1.2 - Solving Equations and Factoring Recap

Solving Linear Equations

To **solve** means to find a value for the variable that satisfies the expression (x = ?)

Ex.1 Solve & check

a. $4(y-2) = 3(y+1)$

$4y - 8 = 3y + 3$

$y = 11$

LS	
$4(y-2)$	$3(y+1)$
$4(11)-8$	$3y+3$
36	36

b. $6\left(\frac{m+2}{2}\right) = \left(\frac{m-1}{3}\right)2$ common denom $= 6$

$3(m+2) = 2(m-1)$

$3m + 6 = 2m - 2$

$m = -8$

c. $\frac{2x-1}{3} - \frac{x+1}{4} = 3$

$12\left(\frac{2x-1}{3}\right) - 12\left(\frac{x+1}{4}\right) = 12(3)$

$8x - 4 - 3x - 3 = 12(3)$

$5x - 7 = 36$

$5x = 36 + 7$

$\frac{5x}{5} = \frac{43}{5}$

$x = \frac{43}{5}$

d. $0.5x - 0.1(x-3) = 4$

$0.5x - 0.1x + 0.3 = 4$

$\frac{0.4x}{0.4} = \frac{3.7}{0.4}$

$x = \frac{3.7}{0.4}$

$x = 9.25$

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$10(0.5x - 0.1x + 0.3 = 4)$

$5x - 1x + 3 = 40$

$\frac{4x}{4} = \frac{37}{4}$

$x = \frac{21}{4}$

Factoring

Recall: There are **4 types** of factoring:

Type 1	Common		
Type 2	Simple Trinomial	(a = 1)	} of the form $ax^2 + bx + c$
Type 3	Complex Trinomial	(a ≠ 1)	
	(Distribution or Australian Method)		
Type 4	Difference of Squares		
Type 5	Grouping		

When factoring, always look for a common factor first and then perform any other factoring, if necessary.

Common Factoring ~ Find the greatest common factor in all of the terms and divide it out of each term

Ex.2 Factor completely

a. $\frac{6x - 18}{6}$

$= 6(x - 3)$

b. $\frac{4xy}{2} - \frac{10xz}{2} + \frac{14yz}{2}$

$2(2xy - 5xz + 7yz)$

c. $2y(y - 7) + 4(y - 7)$

$(y-7) \quad (y-7)$

$= (y-7)(2y+4)$

$= 2(y-7)(y+2)$

d. $4mn(t - 4) - (t - 4)$

$(t-4) \quad (t-4)$

$= (t-4)(4mn-1)$

e. $7x^2 - 21x^6$

f. $4x^3(x + 2) - 8x^2(x + 2)$

Factoring Simple Trinomials [of the form $ax^2 + bx + c$, where $a = 1$]

To factor a simple trinomial, you need to find **two** numbers that will ***multiply to the last term*** and ***add to the middle term***.

$x^2 + 5x + 6$

$= (x+2)(x+3)$

So we need to find to numbers:

$2 \times 3 = 6$

$2 + 3 = 5$

Ex.3 Factor

a. $x^2 + 6x + 9$

$= (x+3)(x+3)$

$= (x+3)^2$

b. $x^2 - 4x - 12$

$(x-6)(x+2)$

c. $x^2 - 12x + 20$

$(x-10)(x-2)$

d. $x^2 - xy - 6y^2$

$(x-3y)(x+2y)$

Factoring Complex Trinomials [of the form $ax^2 + bx + c$, where $a \neq 1$]

To factor this type of trinomial, you may use one of two methods.

Method 1: Product & Sum Method + Decomposition

- ***break up the middle term*** into two numbers that ***add to the middle term***, but ***multiply to the product of the first and last terms***.

$2x^2 + 11x + 12$ ($a = 2$, $b = 11$, $c = 12$)

So we need to find to numbers:

$a \times c = 24$

___ x ___ = 24 ($a \times c$)

___ + ___ = 11 (b)

$= 2x^2 + 8x + 3x + 12$ (2x, 2x, 3, 3)

$= 2x(x+4) + 3(x+4)$

$= (x+4)(2x+3)$

Decomposition – decomposing the middle term into two terms

Product: 2 x 12 = 24		Sum = 11
1	24	
2	12	
3	8	
4	6	

Ex.4 Factor completely using Product & Sum + Decomposition

a. $4x^2 + 25x + 6$

$a \times c = 24$

$\frac{4x^2}{4x} + \frac{24x}{4x} + \frac{1x+6}{1}$

$4x(x+6) + 1(x+6)$

$(x+6)(4x+1)$

b. $\frac{12t^2}{2} - \frac{34t}{2} - \frac{28}{2}$

$a \times c = -84$

$-21 + 4$

$= 2(6t^2 - 17t - 14)$

$2[\frac{6t^2}{2t} + \frac{4t}{2t} - 21t - 14]$

$2[2t(3t+2) - 7(3t+2)]$

$2(3t+2)(2t-7)$

Method 2: Australian Method

1) Multiply a x c.
2) Write all the factors of ac in pairs.
3) Find the pair that adds up to b.
4) Use your results in the binomial product:

$\frac{(ax+\quad)(ax+\quad)}{a}$

5) Simplify. ← down under 

$a \times c = 24$

$3 \quad 8$

$2x^2 + 11x + 12$

$= \frac{(2x+3)(2x+8)}{2}$ (1 · 4)

$= (2x+3)(1x+4)$

Ex. 5 Factor the following using the **Australian** method.

a. $8x^2 - 2x - 1$

$9x\,c = -8$; $-4 + 2$

$= \frac{(8x-4)(8x+2)}{8}$

$= \frac{(8x-4)(8x+2)}{4 \times 2}$

$= (2x-1)(4x+1)$

b. $16x^2 + 8x - 3$

-48 ; $12 \; -4$

$\frac{(16x+12)(16x-4)}{4 \times 4}$

$(4x+3)(4x-1)$

Ex. 6 Factor the following using your choice of method:

a. $6x^2 + 15x + 9$; 54

$\frac{6x^2}{6x} + \frac{18x}{6x} + 3x + 9$

$6x(x+3) + 3(x+3)$

$(x+3)(6x+3)$

b. $16x^2y - 4xy - 2y$; 8 ; $-4 \; 2$

$2[8x^2y - 2xy - y]$

$2[8x^2y - 4xy + 2xy - y]$

$2[4xy(2x-1) + 1y(2x-1)]$

$2 \cdot (2x-1)(4xy+1y)$

c. $3x^2 + 2xy - y^2$; -3 ; $3 \; -1$

$\frac{3x^2}{3x} + \frac{3xy}{3x} - \frac{1xy}{-y} - \frac{y^2}{-y}$

$3x(x+y) + 1y(x+y)$

$(x+y)(3x+y)$

Factoring a Perfect Square Trinomial

Remember the patterns $(a^2 + 2ab + b^2) = (a + b)^2$ and $(a^2 - 2ab + b^2) = (a - b)^2$

Ex.7 Factor

a. $x^2 + 2x + 1$

b. $x^2 + 6x + 9$

c. $9x^2 + 30x + 25$

Factoring a Difference of Squares

Look for the distinct pattern (a squared term subtracting another squared term)

Ex.5 Factor

a. $x^2 - 25$

$= (x+5)(x-5)$

b. $16x^2 - 81$

$(4x+9)(4x-9)$

c. $3m^2 - 27$

$3(m^2 - 9)$

$3(m+3)(m-3)$

d. $a^4b^6 - \frac{1}{49}$

a^2b^3

$= \left(a^2b^3 + \frac{1}{7}\right)\left(a^2b^3 - \frac{1}{7}\right)$

Homework:	Solving Linear Equations & Factoring Worksheet

Solving Linear Equations & Factoring Worksheet

1. Solving linear equations. Solve and check.

a. $2(2r - 1) + 4 = 5(r + 1)$

$4r - 2 + 4 = 5r + 5$

$4r + 2 = 5r + 5$

$\frac{-r}{-} = \frac{3}{-}$

$r = -3$

b. $5(x - 3) - 2x = -6$

$5x - 15 - 2x = -6$

$\frac{3x}{3} = \frac{9}{3}$

$x = 3$

c. $7 - 2(1 - 3x) + 16 = 8x + 11$

$7 - 2 + 6x + 16 = 8x + 11$

$6x - 8x = 11 + 2 - 7$

$\frac{-2x}{2} = \frac{6}{-2}$

$x = -3$

d. $4y - (3y - 1) - 3 + 6(y - 2) = 0$

$4y - 3y + 1 - 3 + 6y - 12 = 0$

$\frac{7y}{7} = \frac{14}{7}$

$y = 2$

e. $4(w - 5) - 2(w + 1) = 3(1 - w)$

$4w - 20 - 2w - 2 = 3 - 3w$

$\frac{5w}{5} = \frac{25}{5}$

$w = 5$

f. $0 = 2(t - 6) + 8 + 4(t + 7)$

$0 = 2t - 12 + 8 + 4t + 28$

$\frac{-6t}{-6} = \frac{24}{-6}$

$t = -4$

2. Solving linear equations. Solve and check.

a. $6\left(\frac{x}{3} + \frac{1}{2}\right) = 0$

$2x + 3 = 0$

$\frac{2x}{2} = \frac{-3}{2}$

b. $\frac{2}{6}\left(\frac{y-1}{3}\right) = 6^{6}$

$2y - 2 = 36$

$\frac{2y}{2} = \frac{38}{2}$

$y = 19$

c. $\frac{3}{6}\left(\frac{w+1}{2} + \frac{2}{6}\left(\frac{w+1}{3}\right) = 5\right)^{6}$

$3w + 3 + 2w + 2 = 30$

$\frac{5w}{5} = \frac{25}{5}$

$w = 5$

d. $\frac{4}{12}\left(\frac{2x+1}{3}\right) - \frac{3}{12}\left(\frac{x+1}{4}\right) = 3^{12}$

$8x + 4 - 3x - 3 = 36$

$\frac{5x}{5} = \frac{35}{5}$

$x = 7$

1

e. $0.4(c - 8) + 3 = 4$

$0.4c - 3.2 + 3 = 4$

$0.4c = 4 - 3 + 3.2$

$\frac{0.4c}{0.4} = \frac{4.2}{0.4}$

$c = 1.05 \times 10$

$c = 10.5$

f. $1.5(a - 3) - 2(a - 0.5) = 10$

$\times 10 \quad 10 \quad 10$

$15a - 45 - 20a + 10 = 100$

$-5a = 100 + 45 - 10$

$\frac{-5a}{-5} = \frac{135}{-5}$

$a = -\frac{27}{1}$

$a = -27$

3. Common factor.

a. $\frac{7t^2}{7t^2} - \frac{14t^3}{7t^2}$

$7t^2(1 - 2t)$

b. $\frac{36x^7}{12x^5} + \frac{24x^5}{12x^5}$

$12x^5(3x^2 + 2)$

c. $\frac{9x^2y}{3xy} + \frac{6xy}{3xy} - \frac{3xy^2}{3xy}$

$3xy \quad (3x + 2 - y^2)$

d. $\frac{10a^2b}{5a} + \frac{5ab}{5a} - \frac{15a}{5a}$

$5a(2ab + b - 3)$

4. Factoring $ax^2 + bx + c$, $a = 1$

a. $d^2 + 3d - 10 \quad \frac{-10}{5 \quad -2}$

$d^2 + 5d - 2d - 10$

$d(d + 5) - 2(d + 5)$

$(d + 5)(d - 2)$

b. $w^2 - 81$

$(w + 9)(w - 9)$

c. $y^2 - 2y - 8 \quad \frac{-8}{-4 \quad 2}$

$y^2 - 4y \; 2y - 8$

$y(y - 4) \; 2(y - 4)$

$(y - 4)(y + 2)$

d. $t^2 - 4t$

$t(t + 2)(t - 2)$

5. Factoring $ax^2 + bx + c$, $a \neq 1$

a. $2x^2 + 7x + 3$ $\quad 6, 1$

$2x^2+6x+1x+3$

$2x(x+3)+1(x+3)$

$(x+3)(2x+1)$

b. $6x^2 + x - 1$ $\quad -6, -2, 3$

$6x^2-2x+3x-1$

$2x(3x-1)+1(3x-1)$

$(3x-1)(2x+1)$

c. $3x^2 + 7x - 20$ $\quad -60, 12, -5$

$3x^2+12x-5x-20$

$3x(x+4)-5(x+4)$

$(x+4)(3x-5)$

d. $6s^2 - 7s - 3$ $\quad -18, -9, 2$

$6s^2-9s+2s-3$

$3s(2s-3)+1(2s-3)$

$(2s-3)(3s+1)$

Solutions

1.a. -3 b. 3 c. 5 d. 2 e. 5 f. -4

2.a. $\frac{-3}{2}$ b. 19 c. 5 d. 7 e. 10.5

f. -27

3. a. $7t^2(1 - 2t)$ b. $12x^5(3x^2 + 2)$

c. $3xy(3x + 2 - y)$

d. $5a(2ab + b - 3)$

4. a. $(d+5)(d - 2)$ b. $(w - 9)(w + 9)$