1.7 - Adding and Subtracting Rational Expressions Part 2

Method:

1 – Factor every denominator (if possible)

2 – Determine a LCD

3 – Re-write the expression over one single LCD (remember to multiply the numerator when required)

4 – State restrictions

5 – Simplify the numerator

6 – Simplify the result (if possible)

Examples: State any restrictions on the variable and simplify

a. $\frac{2a-5}{4a} - \frac{7a+2}{6b}$

LCD = 12ab

$\frac{6ab-15b}{12ab} - \frac{14a^2-4a}{12ab}$

$= \frac{-14a^2+6ab-4a-15b}{12ab}$

restriction: $a \neq 0$, $b \neq 0$

b. $\frac{4}{(x+4)} - \frac{6}{(x+3)}$

LCD = $(x+4)(x+3)$, $x \neq -4, -3$

$\frac{4(x+3) - 6(x+4)}{(x+4)(x+3)}$

$= \frac{4x-12-6x-24}{(x+4)(x+3)}$

$= \frac{-2x-12}{(x+4)(x+3)}$

$= \frac{-2(x+6)}{(x+4)(x+3)}$, $x \neq -4$, $x \neq -3$

c. $\frac{x}{6x+6} + \frac{5}{4x-12}$

* factor first!

LCD = $12(x+1)(x-3)$

$\frac{2(x-3)}{2(x-3)} \cdot \frac{x}{6(x+1)} + \frac{3(x+1)}{3(x+1)} \cdot \frac{5}{4(x-3)}$

$= \frac{2x(x-3) + 15(x+1)}{12(x+1)(x-3)}$

$= \frac{2x^2-6x+15x+15}{12(x+1)(x-3)}$

$= \frac{2x^2+9x+15}{12(x+1)(x-3)}$, $x \neq -1, -3$

c. $\dfrac{x}{6x+6}+\dfrac{5}{4x-12}$

LCD $= 12(x+1)(x-3)$

$2(x-3)$ $\quad 3(x+1)$

$\dfrac{x+5}{6(x+1)+4(x-3)}$ $\quad 2(x-3)$ $\quad 3(x+1)$

$\dfrac{2x(x-3)+15(x+1)}{12(x+1)+12(x-3)} = \dfrac{2x^2-6x+15x+15}{12(x+1)(x-3)}$

$= \dfrac{2x^2+9x+15}{12(x+1)(x-3)}$

d. $\dfrac{2}{t^2+3t+2}-\dfrac{1}{t^2+t-2}$

factor first!

e. $\dfrac{x+1}{3x^2+4x+1}+\dfrac{2x-1}{3x^2-5x-2}$ $\quad -c$ $\quad -c+1$

$(3x+3)(3x+1)$ $\quad 3$

$(x+1)(3x+1)$ $\quad \dfrac{(3x-6)(3x+1)}{3}$

$(x-2)(3x+1)$

LCD $= (3x+1)(x-2)(x+1)$

$(x-2)$ $\dfrac{x+1}{(x+1)(3x+1)}+\dfrac{2x-1}{(x-2)(3x+1)}$ $(x+1)$

$(x-2)$ $\quad (x+1)$

$= \dfrac{(x-2)(x+1)+(2x-1)(x+1)}{(3x+1)(x+1)(x-2)}$, $x \neq -1, -\frac{1}{3}, 2$

$= \dfrac{x-2+2x-1}{(3x+1)(x-2)}$

$= \dfrac{3(x-1)}{(3x+1)(x-2)}$, $x \neq 2, -1, -\frac{1}{3}$

Homework:	Adding & Subtracting Rational Expressions ~ Part 2 Worksheet
	[Nelson pg 129] # 5(b,d), 6(b,e,f), 7(a,c,e), 8b, 9c, 12
	Optional Worksheet #3(d, f, g, h) 5, 6, 8

Adding & Subtracting Rational Expressions ~ Part 2 Worksheet

5. Simplify. State any restrictions on the variables.

a) $\frac{2x}{3} + \frac{3x}{4} - \frac{x}{6}$

b) $\frac{3}{t^4} + \frac{1}{2t^2} - \frac{3}{5t}$

c) $\frac{2x}{3y} - \frac{x^2}{4y^3} + \frac{3}{5y^4}$

d) $\frac{n}{m} + \frac{m}{n} - m$

6. Simplify. State any restrictions on the variables.

a) $\frac{7}{a-4} + \frac{2}{a}$

b) $\frac{4}{3x-2} + 6$

c) $\frac{5}{x+4} + \frac{7}{x+3}$

d) $\frac{6}{2n-3} - \frac{4}{n-5}$

e) $\frac{7x}{x+4} + \frac{3x}{x-6}$

f) $\frac{7}{2x-6} + \frac{4}{10x-15}$

2(x-3) 5(2x-3)

7. Simplify. State any restrictions on the variables.

a) $\frac{3}{x+1} + \frac{4}{x^2-3x-4}$

b) $\frac{2t}{t-4} - \frac{5t}{t^2-16}$

c) $\frac{3}{t^2+t-6} + \frac{5}{(t+3)^2}$

d) $\frac{4x}{x^2+6x+8} - \frac{3x}{x^2-3x-10}$

e) $\frac{x-1}{x^2-9} + \frac{x+7}{x^2-5x+6}$

f) $\frac{2t+1}{2t^2-14t+24} + \frac{5t}{4t^2-8t-12}$

8. Simplify. State any restrictions on the variables.

a) $\frac{3}{4x^2+7x+3} - \frac{5}{16x^2+24x+9}$

b) $\frac{a-1}{a^2-8a+15} - \frac{a-2}{2a^2-9a-5}$

c) $\frac{3x+2}{4x^2-1} + \frac{2x-5}{4x^2+4x+1}$

9. Simplify. State any restrictions on the variables. Remember the order of operations.

a) $\frac{2x^3}{3y^2} \times \frac{9y}{10x} - \frac{2y}{3x}$

b) $\frac{x+1}{2x-6} \div \frac{2(x+1)^2}{2-x} + \frac{11}{x-2}$

c) $\frac{p+1}{p^2+2p-35} + \frac{p^2+p-12}{p^2-2p-24} \times \frac{p^2-4p-12}{p^2+2p-15}$

d) $\frac{5m-n}{2m+n} - \frac{4m^2-4mn+n^2}{4m^2-n^2} \div \frac{6m^2-mn-n^2}{3m+15n}$

12. Fred drove his car a distance of $2x$ km in 3 h. Later, he drove a distance of $x + 100$ km in 2 h. Use the equation speed $= \frac{\text{distance}}{\text{time}}$.

a) Write a simplified expression for the difference between the first speed and the second speed.

b) Determine the values of x for which the speed was greater for the second trip.

1st speed = 2x $= \frac{2(2x) - 3(x+100)}{}$

Answers

Lesson 2.7, pp. 128–130

5. a) $\frac{5x}{4}$

b) $\frac{30 + 5t^2 - 6t^3}{10t^4}, t \neq 0$

c) $\frac{40xy^3 - 15x^2y + 36}{60y^4}, y \neq 0$

d) $\frac{n^2 + m^2 - m^2n}{mn}, m \neq 0, n \neq 0$

6. a) $\frac{9a-8}{a(a-4)}, a \neq 0, 4$

b) $\frac{18x-8}{3x-2}, x \neq \frac{2}{3}$

c) $\frac{12x+43}{(x+4)(x+3)}, x \neq -4, -3$

d) $\frac{-2n-18}{(2n-3)(n-5)}, n \neq \frac{3}{2}, 5$

e) $\frac{10x^2-30x}{(x+4)(x-6)}, x \neq -4, 6$

f) $\frac{78x-129}{10(x-3)(2x-3)}, x \neq \frac{3}{2}, 3$

7. a) $\frac{3x-8}{(x+1)(x-4)}, x \neq -1, 4$

b) $\frac{2t^2+3t}{(t-4)(t+4)}, t \neq -4, 4$

c) $\frac{8t-1}{(t+3)^2(t-2)}, t \neq -3, 2$

d) $\frac{x^2-32x}{(x+2)(x+4)(x-5)}, x \neq 5, -2, -4$

e) $\frac{2x^2+7x+23}{(x-3)(x+3)(x-2)}, x \neq -3, 2, 3$

f) $\frac{9t^2-14t+2}{4(t-3)(t-4)(t+1)}, t \neq -1, 3, 4$

8. a) $\frac{7x+4}{(x+1)(4x+3)^2}, x \neq -1, -\frac{3}{4}$

b) $\frac{a^2+4a-7}{(a-3)(a-5)(2a+1)}, a \neq -\frac{1}{2}, 3, 5$

c) $\frac{10x^2-5x+7}{(2x-1)(2x+1)^2}, a \neq -\frac{1}{2}, \frac{1}{2}$

9. a) $\frac{9x^3-10y^2}{15xy}, x \neq 0, y \neq 0$

b) $43x^2 - 84x - \frac{136}{4(x-3)(x+1)(x-2)}, x \neq -1, 2, 3$

c) $\frac{p^3+5p^2-25p-65}{(p+5)(p+7)(p-5)}, p \neq -7, -5, -4, 3, 5, 6$

d) $\frac{15m^2-2mn-3m-n^2-15n}{(2m+n)(3m+n)}, m \neq -\frac{1}{2}n, -\frac{1}{3}n, -5n, \frac{1}{2}n$

10. a) $\frac{23m+20}{10}$

b) $\frac{20x-3}{4x^3}, x \neq 0$

c) $\frac{-y-7}{(y+1)(y-2)}, y \neq -1, 2$

d) $\frac{2x^2+13x+15}{(x+3)(x-2)(x+4)}, x \neq -4, -3, 2$

11. $\frac{-s+t-1}{t+s}, t \neq -s$

12. a) $\frac{x-300}{6}$ b) $0 \leq x < 300$

2nd Speed = $\frac{x+100}{2}$ $= \frac{4x-3x-300}{6}$

$\frac{x+100}{2} \times \frac{2x}{3}$

(2) $\frac{2x}{3} - \frac{x+100}{2}$ LCD = 6

(3) $= \frac{x-300}{6}$

$3x+300 > 4x$

$300 > x$

x has to be between 0 and 300.

Optional Worksheet

In each of the following, state any restrictions on the variables.

A

1. Write an equivalent expression with a denominator of $12x^2y^2$.

a) $\frac{2}{xy}$ b) $\frac{x}{y}$

c) $\frac{5}{3xy^2}$ d) $\frac{-y}{6x^2}$

2. Find the LCM.

a) $10a^2b, 4ab^3$ b) $3m^2n, 2mn^2, 6mn$

c) $2x^3, 6xy^2, 4y$ d) $10s^2t^2, 20s^2t, 15st^2$

3. Simplify.

a) $\frac{3}{2x} + \frac{4}{5x}$

b) $\frac{2}{4y} + \frac{3}{3y} - \frac{1}{2y}$

c) $\frac{1}{2x^2} + \frac{3}{3x} - \frac{2}{x^3}$

d) $\frac{3}{2m^2n} - \frac{1}{m^2n^3} + \frac{4}{5mn}$

e) $x - \frac{2}{x} + 5$

f) $\frac{3m+4}{mn} - \frac{1}{m} - 2$

g) $\frac{4x-1}{3x^2} - \frac{2x+3}{x} + \frac{5x+2}{5x^2}$

h) $\frac{x-2y}{x} - \frac{4x+y}{xy} - \frac{3x-4y}{y}$

4. Find the LCM of each of the following. Leave answers in factored form.

a) $3m+6, 2m+4$

b) $3y-3, 5y+10$

c) $4m-8, 6m-18$

d) $8x-12, 10x-15$

5. Simplify.

a) $\frac{4}{x+3} + \frac{5}{4x+12}$ b) $\frac{1}{3y-15} - \frac{2}{y-5}$

c) $\frac{t}{t-4} - \frac{2t}{3t-12}$ d) $\frac{2}{2m+2} + \frac{5}{3m+3}$

e) $\frac{3}{4y-8} - \frac{2}{3y-6}$ f) $\frac{1}{4a+2} + \frac{4}{6a+3}$

6. Simplify.

a) $\frac{2}{x+1} + \frac{3}{x+2}$ b) $\frac{m}{m-3} - \frac{5}{m+2}$

c) $\frac{3}{x} + \frac{5}{x-1}$ d) $\frac{2}{t-1} + \frac{1}{5} + 2$

e) $\frac{2x}{x-2} - \frac{3x}{x+2}$ f) $\frac{4}{3n-1} - \frac{3}{2n+3}$

g) $\frac{1}{2x-2} + \frac{3}{4x-8}$ h) $\frac{t}{3t+15} - \frac{1}{6t-24}$

i) $\frac{4}{2s-12} - \frac{s}{5s-5}$ j) $\frac{2m}{3m-15} + \frac{m}{4m-8}$

7. State the LCM in factored form.

a) $x+2, x^2+4x+4$

b) y^2+6y+8, y^2-4

c) t^2-t-12, t^2-3t-4

d) $2x-4, x^2-3x-4$

e) $m^2+6m+9, m^2-2m-15$

8. Simplify.

a) $\frac{2}{x+3} + \frac{3}{x^2+5x+6}$

b) $\frac{y}{y^2-16} - \frac{4}{y+4}$

c) $\frac{3x}{x-5} + \frac{2x}{x^2-4x-5}$

LCD: $12(m-5)(m-2)$

j) $\frac{2m}{3m-15} + \frac{m}{4m-8}$

$3(m-5)$ $4(m-2)$

$\frac{8m(m-2)+3m(m-5)}{12(m-5)(m-2)}$

$8m^2-16m+3m^2-15m = \frac{11m^2-31m}{12(m-5)(m-2)}$

d) $\frac{3}{2m^2n} - \frac{1}{m^2n^3} + \frac{4}{5mn}$

LCD: $10m^2n^3$

$(5n^2)$ 10 $(2mn^2)$

$\frac{15n^2}{10m^2n^3} - \frac{10}{10m^2n^3} + \frac{8mn^2}{10m^2n^3} = \frac{15n^2+8mn^2-10}{10m^2n^3}$

Adding & Subtracting Rational Expressions ~ Part 2 Worksheet

Solutions

1. a) $\frac{24xy}{12x^2y^2}$, $x, y \neq 0$ **b)** $\frac{12x^3y}{12x^2y^2}$, $x, y \neq 0$ **c)** $\frac{20x}{12x^2y^2}$, $x, y \neq 0$

d) $\frac{-2y^3}{12x^2y^2}$, $x, y \neq 0$ **2. a)** $20s^2t^3$ **b)** $6m^2n^2$ **c)** $12x^3y^2$ **d)** $60s^2t^2$

3. a) $\frac{23}{10x}$, $x \neq 0$ **b)** $\frac{1}{y}$, $y \neq 0$ **c)** $\frac{2x^2 + x - 4}{2x^3}$, $x \neq 0$

d) $\frac{15n^2 + 8mn^4 - 10}{10m^2n^3}$, $m, n \neq 0$ **e)** $\frac{x^2 + 5x - 2}{x}$, $x \neq 0$

f) $\frac{3m - 2mn - n + 4}{mn}$, $m, n \neq 0$ **g)** $\frac{-30x^2 - 10x + 1}{15x^2}$, $x \neq 0$

h) $\frac{-3x^2 - 2y^2 + 5xy - 4x - y}{xy}$, $x, y \neq 0$ **4. a)** $6(m + 2)$

b) $15(y - 1)(y + 2)$ **c)** $12(m - 2)(m - 3)$ **d)** $20(2x - 3)$

5. a) $\frac{2}{4(x + 3)}$, $x \neq -3$ **b)** $\frac{-5}{3(y - 5)}$, $y \neq 5$ **c)** $\frac{t}{3(t - 4)}$, $t \neq 4$

d) $\frac{8}{3(m + 1)}$, $m \neq -1$ **e)** $\frac{1}{12(y - 2)}$, $y \neq 2$ **f)** $\frac{11}{6(2a + 1)}$, $a \neq -\frac{1}{2}$

6. a) $\frac{5x + 7}{(x + 1)(x + 2)}$, $x \neq -1, -2$ **b)** $\frac{m^2 - 3m + 15}{(m - 3)(m + 2)}$, $m \neq -2, 3$

c) $\frac{8x - 3}{x(x - 1)}$, $x \neq 0, 1$ **d)** $\frac{11t - 1}{5(t - 1)}$, $t \neq 1$ **e)** $\frac{10x - x^2}{(x - 2)(x + 2)}$,

$x \neq \pm 2$ **f)** $\frac{15 - n}{(3n - 1)(2n + 3)}$, $n \neq -\frac{3}{2}, \frac{1}{3}$ **g)** $\frac{5x - 7}{4(x - 1)(x - 2)}$ $x \neq 1, 2$ **h)** $\frac{2t^2 - 9t - 5}{6(t + 5)(t - 4)}$, $t \neq -5, 4$ **i)** $\frac{-s^2 + 16s - 10}{5(s - 6)(s - 1)}$, $s \neq 1, 6$

j) $\frac{11m^2 - 31m}{12(m - 5)(m - 2)}$, $m \neq 2, 5$ **7. a)** $(x + 2)^2$

b) $(y - 2)(y + 2)(y + 4)$ **c)** $(t + 3)(t - 4)(t + 1)$

d) $2(x - 2)(x + 1)(x - 4)$ **e)** $(m + 3)^2(m - 5)$

8. a) $\frac{2z + 7}{(z + 3)(z + 2)}$, $z \neq -3, -2$ **b)** $\frac{-3y + 16}{(y - 4)(y + 4)}$, $y \neq \pm 4$

c) $\frac{3x^2 + 5x}{(x - 5)(x + 1)}$, $x \neq -1, 5$