1.4 – Simplifying Rational Expressions

Simplifying Rational Expressions

A rational expression is any expression in the form: $\frac{a}{b}$ where both a and b are polynomial functions

Examples: $\frac{x+1}{3x^2+5}$ or $\frac{5x^2+8x+9}{8x^4+8x+9}$ or $\frac{2y}{5y+1}$ or 5x

$\frac{0}{8}=0$ $\frac{8}{0}=$ error

Method of Simplifying Rational Expressions

1. Factor the numerator and denominator **fully** – FACTOR, FACTOR, FACTOR
2. Write any RESTRICTIONS!
   - Recall that we cannot divide by zero, therefore we must ensure that **any and all** factors that are in the denominator **throughout** our solution do not equal zero.
   - $\frac{5}{0}$ evaluated on a calculator equals ERROR and is therefore undefined
3. Reduce (through division) common factors in both the numerator and denominator (aka canceling out)

Examples of restrictions:

a. $\frac{5x}{x+3}$, $x+3\neq 0$, $x\neq -3$

b. $\frac{7x^2+x+3}{x}$, $x\neq 0$

c. $\frac{3x+5}{x(x-2)(3x+4)}$, $x\neq 0$, $x\neq 2$, $x\neq \frac{-4}{3}$

$x=2$

$3x+4\neq 0$

$3x\neq -4$

$x\neq -\frac{4}{3}$

Ex.1 Simplify and state all restrictions

a. $\frac{3x-6}{x-2}$, $x \neq 2$

$= \frac{3(x-2)}{x-2}$, $x \neq 2$

$= 3$, $x \neq 2$

b. $\frac{2x}{2x+6}$

$= \frac{2x}{x(x+3)}$, $x \neq -3$

$\frac{x}{x+3}$, $x \neq -3$

c. $\frac{5-x}{x-5}$, $x \neq 5$

$\frac{-(-5+x)}{x-5}$, $x \neq 5$

$= \frac{-1(x-5)}{x-5}$, $x \neq 5$

d. $\frac{x^2-7x+6}{x-1}$ $\quad 6, -6$ $\quad x \neq 1$

$x^2 - 1x - 6x + 6$

$x(x-1) - 6(x-1)$

$\frac{(x-1)(x-6)}{x-1}$

$= x - 6$

e. $\frac{3t^4+9t^3-18t^2}{3t} = t \neq 0$

$\frac{3t^2(t^2+3t-6)}{3t}$ $\quad -6$

$= t(t^2+3t-6)$, $t \neq 0$

f. $\frac{x^2-7x+6}{36-x^2}$ $\quad 6, -6$ $\quad x \neq 6, -6$

$= \frac{(x-6)(x-1)}{(6+x)(6-x)}$

$= \frac{(x-6)(x-1)}{-(6+x)(x-6)}$

$= -\frac{x-1}{(x+6)}$

g. $\frac{2y^2-y-15}{3y^2-7y-6}$ $\quad -30, -65$ $\quad -18, -12$

$y \neq 3$

$y \neq -\frac{2}{3}$

$3y^2 - 9y + 2y - 6$

$3y(y-3) + 2(y-3)$

$(y-3)(3y+2)$

$= \frac{(y-3)(2y+5)}{(y-3)(3y+2)}$

$= \frac{2y+5}{3y+2}$

$(2y-5)(y+3)$

$(y-3)(y+3)$

NEED EXTRA HELP?? Check out this video: [Simplifying Rational Expressions - YouTube]

Homework: Simplifying Rational Expressions Worksheet

Simplifying Rational Expressions ~ Worksheet

Practise

In each of the following, state any restrictions on the variables.

A

1. Simplify.

a) $\frac{3t^3+6t^2-15t}{3t}$

b) $\frac{6a^2+9a}{12a^2}$

c) $\frac{10y^4+5y^3-15y^2}{5y}$

d) $\frac{14n^4-4n^3+6n^2+8n}{2n^2}$

e) $\frac{4m^2-8mn}{4mn}$

f) $\frac{-6x^2y^3}{-18x^3y}$

g) $\frac{16a^2bc}{4a^2b^2c^2}$

h) $\frac{-4x^4y^2z}{20x^3y^3z}$

i) $\frac{21m(m-4)}{7m^2}$

2. Express in simplest form.

a) $\frac{5x}{5(x+4)}$

b) $\frac{8t^2(t+5)}{4t(t-5)}$

c) $\frac{7x(x-3)}{14x^2(x-3)}$

d) $\frac{(m-1)(m+2)}{(m+4)(m-1)}$

e) $\frac{2x}{2x+8}$

f) $\frac{y^2}{y^2+2y}$

g) $\frac{10x}{5x^2-15x}$

h) $\frac{4x}{16x^3-12x}$

i) $\frac{3xy}{6x^2y-12xy^2}$

3. Simplify.

a) $\frac{6t-36}{t-6}$

b) $\frac{4m+24}{8m-24}$

c) $\frac{5x-10}{3x-6}$

d) $\frac{a^2+2a}{a^2-3a}$

e) $\frac{8x^2+4x}{6x^2+3x}$

f) $\frac{2x^2-2x}{2x^2+2x}$

g) $\frac{4x+4y}{5x+5y}$

h) $\frac{4a^2b+8ab}{6a^2-6a}$

i) $\frac{5xy+10x}{2y^2+4y}$

4. Express in simplest equivalent form.

a) $\frac{m-2}{m^2-5m+6}$

b) $\frac{y^2+10y+25}{y+5}$

c) $\frac{2x+6}{x^2-6x-27}$

d) $\frac{r^2-4}{5r+10}$

e) $\frac{a^2+a}{a^2+2a+1}$

f) $\frac{x^2-9}{2x^2y-6xy}$

g) $\frac{2w+2}{2w^2+3w+1}$

h) $\frac{3t^2-8t+4}{6t^2-4t}$

i) $\frac{8z+6z^2}{9z^2-16}$

j) $\frac{5x^2+3xy-2y^2}{3x^2+3xy}$

5. Simplify.

a) $\frac{y-2}{2-y}$

b) $\frac{3-x}{x-3}$

c) $\frac{2t-1}{4-8t}$

d) $\frac{6-10w}{15w-9}$

e) $\frac{x^2-1}{1-x^2}$

f) $\frac{1-4y^2}{8y^2-2}$

6. Simplify.

a) $\frac{x^2+4x+4}{x^2+5x+6}$

b) $\frac{a^2-a-12}{a^2-9a+20}$

c) $\frac{m^2-5m+6}{m^2+2m-15}$

d) $\frac{y^2-8y+15}{y^2-25}$

e) $\frac{x^2-10x+24}{x^2-12x+36}$

f) $\frac{n^2-n-2}{n^2+n-6}$

g) $\frac{p^2+8p+16}{p^2-16}$

h) $\frac{2t^2-t-1}{t^2-3t+2}$

i) $\frac{6v^2+11v+3}{4v^2+8v+3}$

j) $\frac{6x^2-13x+6}{8x^2-6x-9}$

k) $\frac{3z^2-7z+2}{9z^2-6z+1}$

l) $\frac{2m^2-mn-n^2}{4m^2-4mn-3n^2}$

Simplifying Rational Expressions ~ Worksheet Solutions

1. a) $t^2 + 2t - 5,\ t \neq 0$ **b)** $\frac{2a+3}{4a},\ a \neq 0$ **c)** $2y^3 + y^2 - 3y,\ y \neq 0$

d) $\frac{7n^3 - 2n^2 + 3n + 4}{n},\ n \neq 0$ **e)** $\frac{m-2n}{n},\ m,\ n \neq 0$

f) $\frac{y^2}{3x};\ x,\ y \neq 0$ **g)** $\frac{4}{bc},\ a,\ b,\ c \neq 0$ **h)** $-\frac{x}{5y},\ x,\ y,\ z \neq 0$

i) $\frac{3(m-4)}{m},\ m \neq 0$ **2. a)** $\frac{x}{x+4},\ x \neq -4$ **b)** $\frac{2t(t+5)}{t-5},$

$t \neq 0,\ 5$ **c)** $\frac{1}{2x},\ x \neq 0,\ 3$ **d)** $\frac{m+2}{m+4},\ m \neq 1,\ -4$ **e)** $\frac{x}{x+4},\ x \neq -4$

f) $\frac{y}{y+2},\ y \neq 0,\ -2$ **g)** $\frac{2}{x-3},\ x \neq 0,\ 3$ **h)** $\frac{1}{4x^2 - 3},\ x \neq 0,\ \pm\frac{\sqrt{3}}{2}$

i) $\frac{1}{2x-4y},\ x,\ y \neq 0,\ x \neq 2y$ **3. a)** $6,\ t \neq 6$ **b)** $\frac{m+6}{2m-6},\ m \neq 3$

c) $\frac{5}{3},\ x \neq 2$ **d)** $\frac{a+2}{a-3},\ a \neq 0,\ 3$ **e)** $\frac{4}{3},\ x \neq 0,\ -\frac{1}{2}$ **f)** $\frac{x-1}{x+1},$

$x \neq 0,\ -1$ **g)** $\frac{4}{5},\ x \neq -y$ **h)** $\frac{2ab+4b}{3a-3},\ a \neq 0,\ 1$ **i)** $\frac{5x}{2y},\ y \neq 0,\ -2$

4. a) $\frac{1}{m-3},\ m \neq 2,\ 3$ **b)** $y + 5,\ y \neq -5$ **c)** $\frac{2}{x-9},\ x \neq 9,\ -3$

d) $\frac{r-2}{5},\ r \neq -2$ **e)** $\frac{a}{a+1},\ a \neq -1$ **f)** $\frac{x+3}{2xy},\ x \neq 0,\ 3,\ y \neq 0$

g) $\frac{2}{2w+1},\ w \neq -1,\ -\frac{1}{2}$ **h)** $\frac{t-2}{2t},\ t \neq 0,\ \frac{2}{3}$ **i)** $\frac{2z}{3z-4},\ z \neq \pm\frac{4}{3}$

j) $\frac{5x-2y}{3x},\ x \neq 0,\ -y$ **5. a)** $-1,\ y \neq 2$ **b)** $-1,\ x \neq 3$ **c)** $-\frac{1}{4},\ t \neq \frac{1}{2}$

d) $-\frac{2}{3},\ w \neq \frac{3}{5}$ **e)** $-1,\ x \neq \pm 1$ **f)** $-\frac{1}{2},\ y \neq \pm\frac{1}{2}$ **6. a)** $\frac{x+2}{x+3},$

$x \neq -2,\ -3$ **b)** $\frac{a+3}{a-5},\ a \neq 4,\ 5$ **c)** $\frac{m-2}{m+5},\ m \neq 3,\ -5$ **d)** $\frac{y-3}{y+5},$

$y \neq \pm 5$ **e)** $\frac{x-4}{x-6},\ x \neq 6$ **f)** $\frac{n+1}{n+3},\ n \neq 2,\ -3$ **g)** $\frac{p+4}{p-4},\ p \neq \pm 4$

h) $\frac{2t+1}{t-2},\ t \neq 1,\ 2$ **i)** $\frac{3v+1}{2v+1},\ v \neq -\frac{1}{2},\ -\frac{3}{2}$ **j)** $\frac{3x-2}{4x+3},$

$x \neq -\frac{3}{4},\ \frac{3}{2}$ **k)** $\frac{z-2}{3z-1},\ z \neq \frac{1}{3}$ **l)** $\frac{m-n}{2m-3n},\ m \neq -\frac{1}{2}n,\ \frac{3}{2}n$