/8	0.65	-.125*π

(other components are 0 or redundant)

The DFT calculates the amplitudes and phases of each frequency component

The Discrete Fourier Transform

- The DFT calculates the amplitudes and phases of each sinusoidal component
  - The frequencies are linearly spaced between 0 and 2π: $\frac{2\pi k}{N}$ for $0 \leq k < N$
  - Each amplitude/phase measurement is called a bin
  - Collectively, the N bins are called the frequency domain representation of the signal



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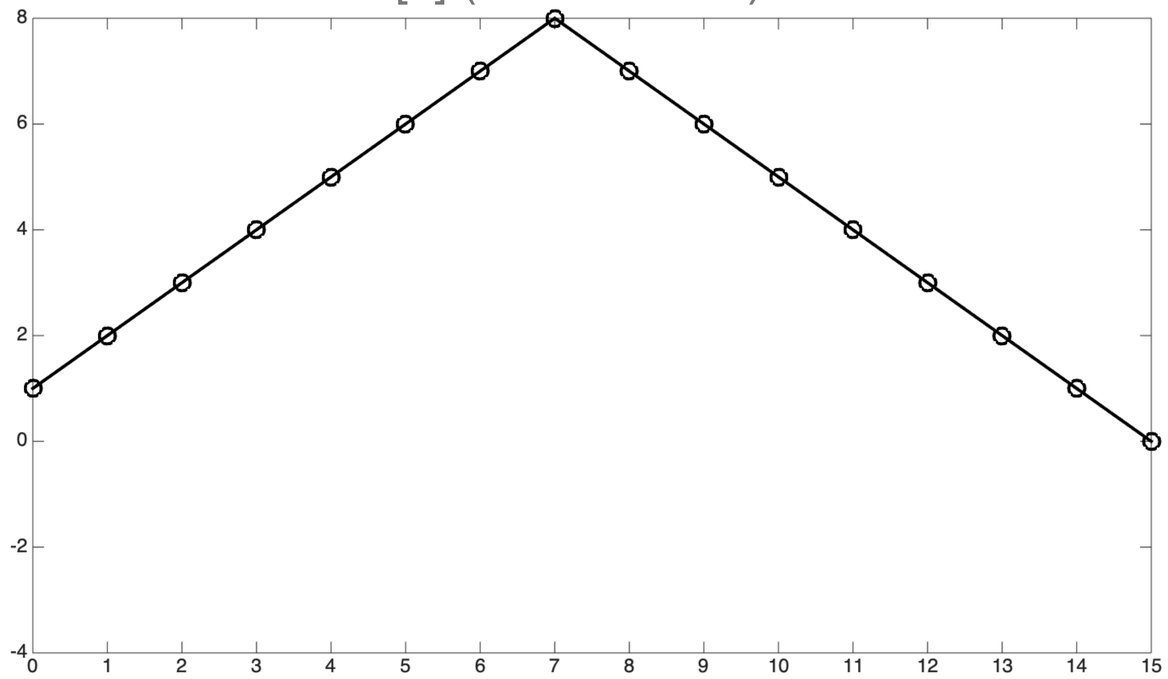
The Discrete Fourier Transform

- Key point: N time samples ↔ N frequency samples
  - Two different mathematical perspectives on the same signal
  - The Inverse Discrete Fourier Transform (IDFT) converts from frequency domain to time domain
  - The DFT+IDFT is an exact reconstruction as long as signal length $M \leq$ DFT length N

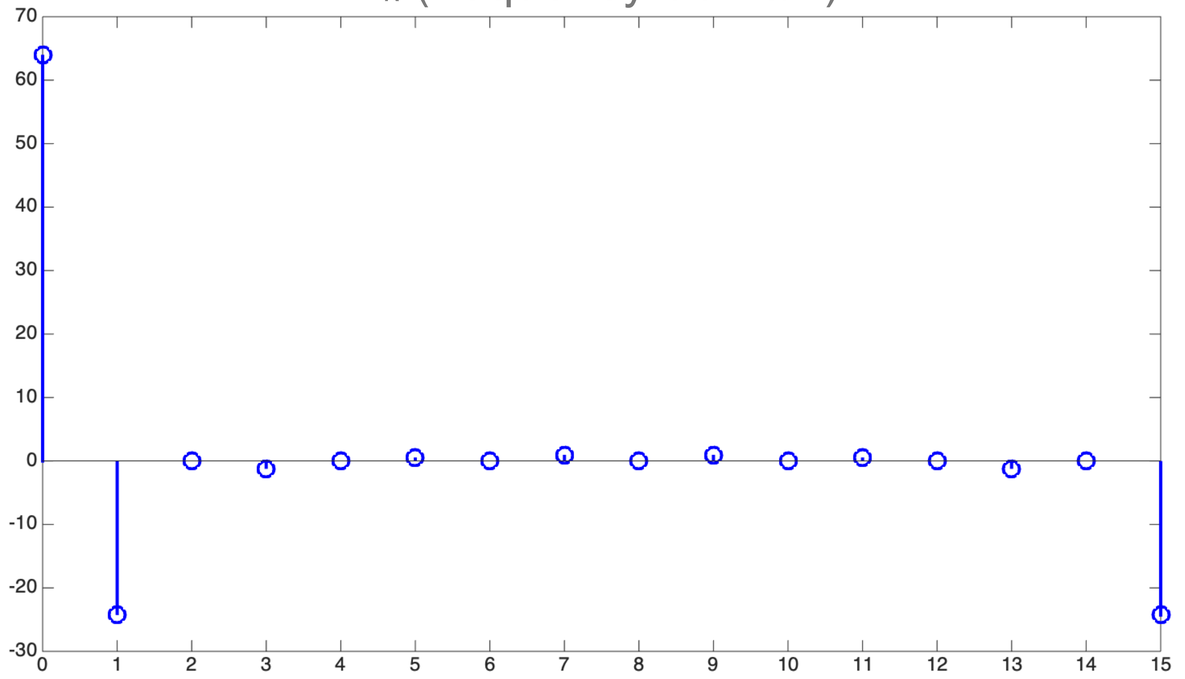
$x[n]$ (time domain)



DFT

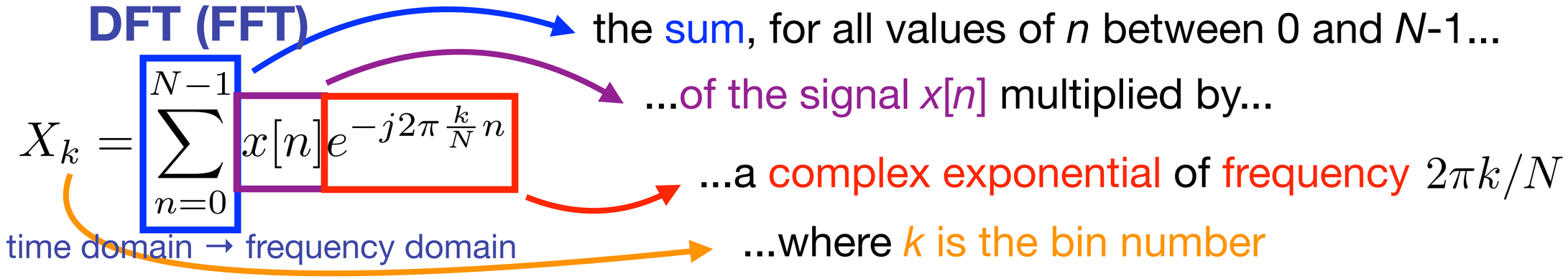
IDFT

X_k (frequency domain)



The Discrete Fourier Transform

- The mathematical basis of the Discrete Fourier Transform:



- N is the size of the DFT
  - For the FFT, N is almost always a power of 2 (the Cooley-Tukey algorithm)
- Each time sample *x*[*n*] is a real number
- Each frequency bin X_k is a complex number: $a + bj$ where $j = \sqrt{-1}$
  - Magnitude of the bin is given by: $\sqrt{a^2 + b^2}$
    - We usually use magnitude to plot a spectrum or spectrogram of a signal
  - Phase of the bin is given by: $\arctan(b/a)$

The Discrete Fourier Transform

- The mathematical basis of the Discrete Fourier Transform:

DFT (FFT)

$$X_k = \sum_{n=0}^{N-1} x[n] e^{-j2\pi \frac{k}{N} n}$$

time domain → frequency domain

IDFT (IFFT)

$$x[n] = \frac{1}{N} \sum_{k=0}^{N-1} X_k e^{+j2\pi \frac{k}{N} n}$$

frequency domain → time domain

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- Each time sample *x*[*n*] is a real number
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DFT magnitude and phase

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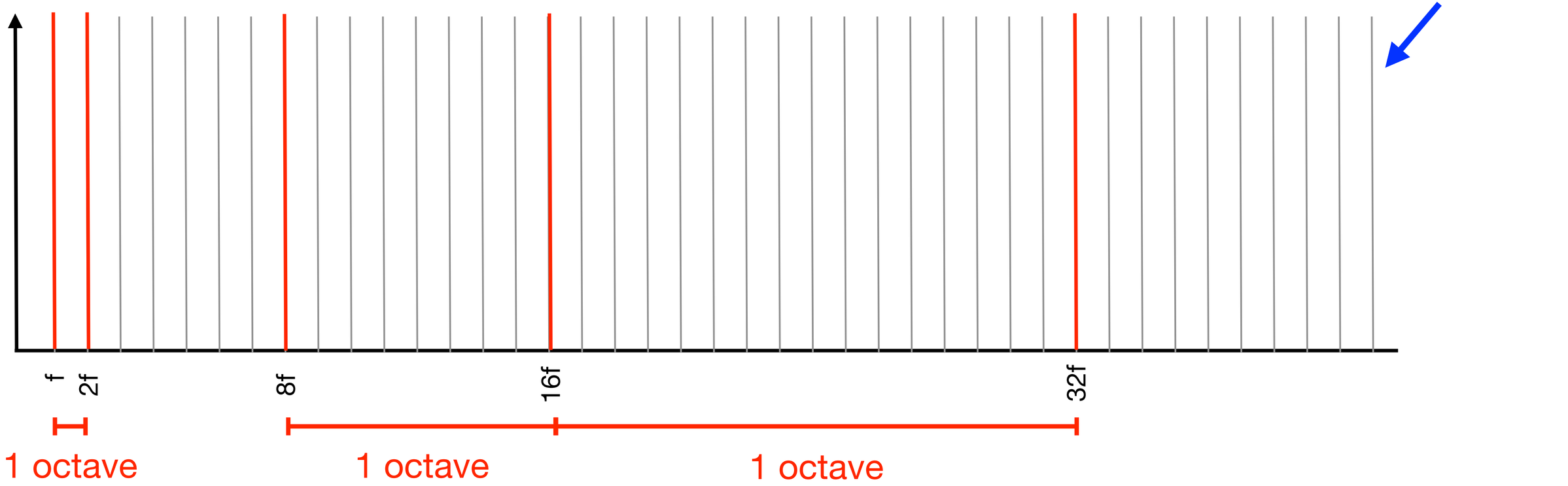
  - Magnitude of the bin is given by: $\sqrt{a^2 + b^2}$
    - We usually use magnitude to plot a spectrum or spectrogram of a signal
  - Phase of the bin is given by: $\arctan(b/a)$
- We can also turn the calculation around:
  - Given a magnitude M and a phase ϕ, calculate real and imaginary components
  - Real component: $a = M\cos(\phi)$ Imaginary component: $b = M\sin(\phi)$
- Remember, the frequency resolution depends on the DFT size
  - Bin frequencies are spaced 2π/N apart, where 2π corresponds to the sample rate
  - For example, N=1024, f_s = 44.1kHz: bin spacing is 43.1Hz

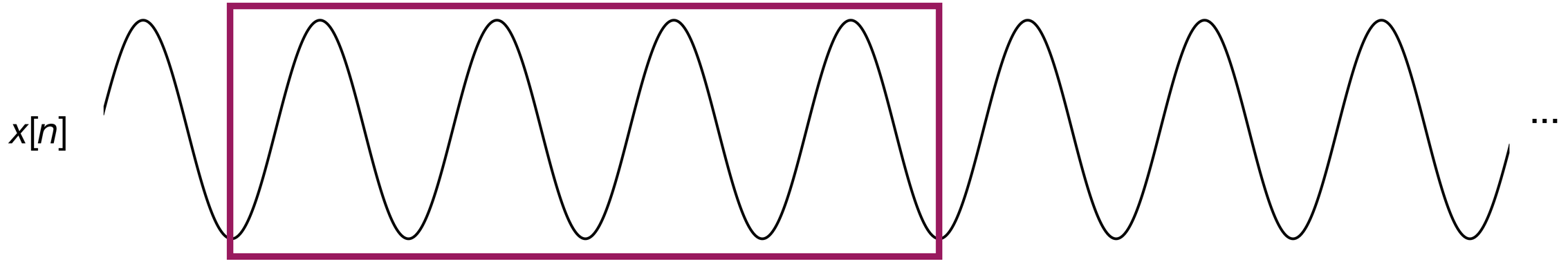
DFT frequency spacing

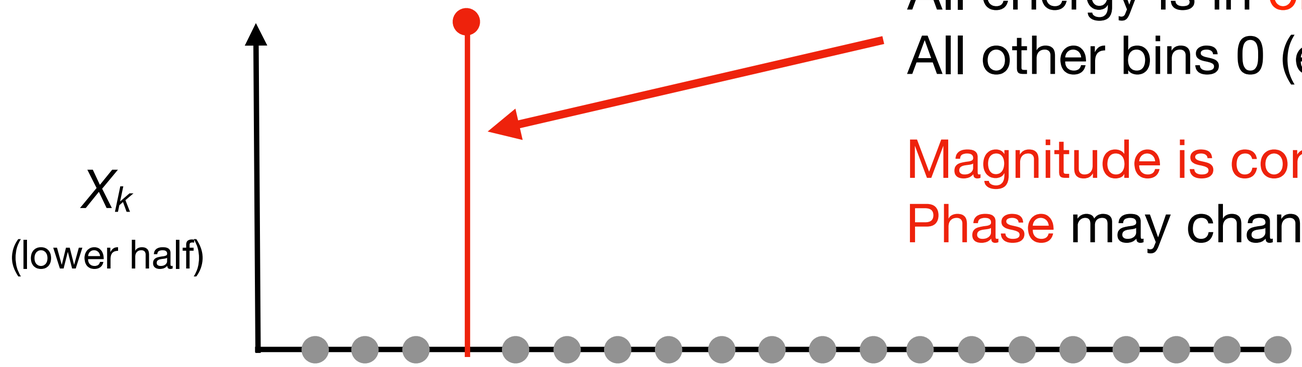
- Discrete Fourier Transform bins are spaced linearly in frequency
  - By contrast, musical intervals are defined as ratios in frequency (e.g. 1 octave = 2x)
  - This means that pitch resolution of the DFT is better for higher bins
    - This can be a headache for many musical applications



Sine wave in the frequency domain

- What does the FFT of a single sine wave look like?
  - Let's suppose its frequency exactly matches a bin: $2\pi k/N$ for some k
  - This means that an integer number of oscillations fit in the window



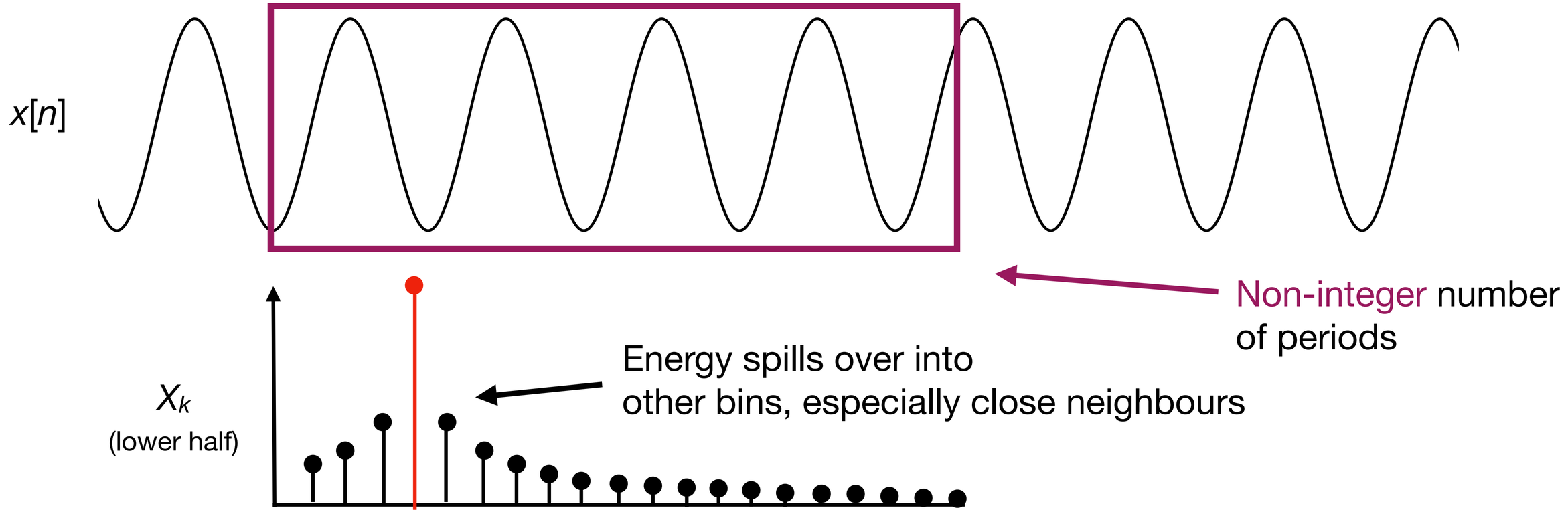


All energy is in one frequency bin
All other bins 0 (except mirror image above Nyquist)

Magnitude is constant no matter where the window starts
Phase may change depending on window location

Sine wave in the frequency domain

- What if the frequency doesn't exactly match any bin? We'll see two effects:
  1. Energy smears out across all the other frequency bins (we'll see why shortly)
  2. Phase is likely to be different on each successive hop
     - If we're clever, we can reconstruct the exact frequency based on the phase changes!



Sine wave in the frequency domain: task

- Task: run the example project fft-sine
  - This project takes the FFT of a sine wave and displays the spectrum in the GUI
    - Block-based processing in `render.cpp` follows Lecture 17
  - Adjust the frequency of the oscillator in the GUI: what happens to the spectrum?
    - By default, the sine is exactly on an FFT bin frequency
    - After you adjust it, the frequency will fall between FFT bins
  - You can change the FFT size at the top of `render.cpp`

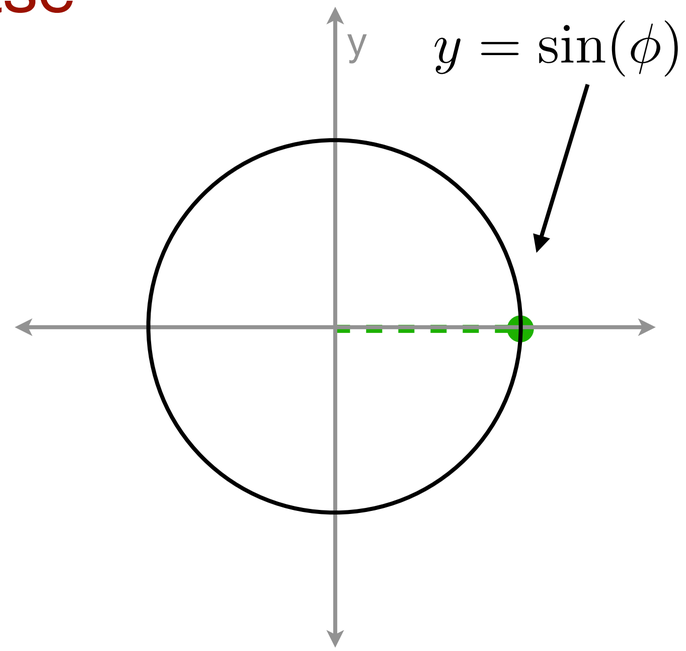
Reconstructing exact frequency

- We can estimate the exact frequency of a sine wave from the FFT
  - Even though the FFT only has bins at fixed, regularly spaced frequencies!
- To do this, we will need two hops
  - Two snapshots of the signal taken a known length of time apart
- Key point: frequency is the derivative of phase
  - $\omega(t) = \frac{d\phi}{dt}$ or in discrete time: $\omega[n] = \frac{\Delta\phi}{\Delta n}$
  - We can write out the first-order difference:

$$\omega[n] = \frac{\phi[n] - \phi[n-1]}{(n) - (n-1)} = \phi[n] - \phi[n-1]$$

  - Or generalising over a hop of *H* samples:

$$\omega[n] = \frac{\phi[n] - \phi[n-H]}{(n) - (n-H)} = \frac{\phi[n] - \phi[n-H]}{H}$$



Reconstructing exact frequency

- Calculating frequency from phase in discrete time:

$$\omega[n] = \frac{\phi[n] - \phi[n-H]}{(n) - (n-H)} = \frac{\phi[n] - \phi[n-H]}{H}$$

  - In discrete time, frequency ω is normalised between 0 and 2π;
  - Assumes unwrapped phase (i.e. always increasing, not constrained 0-2π)
- We can invert this formula to calculate phase shift based on frequency:

$$\phi[n] - \phi[n-H] = H\omega[n]$$

  - For a DFT of size *N*, we know bin *k* has a frequency of $2\pi k/N$
  - For a sine wave of exactly the bin frequency, this is the expected phase shift over 1 hop:

$$\phi[n] - \phi[n-H] = \frac{2\pi k H}{N}$$

  - By subtracting off this expected phase shift, the remainder will tell us whether the actual frequency is higher or lower than the bin frequency

Reconstructing exact frequency

- First we calculate the phase remainder: $\phi_r[n] = (\phi[n] - \phi[n-H]) - \frac{2\pi k H}{N}$

  measured **phase values** from bin *k* on **two successive hops**

  centre frequency of bin *k* multiplied by the **hop size**

- Then use remainder to calculate the deviation from the bin centre frequency:
  - We should wrap the phase to be between -π and π (i.e. the principal argument)

$$\omega[n] - \omega_k = \frac{\text{wrap}(\phi_r[n])}{H}$$

centre frequency of bin *k* (= 2π*k*/*N*)

- Finally, we can rearrange to get the exact frequency of our sine wave:

$$\omega[n] = \frac{\text{wrap}(\phi_r[n])}{H} + \omega_k$$

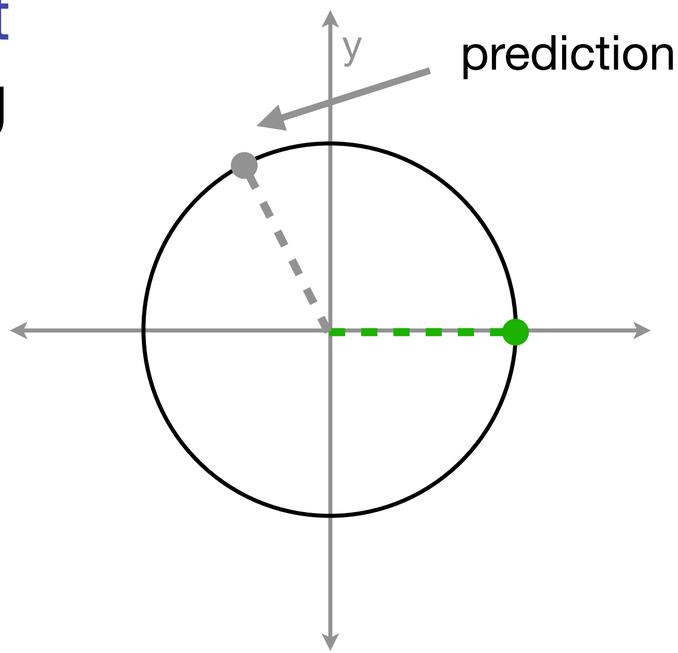
  - We could also calculate in terms of (fractional) FFT bins
    - Use the fact that bins are spaced 2π/*N* apart in frequency

fractional bin number of the sine wave

$$b[n] - k = \frac{\text{wrap}(\phi_r[n])N}{2\pi H} \rightarrow b[n] = \frac{\text{wrap}(\phi_r[n])N}{2\pi H} + k$$

Reconstructing exact frequency: code

- Effectively, we are predicting how far the phase ought to advance from one hop to the next, then comparing the actual measured values
  - Phase advancing farther means a higher frequency (compared to the bin frequency)
  - Phase advancing less far means a lower frequency
- What do we need to implement this in code?
  - A global (or static) variable to hold the phase of each bin
    - Saving each bin implies an array
    - This is very similar to saving previous values of $x[n]$ or $y[n]$ for a filter (see Lecture 8)
  - A function to wrap the phase to the range -π to π
  - Implement the calculations inside of our `process_fft()` function
    - Write a `for()` loop iterating across each bin



Reconstructing exact frequency: task

- Task: in fft-sine, write the code to work out the exact frequency of each bin
  - Implement these formulas:

$$\phi_r[n] = (\phi[n] - \phi[n-H]) - \frac{2\pi k H}{N}$$

bin phase this hop; bin phase last hop; FFT size; bin number; hop size

$$\omega[n] = \frac{\text{wrap}(\phi_r[n])}{H} + \omega_k$$

`wrapPhase()` written for you; bin centre frequency ($2\pi k/N$)

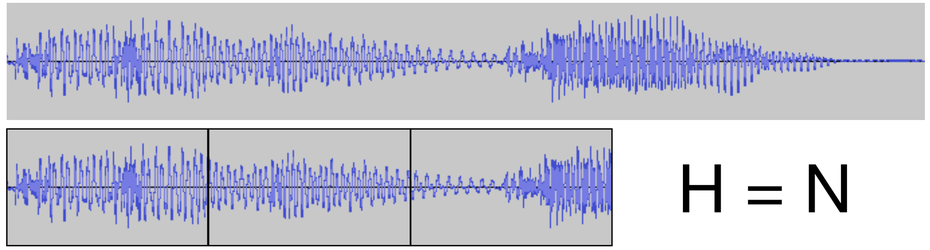
  - Remember you will need a global or static array to hold the previous phases
  - The GUI will display the frequency of the bin with highest magnitude
  - However, the same formula can apply to every bin regardless of magnitude
    - In practice, we only look at the first half of bins because the DFT is conjugate symmetric

Choosing an FFT size

- The previous formulas gave us one frequency component for each bin
  - An FFT of size *N* has (*N*/2+1) unique bins after accounting for symmetry
  - This tells us the maximum number of frequency components we can represent
- Hence, the larger the FFT, the greater the frequency resolution
  - Bins are spaced more closely together in frequency: $2\pi k/N$
  - Easier to resolve two closely-spaced frequency components of a signal
- What's the tradeoff? Poor time resolution
- N time-domain samples ↔ N frequency-domain bins
  - Resolving short transient events within a long time window is difficult
  - Frequency-domain processing tends to smear transients, distorting their shape
  - Another tradeoff to large FFTs: longer latency
  - These are fundamental mathematical limitations

Choosing a hop size

- How often should we take an FFT?
  - Assuming we want to reconstruct the signal, the hop size H must be no more than the FFT size N
- Advantages of smaller hop sizes
  - We get more frequent time snapshots of the signal
  - Phase of each bin advances less from one hop to the next, improving frequency estimation
  - Some types of window need a hop size of N/2, N/3 or even less to reconstruct the signal
  - Lower latency for phase vocoder effects
- Disadvantages of smaller hop sizes
  - More computationally expensive
  - Hop sizes that aren't an integer division of the FFT size can lead to artefacts on reconstruction



H = N

Choosing a hop size

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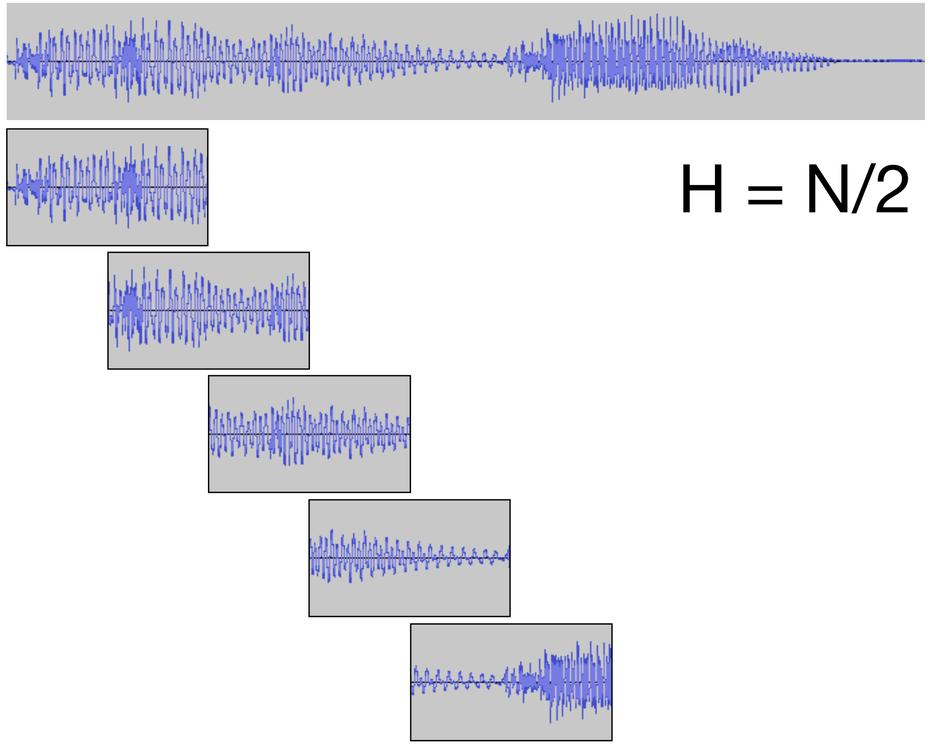

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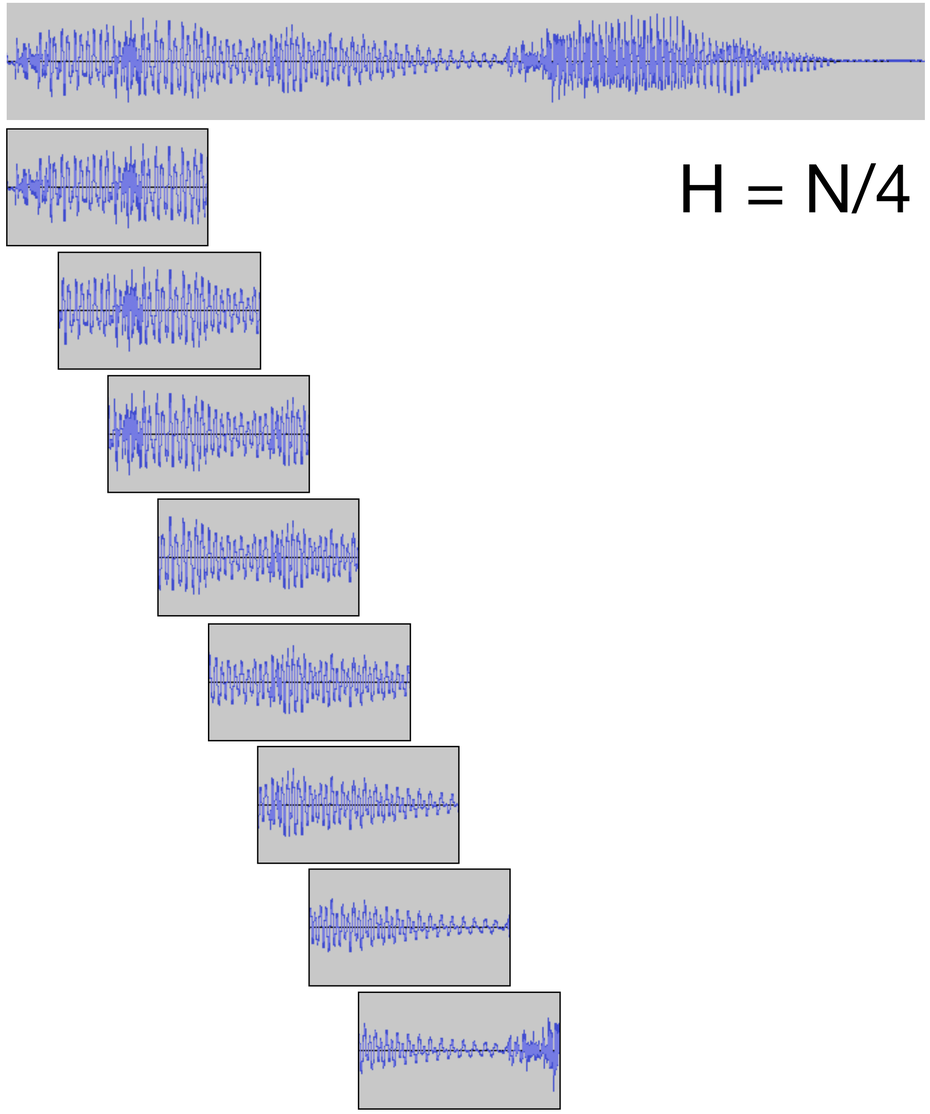
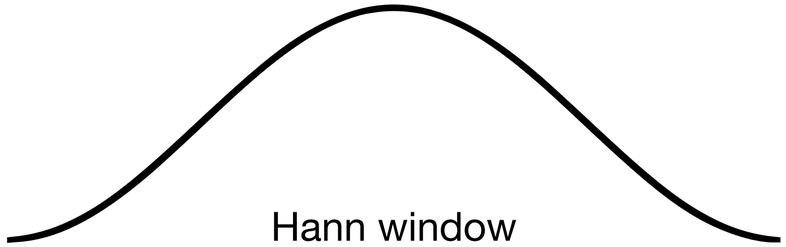
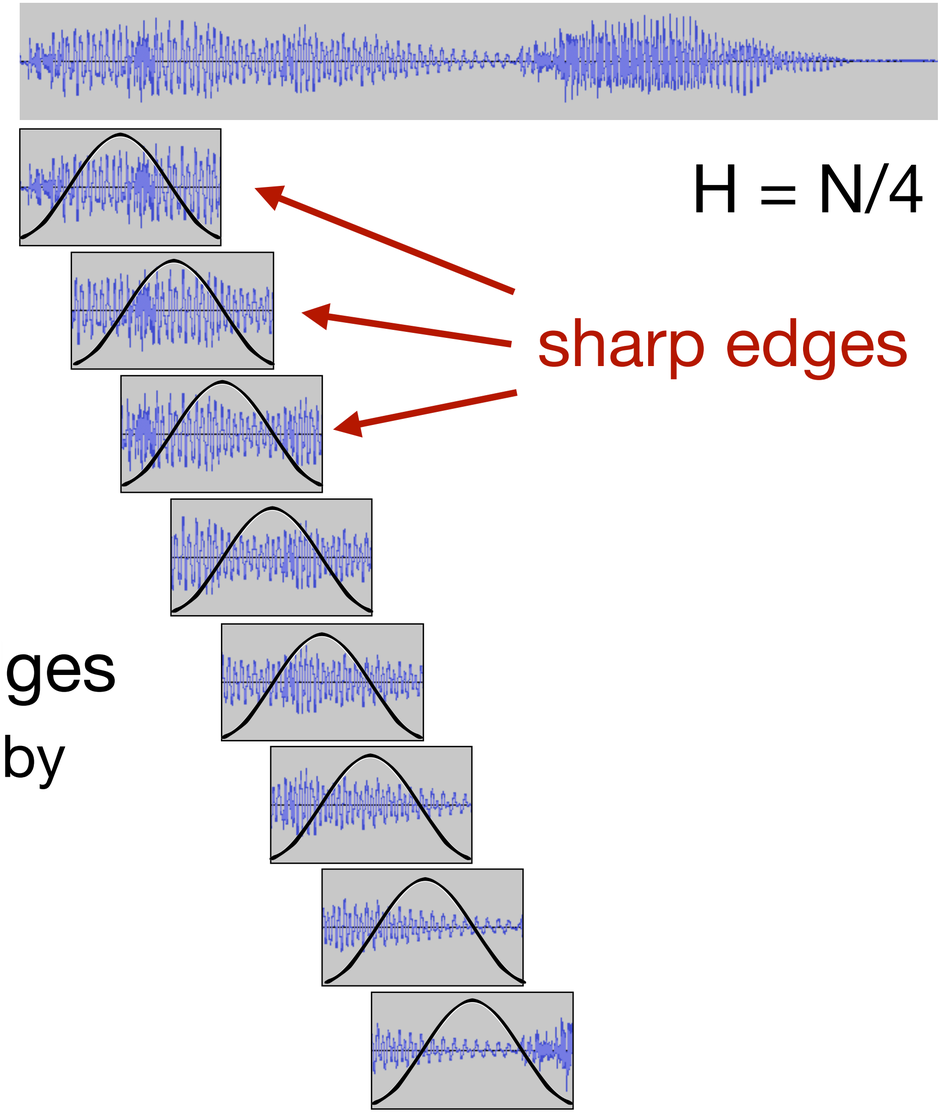


Windowing

- Overlap-Add process segments the signal into finite-length windows
  - These segments can have discontinuities at the edges (assuming 0 outside the window)
  - We might not notice this if we just reconstruct the signal with no frequency-domain processing
  - If we modify the signal, these sharp edges can lead to crackling or buzzing sounds
- We can use a window function to taper the edges
  - Pre-calculated shape that we multiply the segment by before FFT



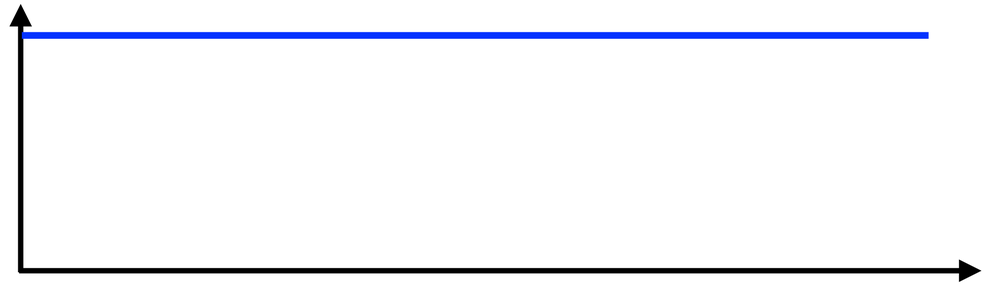
Hann window



Types of window function

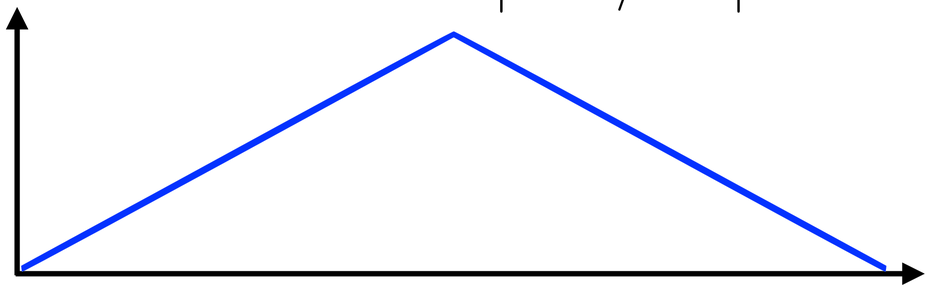
Rectangular

$$w[n] = 1 \quad \text{for } 0 \leq n < N$$



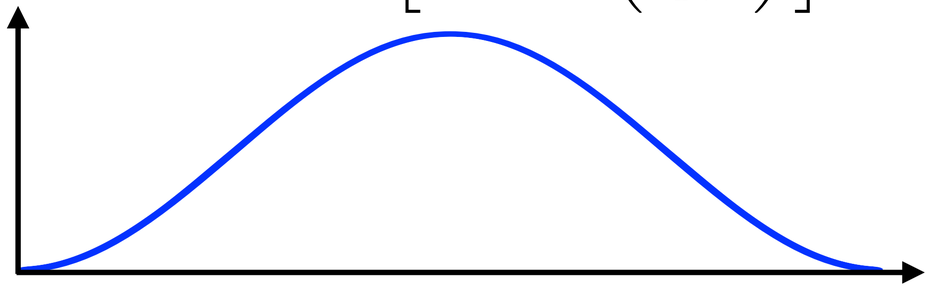
Triangular (Bartlett)

$$w[n] = 1 - \left| \frac{n - N/2}{N/2} \right|$$



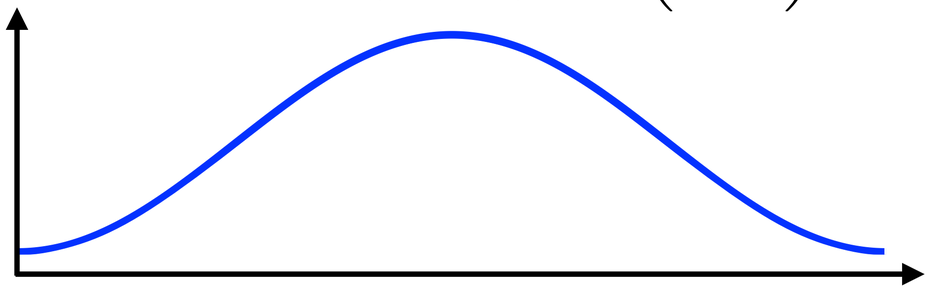
Hann

$$w[n] = 0.5 \left[1 - \cos\left(\frac{2\pi n}{N} \right) \right]$$



Hamming

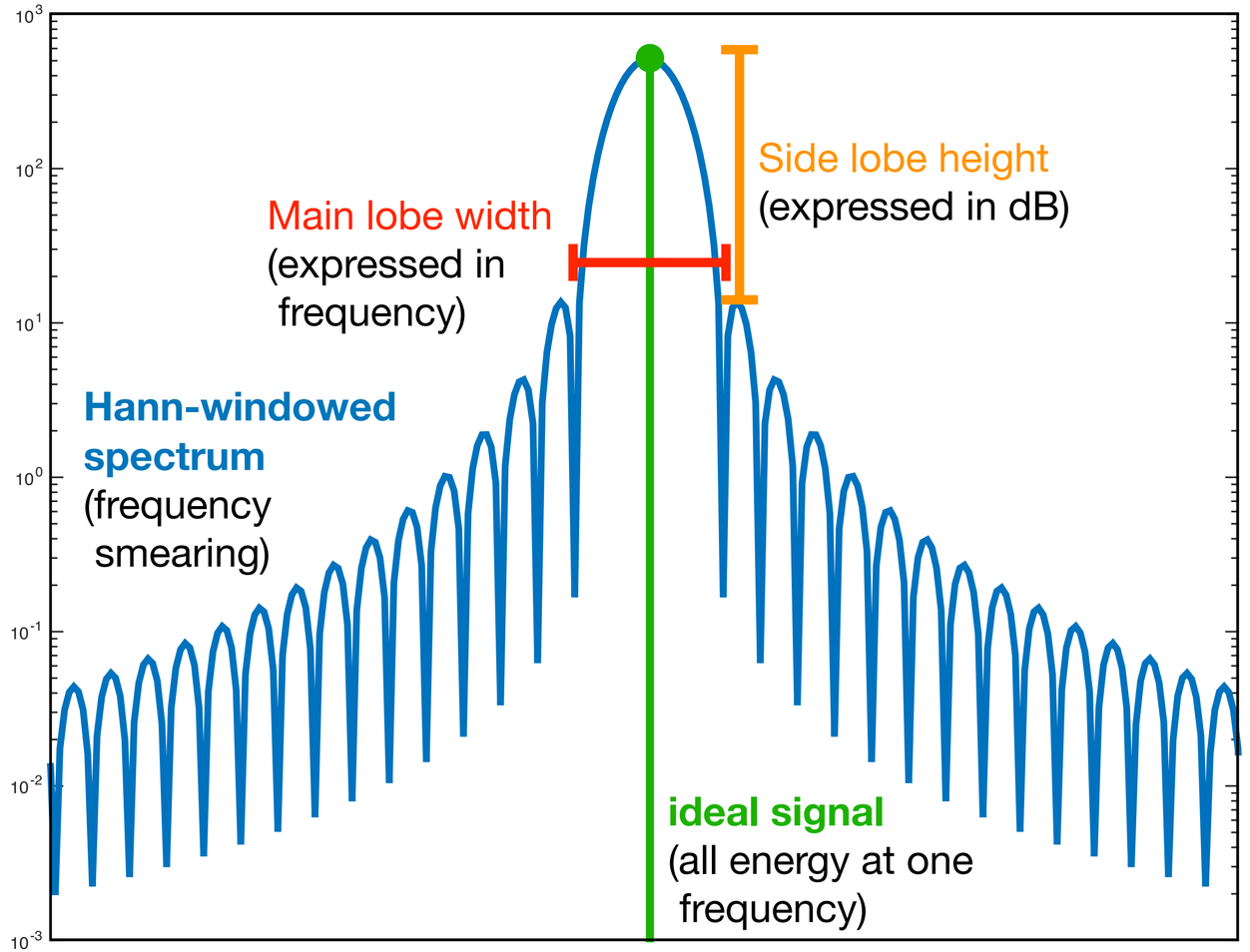
$$w[n] = \frac{25}{46} - \frac{21}{46} \cos\left(\frac{2\pi n}{N} \right)$$



And more types: Blackman, Kaiser, Flat Top, ...

Effects of windowing

- Windowing lets us isolate a particular time segment of our signal
- The cost is smearing of energy in the frequency domain
  - Energy appears at frequencies where it doesn't exist in the original signal
- Each frequency component of the signal appears as a series of lobes
  - Main lobe: a broad peak centred around the actual frequency
  - Side lobes: secondary peaks at other frequencies
- We typically want to minimise:
  - Main lobe width
  - Side lobe height

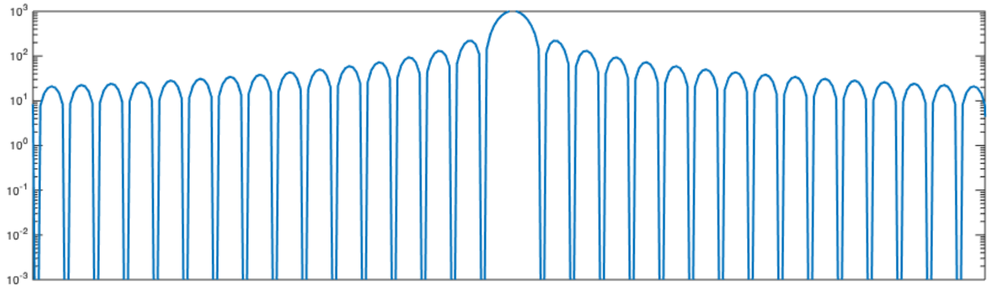


Window spectra

Rectangular

Main lobe width: 4π/N

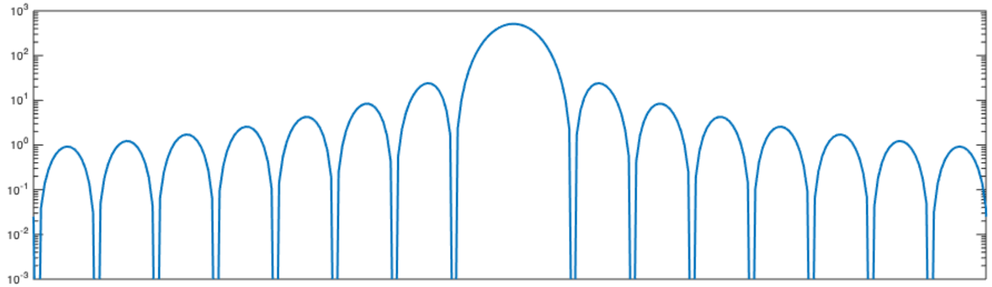
Side lobe height: -13dB



Triangular (Bartlett)

Main lobe width: 8π/N

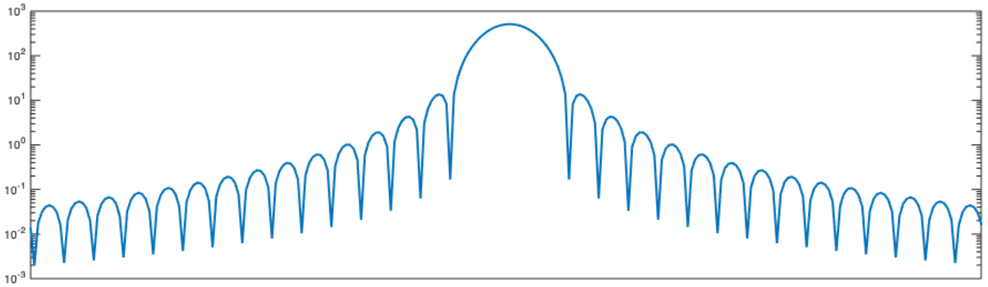
Side lobe height: -26.5dB



Hann

Main lobe width: 8π/N

Side lobe height: -31.5dB

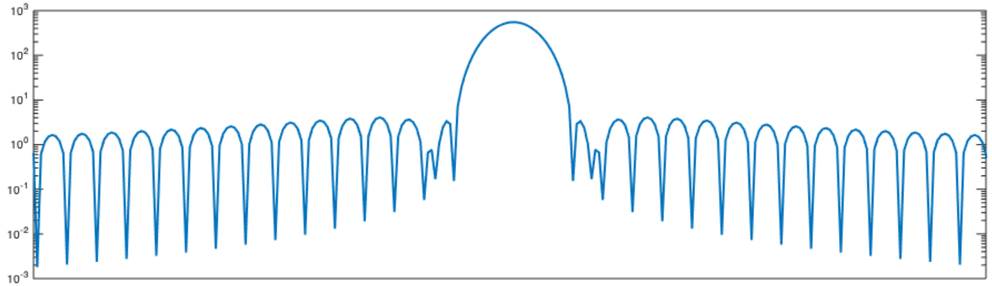


→ When in doubt: use a Hann window

Hamming

Main lobe width: 8π/N

Side lobe height: -42.8dB



Coding a window

- It is most efficient to pre-calculate the window when the program starts
  - Then recalculate when settings change: window size, window type etc.
- Store the window in an array (typically a global variable)
  - Length of the array is equal to the window size
- To calculate, use a `for()` loop to iterate over each sample of the window

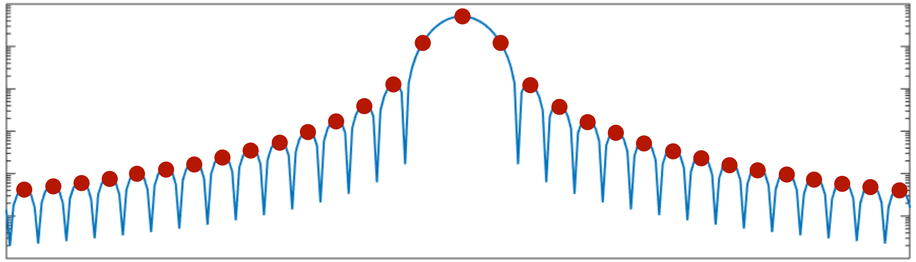
```
const int windowSize = 1024;
float windowBuffer[gWindowSize];

for(unsigned int n = 0; n < windowSize; n++) {
    windowBuffer[n] = equation for window
}
```

- To apply, multiply the signal by the window buffer frame-by-frame
  - Could do this while unwrapping a circular buffer (see Lecture 18)

```
for(unsigned int n = 0; n < windowSize; n++) {
    windowedSignal[n] = originalSignal[n] * windowBuffer[n];
}
```

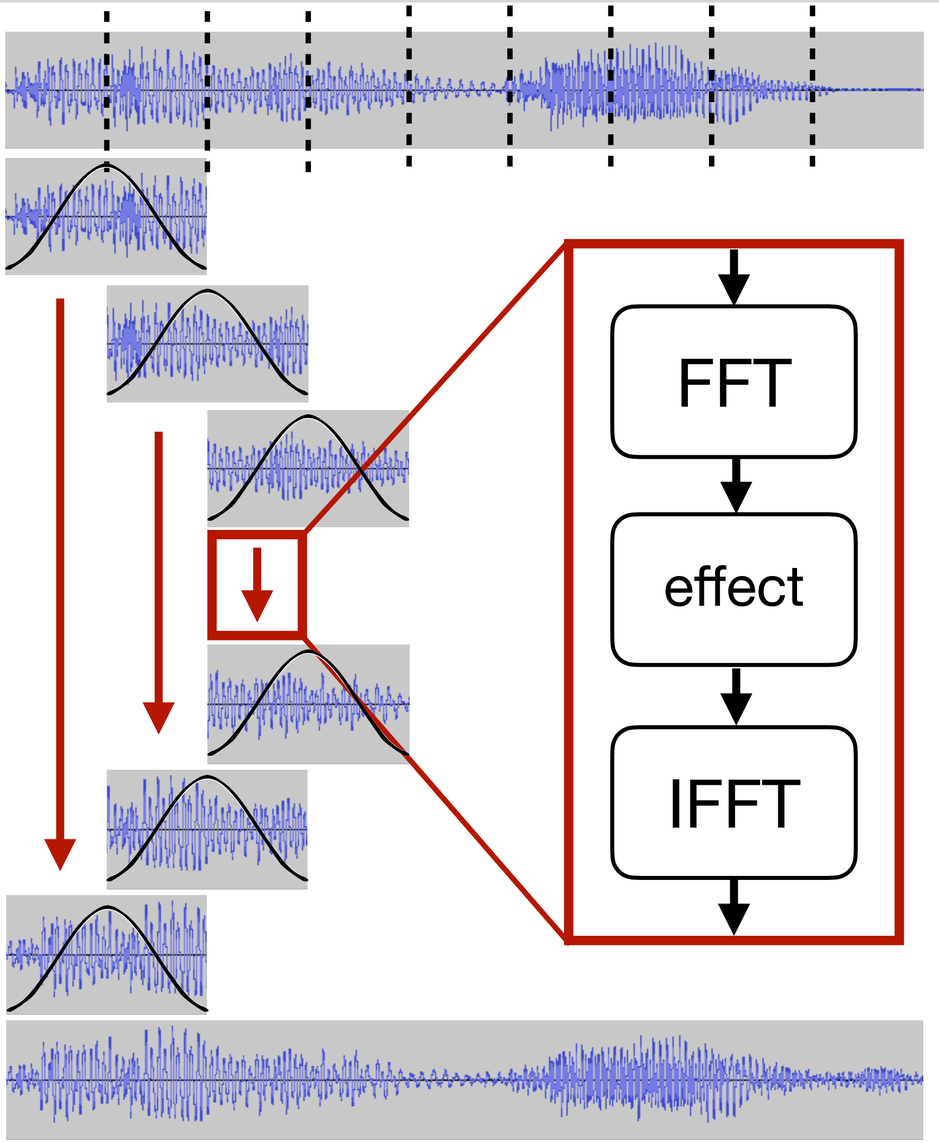
Visualising the effects of windowing

- Let's go back to our fft-sine project and see the window in action
- Task: in fft-sine, implement a Hann window
  - Change the frequency in the GUI so it doesn't align with any bin
  - What happens to the width of the main lobe?
  - What happens to the side lobes?
  - As a bonus, the Hann window should improve the frequency estimation!
- Why didn't we see the classic comb filter pattern?
  - Because the window size is the same as the FFT size
  - The FFT is effectively sampling the peaks of each lobe
- Task 2: in `setup()`, change the window size to be a fraction of the FFT length
  - Say 1/4 or 1/8 of the FFT length
  - Now the lobes should be visible



Next lecture: Phase vocoder, part 3

- Next time, we will bring everything together to create phase vocoder effects:
  - Robotisation (phase zeroing)
  - Whisperisation (phase randomisation)
  - Pitch shifting



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