1.9 – Domain, Range and Function Notation

Recall: **SET NOTATION** $\{ x \in \mathbb{R} \mid \ldots \}$

x *is an element of the set of all real numbers such that …*

Interval Notation

- All denote a set of real numbers

Bracket

- [ ] Square brackets indicate that the end value is included in the interval
- ( ) Round brackets indicate that the end value is not included
- Infinite intervals are expressed by $(-\infty, \infty)$ as they are without bound

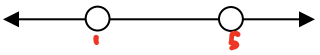
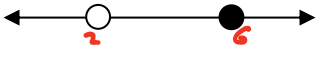
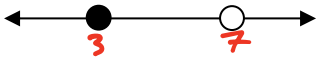
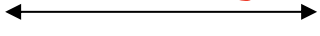
Inequality

- ≤ less than or equal to *and* ≥ greater than or equal to — indicate the end value is included in the interval
- < less than *and* > greater than — indicate end value is not included

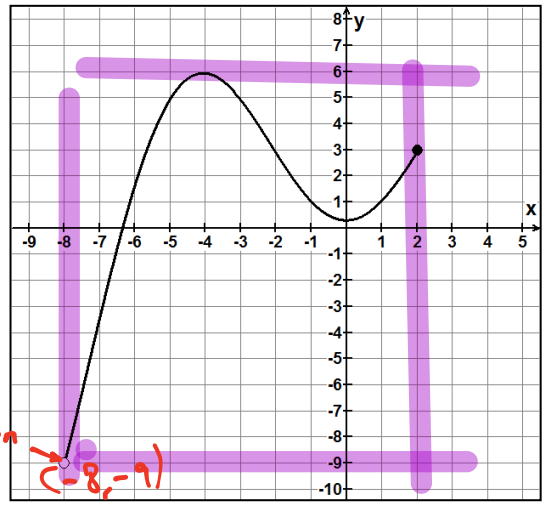
Graph/Number Line

- ● closed dots indicate the end value is included in the interval
- ○ open dots indicate the end value is not included

Examples:

Interval/Bracket	Inequality	Graph/Number Line
(1, 5)	$1 < x < 5$	
(2, 6]	$2 < x \leq 6$	
[3, 7)	$3 \leq x < 7$	
[4, 8]	$4 \leq x \leq 8$	
$(-\infty, \infty)$	$-\infty < x < \infty$	

Ex. 1 State the domain and range of the graph using set notation



smallest x: ____ smallest y: ____

largest x: ____ largest y: ____

(-8, 2]

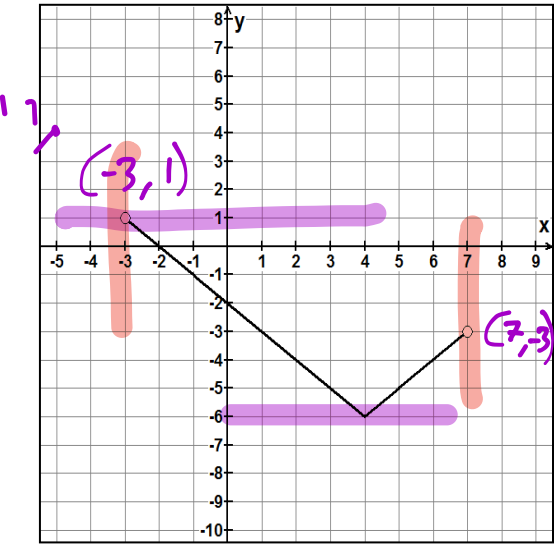
Domain : $\{x \in R \mid -8 < x \leq 2\}$ open

(-8, -9)

Range : $\{y \in R \mid -9 < y \leq 6\}$

(-9, 6]

Practice:



(-3, 1)

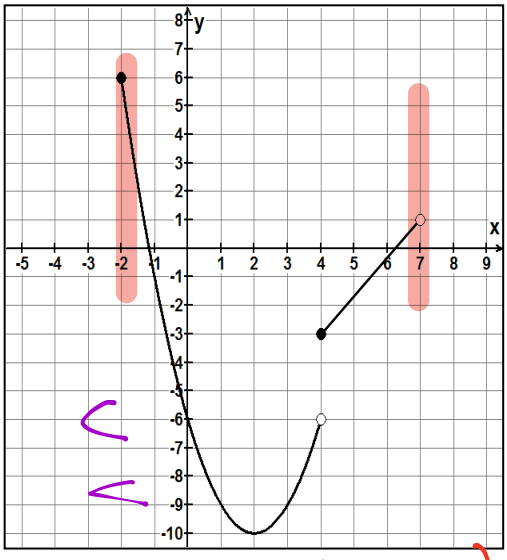
(7, -3)

(-3, 7)

Domain: $\{x \in R \mid -3 < x < 7\}$

Range: $\{y \in R \mid -6 < y \leq 1\}$

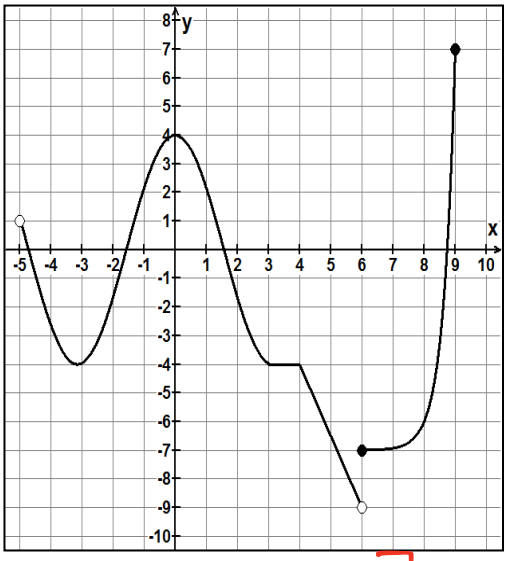
(-6, 1)



[-2, 7)

Domain: $\{x \in R \mid -2 \leq x < 7\}$

Range: $\{y \in R \mid -10 \leq y \leq 6\}$



(-5, 9]

Domain: $\{x \in R \mid -5 < x \leq 9\}$

Range: $\{y \in R \mid -9 < y \leq 7\}$

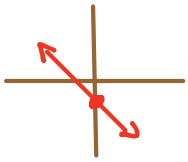
(-9, 7]

Determining the Domain and Range from the Function Equation

$y = mx + b$ (slope (ROC) = rise/run)

Ex. 2 Determine the domain and range of each function:

i. $y = -x - 5$ ⇒ Linear



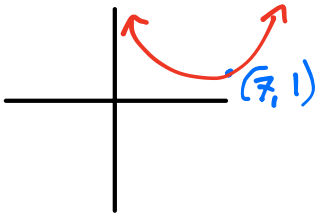
D: $\{x \in R\}$ or $(-\infty, \infty)$

R: $\{y \in R\}$ or $(-\infty, \infty)$

ii. $y = \frac{3}{4}(x-7)^2 + 1$ ⇒ Quadratic

vertex: (7, 1)

opens up

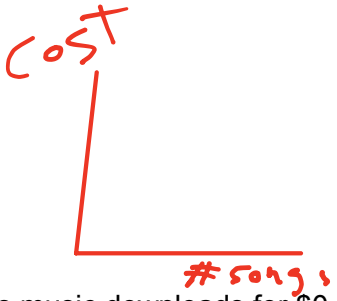


D: $\{x \in R\}$ or $(-\infty, \infty)$

R: $\{y \in R \mid y \geq 1\}$

$[1, \infty)$

iii. $y = \sqrt{6 - x}$



Consider:

E-tunes sells music downloads for $0.50 per song, plus a one-time membership fee of $10.00. Write an equation to model this situation.

i. using traditional variables

n = # songs

$C = 0.50n + 10$

$y = 0.50n + 10$

ii. using function notation

$C(n) = 0.50n + 10$

$C(5) = 0.50(5) + 10$

$C(5) = \$12.50$

$y(x) = 0.50x + 10$

Same as "y"

Function Notation

$f(x)$

In general,

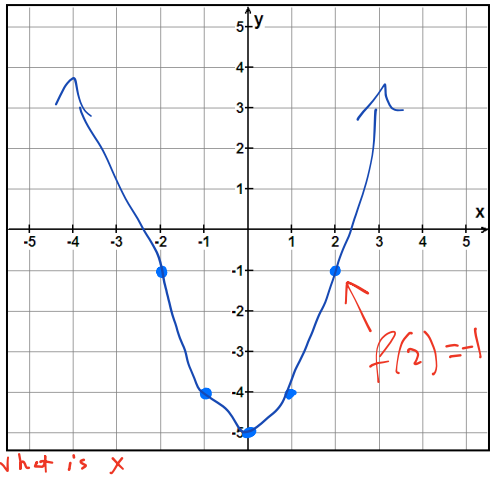
- represents the dependent variable...we used to use y
- means the value of the function (output) for a given x value (input)
- reads " **f *of* x** " *or* " **f *at* x** "
- finding a **y**-value given an **x**-value simply requires ***substitution***
- thus, we can write ordered pairs $(x, f(x))$ which is the same as (x, y)

Over the semester we will use different notations [f(x), g(x), h(x), v(x), etc...]. These are all function notations. Different letters are used to distinguish between different functions.

Ex. 2 Complete a *table of values* for the **function** $f(x) = x^2 - 5$ and **graph**.

$y = x^2 - 5$

x	$f(x) = x^2 - 5$	$(x, f(x))$
-2	$f(-2) = (-2)^2 - 5 = -1$	$(-2, -1)$
-1	$f(-1) = (-1)^2 - 5 = -4$	$(-1, -4)$
0	$f(0) = (0)^2 - 5 = -5$	$(0, -5)$
1	$f(1) = (1)^2 - 5 = -4$	$(1, -4)$
2	$f(2) = (2)^2 - 5 = -1$	$(2, -1)$



What is x when $f(x) = -5$

$x = 0$

Ex. 3 Find $f(2)$ if $f(x) = x^2 - 2x + 1$. (Here, we are looking for the "y-value" when $x = 2$.)

$f(2) = 2^2 - 2(2) + 1$

$= 4 - 4 + 1$

$f(2) = 1$

When $x = 2$

$y = -1$

Ex. 4 Given the function $f(x) = x^2 - 3x$ and $g(x) = 1 - 2x$, find…

2 times g(x)

a. $2g(x)$

$= 2(1-2x)$

$= 2(1-2x$

$= 2 - 4x$

b. $f(5) - g(5)$

$= [5^2 - 3(5)] - [1-2(5)]$

$= 25 - 15] - [1 - 10]$

$= 10 - (-9)$

$= 19$

$g(x) = 1-2x$

Sub in $a+2$ for x

c. $g(a+2)$

$= 1 - 2(a+2)$

$= 1 - 2a - 4$

$= -3 - 2a$

$= -2a - 3$

d. $f(x) + g(x)$

$x^2 - 3x + 1 - 2x$

$= x^2 - 5x + 1$

Classwork / Homework:

pg 22 #1, 2, 5, 6, 8b pg 35 # 2, 3, 5, 9

Domain & Range and Function Notation ~ Worksheet pg 22 #1, 2, 5, 6, 8b

1. Evaluate, where $f(x) = 2 - 3x$.

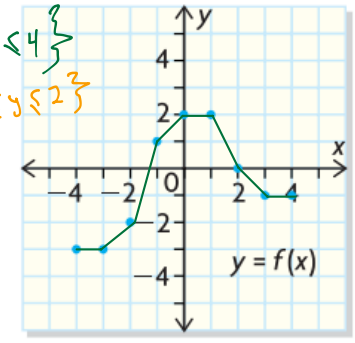
a) $f(2) = -4$ c) $f(-4) = 14$ e) $f(a) = -3a + 2$

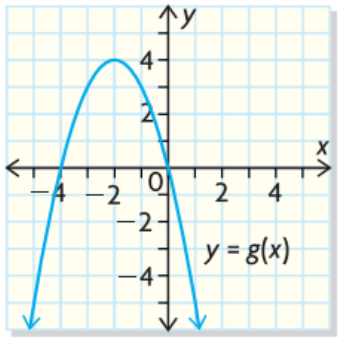
b) $f(0) = 2$ d) $f\left(\frac{1}{2}\right) = 0.5$ f) $f(3b) = -9b + 2$

1, 2, 5, 6, 8b

2. The graphs of $y = f(x)$ and $y = g(x)$ are shown.

D: $\{x \in R \mid -4 \le x \le 4\}$
R: $\{y \in R \mid -3 \le y \le 2\}$





D: $\{x \in R \mid -4 \le x \le 0\}$
R: $\{y \in R\}$

y =

Using the graphs, evaluate

a) $f(1)$ c) $f(4) - g(-2)$

b) $g(-2)$ d) x when $f(x) = -3$

5. For $f(x) = \frac{1}{2x}$, determine

a) $f(-3)$ b) $f(0)$ c) $f(1) - f(3)$ d) $f\left(\frac{1}{4}\right) + f\left(\frac{3}{4}\right)$

$f(-3) = \frac{1}{2(-3)}$

$f(-3) = -\frac{1}{6}$

undefined

$\frac{3}{6} - \frac{1}{6}$

$\frac{2}{6} = \frac{1}{3}$

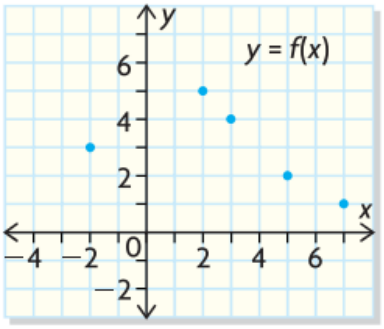
6. The graph of $y = f(x)$ is shown at the right.

a) State the domain and range of f.

b) Evaluate.

i) $f(3)$ **iii)** $f(5 - 3)$

ii) $f(5)$ **iv)** $f(5) - f(3)$



8. Consider the function $g(t) = 3t + 5$.

a) Create a table of values and graph the function.

b) Determine each value.

i) $g(0)$ = 5

ii) $g(3)$ = 14

iii) $g(1) - g(0)$

8 − 5

= 3

iv) $g(2) - g(1)$ 11 − 8 = 3

v) $g(1001) - g(1000)$ = 3

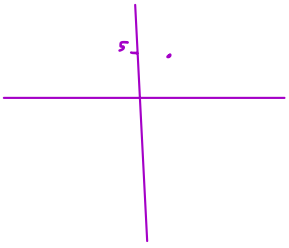
vi) $g(a + 1) - g(a)$

3a+3+5

3a+8 − 3a+5

= 3

x	y



ANSWERS:

Lesson 1.2, pp. 22–24

1. a) -4 c) 14 e) $2 - 3a$

b) 2 d) $\frac{1}{2}$ f) $2 - 9b$

2. a) 2 b) 4 c) -5 d) -3 or -4

3. a) $f(x) = 1200 - 3x$

b) 840 mL c) 3:10 pm

4. a) 8, 0, -0.75 b) $-5, -25, -2.5$

5. a) $-\frac{1}{6}$ b) undefined c) $\frac{1}{3}$ d) $2\frac{2}{3}$

6. a) domain = $\{-2, 0, 2, 3, 5, 7\}$, range = $\{1, 2, 3, 4, 5\}$

b) i) 4 ii) 2 iii) 5 iv) -2

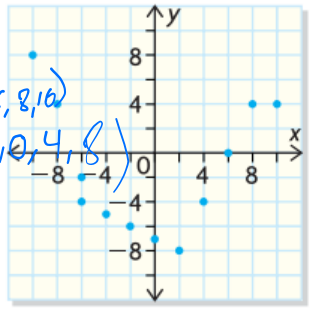
7. a) $2a - 5$ b) $2b - 3$ c) $6c - 7$ d) $-10x - 1$

8. b) i) 5 ii) 14 iii) 3 iv) 3 v) 3 vi) 3

Domain & Range and Function Notation ~ Worksheet pg 35 # 2, 3, 5, 9

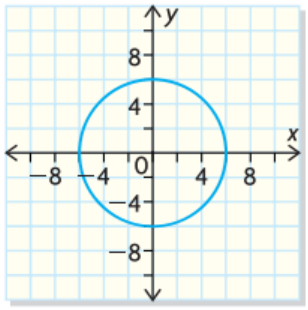
2. State the domain and range of each relation.

a)



x={-10, -8, -6, -4, -2, 0, 2, 4, 6, 8, 10}
y{-8, -7, -6, -5, -4, -2, 0, 4, 8}

d)

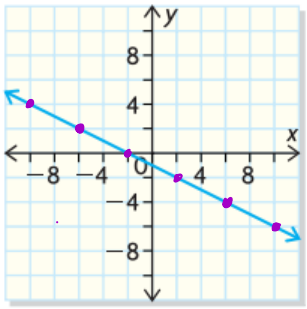


$\{x \in R \mid -6 \leq x \leq 6\}$

$\{y \in R \mid -6 \leq y \leq 6\}$

b)

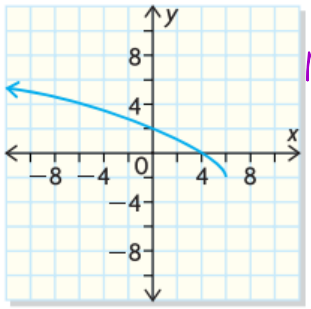
Linear



D: $\{x \in R\}$

R: $\{y \in R\}$

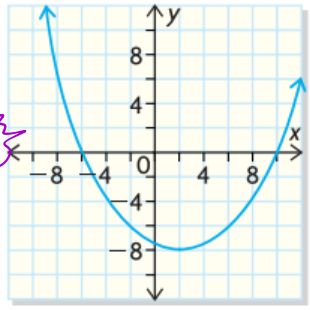
e)



D: $\{x \in R \mid x < 6\}$

R: $\{y \in R \mid y > 6\}$

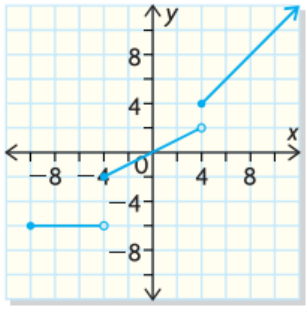
c)



D: $\{x \in R\}$

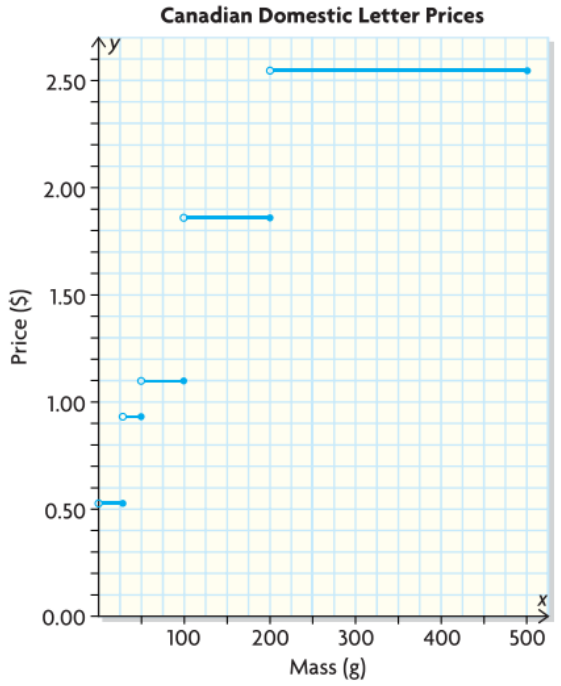
R: $\{y \in R \mid -8 \geq\}$

f)



3. Identify which of the relations in questions 1 and 2 are functions.

5. The graph shows how 2007 prices for mailing letters in Canada vary with mass.



a) Explain why this relation is a function. Why is it important for this to be so?

b) State the domain and range of the function.

9. Determine the domain and range of each function.

a) $f(x) = -3x + 8$

b) $g(x) = -0.5(x + 3)^2 + 4$

c) $h(x) = \sqrt{x - 1}$

d) $p(x) = \frac{2}{3}(x - 2)^2 - 5$

e) $q(x) = 11 - \frac{5}{2}x$

f) $r(x) = \sqrt{5 - x}$

ANSWERS:

Lesson 1.4, pp. 35–37

1. a) domain = {1900, 1920, 1940, 1960, 1980, 2000},
range = {47.3, 54.1, 62.9, 69.7, 73.7, 77.0}
b) domain = {−5, −1, 0, 3}, range = {9, 15, 17, 23}
c) domain = {−4, 0, 3, 5}, range = {−1, 0, 3, 5, 7}

2. a) domain = {0, ±2, ±4, ±6, ±8, ±10},
range = {−8, −7, −6, −5, −4, −2, 0, 4, 8}
b) domain = $\{x \in \mathbf{R}\}$, range = $\{y \in \mathbf{R}\}$
c) domain = $\{x \in \mathbf{R}\}$, range = $\{y \in \mathbf{R} \mid y \geq -8\}$
d) domain = $\{x \in \mathbf{R} \mid -6 \leq x \leq 6\}$,
range = $\{y \in \mathbf{R} \mid -6 \leq y \leq 6\}$

3. 1. (a), (b); 2. (b), (c), (e), (f)

4. domain = $\{x \in \mathbf{R}\}$, range = $\{y \in \mathbf{R} \mid y \geq -3\}$

5. a) Even at masses when the price changes, a single price (the lower one) is assigned. It would not make sense to assign two or more prices to the same mass.
b) domain = $\{x \in \mathbf{R} \mid 0 < x \leq 500\}$,
range = {0.52, 0.93, 1.20, 1.86, 2.55}

9. a) domain = $\{x \in \mathbf{R}\}$, range = $\{y \in \mathbf{R}\}$
b) domain = $\{x \in \mathbf{R}\}$, range = $\{y \in \mathbf{R} \mid y \leq 4\}$
c) domain = $\{x \in \mathbf{R} \mid x \geq 1\}$, range = $\{y \in \mathbf{R} \mid y \geq 0\}$
d) domain = $\{x \in \mathbf{R}\}$, range = $\{y \in \mathbf{R} \mid y \geq -5\}$
e) domain = $\{x \in \mathbf{R}\}$, range = $\{y \in \mathbf{R}\}$
f) domain = $\{x \in \mathbf{R} \mid x \leq 5\}$, range = $\{y \in \mathbf{R} \mid y \geq 0\}$