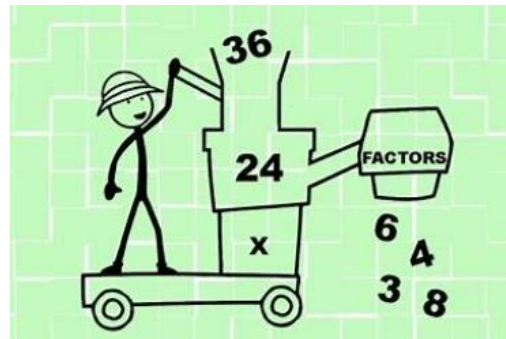


1.3 – More Solving and Factoring

Solving Quadratics by Factoring

Solve $x = \dots$

With **quadratics** there will be **two possible solutions** for x . These represent the points where the parabola crosses the x -axis. They are also called **roots** or **zeros**.



quadratic $x^2 - 4x = -4$ must be 0.

$$x^2 - 4x + 4 = 0$$

$$(x-2)(x-2) = 0$$

$$x-2=0 \quad x-2=0$$

$$\boxed{x=2} \quad \boxed{x=2}$$

(Solutions are x -intercepts zeros/roots)

Note: If $a \times b = 0$ then
 $a = 0$ and/or $b = 0$

Ex.1 Solve

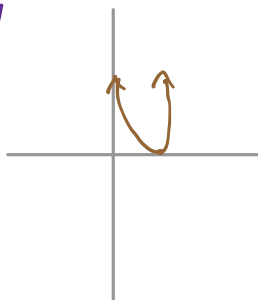
a. $2x^2 - 5x + 2 = 0$ $\frac{a \times c = 4}{-4 - 1}$

$$(2x^2 - 4)(x - 1)$$

$$(x-2)(2x-1)$$

$$x-2=0 \quad 2x-1=0$$

$$x=2 \quad x=\frac{1}{2}$$



b. $\frac{m^2}{m} - \frac{7m}{m} = 0$

$$m(m-7) = 0$$

$$\boxed{m=0} \quad m-7=0$$

$$m=7$$

More Factoring & Special Cases

Ex.2 Factor the following

a. $\frac{7m^3n^2}{7mn} - \frac{14m^2n^3}{7mn} - \frac{21m^5n}{7mn}$

$$7m^2(n-2n^2-3m^3)$$

b. $6x^2 - xy - 35y^2$ $\frac{a \times c = -210}{-15 + 14}$

$$(6x-15y)(x+7y)$$

$$(2x-5)(3x+7)$$

Decomposition:

$$= 6x^2 + 14xy - 15xy - 35y^2$$

$$= 2x(3x+7y) - 5y(3x+7y)$$

$$= (3x+7)(2x-5y)$$

c. $6a^2(x-y) + 13a(x-y) - 5(x-y)$

$$(x-y) (6a^2 + 13a - 5) \xrightarrow{-30} \frac{(6a+5)(a-1)}{3 \quad 6 \quad 2}$$

$$(2a+5)(3a-1)$$

d. $(y-4)^2 - 36$

$$(y-4+6)(y-4-6)$$

$$= (y+2)(y-10)$$

$$= (x-y)(2a+5)(3a-1)$$

e. $27u - 90ut + 75ut^2$

$$3u(25t^2 - 30t + 9)$$

$$(25t^2 - 30t + 9) \xrightarrow{225} (5t-3)^2$$

$$3u(5t-3)^2$$

f. $am - bn + an - bm$

$$\frac{am + an - bn - bm}{a} = \frac{a(m+n) - b(n+m)}{a}$$

$$(m+n)(a-b)$$



Solving Quadratics Worksheet

1. Solving quadratic equations by factoring. Solve by factoring and check your solutions.

$$\begin{array}{l} \text{a. } x^2 - x - 2 = 0 \quad \frac{-2}{-2 \mid} \\ x^2 - 2x + 1x - 2 \\ \underline{x(x-2) \quad 1(x-2)} \\ (x-2)(x+1) \\ x=2 \quad x=-1 \end{array}$$

$$\text{b. } y^2 - 9 = 0$$

$$\begin{array}{l} y-3=0 \\ y=3 \quad y=-3 \end{array}$$

$$\text{c. } n^2 - 7n = 0$$

$$\begin{array}{l} n(n-7) = 0 \\ n=0 \quad n=7 \end{array}$$

$$\text{d. } 6x + 8 = -x^2$$

$$\begin{array}{l} x^2 + 6x + 8 = 0 \quad \frac{8}{-2 \mid} \\ x^2 + 2x + 4x + 8 \\ \underline{x(x+2) \quad 4(x+2)} \\ (x+2)(x+4) \\ x=-2 \quad x=-4 \end{array}$$

$$\text{e. } 2x^2 - 5x + 2 = 0 \quad \frac{4}{-4 \mid}$$

$$\begin{array}{l} 2x^2 - 4x - 1x + 2 \\ \underline{2x(x-2) - 1(x-2)} \\ (x-2)(2x-1) \\ x=2 \quad \frac{2x}{2} = \frac{1}{2} \\ x=2 \quad x=\frac{1}{2} \end{array}$$

$$\text{f. } 2y^2 + 7y + 3 = 0 \quad \frac{6}{1 \mid}$$

$$\begin{array}{l} 2y^2 + 1y + 6y + 3 \\ \underline{y(2y+1) \quad 3(2y+1)} \\ (2y+1)(y+3) \\ \frac{2y}{2} = -\frac{1}{2} \quad y = -3 \\ y = -\frac{1}{2} \end{array}$$

$$\text{g. } 15x^2 - 7x - 2 = 0 \quad \frac{-30}{-10 \mid}$$

$$\begin{array}{l} 15x^2 - 10x + 3x - 2 \\ \underline{5x(3x-2) + 1(3x-2)} \\ (3x-2)(5x+1) \\ \frac{3x}{3} = \frac{2}{3} \quad \frac{5x}{5} = -\frac{1}{5} \\ x = \frac{2}{3} \quad x = -\frac{1}{5} \end{array}$$

$$\text{h. } z^2 + 12 = -z$$

$$\begin{array}{l} z^2 + z + 12 = 0 \\ b^2 - 4ac \\ = (1)^2 - 4(1)(12) \\ = 1 - 48 \\ = -47 \end{array}$$

Solutions

1 a. $x = -1, x = 2$

b. $y = -3, y = 3$

c. $n = 0, n = 7$

d. $x = -4, x = -2$

e. $x = \frac{1}{2}, x = 2$

f. $y = -3, y = -\frac{1}{2}$

g. $x = \frac{-1}{5}, x = \frac{2}{3}$

h. No real roots

Special Cases Factoring Worksheet

1. Factor each of the following completely.

a. $9x^2y - 6xy^2 + 3xy$

$$3xy(3x - 2y + 1)$$

b. $x^2 + 3xy - 10y^2$ $\frac{-10}{-2}$

$$x^2 + 5xy - 2xy - 10y^2$$

$$x(x + 5y) - 2y(x + 5y)$$

$$(x + 5y)(x - 2y)$$

c. $x^4 + 7x^2 + 12$ $\frac{12}{4+3}$

$$x^4 + 4x^2 + 3x^2 + 12$$

$$x^2(x^2 + 4) + 3(x^2 + 4)$$

$$(x^2 + 4)(x^2 + 3)$$

d. $x^2y^2 - 6xy - 7$ $\frac{-7}{-7}$ 1

$$x^2y^2 - 7xy + 1xy - 7$$

$$xy(xy - 7) + 1(xy - 7)$$

$$(xy - 7)(xy + 1)$$

e. $9x^2 - 12xy + 4y^2$ $\frac{36}{-6}$ -6

$$9x^2 - 6xy - 6xy + 4y^2$$

$$3x(3x - 2y) - 2y(3x - 2y)$$

$$(3x - 2y)(3x - 2y)$$

$$(3x - 2y)^2$$

f. $\frac{5x^2 - 45}{5}$

$$5(x^2 - 9)(x^2 + 9)$$

g. $-x^2 - x + 12$ $\frac{12}{-4+3}$

$$-x^2 - 4x + 3x + 12$$

$$-1x(x + 4) + 3(x + 4)$$

$$(x + 4)(-1x + 3)$$

$$= -1(x^2 + x - 12)$$

$$= -1(x + 4)(x - 3)$$

h. $\frac{2y^3 - 40y^2 + 72y}{2y}$ $\frac{36}{-18-2}$

$$y^2 - 20y + 36$$

$$y^2 - 18y - 2y + 36$$

$$y(y - 18) - 2(y - 18)$$

$$2y(y - 18)(y - 2)$$

Solutions

1 a. $3xy(3x + 1 - 2y)$

b. $(x + 5y)(x - 2y)$

c. $(x^2 + 3)(x^2 + 4)$

d. $(xy - 7)(xy + 1)$

e. $(3x - 2y)^2$

f. $5(x + 3)(x - 3)$

$$2y^3 - 40y^2 + \underline{72y}$$

$$2y(y^2 - 20y + 36)$$

$$2y(y-18)(y-2)$$

$$9x^2 - 12xy + 4y^2 - \frac{36}{3} - 6$$

$$\frac{(9x-6y)(9x-6y)}{3 \cdot 3 \cdot 3}$$

$$(3x-2y)(3x-2y)$$

$$= (3x-2y)^2$$