When Multiplying & Dividing Rational Expressions…

Step 1 – Factor all numerators and denominators

Step 2 – State any restrictions

Step 3 – Reduce any common factors that are in both the numerator and denominator to simplify the expression

Step 4 – $\dfrac{Top \ x \ Top}{Bottom \ x \ Bottom}$ for multiplication **or** invert and multiply for division

– If **division**: make sure to state new restrictions for 'new' denominator

Step 5 – Simplify further (if possible)

Ex.1   State any restrictions on the variable and simplify

a. $\dfrac{3x^3}{2y^2} \times \dfrac{10y^3}{9x^2}$, $x \neq 0$
$y \neq 0$

$\dfrac{x}{1} \times \dfrac{5y}{3}$

$= \dfrac{5xy}{3}$, $x \neq 0$
$y \neq 0$

b. $\dfrac{2ab}{5c} \div \dfrac{14a^2b^2}{15c^2}$, $a \neq 0$
$b \neq 0$
$c \neq 0$

$\dfrac{2ab}{5c} \times \dfrac{15c^2}{14a^2b^2} = \dfrac{3c}{7ab}$

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$a \neq 0$
$b \neq 0$
$c \neq 0$

c. $\dfrac{2a^2 - a - 1}{a^2 - 1} \times \dfrac{4a^2 + 4a}{2a^2 + 7a + 3}$ ⌐ $a \neq 0$

$= \dfrac{(a-1)(2a+1)}{(a-1)(a+1)} \times \dfrac{4(a+1)}{(2a+1)(a+3)}$

$\dfrac{2(2a+1)}{(a+1)} \times \dfrac{4}{a+3}$, $a \neq \pm 1$

$a \neq -3$

$a \neq -\dfrac{1}{2}$

$= \dfrac{4a}{a+3}$, $a \neq \pm 1$

$a = -3$

$a = -\dfrac{1}{2}$

d. $\dfrac{2x^2 - 6x}{x^2 - 9} \div \dfrac{4x^3 + 28x^2}{x^2 + 10x + 21}$

$\dfrac{2x(x-3)}{(x-3)(x+3)} \div \dfrac{4x^2(x+7)}{(x+7)(x+3)}$ ⌐ $x \neq \pm 3$
$x \neq -7$

$\dfrac{2x(x-3)}{(x-3)(x+3)} \times \dfrac{(x+7)(x+3)}{4x^2(x+7)}$ • $x \neq \pm 3$
$x \neq -7$
→ $x \neq 0$

$\dfrac{1}{1} \times \dfrac{1}{2x} = \dfrac{1}{2x}$

$x \neq \pm 3, -7, 0$

e. $\dfrac{k^2 + 6k + 8}{8k^2 - 24k} \times \dfrac{k^2 - 2k - 3}{k^2 - 4} \div \dfrac{k^2 + 5k + 4}{4k^2 - 8k}$, $k \neq 0$

$\dfrac{(k+4)(k+2)}{8k(k-3)} \times \dfrac{(k-3)(k+1)}{(k-2)(k+2)} \times \dfrac{4k(k-2)}{(k+4)(k+1)}$ ↗ $k \neq \pm 1, \pm 2, +3, 0, -4$

$\dfrac{1}{2}$

IT'S YOUR TURN!

Homework:

Text pg.122 - 23 # 4d, 5(c,d), 6(c,d), 7(a,d), 8, 9, 10

Multiplying & Dividing Rational Expressions ~ Worksheet

[pg.122 - 23 # 4d, 5(c,d), 6(c,d), 7(a,d), 8, 9, 10]

4. Simplify. State any restrictions on the variables. $x \neq 0$, $y \neq 0$

$x \neq 0$

a) $\dfrac{2x^2}{7} \times \dfrac{21}{x} = \dfrac{21x}{1}$

c) $\dfrac{2x^3y}{3xy^2} \times \dfrac{9x}{4x^2y}$ $\dfrac{x}{y^2} \times \dfrac{3}{2}$ $\dfrac{3x}{2y^2}$

b) $\dfrac{7a}{3} \div \dfrac{14a^2}{5}$

d) $\dfrac{3a^2b^3}{2ab^2} \div \dfrac{9a^2b}{14a^2}$ $a \neq 0$, $b \neq 0$

$\dfrac{7a}{3} \quad \dfrac{5}{14a^2} \quad a \neq 0$

$\dfrac{3a^2b^3}{2ab^2} \times \dfrac{14a^2}{9a^2b}$ $b \neq 0$

$\dfrac{1}{3} \times \dfrac{5}{7a} = \dfrac{5}{21a}$

$\dfrac{b^2}{b^2} \quad \dfrac{7a}{3} \quad \dfrac{7ab^2}{3b^2}$

$x \neq -1$

5. Simplify. State any restrictions on the variables. $x \neq 0, 2$

a) $\dfrac{2(x+1)}{3} \times \dfrac{x-1}{6(x+1)}$ $\dfrac{x-1}{9}$

c) $\dfrac{2(x-2)}{9x^3} \times \dfrac{12x^4}{-2+x}$ $\dfrac{2(x-2)}{9x^3} \times \dfrac{12x^4}{-(x-2)}$ $\dfrac{2}{3}x \cdot \dfrac{4}{-1}$ $\dfrac{4x}{3}$

b) $\dfrac{3a-6}{a+2} \div \dfrac{a-2}{a+2}$

d) $\dfrac{3(m+4)^2}{2m+1} \div \dfrac{5(m+4)}{7m+14}$ $m \neq 0$ $= \dfrac{8}{-3}$

$a \neq -2$

$a \neq 2$ $\dfrac{3(a-2)}{a+2} \quad \dfrac{a+2}{a-2}$

3

$\dfrac{3(m+4)(m+4)}{2m+1} \times \dfrac{7m+14}{5(m+4)}$ $m \neq -4, 0, -\frac{1}{2}, -2$

$\dfrac{3(m+4)}{2m+1} \times \dfrac{7(m+2)}{5}$ $(m+2)$

$= \dfrac{21(m+4)(m+2)}{5(2m+1)}$

$$x \neq -2, \pm 3$$

6. Simplify. State any restrictions on the variables.

a) $\dfrac{(x + 1)(x - 3)}{(x + 2)(x-3)} \times \dfrac{2(x + 2)}{(x-3)(x + 3)}$ $\quad \dfrac{(x+1)}{(x+2)} \times \dfrac{2}{(x+3)} \quad \dfrac{2(x+1)}{(x+2)(x+3)}$

$n \neq 4, 3, -2, 2$

b) $\dfrac{2(n^2 - 7n + 12)}{n^2 - n - 6} \div \dfrac{5(n - 4)}{n^2 - 4}$ $\quad \dfrac{2(n^2-7n+12)}{n^2-n-6} \times \dfrac{(n-2)(n+2)}{5(n-4)} \left| \dfrac{2(n-4)(n-3)}{(n-3)(n+2)} \times \dfrac{(n-2)(n+2)}{5(n-4)} = \dfrac{2(n-2)}{5}\right.$

$x \neq 3, -2, -\tfrac{5}{4}, -\tfrac{1}{2}$

c) $\dfrac{2x^2 - x - 1}{x^2 - x - 6} \times \dfrac{6x^2 - 5x + 1}{8x^2 + 14x + 5}$ $\quad \dfrac{(x-1)(2x+1)}{(x-3)(x+2)} \times \dfrac{(2x-1)(3x-1)}{(4x+5)(2x+1)} = \dfrac{(x-1)(2x-1)(3x-1)}{(x-3)(x+2)(4x+5)}$

$y \neq 0$

d) $\dfrac{9y^2 - 4}{4y - 12} \div \dfrac{9y^2 + 12y + 4}{18 - 6y}$ $\quad \dfrac{36}{+6+6} \quad \dfrac{(9y+6)(9y+6)}{3 \wedge 3}$

$y \neq 3, -\tfrac{2}{3}$ $\quad \dfrac{(3y-2)(3y+2)}{2\,4(y-3)} \times \dfrac{-6(y-3)}{(3y+2)(3y+2)}$

$= -\dfrac{3(3y-2)}{2(3y+2)}$

7. Simplify. State any restrictions on the variables.

a) $\dfrac{x^2 - 5xy + 4y^2}{x^2 + 3xy - 28y^2} \times \dfrac{x^2 + 2xy + y^2}{x^2 - y^2}$

b) $\dfrac{2a^2 - 12ab + 18b^2}{a^2 - 7ab + 10b^2} \div \dfrac{4a^2 - 12ab}{a^2 - 7ab + 10b^2}$

c) $\dfrac{10x^2 + 3xy - y^2}{9x^2 - y^2} \div \dfrac{6x^2 + 3xy}{12x + 4y}$

d) $\dfrac{15m^2 + mn - 2n^2}{2n - 14m} \times \dfrac{7m^2 - 8mn + n^2}{5m^2 + 7mn + 2n^2}$

$\dfrac{-30}{+6-5}$

$\dfrac{(15mn + 2n^{\overset{2}{}})(18m - 8^{\overset{1}{}})}{15 \quad 5 \times 3}$

$(5m + 2n)(3n - 1n)$

$= \dfrac{(5m + 2m)(3m - 1n)}{2\,(n - 7m)} \times \dfrac{(m - n)(7m - n)}{(m + n)(5m + 2n)}$

$\dfrac{7}{-7 - 1}$

$\dfrac{(7m - 7n)(7m - n)}{7}$

$(m - n)(7m - n)$

$\overset{-1}{}\overset{-1}{}$

$= \dfrac{(3m - n)(m - n)}{-2(7m - n)(m + n)}$

$= \dfrac{(3m - n)(m - n)}{-2(m + n)}$

8. Simplify. State any restrictions on the variables.

K

$\dfrac{x^2 + x - 6}{(2x - 1)^2} \times \dfrac{x(2x - 1)^2}{x^2 + 2x - 3} \div \dfrac{x^2 - 4}{3x}$

$x \neq 0, \tfrac{1}{2},$

$x \neq \tfrac{1}{2}, -3, 1, 2 \quad \dfrac{(x + 3)(x - 2)}{(2x - 1)(2x - 1)} \times \dfrac{1x(2x - 1)(2x - 1)}{(x + 3)(x - 1)} \times \dfrac{3x}{(x - 2)(x - 2)} = \dfrac{3x^2}{(x - 1)(x - 2)}$
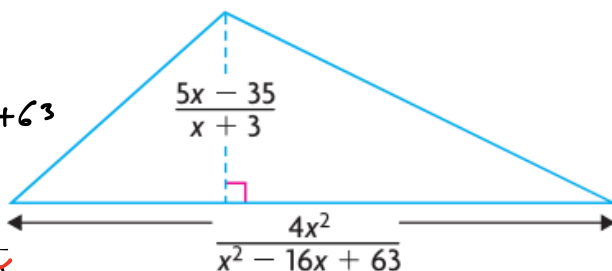
$x \neq 0, \tfrac{1}{2}, -3, 1, 2$

9. Determine the area of the triangle in simplified form. State the restrictions.

A

$x \neq -3$

$\dfrac{5x-35}{x+3} \times \dfrac{4x^2}{x^2-16x+63}$

$\dfrac{5x-35}{x+3}$

$\dfrac{4x^2}{x^2-16x+63}$

$x \neq 3, 9, 7 \quad \dfrac{5(x-7)}{x+3} \times \dfrac{4x^2}{(x-9)(x-7)}$

$\dfrac{20x^2-2}{(x+3)(x-9)} = \dfrac{10x^2}{(x+3)(x-9)}$

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10. An object has mass $m = \dfrac{p+1}{3p+1}$ and density $\rho = \dfrac{p^2-1}{9p^2+6p+1}$.

Determine its volume v, where $\rho = \dfrac{m}{v}$. State the restrictions on any variables.

$\dfrac{p^2-1}{9p^2+6p+1} = \dfrac{\frac{p+1}{3p+1}}{v}$

$p \neq -\frac{1}{3}, 1$

$V = \dfrac{p+1}{3p+1} \div \dfrac{p^2-1}{9p^2+6p+1}$

$p \neq 1, -\frac{1}{3}, 1$

$V = \dfrac{p+1}{3p+1} \times \dfrac{9p^2+6p+1}{(p-1)(p+1)}, \dfrac{p+1}{3p+1} \times \dfrac{(3p+1)(3p+1)}{(p-1)(p+1)}$

$V = \dfrac{(3p+1)}{(p-1)} \qquad , p \neq -\frac{1}{3}, \pm 1$

Solutions:

4. a) $6x, x \neq 0$ c) $\dfrac{3x}{2y^2}, x \neq 0, y \neq 0$

   b) $\dfrac{5}{6a}, a \neq 0$ d) $\dfrac{7a}{3}, a \neq 0, b \neq 0$

5. a) $\dfrac{(x-1)}{9}, x \neq -1$ c) $-\dfrac{8x}{3}, x \neq 0, 2$

   b) $3, a \neq -2, 2$ d) $\dfrac{21(m+4)(m+2)}{5(2m+1)}, m \neq -4, -2, -\dfrac{1}{2}$

6. a) $\dfrac{2(x+1)}{(x+2)(x+3)}, x \neq -3, -2, 3$

   b) $\dfrac{2(n-2)}{5}, n \neq -2, 2, 3, 4$

   c) $\dfrac{(x-1)(3x-1)(2x-1)}{(x-3)(x+2)(4x+5)}, x \neq -2, -\dfrac{5}{4}, -\dfrac{1}{2}, 3$

   d) $\dfrac{-3(3y-2)}{2(3y+2)}, y \neq -\dfrac{2}{3}, 3$

7. a) $\dfrac{x+y}{x+7y}, x \neq -y, -7y, y, 4y$

   b) $\dfrac{(a-3b)}{2a}, a \neq 0, 2b, 3b, 5b$

   c) $\dfrac{4(5x-y)}{3x(3x-y)}, x \neq 0, -\dfrac{1}{2}y, -\dfrac{1}{3}y, \dfrac{1}{3}y$

   d) $\dfrac{-(3m-n)(m-n)}{2(m+n)}, m \neq \dfrac{1}{7}n, -\dfrac{2}{5}n, -n$

8. $\dfrac{3x^2}{(x-1)(x+2)}$, restrictions: $x \neq -3, -2, 0, \dfrac{1}{2}, 1, 2$

9. $\dfrac{10x^2}{(x-9)(x+3)}, x \neq 9, 7, -3$

10. $\dfrac{3p+1}{p-1}, p \neq -1, -\dfrac{1}{3}, 1$