many

1.1 - Working With Polynomials

Polynomials

A polynomial is of the form $a_0x^0 + a_1x^1 + a_2x^2 + \ldots + a_nx^n$,

where x^m is a term

x is a variable

m is an exponent (integer)

a is a coefficient (real number)

Monomials: 3, 4x, $-5x^4$

one term

Binomials: $5x + 3$, $3x^2 - 4x$, $1001x^{24} - 5$

two terms

Trinomials: $2x^2 + 6x + 3$

three terms

Operations with Polynomials

To **Add** polynomials, collect like terms.

$-13x^2$
$2x^2$
$15x^2y$
$7x^2y$

To **Subtract** polynomials, add the opposite.

Ex. Simplify: $(5m-3n)-(2m-7n+4)$ → distributive property

$5m - 3n - 2m + 7n - 4$

$3m + 4n - 4$

To **Multiply** polynomials, multiply each term of one polynomial by each term of the other polynomial (the distributive property)

Ex.1 Expand and simplify: $2m(1-2m)-(2m-3)+m$

$2m - 4m^2 - 2m + 3 + m$

$-4m^2 + m + 3$

FOIL – First, Outer, Inner, Last

* make sure to multiply the coefficients (the number in front of the letter)

Ex. 2 Expand and simplify: $(2t-1)(t+4)-(t+6)(3t+2)$

$(2t^2+8t-t-4)-(3t^2+2t+18+12$

$-t^2-13t-16$

$(a+b)(a+b) = a^2+ab+ab+b^2$

$= a^2 + 2ab + b^2$

Perfect Square $(a + b)^2 = a^2 + 2ab + b^2$

$(a - b)^2 = a^2 – 2ab + b^2$

– Square the first term

– Multiply both terms, then multiply by 2

– Square the last term

Ex. 3 Expand: $(2x – 5)^2$

$(4x^2-20x+25)$

Difference of Squares $(a + b)(a – b) = a^2 – b^2$

– Square the first term
– Insert a subtraction sign (the “difference”)
– Square the last term

$a^2 - b^2$

Ex. 4 Expand: $(6x + 2y)(6x - 2y)$

$= (6x)^2 - (2y)^2$

$= 36x^2 - 4y^2$

Ex. 5 Expand and simplify: $2[(x - 5)(2x + 3) - 2(x - 1)^2]$

Ex 6. Expand and simplify: $(2x^2 - 5x + 3)(x^3 + 4x^2 - 6)$

$2x^5 + 8x^4 - 12x^2 - 5x^4 - 20x^3 + 30x + 3x^3 + 12x^2 - 18$

$2x^5 - 3x^4 - 29x^3 - 30x - 18$

Homework:	p. 88 # 2, 4–6 (eo), 8(eo), 10–12 p. 95 # 4(b,d,f), 5(b,d,f), 11 **Challenge worksheet**

Textbook pg. 88 - # 4–6 (eo), 8(eo), 10–12

4. Simplify.

a) $(2a + 4c + 8) + (7a - 9c - 3)$
b) $(3x + 4y - 5z) + (2x^2 + 6z)$
c) $(6x + 2y + 9) + (-3x - 5y - 8)$
d) $(2x^2 - 7x + 6) + (x^2 - 2x - 9)$
e) $(-4x^2 - 2xy) + (6x^2 - 3xy + 2y^2)$
f) $(x^2 + y^2 + 8) + (4x^2 - 2y^2 - 9)$

5. Simplify.

a) $(m - n + 2p) - (3n + p - 7)$
b) $(-6m - 2q + 8) - (2m + 2q + 7)$
c) $(4a^2 - 9) - (a^3 + 2a - 9)$
d) $(2m^2 - 6mn + 8n^2) - (4m^2 - mn - 7n^2)$
e) $(3x^2 + 2y^2 + 7) - (4x^2 - 2y^2 - 8)$
f) $5x^2 - (2x^2 - 30) - (-20)$

6. Simplify.

a) $(2x - y) - (-3x + 4y) + (6x - 2y)$

b) $(3x^2 - 2x) + (x^2 - 7x) - (7x + 3)$

c) $(2x^2 + xy - y^2) - (x^2 - 4xy - y^2) + (3x^2 - 5xy)$

d) $(xy - xz + 4yz) + (2x - 3yz) - (4y - xz)$

e) $\left(\frac{1}{2}x + \frac{1}{3}y\right) - \left(\frac{1}{5}x - y\right)$

f) $\left(\frac{3}{4}x + \frac{1}{2}y\right) - \left(\frac{2}{3}x + \frac{1}{4}y - 1\right)$

8. Determine whether each pair of functions is equivalent.

a) $f(x) = (2x^2 + 7x - 2) - (3x + 7)$ and
$g(x) = (x^2 + 12) + (x^2 + 4x - 17)$

b) $s_1(t) = (t + 2)^3$ and $s_2(t) = t^3 + 8$

c) $y_1 = (x - 1)(x)(x + 2)$ and $y_2 = 3x(x^2 - 1)$

d) $f(n) = 0.5n^2 + 2n - 3 + (1.5n^2 - 6)$ and
$g(n) = n^2 - n + 1 - (-n^2 - 3n + 10)$

e) $y_1 = 3p(q - 2) + 2p(q + 5)$ and $y_2 = p(q + 4)$

f) $f(m) = m(5 - m) - 2(2m - m^2)$ and
$g(m) = 4m^2(m - 1) - 3m^2 + 5m$

$P = 6x - 2y + 1$

$6(13) - 2(7)$

$78 - 14$

$P = 65$

10. Kosuke wrote a mathematics contest consisting of 25 multiple-choice questions. The scoring system gave 6 points for a correct answer, 2 points for not answering a question, and 1 point for an incorrect answer. Kosuke got x correct answers and left y questions unanswered.

1 question

a) Write an expression for the number of questions he answered incorrectly.

b) Write an expression, in simplified form, for Kosuke's score.

c) Use the expressions you wrote in parts (a) and (b) to determine Kosuke's score if he answered 13 questions correctly and 7 incorrectly.

x = correct

y = unanswered

$x = 3$

$y = 5$

7 incorrect

b) Score $= 6x + 2y + 1(25 - x - y)$

$= 6x + 2y + 25 - x - y$

$= 5x + y + 25$

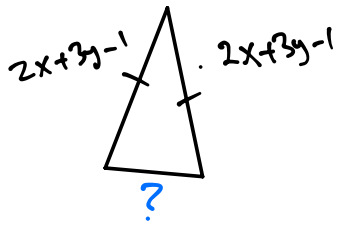
c) Score $= 5(13) + 5 + 25$

$= 65 + 30$

$= 95$ points

11. The two equal sides of an isosceles triangle each have a length of $2x + 3y - 1$. The perimeter of the triangle is $7x + 9y$. Determine the length of the third side.

A



$P = 7x + 9y$

$P = S_1 + S_2 + S_3$

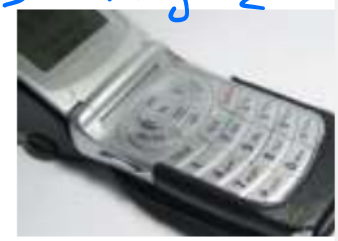
$S_3 = P - S_1 - S_2$

$= (7x + 9y) - 2(2x + 3y - 1)$

$= 7x + 9y - 4x - 6y - 2$

$= 3x + 3y - 2$

12. Tino owns a small company that produces and sells cellphone cases. The revenue and cost functions for Tino's company are shown below, where x represents the selling price in dollars.



$$\text{Revenue: } R(x) = -50x^2 + 2500x$$

$$\text{Cost: } C(x) = 150x + 9500$$

a) Write the simplified form of the profit function, $P(x) = R(x) - C(x)$.

b) What profit will the company make if it sells the cases for \$12 each?

$P(x) = R(x) - C(x)$

$= (-50x^2 + 2500x) - (150x + 9500)$

$= -50x^2 + 2500x - 150x - 9500$

$P(x) = -50(12)^2 + 2350(12) - 9500$

$= 11500$

Lesson 2.1, pp. 88–90

1. a) $4x^2 - 8x + 8$ b) $2x^2 - 4$ c) $2x^2 - x$
2. $f(x) = 7x - 2$
   $g(x) = 7x - 2$
3. Answers may vary. For example, $f(1) = -10$; $g(1) = -20$
4. a) $9a - 5c + 5$
   b) $2x^2 + 3x + 4y + z$
   c) $3x - 3y + 1$
   d) $3x^2 - 9x - 3$
   e) $2x^2 - 5xy + 2y^2$
   f) $5x^2 - y^2 - 1$
5. a) $m - 4n + p + 7$
   b) $-8m - 4q + 1$
   c) $-a^3 + 4a^2 - 2a$
   d) $-2m^2 - 5mn + 15n^2$
   e) $-x^2 + 4y^2 + 15$
   f) $3x^2 + 50$
6. a) $11x - 7y$
   b) $4x^2 - 16x - 3$
   c) $4x^2$
   d) $2x + xy - 4y + yz$
   e) $\frac{3}{10}x + \frac{4}{3}y$
   f) $\frac{1}{12}x + \frac{1}{4}y + 1$
7. i) $(3x^2 - x) - (5x^2 - x)$
   $= -2x^2$
   $\neq -2x^2 - 2x$
   ii) Answers will vary. For example, if $x = 1$,
   $(3x^2 - x) - (5x^2 - x)$
   $= (3 - 1) - (5 - 1)$
   $= -2$
   but $-2x^2 - 2x$
   $= -2 - 2$
   $= -4$
8. a) $f(x) = 2x^2 + 4x - 9$ and $g(x) = 2x^2 + 4x - 5 \therefore f(x) \neq g(x)$
   b) $s_1(1) = 27$ and $s_1(1) = 9 \therefore s_1(t) \neq s_2(t)$
   c) e.g., if $x = -1$, then $y_1 = 2$ and $y_2 = 0$
   $\therefore y_1 \neq y_2$
   d) $f(n) = 2n^2 + 2n - 9$ and $g(n) = 2n^2 + 2n - 9$
   $\therefore f(n) = g(n)$
   e) $p = 1, q = 1, y_1 = 9; y_2 = 5 \therefore y_1 \neq y_2$
   f) $f(2) = 6$
   $g(2) = 14$
   $\therefore f(m) \neq g(m)$
9. Answers will vary. For example, $f(x) = 2x$ and $g(x) = x^2$
10. a) $25 - x - y$ b) $5x + y + 25$ c) 95
11. $3x + 3y + 2$
12. a) $P(x) = -50x^2 + 2350x - 9500$ b) \$11 500

Homework: p. 95 # 4(b,d,f), 5(b,d,f), 11

4. Expand and simplify.

a) $5x(5x^2 + 3x - 4)$

b) $(x - 6)(2x + 5)$

c) $(x + 3)(x - 3) + (5x - 6)(3x - 7)$

d) $4(n - 4)(3 + n) - 3(n - 5)(n + 8)$

e) $3(2x - 1)^2 - 5(4x + 1)^2$

f) $2(3a + 4)(a - 6) - (3 - a)^2 + 4(5 - a)$

f) $5a^2 - 26a - 37$

5. Expand and simplify.

a) $4x(x + 5)(x - 5)$

b) $-2a(a + 4)^2$

c) $(x + 2)(x - 5)(x - 2)$

d) $(2x + 1)(3x - 5)(4 - x)$

e) $(9a - 5)^3$

f) $(a - b + c - d)(a + b - c - d)$

$x^6 - x^5 - x^4 - x^3 + x^5 - x^4 - x^3 - x^2 + x^4 - x^3 - x^2 - x + x^3 - x^2 - x - 1$

$x^6 - x^4 - 2x^3 - 3x^2 - 2x - 1$

11. Expand and simplify.

a) $(x^2 + 2x - 1)^2$

b) $(2 - a)^3$

c) $(x^3 + x^2 + x + 1)(x^3 - x^2 - x - 1)$

d) $2(x + 1)^2 - 3(2x - 1)(3x - 5)$

Lesson 2.2, pp. 95–97

1. **a)** $6x^2 - 10x^3 + 8xy$ **b)** $6x^2 + 7x - 20$ **c)** $x^2 + 8x + 16$ **d)** $x^3 + 3x^2 - x - 3$

2. **a)** no; for $x = 1$, left side is 25, right side is 13 **b)** $9x^2 + 12x + 4$

3. **a)** $6x^3 + 24x^2 + 14x - 20$ **b)** same as (a)

4. **a)** $25x^3 + 15x^2 - 20x$ **b)** $2x^2 - 7x - 30$ **c)** $16x^2 - 53x + 33$ **d)** $n^2 - 13n + 72$ **e)** $-68x^2 - 52x - 2$ **f)** $5a^2 - 26a - 37$

5. **a)** $4x^3 - 100x$ **b)** $-2a^3 - 16a^2 - 32a$ **c)** $x^3 - 5x^2 - 4x + 20$ **d)** $-6x^3 + 31x^2 - 23x - 20$ **e)** $729a^3 - 1215a^2 + 675a - 125$ **f)** $a^2 - 2ad - b^2 + 2bc - c^2 + d^2$

6. **a)** yes **b)** yes **c)** no **d)** yes **e)** no **f)** yes

7. All real numbers. Expressions are equivalent.

8. **a)** Both methods give $285x^2 + 209x - 266$.

b) Answer may vary. For example, I preferred multiplying the last two factors together first. Multiplying the first two factors together first meant that I had to multiply larger numbers in the second step.

9. **a)** $16x^2 + 8\pi x$ **b)** $8\pi x^3 + 4\pi x^2 - 2\pi x - \pi$

10. **a)** yes

b) no, $x - 3 = -(3 - x)$. A negative number squared is positive (the same); a negative number cubed is negative (different).

11. **a)** $x^4 + 4x^3 + 2x^2 - 4x + 1$ **b)** $8 - 12a + 6a^2 - a^3$ **c)** $x^6 - x^4 - 2x^3 - 3x^2 - 2x - 1$ **d)** $-16x^2 + 43x - 13$

12. 0

13. **a)** $\frac{1}{2}mv^2 + \frac{1}{2}xv^2$ **b)** $\frac{1}{2}mv^2 + mvy + \frac{1}{2}my^2$

14. **a)** 6 $2 \times 3 = 6$; $(x^7 + x^6)(x^9 + x^4 + 1)$ has 6 terms

b) Multiply the number of terms in each polynomial

15. **a)** **i)** 8 **ii)** 12 **iii)** 6 **iv)** 1

b) **i)** 8 **ii)** 96 **iii)** 384 **iv)** 512

c) **i)** 8 **ii)** $12(n - 2)$ **iii)** $6(n - 2)^2$ **iv)** $(n - 2)^3$

d) same answers

16. **a)** Answers may vary. For example,
115: $11^2 + 11 = 132$
$115^2 = 13\,225$

b) $(10x + 5)^2 = 100x^2 + 100x + 25$ and $(x^2 + x)100 + 25$ are both the same

Operations with Polynomials ~ Challenge Worksheet

1. Expand and simplify

a. $3[5 + 4(x – 7)]$

$3[5+4x-28]$

$3[4x-23]$

$12x-69$

b. $2[3(2t – 4) + 5(t+3)]$

c. $2x[x+2(x-3)] – x(3x-4)$

2. Expand and simplify.

a. $(x – 7)(x + 1) + (x + 6)(x + 2)$

b. $2(x – 4)(x+3) + 5(2x -1)(x +6)$

c. $2(m - 3)(m – 4) – 3(m + 5)^2$

d. $5(2y – 5)(2y + 5) – 4(y -2)(y + 3) – (2y +1)^2$

$(2y+1)(2y+1)$

$4y^2+2y+2y+1$

$4y^2+4y+1$

$(10y-25)(2y+5)-(4y+8)(y+3)-4y^2-4y-1$

$20y^2+50y-50y-125$

$20y^2-125-4y^2-12y+8y+24-4y^2-4y-1$

$20y^2-4y^2-4y^2-12y+8y-4y-125+24-1$

$12y^2-8y-102$

$(x+y)(x+y)$

$x^2+xy+xy+y^2$

$x^2+2xy+y^2$

d. $4(x^2 - 3xy) - (x + y)^2 - 2(x - y)(x + y) + 5$

$4x^2 - 12xy - x^2 - 2xy - y^2 - (2x+2y)(x+y) + 5$

$-2x^2 - 2xy + 2xy + 2y^2$

$4x^2 - x^2 - 2x^2 - 12xy - 2xy - 2xy + 2xy - y^2 + 2y^2 + 5$

$x^2 - 14xy + y^2 + 5$

Solutions

1. a. $12x - 69$ b. $22t + 6$ c. $3x^2 - 8x$

2. a. $2x^2 + 2x + 5$ b. $12x^2 + 53x - 54$

   c. $-m^2 - 44m - 51$ d. $12y^2 - 8y - 102$

   e. $x^2 - 14xy + y^2 + 5$