Chapter 2 **Derivatives**

Chapter 2 Prerequisite Skills

Chapter 2 Prerequisite Skills **Question 1 Page 70**

Justifications may vary. For example:

a) polynomial; The variables and constants in the function are either added or subtracted together.

b) sinusoidal; The sine function is a sinusoidal function.

c) polynomial; The variables and constants in the function are either added or subtracted together.

d) root; The function is the square root of a variable.

e) exponential; The function consists of a constant raised to the power of a variable.

f) rational; The function is expressed as the ratio of two polynomials.

g) logarithmic; The function is logarithmic since it is of the form $\log_b(x)$, where $b = 3$ is the base. The function can be written implicitly as $3^y = x$.

h) polynomial; After expanding the function, all the terms are variables or constants that are either added or subtracted together.

Chapter 2 Prerequisite Skills **Question 2 Page 70**

a) $m = 2$ so the perpendicular slope to m is $-\frac{1}{2}$.

b) $m = -5$ so the perpendicular slope to m is $-\left(-\frac{1}{5}\right) = \frac{1}{5}$.

c) $y = \frac{2}{3}x - 6$, so $m = \frac{2}{3}$ and the slope perpendicular to m is $-\frac{3}{2}$.

d) $m = 0$ so the perpendicular slope to m is undefined.

e) $m = 1$ so the perpendicular slope to m is -1.

f) m is undefined so the perpendicular slope to m is 0.

Chapter 2 Prerequisite Skills **Question 3 Page 70**

a) $x^{\frac{1}{2}}$

b) $x^{\frac{1}{3}}$

c) $x^{\frac{3}{4}}$

d) $x^{\frac{2}{5}}$

Chapter 2 Prerequisite Skills **Question 4 Page 70**

a) x^{-1}

b) $-2x^{-4}$

c) $x^{-\frac{1}{2}}$

d) $x^{-\frac{2}{3}}$

Chapter 2 Prerequisite Skills **Question 5 Page 70**

a) $(x^3 - 1)(5x + 2)^{-1}$

b) $(3x^4)(5x+6)^{-\frac{1}{2}}$

c) $(9 - x^2)^3(2x + 1)^{-4}$

d) $(x+3)^2(1-7x^2)^{-\frac{1}{3}}$

Chapter 2 Prerequisite Skills **Question 6 Page 70**

a) $x^{-6} = \dfrac{1}{x^6}$

b) $\dfrac{2x^3}{x^3} - \dfrac{x^2}{x^3} + \dfrac{3x}{x^3} = 2 - \dfrac{1}{x} + \dfrac{3}{x^2}$

c) $x^{-3} = \dfrac{1}{x^3}$

d) $x^{\frac{1}{2}} - x^{-\frac{1}{2}} = x^{\frac{1}{2}} - \dfrac{1}{x^{\frac{1}{2}}}$

e) $c^6c^3 = c^9$

f) $\dfrac{(4x-3)^2}{(x^2+3)^{\frac{3}{2}}}$

Chapter 2 Prerequisite Skills Question 7 Page 70

a) i) increasing: $(-\infty, -1)$, $(2.5, \infty)$; decreasing: (–1, 2.5)

ii) positive: (–3, 2), $(3, \infty)$; negative: $(-\infty, -3)$, (2, 3)

iii) zero slope: $x = -1$, $x = 2.5$; positive slope: $(-\infty, -1)$, $(2.5, \infty)$; negative slope: (–1, –2.5)

b) i) increasing: (–2.5, 1.5), $(4.5, \infty)$; decreasing: $(-\infty, -2.5)$, (1.5, 4.5)

ii) positive: $(-\infty, -4)$, (–0.5, 4), $(5, \infty)$; negative: (–4, –0.5), (4, 5)

iii) zero slope: $x = -2.5$, $x = 1.5$, $x = 4.5$; positive slope: (–2.5, 1.5), $(4.5, \infty)$;
negative slope: $(-\infty, -2.5)$, (1.5, 4.5)

Chapter 2 Prerequisite Skills Question 8 Page 71

a)
$$x^2 - 8x + 12 = 0$$
$$(x-2)(x-6) = 0$$
$$x = 2, x = 6$$

b)
$$4x^2 - 16x - 84 = 0$$
$$x^2 - 4x - 21 = 0$$
$$(x+3)(x-7) = 0$$
$$x = -3, x = 7$$

c)
$$5x^2 - 14x + 8 = 0$$
$$(5x-4)(x-2) = 0$$
$$x = \frac{4}{5}, x = 2$$

d)
$$6x^2 - 5x - 6 = 0$$
$$(3x+2)(2x-3) = 0$$
$$x = -\frac{2}{3}, x = \frac{3}{2}$$

e) $x = \dfrac{-5 \pm \sqrt{5^2 - 4(1)(-4)}}{2(1)}$

$x = \dfrac{-5 \pm \sqrt{41}}{2}$

f) $x = \dfrac{-13 \pm \sqrt{13^2 - 4(2)(-6)}}{2(2)}$

$x = \dfrac{-13 \pm \sqrt{217}}{4}$

g) $x = \dfrac{-(-9) \pm \sqrt{(-9)^2 - 4(4)(3)}}{2(4)}$

$x = \dfrac{9 \pm \sqrt{33}}{8}$

h) $x = \dfrac{-7 \pm \sqrt{7^2 - 4(-1)(-1)}}{2(-1)}$

$x = \dfrac{7 \pm \sqrt{45}}{2}$

$x = \dfrac{7 \pm 3\sqrt{5}}{2}$

Chapter 2 Prerequisite Skills Question 9 Page 71

The factor theorem requires using trial and error to find all values of x that satisfy the given equation. Use long or synthetic division to help find the factors of each polynomial.

a) $(x + 1)$ is a factor of $x^3 + 3x^2 - 6x - 8 = 0$

$(x + 1)(x^2 + 2x - 8) = 0$

$(x + 1)(x + 4)(x - 2) = 0$

$x = -4, x = -1, x = 2$

b) $(x + 1)$ is a factor of $2x^3 - x^2 - 5x - 2 = 0$

$(x + 1)(2x^2 - 3x - 2) = 0$

$(x + 1)(2x + 1)(x - 2) = 0$

$x = -1, x = -\dfrac{1}{2}, x = 2$

c) $(x-3)$ is a factor of $3x^3+4x^2-35x-12=0$

$$(x-3)(3x^2+13x+4)=0$$
$$(x-3)(3x+1)(x+4)=0$$
$$x=-4,\ x=-\frac{1}{3},\ x=3$$

d) $(x-1)$ is a factor of $5x^3+11x^2-13x-3=0$

$$(x-1)(5x^2+16x+3)=0$$
$$(x-1)(5x+1)(x+3)=0$$
$$x=-3,\ x=-\frac{1}{5},\ x=1$$

e) $(x-1)$ is a factor of $3x^3+2x^2-7x+2=0$

$$(x-1)(3x^2+5x-2)=0$$
$$(x-1)(3x-1)(x+2)=0$$
$$x=-2,\ x=\frac{1}{3},\ x=1$$

f) $(x+1)$ is a factor of

$$x^4-2x^3-13x^2+14x+24=0$$
$$(x+1)(x^3-3x^2-10x+24)=0$$

$(x-2)$ is a factor of $x^3-3x^2-10x+24=0$

$$(x+1)(x-2)(x^2-x-12)=0$$
$$(x+1)(x-2)(x+3)(x-4)=0$$
$$x=-3,\ x=-1,\ x=2,\ x=4$$

Chapter 2 Prerequisite Skills Question 10 Page 71

a) $$\begin{aligned}(x^2+4)(5)+2x(5x-7)&=5x^2+20+10x^2-14x\\&=15x^2-14x+20\end{aligned}$$

b) $$\begin{aligned}(9-5x^3)(14x)+(-15x^2)(7x^2+2)&=126x-70x^4-105x^4-30x^2\\&=-175x^4-30x^2+126x\end{aligned}$$

c) $$\begin{aligned}(3x^4-6x)(6x^2+5)+(12x^3-6)(2x^3+5x)&=18x^6+15x^4-36x^3-30x+24x^6+60x^4-12x^3-30x\\&=42x^6+75x^4-48x^3-60x\end{aligned}$$

Chapter 2 Prerequisite Skills Question 11 Page 71

a) $$\begin{aligned}8(x^3-1)^5(2x+7)^3+15x^2(x^3-1)^4(2x+7)^4&=(x^3-1)^4(2x+7)^3[8(x^3-1)+15x^2(2x+7)]\\&=(x^3-1)^4(2x+7)^3(8x^3-8+30x^3+105x^2)\\&=(x^3-1)^4(2x+7)^3(38x^3+105x^2-8)\end{aligned}$$

b) $6(x^3+4)^{-1}-3x^2(6x-5)(x^3+4)^{-2} = 3(x^3+4)^{-2}[2(x^3+4)-x^2(6x-5)]$

$$= 3(x^3+4)^{-2}(2x^3+8-6x^3+5x^2)$$
$$= (x^3+4)^{-2}(-12x^3+15x^2+24)$$

c) $2x^{\frac{7}{2}}-2x^{\frac{1}{2}} = 2x^{\frac{1}{2}}(x^3-1)$

$$= 2\sqrt{x}(x-1)(x^2+x+1)$$

d) $1+2x^{-1}+x^{-2} = x^{-2}(x^2+2x+1)$

$$= x^{-2}(x+1)(x+1)$$
$$= \frac{(x+1)^2}{x^2}$$

Chapter 2 Prerequisite Skills Question 12 Page 71

a) $u = \sqrt{4}$ which is 2 so $y = 6(2)^2 - 1$
$= 23$

b) $u = 9 - 2(4)$ which is 1 so $y = -\dfrac{5}{(1)^3}$
$= -5$

c) $u = 5(4) - 18$ which is 2 so $y = -(2)^2 + 3(2) + 1$
$= 3$

Chapter 2 Prerequisite Skills Question 13 Page 71

a) $f \circ g(x) = f\left[\dfrac{1}{x-2}\right]$

$$= \left(\frac{1}{x-2}\right)^3 + 1$$

b) $g \circ h(x) = g\left(\sqrt{1-x^2}\right)$

$$= \frac{1}{\sqrt{1-x^2}-2}$$

c) $h[f(x)] = h[x^3+1]$

$$= \sqrt{1-(x^3+1)^2}$$
$$= \sqrt{-x^6-2x^3}$$

d) $g[f(x)] = g[x^3 + 1]$

$$= \frac{1}{(x^3+1)-2}$$

$$= \frac{1}{x^3-1}$$

Chapter 2 Prerequisite Skills **Question 14 Page 71**

Answers may vary. For example:

a) Set $f(x) = x^2$ and $g(x) = 2x - 3$.
Then $f[g(x)] = (2x - 3)^2$ so $h(x) = f[g(x)]$.

b) Set $f(x) = \sqrt{x}$ and $g(x) = 2 + 4x$.
Then $f[g(x)] = \sqrt{2+4x}$ so $h(x) = f[g(x)]$.

c) Set $f(x) = \dfrac{1}{x}$ and $g(x) = 3x^2 - 7x$.
Then $f[g(x)] = \dfrac{1}{3x^2-7x}$ so $h(x) = f[g(x)]$.

d) Set $f(x) = \dfrac{1}{x^2}$ and $g(x) = x^3 - 4$.
Then $f[g(x)] = \dfrac{1}{(x^3-4)^2}$ so $h(x) = f[g(x)]$.

Chapter 2 Section 1 **Derivative of a Polynomial Function**

Chapter 2 Section 1 **Question 1 Page 83**

A, B, E, G, and H all have a derivative of zero since they are all constants.

Chapter 2 Section 1 **Question 2 Page 83**

a) $$\begin{aligned}\frac{dy}{dx} &= 1(x^{1-1}) \\ &= 1\end{aligned}$$

b) $$\begin{aligned}\frac{dy}{dx} &= \frac{1}{4}(2x^{2-1}) \\ &= \frac{1}{2}x\end{aligned}$$

c) $$\begin{aligned}\frac{dy}{dx} &= 5x^{5-1} \\ &= 5x^4\end{aligned}$$

d) $$\begin{aligned}\frac{dy}{dx} &= -3(4x^{4-1}) \\ &= -12x^3\end{aligned}$$

e) $$\begin{aligned}\frac{dy}{dx} &= 1.5(3x^{3-1}) \\ &= 4.5x^2\end{aligned}$$

f) $$\begin{aligned}y &= x^{\frac{3}{5}} \\ \frac{dy}{dx} &= \frac{3}{5}x^{\frac{3}{5}-1} \\ &= \frac{3}{5}x^{-\frac{2}{5}} \\ &= \frac{3}{5\sqrt[5]{x^2}}\end{aligned}$$

g) $$\begin{aligned}y &= 5x^{-1} \\ \frac{dy}{dx} &= 5(-x^{-1-1}) \\ &= -\frac{5}{x^2}\end{aligned}$$

h) $y = 4x^{-\frac{1}{2}}$

$$\frac{dy}{dx} = 4(-\frac{1}{2}x^{-\frac{1}{2}-1})$$
$$= -2x^{-\frac{3}{2}}$$
$$= -\frac{2}{\sqrt{x^3}}$$

Chapter 2 Section 1 **Question 3 Page 83**

a) $y = 6$ is a constant so $\dfrac{dy}{dx} = 0$.

The slope of the tangent line is 0 at all values of x so the slope is 0 at $x = 12$.

b) $f'(x) = 2(5x^{5-1})$

$$= 10x^4$$

The slope of the tangent to f at $x = \sqrt{3}$ is the derivative of f at $x = \sqrt{3}$, so find $f'\left(\sqrt{3}\right)$.

$$f'(\sqrt{3}) = 10\left(\sqrt{3}\right)^4$$
$$= 90$$

c) $g(x) = -3x^{-\frac{1}{2}}$

$$g'(x) = -3\left(-\frac{1}{2}x^{-\frac{1}{2}-1}\right)$$
$$= \frac{3}{2\sqrt{x^3}}$$

The slope of the tangent to g at $x = 4$ is the derivative of g at $x = 4$, so find $g'(4)$.

$$g'(4) = \frac{3}{2\sqrt{4}^3}$$
$$= \frac{3}{16} \text{ or } 0.1875$$

d) $h'(t) = -4.9\left(2t^{2-1}\right)$

$$= -9.8t$$

The slope of the tangent to h at $t = 3.5$ is the derivative of h at $t = 3.5$, so find $h'(3.5)$.

$$h'(3.5) = -9.8(3.5)$$
$$= -34.3$$

e) $A'(r) = \pi(2r^{2-1})$
$= 2\pi r$

The slope of the tangent to A at $r = \frac{3}{4}$ is the derivative of A at $r = \frac{3}{4}$, so find $A'\left(\frac{3}{4}\right)$.

$$A'\left(\frac{3}{4}\right) = 2\pi\left(\frac{3}{4}\right)$$
$$= \frac{3\pi}{2} \text{ or } 4.712\,389$$

f) $y = \frac{1}{3}x^{-1}$

$$\frac{dy}{dx} = \frac{1}{3}(-x^{-1-1})$$
$$= -\frac{1}{3x^2}$$

The slope of the tangent to y at $x = -2$ is the derivative of y at $x = -2$, so find $\left.\frac{dy}{dx}\right|_{x=-2}$

$$\left.\frac{dy}{dx}\right|_{x=-2} = -\frac{1}{3(-2)^2}$$
$$= -\frac{1}{12}$$

Chapter 2 Section 1 **Question 4 Page 83**

a) $f'(x) = 2(2x^{2-1}) + 3x^{3-1}$
$= 4x + 3x^2$
The sum rule, power rule, and constant multiple rule were used.

b) $\frac{dy}{dx} = \frac{4}{5}(5x^{5-1}) - 3x^{1-1}$
$= 4x^4 - 3$
The difference rule, power rule, and constant multiple rule were used.

c) $h'(t) = -1.1(4t^{4-1}) + 0$
$= -4.4t^3$
The sum rule, power rule, constant rule, and constant multiple rule were used.

d) $V'(r) = \frac{4}{3}\pi(3r^{3-1})$
$= 4\pi r^2$
The power rule and constant multiple rule were used.

e) $p'(a) = \frac{1}{15}(5a^{5-1}) - 2(\frac{1}{2}a^{\frac{1}{2}-1})$

$= \frac{1}{3}a^4 - \frac{1}{\sqrt{a}}$

The difference rule, power rule and constant multiple rule were used.

f) $k'(s) = -1(-2s^{-2-1}) + 7(4s^{4-1})$

$= \frac{2}{s^3} + 28s^3$

The sum rule, power rule, and constant multiple rule were used.

Chapter 2 Section 1 **Question 5 Page 84**

a) i) The slope is given by y'.

$y' = 6(2x^{2-1}) - 3x^{1-1} + 0$

$= 12x - 3$

Set $y' = 0$.

$0 = 12x - 3$

$x = \frac{1}{4}$ or 0.25

At $x = 0.25$, $y = 6(0.25)^2 - 3(0.25) + 4 = 3.625$, so the point is (0.25, 3.625).

ii) The slope is given by y'.

$y' = -(2x^{2-1}) + 5x^{1-1} + 0$

$= -2x + 5$

Set $y' = 0$.

$0 = -2x + 5$

$x = \frac{5}{2}$ or 2.5

At $x = 2.5$, $y = -(2.5)^2 + 5(2.5) - 1 = 5.25$, so the point is (2.5, 5.25).

iii) The slope is given by y'.

$y = \frac{15}{4}x^2 + 4x + 3$

$y' = \frac{15}{4}(2x^{2-1}) + 4x^{1-1} + 0$

$= \frac{15}{2}x + 4$

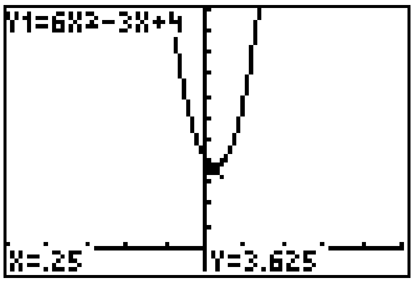
Set $y' = 0$.

$0 = \frac{15}{2}x + 4$

$x = -\frac{8}{15}$ or $-0.5\overline{3}$

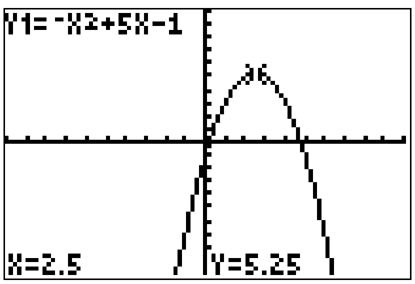
At $x = -0.5\overline{3}$, $y = \frac{15}{4}(-0.5\overline{3})^2 + 4(-0.5\overline{3}) + 3$ or $1.9\overline{3}$, so the point is $(-0.5\overline{3}, -1.9\overline{3})$.

b) i)



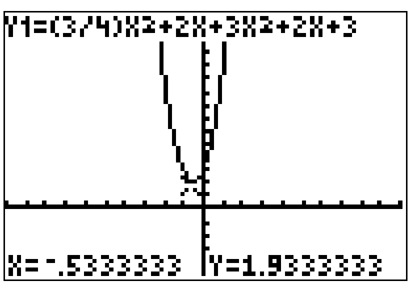
The point (0.25, 3.625) is a local minimum.

ii)



The point (2.5, 5.25) is a local maximum.

iii)



The point $(-0.5\overline{3}, 1.9\overline{3})$ is a local minimum.

Chapter 2 Section 1 **Question 6 Page 84**

a) $f(x) = 5x^2 - 3x$

$$\begin{aligned} f'(x) &= 5(2x^{2-1}) - 3x^{1-1} \\ &= 10x - 3 \end{aligned}$$

b) $g(x) = 6x^2 + 5x - 4$

$$\begin{aligned} g'(x) &= 6(2x^{2-1}) + 5x^{1-1} + 0 \\ &= 12x + 5 \end{aligned}$$

c) $p(x) = \frac{1}{4}x^5 - x^3 + \frac{1}{2}$

$$\begin{aligned} p'(x) &= \frac{1}{4}(5x^{5-1}) - 3x^{3-1} + 0 \\ &= \frac{5}{4}x^4 - 3x^2 \end{aligned}$$

d) $f(x) = 25x^2 + 20x + 4$

$$\begin{aligned} f'(x) &= 25(2x^{2-1}) + 20x^{1-1} + 0 \\ &= 50x + 20 \end{aligned}$$

Chapter 2 Section 1 **Question 7 Page 84**

Answers may vary. For example:

a) Use a graphing calculator to graph the curve, and then draw the tangent to the curve and determine the equation of the tangent to the curve at the given x-value.

b) The derivative is equal to the slope of the tangent at the given point. The equation of the derivative can then be solved to find the x-value of the tangent point. The x-value is then substituted into the equation of the function to find the y-value of the tangent point.

Chapter 2 Section 1 **Question 8 Page 84**

a) Answers may vary. For example:

The rules in this chapter require that the given function be expanded and then differentiated using the sum, power, and constant multiple rules. This question needs to be in a more expanded form to solve via these methods.

b) $f(x)$ would need to be expanded and simplified in order to differentiate using the rules of this section.

c) $f(x) = (4x^2 - 4x + 1)(x + 3)$

$$= 4x^3 + 12x^2 - 4x^2 - 12x + x + 3$$

$$= 4x^3 + 8x^2 - 11x + 3$$

$$f'(x) = 4(3x^{3-1}) + 8(2x^{2-1}) - 11x^{1-1} + 0$$

$$= 12x^2 + 16x - 11$$

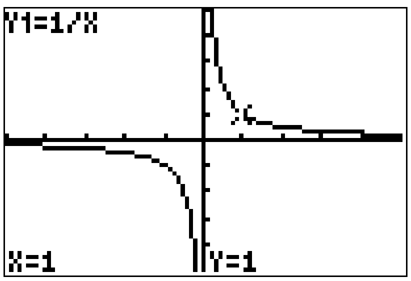
Chapter 2 Section 1 **Question 9 Page 84**

Answers may vary. The question is done **algebraically** in Section 1, Example 2 on page 79.

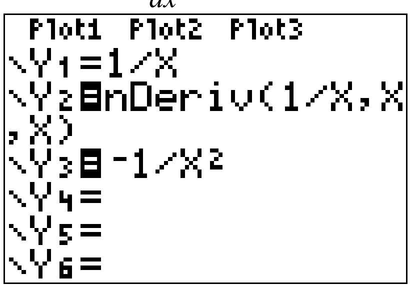
Graphically: Since $y = x^{-1}$, need to show that $\frac{dy}{dx} = -x^{-2}$.

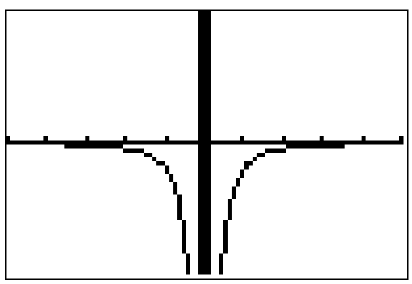
Following the procedure in Section 2.1, Example 1 on page 78:

Graph $y = x^{-1}$:



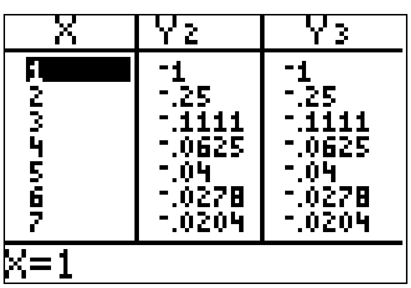
Graph $y_2 = \frac{dy}{dx}$ and $y_3 = x^2$ together:





The two graphs are the same, so the derivative of y is $-\frac{1}{x^2}$.

Numerically: the table of values for $y_2 = \frac{dy}{dx}$ and $y_3 = -\frac{1}{x^2}$ are the same:



X	Y2	Y3
1	-1	-1
2	-.25	-.25
3	-.1111	-.1111
4	-.0625	-.0625
5	-.04	-.04
6	-.0278	-.0278
7	-.0204	-.0204

X=1

Chapter 2 Section 1 **Question 10 Page 84**

$f'(t)$ represents the rate of change of the amount of water flowing into the first barrel, and $g'(t)$ represents the rate of change of the amount of water flowing into the second barrel. So, $f'(t)+g'(t)$ represents the sum of the rates of change of water flowing into the two barrels and by the sum rule, $(f+g)'(t)$ also represents the rate of change of water flowing into the two barrels.

Chapter 2 Section 1 **Question 11 Page 84**

a) Find $h'(5)$.

$$\begin{aligned} h'(t) &= 0-4.9(2t^{2-1}) \\ &= -9.8\text{t} \end{aligned}$$

The rate of change of the skydiver at 5 s is –9.8(5) = –49 m/s.

b) Find *t* for $h'(t)$ = 1000 m.

$$\begin{aligned} 1000 &= 2500-4.9t^2 \\ t &= \sqrt{\frac{1500}{4.9}} \\ t &\doteq 17.5 \end{aligned}$$

The skydiver's parachute opens at time *t* = 17.5 s.

c) Find $h'\left(17.5\right)$.

$$\begin{aligned} h'\left(17.5\right) &= -9.8\left(17.5\right) \\ &= 171.5 \end{aligned}$$

The rate of change of the height of the skydiver is 171.5 m/s.

Chapter 2 Section 1 **Question 12 Page 84**

$$\begin{aligned} h(t) &= -0.5gt^2+250 \\ h'(t) &= -0.5g(2t^{2-1})+0 \\ &= -gt \end{aligned}$$

Earth:

$$\begin{aligned} h'(t) &= -9.8t \\ h'(4) &= -9.8(4) \\ &= -39.2 \text{ m/s} \end{aligned}$$

Venus:

$$\begin{aligned} h'(t) &= -8.9t \\ h'(4) &= -8.9(4) \\ &= -35.6 \text{ m/s} \end{aligned}$$

Mars:

$$\begin{aligned} h'(t) &= -3.7t \\ h'(4) &= -3.7(4) \\ &= -14.8 \text{ m/s} \end{aligned}$$

Saturn:

$$\begin{aligned} h'(4) &= -10.5t \\ h'(4) &= -10.5(4) \\ &= -42.0 \text{ m/s} \end{aligned}$$

Neptune:

$$\begin{aligned} h'(4) &= -11.2t \\ h'(4) &= -11.2(4) \\ &= -44.8 \text{ m/s} \end{aligned}$$

Chapter 2 Section 1 **Question 13 Page 84**

a) $\frac{dy}{dx} = -6(4x^{4-1}) + 2(3x^{3-1}) + 0$

$= -24x^3 + 6x^2$

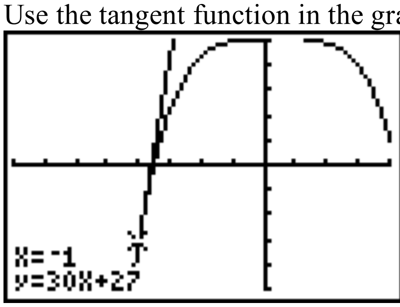
$m = -24(-1)^3 + 6(-1)^2$

$= 30$

b) Substitute the point (–1, –3) into the equation of the tangent line $y = 30x + b$ to solve for b.
$-3 = 30(-1) + b$
$b = 27$
The equation of the tangent line is $y = 30x + 27$.

c) Answers may vary. For example:

Use the tangent function in the graphing calculator to find the equation of the tangent:



Chapter 2 Section 1 **Question 14 Page 85**

a) $\frac{dy}{dx} = -1.5(3x^{3-1}) + 3x^{1-1} - 2$

$= -4.5x^2 + 3$

At $x = 2$, the slope of the tangent is $4.5(2)^2 + 3 = -15$.

b) Substitute the point (2, –8) into the equation of the tangent line $y = -15x + b$ to solve for b.

$y = -15x + b$

$-8 = -15(2) + b$

$b = 22$

The equation of the tangent at (2, –8) is $y = -15x + 22$.

c) Answers may vary. For example:

Use the tangent function in the graphing calculator to find the equation of the tangent:

Chapter 2 Section 1 **Question 15 Page 85**

a) Find $h(2)$.

$$\begin{aligned} h(2) &= -4.9(2)^2 + 24.5(2) + 2 \\ &= 31.4 \end{aligned}$$

The height is 31.4 m.

b)
$$\begin{aligned} h'(t) &= -4.9(2t^{2-1}) + 24.5t^{1-1} \\ &= -9.8t + 24.5 \end{aligned}$$

$h'(1) = 14.7$ m/s; $h'(2) = 4.9$ m/s; $h'(4) = -14.7$ m/s; $h'(5) = -24.5$ m/s

c) After 5 s, the arrow is at the initial launch height of 2 m off the ground.

d) Solve $0 = -4.9t^2 + 24.5t + 2$ using the quadratic formula.

$$t = \frac{-24.5 \pm \sqrt{(24.5)^2 - 4(-4.9)(2)}}{2(-4.9)}$$

$t = -0.08$ s or $t = 5.08$ s

Since the time cannot be negative, the arrow hits the ground after 5.08 s.

e) The speed of the arrow at any time t is given by $h'(t)$. Solve for $h'(5.08)$.

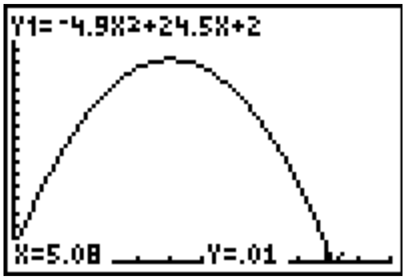
$$\begin{aligned} h'(5.08) &= -9.8(5.08) + 24.5 \\ &= -25.28 \end{aligned}$$

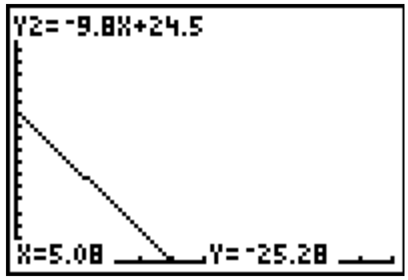
The speed of the arrow is –25.28 m/s.

Explanations may vary. For example:
I plugged the time when the arrow hits the ground into the derivative function $h'(t)$. Also, the arrow is travelling in a negative (downwards) direction at this time so the speed is 25.28 m/s.

f) Answers may vary. For example:

Draw the graphs for $h(t)$ and $h'(t)$ from 0 to about 5.08 s and read values off of the plots.





Chapter 2 Section 1 **Question 16 Page 85**

a $f'(x) = 3x^{3-1} - 7$

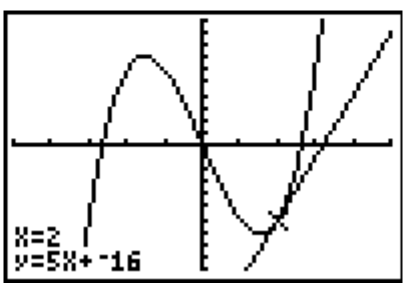
$5 = 3x^2 - 7$

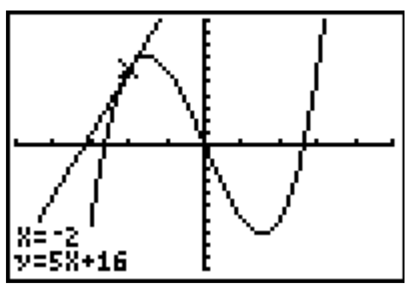
$x = -2$ or $x = 2$

The corresponding values of y are $f(2) = (2)^3 - 7(2)$, which is –6, and $f(-2) = (-2)^3 - 7(-2)$, which is 6. Therefore, the points are (2, –6) and (–2, 6).

b) If the tangent equation is $y = mx + b$ where m = 5, x = 2 and y = –6, then b = –6 – 5(2) = 16.
Similarly, when m = 5, x = –2 and y = 6, then b = 6 – 5(–2) = 16.
So, the equations of the tangents are $y = 5x - 16$ at (2, –6) and $y = 5x + 16$ at (–2, 6).

c) A CAS solution is shown in the **Technology Extension** on page 87.





Chapter 2 Section 1 **Question 17 Page 85**

a) The slope is given by $f'(x) = 4x$ and $g'(x) = 3x^2$.

If $4x = 3x^2$, then x = 0 or $x = \frac{4}{3}$.

b) When $x = 0$, $f(x) = 0$ and $g(x) = 0$ so the equation of the tangent line to f and g at this point is $y = 0$.

For the tangent of f at $x = \frac{4}{3}$, the y-coordinate is $f\left(\frac{4}{3}\right) = \frac{32}{9}$ and the slope, m, is $f'\left(\frac{4}{3}\right) = \frac{16}{3}$.

Substitute these values into $y = mx + b$ to find b.

$$\frac{32}{9} = \left(\frac{16}{3}\right)\left(\frac{4}{3}\right) + b$$

$$b = -\frac{32}{9}$$

The tangent equation for $f(x) = 2x^2$ at $x = \frac{4}{3}$ is $y = \frac{16}{3}x - \frac{32}{9}$.

For the tangent of g at $x = \frac{4}{3}$, the y-coordinate is $g\left(\frac{4}{3}\right) = \frac{64}{27}$ and the slope, m, is also $g'\left(\frac{4}{3}\right) = \frac{16}{3}$.

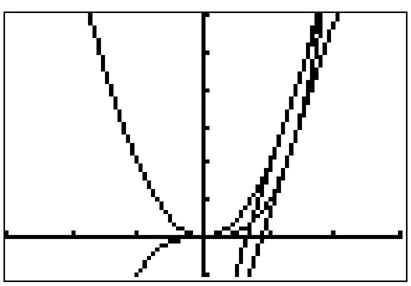
Substitute these values into $y = mx + b$ to find b.

$$\frac{64}{27} = \left(\frac{16}{3}\right)\left(\frac{4}{3}\right) + b$$

$$b = -\frac{128}{27}$$

The tangent equation for $g(x) = x^3$ at $x = \frac{4}{3}$ is $y = \frac{16}{3}x - \frac{128}{27}$.

c)



Chapter 2 Section 1 **Question 18 Page 85**

Use the definition of the derivative to prove that if $h(t)=f(t)+g(t)$, then $h'(t)=f'(t)+g'(t)$.

$$\begin{aligned}
h'(t) &= \lim_{x\to 0}\frac{h(t+x)-h(t)}{x} \\
&= \lim_{x\to 0}\frac{\left(f(t+x)+g(t+x)\right)-\left(f(t)+g(t)\right)}{x} \\
&= \lim_{x\to 0}\frac{\left(f(t+x)-f(t)\right)+\left(g(t+x)-g(t)\right)}{x} \\
&= \lim_{x\to 0}\frac{f(t+x)-f(t)}{x}+\lim_{x\to 0}\frac{g(t+x)-g(t)}{x} \\
&= f'(t)+g'(t)
\end{aligned}$$

Chapter 2 Section 1 **Question 19 Page 85**

a)

$$\begin{aligned}
C(1000) &= 3450+1.5(1000)-0.0001(1000)^2 \\
&= \$4850 \\
R(1000) &= 3.25(1000) \\
&= \$3250
\end{aligned}$$

b)

$$\begin{aligned}
C'(x) &= 1.5-0.0002x \\
C'(1000) &= 1.5-0.0002(1000) \\
&= 1.3 \\
C'(3000) &= 1.5-0.0002(3000) \\
&= 0.9
\end{aligned}$$

The values give the rate of change of the cost when 1000 and 3000 yogurt bars are produced, which is the marginal cost of production per unit.

c) If $C'(x)=0$, then $1.5 - 0.0002x = 0$ with solution $x = 7500$.
This is outside the given domain, $0 \le x \le 5000$, so $x \neq 7500$.

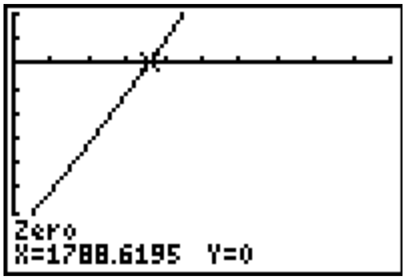
d) $R'(x)=3.25$
This is the rate of change of revenue per yogurt bar produced.

e) The profit function is $P(x)=R(x)-C(x)$.

$$P(x)=(3.25x)-(3450+1.5x-0.0001x^2)$$
$$P(x)=0.0001x^2+1.75x-3450$$

f) Answers may vary. For example:

Use a graphing calculator to plot the function $P(x)$ and find points on the graph that are positive.



$P(x)$ is positive for $x \geq 1789$ and negative for $0 \leq x \leq 1788$, since x cannot be negative; This information tells the owners whether they are going to make or lose money based on how many yogurt bars they are producing and it shows the break even point.

Chapter 2 Section 1 **Question 20 Page 85**

a) The slope of the tangent to $f(x)$ is $f'(x)$.

$$\begin{aligned} f'(x) &= -2(3x^{3-1}) + 5(2x^{2-1}) - x^{1-1} + 0 \\ &= -6x^2 + 10x - 1 \end{aligned}$$

The slope of the tangent to the curve at $x = 2$ is $f'(2) = -6(2)^2 + 10(2) - 1$, which is –5.

b) The slope of the normal is perpendicular to the slope of the tangent. Use $y = mx + b$ for the normal.

$m = \dfrac{-1}{f'(2)}$, which evaluates to $\dfrac{-1}{-5} = \dfrac{1}{5}$ or 0.2.

The y-coordinate at $x = 2$ is $f(2) = 5$ so (2, 5) lies on the normal. Use (2, 5) and $m = 0.2$ to find b.

$5 = 0.2(2) + b$ so $b = 4.6$

The equation of the normal to $f(x)$ at $x = 2$ is given by $y = 0.2x + 4.6$.

Chapter 2 Section 1 **Question 21 Page 85**

a) $f(x) = -4x^3 + 3x^{-1} + x^{\frac{1}{2}} - 2$

The slope of the tangent to $f(x)$ is $f'(x)$.

$$\begin{aligned} f'(x) &= -4(3x^{3-1}) + 3(-x^{-1-1}) + \frac{1}{2}x^{\frac{1}{2}-1} + 0 \\ &= -12x^2 - \frac{3}{x^2} + \frac{1}{2\sqrt{x}} \end{aligned}$$

The slope of the tangent to the curve at $x = 1$ is $f'(1) = -12(1)^2 - \dfrac{3}{(1)^2} + \dfrac{1}{2\sqrt{1}}$, which is $-\dfrac{29}{2}$.

b) The slope of the normal is perpendicular to the slope of the tangent. Use $y = mx + b$ for the normal.

$m = \dfrac{-1}{f'(1)}$, which evaluates to $\dfrac{-1}{\left(-\dfrac{29}{2}\right)} = \dfrac{2}{29}$.

The y-coordinate at $x = 1$ is $f(1) = -2$ so $(1, -2)$ lies on the normal. Use $(1, -2)$ and $m = \dfrac{2}{29}$ to find b.

$-2 = \dfrac{2}{29}(1) + b$ so $b = -\dfrac{60}{29}$

The equation of the normal to $f(x)$ at $x = 1$ is given by $y = \dfrac{2}{29}x - \dfrac{60}{29}$.

Chapter 2 Section 1 **Question 22 Page 86**

a) $f(x) = 4 - 4x^{\frac{1}{2}} + x$

The slope of the tangent to $f(x)$ is $f'(x)$.

$$f'(x) = 0 - 4\left(\frac{1}{2}x^{\frac{1}{2}-1}\right) + x^{1-1}$$
$$= -\frac{2}{\sqrt{x}} + 1$$

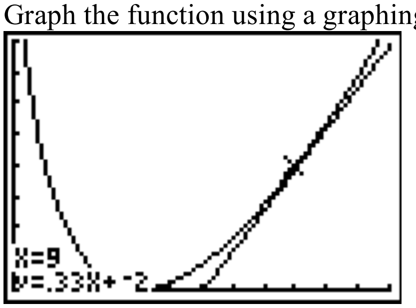
The slope of the tangent to the curve at $x = 9$ is $f'(9) = -\dfrac{2}{\sqrt{9}} + 1$, which is $\dfrac{1}{3}$.

The y-coordinate at $x = 9$ is $f(9) = 1$ so $(9, 1)$ lies on the tangent. Use $(9, 1)$ and $m = \dfrac{1}{3}$ to find b.

$1 = \dfrac{1}{3}(9) + b$ so $b = -2$.

The equation of the tangent to $f(x)$ at $x = 9$ is given by $y = \dfrac{1}{3}x - 2$.

b) Graph the function using a graphing calculator and use the tangent function.



Chapter 2 Section 1 **Question 23 Page 86**

a) $g(x) = 4x^{-2} - 12x^{-3} + x - 3$

The slope of the tangent to $g(x)$ is $g'(x)$.

$$\begin{aligned} g'(x) &= 4(-2x^{-2-1}) - 12(-3x^{-3-1}) + x^{1-1} + 0 \\ &= -\frac{8}{x^3} + \frac{36}{x^4} + 1 \end{aligned}$$

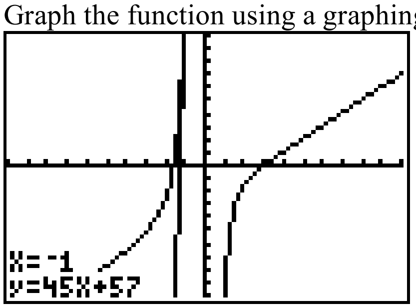
The slope of the tangent to the curve at $x = -1$ is $g'(-1) = -\frac{8}{(-1)^3} + \frac{36}{(-1)^4} + 1$, which is 45.

The y-coordinate at $x = -1$ is $g(-1) = 12$ so (–1, 12) lies on the tangent. Use (–1, 12) and $m = 45$ in $y = mx + b$ to find b.

$12 = 45(-1) + b$ so $b = 57$.

The equation of the tangent to $g(x)$ at $x = -1$ is given by $y = 45x + 57$.

b) Graph the function using a graphing calculator and use the tangent function.



Chapter 2 Section 1 **Question 24 Page 86**

a) The slope of the tangents will be 0 since the slope of the line $x = 1$ is undefined or infinite.

$$\begin{aligned} \frac{dy}{dx} &= -4x^{4-1} + 8(2x^{2-1}) \\ &= -4x^3 + 16x \end{aligned}$$

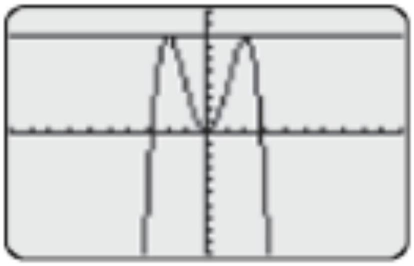
Now, find the values of x such that $\frac{dy}{dx} = 0$.

$$\begin{aligned} -4x^3 + 16x &= 0 \\ -4x(x^2 - 4) &= 0 \\ -4x(x+2)(x-2) &= 0 \end{aligned}$$

The curve y will have tangents with slope zero at $x = 0$, $x = -2$, $x = 2$.
When $x = 0$, $y = 0$.
When $x = -2$ or $x = 2$, $y = 16$.

The equations of the tangent lines to y that are perpendicular to $x = -1$ are $y = 0$ at the point (0, 0) and $y = 16$ at the point (2, 16).

b)



Chapter 2 Section 1 **Question 25 Page 86**

a) $\frac{dy}{dx} = 6(3x^{3-1}) + 2(2x^{2-1})$

$= 18x^2 + 4x$

Show that there are no points on the curve where the tangents have slope –5, so set $\frac{dy}{dx} = -5$.

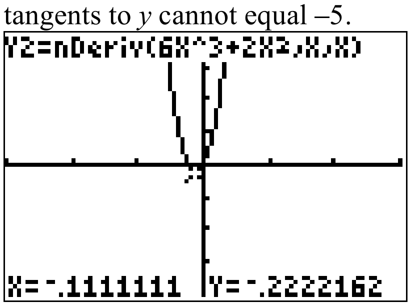
$-5 = 18x^2 + 4x$
$18x^2 + 4x + 5 = 0$
Use the quadratic formula to solve for x.

$$x = \frac{-4 \pm \sqrt{(4)^2 - 4(18)(5)}}{2(18)}$$

But the discriminant $4^2 - 4(18)(5) = -344$, which is less than 0.
Therefore, since the equation cannot be solved for real values of x, there are no tangents to the curve with a slope of –5.

b) Answers may vary. For example:

Plot the first derivative. Notice that the value of $\frac{dy}{dx}$ cannot be less than $-\frac{2}{9}$, so the slopes of the tangents to y cannot equal –5.



Chapter 2 Section 1 **Question 26 Page 86**

Solutions to the Achievement Checks are shown in the Teacher's Resource.

Chapter 2 Section 1 **Question 27 Page 86**

$$f'(x) = a(3x^{3-1}) + b(2x^{2-1}) + 3x^{1-1} + 0$$
$$= 3ax^2 + 2bx + 3$$

Substitute $f'(-1) = 14$ and $x = -1$ to get $11 = 3a - 2b$ and call this equation (1).

Substitute $f(2) = 10$ and $x = 2$ into $f(x) = ax^3 + bx^2 + 3x - 2$ to get $6 = 8a + 4b$ and call this equation (2).

Either use substitution or elimination to solve for a and b.

For example, solving (2) + 2 × (1) gives $28 = 14a$ so $a = 2$.

Substitution of $a = 2$ into either of the two equations gives $b = -\frac{5}{2}$.

Chapter 2 Section 1 **Question 28 Page 86**

Let (x_1, y_1) represent the point where the tangent meets the curve y.

Find the slope of the tangent to the curve at any value of x, which is $\frac{dy}{dx}$.

$$\frac{dy}{dx} = 2x^{2-1} + 0$$
$$m = 2x$$

Substitute the values $m = 2x_1$, $y_1 = x_1^2 - 2$ into the rise over run formula for the slope to find the values of x_1.

$$m = \frac{y_1 - (-5)}{x_1 - 1}$$
$$2x_1 = \frac{(x_1^2 - 2) + 5}{x_1 - 1}$$
$$2x_1(x_1 - 1) = x_1^2 + 3$$
$$0 = x_1^2 - 2x_1 - 3$$
$$0 = (x_1 + 1)(x_1 - 3)$$

The tangent lines meet y at the points $x_1 = -1$ and $x_1 = 3$.

When $x_1 = -1$, $m = 2(-1)$ or -2. Substituting $(1, -5)$ into $y = -2x + b$ gives $-5 = -2(1) + b$ so $b = -3$.

When $x_1 = 3$, $m = 2(3)$ or 6. Substituting $(1, -5)$ into $y = 6x + b$ gives $-5 = 6(1) + b$ so $b = -11$.

Therefore, the equations of the two tangents are $y = -2x - 3$ and $y = 6x - 11$.

Chapter 2 Section 1 **Question 29 Page 86**

a) Find the slope of the tangent to the curve at any value of x, which is $\frac{dy}{dx}$.

$$\frac{dy}{dx} = 2(3x^{3-1}) - 3(2x^{2-1}) - 11x^{1-1} + 0$$
$$m = 6x^2 - 6x - 11$$

Either use the factor theorem or graph the function to find the solutions to the function when $y = 2$.

$$2 = 2x^3 - 3x^2 - 11x + 8$$
$$0 = 2x^3 - 3x^2 - 11x + 6$$
$$0 = (x+2)(2x^2 - 7x + 3)$$
$$0 = (x+2)(2x-1)(x-3)$$

When $y = 2$, the x-coordinates of the points on the graph are $x = –2$, $x = 0.5$, and $x = 3$.

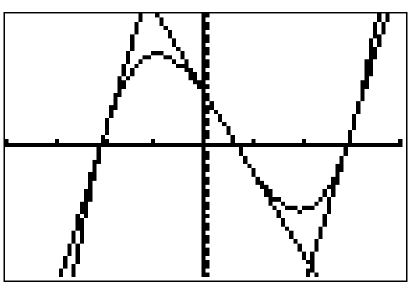
When $x = –2$, $m = 6(–2)^2 – 6(–2) – 11$ or 25.
Substituting (–2, 2) into $y = 25x + b$ gives $2 = 25(–2) + b$ so $b = 52$.

When $x = 0.5$, $m = 6(0.5)^2 – 6(0.5) – 11$ or –12.5.
Substituting (0.5, 2) into $y = –12.5x + b$ gives $2 = –12.5(0.5) + b$ so $b = 8.25$.

When $x = 3$, $m = 6(3)^2 – 6(3) – 11$ or 25.
Substituting (3, 2) into $y = 25x + b$ gives $2 = 25(3) + b$ so $b = –73$.

Therefore, the equations of the three tangents are $y = 25x + 52$, $y = 12.5x + 8.25$, and $y = 25x – 73$.

b)



Chapter 2 Section 1 **Question 30 Page 86**

If the derivative of a polynomial function has an exponent of –1, then the original exponent of the polynomial must be 0. But $x^0 = 1$, and the derivative of 1 is 0, not x^{-1}. Therefore, there does not exist a polynomial function with derivative x^{-1}.

Chapter 2 Section 1 Question 31 Page 86

Consider polynomials $p(x)$ and $q(x)$ of degree $m > 0$ and $n > 0$.
If the equation $p(x) - q(x)$ has $m + n$ real roots, which of the following choices must be true? C
p is of degree m and q is of degree n.
So the equation $p(x) - q(x) = 0$ has degree $\max\{m, n\} < m + n$ since $m \neq 0 \neq n$.
That the equation has more than $\max\{m, n\}$ real roots implies that $p(x) - q(x)$ must be identically zero.
Hence the equation has infinitely many real roots.

Chapter 2 Section 1 Question 32 Page 86

E

Chapter 2 Section 2 **The Product Rule**

Chapter 2 Section 2 **Question 1 Page 93**

a) i) $f(x)=2x^2+7x-4$

$f'(x)=4x+7$

ii) $f'(x)=\left[\frac{d}{dx}\left(x+4\right)\right](2x-1)+(x+4)\left[\frac{d}{dx}\left(2x-1\right)\right]$

$=(1)(2x-1)+(x+4)(2)$

$=4x+7$

b) i) $h(x)=-10x^2+11x-3$

$h'(x)=-20x+11$

ii) $h'(x)=\left[\frac{d}{dx}\left(5x-3\right)\right](1-2x)+(5x-3)\left[\frac{d}{dx}\left(1-2x\right)\right]$

$=(5)(1-2x)+(5x-3)(-2)$

$=-20x+11$

c) i) $h(x)=-3x^2-5x+8$

$h'(x)=-6x-5$

ii) $h'(x)=\left[\frac{d}{dx}\left(-x+1\right)\right](3x+8)+(-x+1)\left[\frac{d}{dx}\left(3x+8\right)\right]$

$=(-1)(3x+8)+(-x+1)(3)$

$=-6x-5$

d) i) $g(x)=-6x^2+11x-4$

$g'(x)=-12x+11$

ii) $g'(x)=\left[\frac{d}{dx}\left(2x-1\right)\right](4-3x)+(2x-1)\left[\frac{d}{dx}\left(4-3x\right)\right]$

$=(2)(4-3x)+(2x-1)(-3)$

$=-12x+11$

Chapter 2 Section 2 **Question 2 Page 93**

a) $f'(x) = \left[\dfrac{d}{dx}\left(5x+2\right)\right](8x-6) + (5x+2)\left[\dfrac{d}{dx}\left(8x-6\right)\right]$

$= (5)(8x-6) + (5x+2)(8)$

$= 80x - 14$

b) $h'(t) = \left[\dfrac{d}{dt}\left(-t+4\right)\right](2t+1) + (-t+4)\left[\dfrac{d}{dt}\left(2t+1\right)\right]$

$= (-1)(2t+1) + (-t+4)(2)$

$= -4t + 7$

c) $p'(x) = \left[\dfrac{d}{dx}\left(-2x+3\right)\right](x-9) + (-2x+3)\left[\dfrac{d}{dx}\left(x-9\right)\right]$

$= (-2)(x-9) + (-2x+3)(1)$

$= -4x + 21$

d) $g'(x) = \left[\dfrac{d}{dx}\left(x^2+2\right)\right](4x-5) + (x^2+2)\left[\dfrac{d}{dx}\left(4x-5\right)\right]$

$= (2x)(4x-5) + (x^2+2)(4)$

$= 12x^2 - 10x + 8$

e) $f'(x) = \left[\dfrac{d}{dx}\left(1-x\right)\right](x^2-5) + (1-x)\left[\dfrac{d}{dx}\left(x^2-5\right)\right]$

$= (-1)(x^2-5) + (1-x)(2x)$

$= -3x^2 + 2x + 5$

f) $h'(t) = \left[\dfrac{d}{dt}\left(t^2+3\right)\right](3t^2-7) + (t^2+3)\left[\dfrac{d}{dt}\left(3t^2-7\right)\right]$

$= (2t)(3t^2-7) + (t^2+3)(6t)$

$= 4t(3t^2+1)$

Chapter 2 Section 2 **Question 3 Page 93**

a) $M'(u) = \left[\dfrac{d}{du}\left(1-4u^2\right)\right](u+2) + (1-4u^2)\left[\dfrac{d}{du}\left(u+2\right)\right]$

$= (-8u)(u+2) + (1-4u^2)(1)$

$= -12u^2 - 16u + 1$

b) $g'(x) = \left[\frac{d}{dx}\left(-x+3\right)\right](x-10)+(-x+3)\left[\frac{d}{dx}\left(x-10\right)\right]$

$= (-1)(x-10)+(-x+3)(1)$

$= -2x+13$

c) $p'(n) = \left[\frac{d}{dn}\left(5n+1\right)\right](-n^2+3)+(5n+1)\left[\frac{d}{dn}\left(-n^2+3\right)\right]$

$= (5)(-n^2+3)+(5n+1)(-2n)$

$= -15n^2-2n+15$

d) $A'(r) = \left[\frac{d}{dr}\left(1+2r\right)\right](2r^2-6)+(1+2r)\left[\frac{d}{dr}\left(2r^2-6\right)\right]$

$= (2)(2r^2-6)+(1+2r)(4r)$

$= 12r^2+4r-12$

e) $b'(k) = \left[\frac{d}{dk}\left(-0.2k+4\right)\right](2-k)+(-0.2k+4)\left[\frac{d}{dk}\left(2-k\right)\right]$

$= (-0.2)(2-k)+(-0.2k+4)(-1)$

$= 0.4k-4.4$

Chapter 2 Section 2 Question 4 Page 93

a) $f(x) = 5x^2+7;\ g(x) = 21-3x$

b) $f(x) = -4x^3+8x;\ g(x) = 2x^2-4x$

c) $f(x) = 2x^3-x;\ g(x) = 0.5x^2+x$

d) $f(x) = -\frac{3}{4}x^4+6x;\ g(x) = 7x-\frac{2}{3}x^2$

Chapter 2 Section 2 Question 5 Page 94

a) $f'(x) = \left[\frac{d}{dx}\left(x^2-2x\right)\right](3x+1)+(x^2-2x)\left[\frac{d}{dx}\left(3x+1\right)\right]$

$= (2x-2)(3x+1)+(x^2-2x)(3)$

$= 9x^2-10x-2$

$f'(-2) = 9(-2)^2-10(-2)-2$

$= 54$

b) $$\begin{aligned} f'(x) &= \left[\frac{d}{dx}\left(1-x^3\right)\right](-x^2+2)+(1-x^3)\left[\frac{d}{dx}\left(-x^2+2\right)\right] \\ &= (-3x^2)(-x^2+2)+(1-x^3)(-2x) \\ &= 5x^4-6x^2-2x \\ f'(-2) &= 5(-2)^4-6(-2)^2-2(-2) \\ &= 60 \end{aligned}$$

c) $$\begin{aligned} f'(x) &= \left[\frac{d}{dx}\left(3x-1\right)\right](2x+5)+(3x-1)\left[\frac{d}{dx}\left(2x+5\right)\right] \\ &= (3)(2x+5)+(3x-1)(2) \\ &= 12x+13 \\ f'(-2) &= 12(-2)+13 \\ &= -11 \end{aligned}$$

d) $$\begin{aligned} f'(x) &= \left[\frac{d}{dx}\left(-x^2+x\right)\right](5x^2-1)+(-x^2+x)\left[\frac{d}{dx}\left(5x^2-1\right)\right] \\ &= (-2x+1)(5x^2-1)+(-x^2+x)(10x) \\ &= -20x^3+15x^2+2x-1 \\ f'(-2) &= -20(-2)^3+15(-2)^2+2(-2)-1 \\ &= 215 \end{aligned}$$

e) $$\begin{aligned} f'(x) &= \left[\frac{d}{dx}\left(2x-x^2\right)\right](7x+4)+(2x-x^2)\left[\frac{d}{dx}\left(7x+4\right)\right] \\ &= (2-2x)(7x+4)+(2x-x^2)(7) \\ &= -21x^2+20x+8 \\ f'(-2) &= -21(-2)^2+20(-2)+8 \\ &= -116 \end{aligned}$$

f) $$\begin{aligned} f'(x) &= \left[\frac{d}{dx}\left(-5x^3+x\right)\right](-x+2)+(-5x^3+x)\left[\frac{d}{dx}\left(-x+2\right)\right] \\ &= (-15x^2+1)(-x+2)+(-5x^3+x)(-1) \\ &= 20x^3-30x^2-2x+2 \\ f'(-2) &= 20(-2)^3-30(-2)^2-2(-2)+2 \\ &= -274 \end{aligned}$$

a) $$f'(x)=\left[\frac{d}{dx}\left(x^2-3\right)\right](x^2+1)+(x^2-3)\left[\frac{d}{dx}\left(x^2+1\right)\right]$$
$$=(2x)(x^2+1)+(x^2-3)(2x)$$
$$=4x^3-4x$$
$$f'(-4)=4(-4)^3-4(-4)$$
$$m=-240$$
The y-coordinate at $x = -4$ is $f(-4) = 221$.

Use the point $(-4, 221)$ and $m = -240$ in the equation $y = mx + b$ to find b.

$221 = -240(-4) + b$ so $b = -739$.

The equation of the tangent is $y = -240x - 739$.

b) $$g'(x)=\left[\frac{d}{dx}\left(2x^2-1\right)\right](-x^2+3)+(2x^2-1)\left[\frac{d}{dx}\left(-x^2+3\right)\right]$$
$$=(4x)(-x^2+3)+(2x^2-1)(-2x)$$
$$=-8x^3+14x$$
$$g'(2)=-8(2)^3+14(2)$$
$$m=-36$$
The y-coordinate at $x = 2$ is $g(2) = -7$.

Use the point $(2, -7)$ and $m = -36$ in the equation $y = mx + b$ to find b.

$-7 = -36(2) + b$ so $b = 65$.

The equation of the tangent is $y = -36x + 65$.

c)
$$h'(x)=\left[\frac{d}{dx}\left(x^4+4\right)\right](2x^2-6)+(x^4+4)\left[\frac{d}{dx}\left(2x^2-6\right)\right]$$
$$=(4x^3)(2x^2-6)+(x^4+4)(4x)$$
$$=12x^5-24x^3+16x$$
$$h'(-1)=12(-1)^5-24(-1)^3+16(-1)$$
$$m=-4$$
The y-coordinate at $x = -1$ is $h(-1) = -20$.

Use the point $(-1, -20)$ and $m = -4$ in the equation $y = mx + b$ to find b.

$-20 = -4(-1) + b$ so $b = -24$

The equation of the tangent is $y = -4x - 24$.

d) $$p'(x) = \left[\frac{d}{dx}\left(-x^3+2\right)\right](4x^2-3)+(-x^3+2)\left[\frac{d}{dx}\left(4x^2-3\right)\right]$$
$$= (-3x^2)(4x^2-3)+(-x^3+2)(8x)$$
$$= -20x^4+9x^2+16x$$
$$p'(3) = -20(3)^4+9(3)^2+16(3)$$
$$m = -1491$$

The y-coordinate at $x = 3$ is $p(3) = -825$.

Use the point (3, –825) and $m = -1491$ in the equation $y = mx + b$ to find b.

$-825 = -1491(3) + b$ so $b = 3648$

The equation of the tangent is $y = -1491x + 3648$.

Chapter 2 Section 2 **Question 7 Page 94**

a) $$\frac{dy}{dx} = \left[\frac{d}{dx}\left(-4x+3\right)\right](x+3)+(-4x+3)\left[\frac{d}{dx}\left(x+3\right)\right]$$
$$= (-4)(x+3)+(-4x+3)(1)$$
$$= -8x-9$$

Find the value of x that satisfies $\frac{dy}{dx} = 0$.

$$0 = -8x-9$$
$$x = -\frac{9}{8}$$

When $x = -\frac{9}{8}$, $y = \frac{225}{16}$ so the point $\left(-\frac{9}{8}, \frac{225}{16}\right)$ on the curve corresponds to the slope $m = 0$.

b) $$\frac{dy}{dx} = \left[\frac{d}{dx}\left(5x+7\right)\right](2x-9)+(5x+7)\left[\frac{d}{dx}\left(2x-9\right)\right]$$
$$= (5)(2x-9)+(5x+7)(2)$$
$$= 20x-31$$

Find the value of x that satisfies $\frac{dy}{dx} = \frac{2}{5}$.

$$\frac{2}{5} = 20x-31$$
$$x = 1.57$$

When $x = 1.57$, $y = -87.021$ so the point (1.57, –87.021) on the curve corresponds to the slope $m = \frac{2}{5}$.

c) $\frac{dy}{dx}=\left[\frac{d}{dx}\left(2x-1\right)\right](-4+x^2)+(2x-1)\left[\frac{d}{dx}\left(-4+x^2\right)\right]$

$=(2)(-4+x^2)+(2x-1)(2x)$

$=6x^2-2x-8$

Find the value of x that satisfies $\frac{dy}{dx}=3$.

$3=6x^2-2x-8$

$0=6x^2-2x-11$

$x=\frac{-(-2)\pm\sqrt{(-2)^2-4(6)(-11)}}{2(6)}$

$x=1.53,\ x=-1.20$

When $x = 1.53$, $y = 3.42$ and when $x = -1.20$, $y = 8.70$ so the points (1.53, 3.42) and (–1.20, 8.70) on the curve correspond to the slope $m = 3$.

d) $\frac{dy}{dx}=\left[\frac{d}{dx}\left(x^2-2\right)\right](2x+1)+(x^2-2)\left[\frac{d}{dx}\left(2x+1\right)\right]$

$=(2x)(2x+1)+(x^2-2)(2)$

$=6x^2+2x-4$

Find the value of x that satisfies $\frac{dy}{dx}=-2$.

$-2=6x^2+2x-4$

$0=6x^2+2x-2$

$x=\frac{-(2)\pm\sqrt{(2)^2-4(6)(-2)}}{2(6)}$

$x=0.43,\ x=-0.77$

When $x = 0.43$, $y = -3.38$ and when $x = -0.77$, $y = 0.76$ so the points (0.43, –3.38) and (–0.77, 0.76) on the curve correspond to the slope $m = -2$.

Chapter 2 Section 2 **Question 8 Page 94**

a) In t years from now, there will be $(120 + 10t)$ trees and the yield will be $(280 + 15t)$ apples per tree.
$Y(t)=(120+10t)(280+15t)$

b) $Y'(t) = \left[\dfrac{d}{dt}\left(120+10t\right)\right](280+15t) + (120+10t)\left[\dfrac{d}{dt}\left(280+15t\right)\right]$

$= (10)(280+15t) + (120+10t)(15)$

$= 4600 + 300t$

$Y'(2) = 4600 + 300(2)$

$= 5200$

The yield is 5200 apples/year. This represents the annual rate of change of apple production after 2 years.

c) $Y'(6) = 4600 + 300(6)$

$= 6400$

The yield is 6400 apples/years. This represents the rate of change of apple production at $t = 6$ years.

Chapter 2 Section 2 **Question 9 Page 94**

a) $\dfrac{dy}{dx} = \left[\dfrac{d}{dx}\left(5x^2 - x + 1\right)\right](x+2) + (5x^2 - x + 1)\left[\dfrac{d}{dx}\left(x+2\right)\right]$

$= (10x - 1)(x+2) + (5x^2 - x + 1)(1)$

$= 15x^2 + 18x - 1$

b) $\dfrac{dy}{dx} = \left[\dfrac{d}{dx}\left(1 - 2x^3 + x^2\right)\right]\left(\dfrac{1}{x^3} + 1\right) + (1 - 2x^3 + x^2)\left[\dfrac{d}{dx}\left(\dfrac{1}{x^3} + 1\right)\right]$

$= (-6x^2 + 2x)\left(\dfrac{1}{x^3} + 1\right) + (1 - 2x^3 + x^2)\left(-\dfrac{3}{x^4}\right)$

$= -6x^2 + 2x - \dfrac{1}{x^2} - \dfrac{3}{x^4}$

c) $\dfrac{dy}{dx} = \left[\dfrac{d}{dx}\left(-x^2\right)\right](4x-1)(x^3+2x+3) + (-x^2)\left[\dfrac{d}{dx}\left(4x-1\right)\right](x^3+2x+3) + (-x^2)(4x-1)\left[\dfrac{d}{dx}\left(x^3+2x+3\right)\right]$

$= (-2x)(4x-1)(x^3+2x+3) + (-x^2)(4)(x^3+2x+3) + (-x^2)(4x-1)(3x^2+2)$

$= (-8x^5 + 2x^4 - 16x^3 - 20x^2 + 6x) + (-4x^5 - 8x^3 - 12x^2) + (-12x^5 + 3x^4 - 8x^3 + 2x^2)$

$= -24x^5 + 5x^4 - 32x^3 - 30x^2 + 6x$

d) $\dfrac{dy}{dx} = 2\left[\dfrac{d}{dx}\left(2x^2 - x^{\frac{1}{2}}\right)\right]\left(2x^2 - x^{\frac{1}{2}}\right)$

$= 2\left(4x - \dfrac{1}{2\sqrt{x}}\right)\left(2x^2 - \sqrt{x}\right)$

$= 16x^3 - 10x^{\frac{3}{2}} + 1$

e) $\dfrac{dy}{dx} = 2\left[\dfrac{d}{dx}\left(-3x^2 + x + 1\right)\right]\left(-3x^2 + x + 1\right)$

$= 2(-6x + 1)(-3x^2 + x + 1)$

$= 36x^3 - 18x^2 - 10x + 2$

Chapter 2 Section 2 **Question 10 Page 94**

a) One method is expanding the function and then differentiating.

$R(n) = (81 - 4n)(6.50 + 0.50n)$

$= 526.50 + 40.50n - 26n - 2n^2$

$= 526.50 + 14.50n - 2n^2$

$R'(n) = 14.50 - 4n$

Another method is differentiating the function using the product rule.

$R'(n) = \left[\dfrac{d}{dn}\left(81 - 4n\right)\right](6.50 + 0.50n) + (81 - 4n)\left[\dfrac{d}{dn}\left(6.50 + 0.50n\right)\right]$

$= (-4)(6.50 + 0.50n) + (81 - 4n)(0.50)$

$= 14.50 - 4n$

b) $R'(4) = 14.5 - 4(4)$

$= -1.5$

This tells the manager that for 4 increases of \$0.50, the miniature golf club is losing revenue at the ticket price of \$8.50.

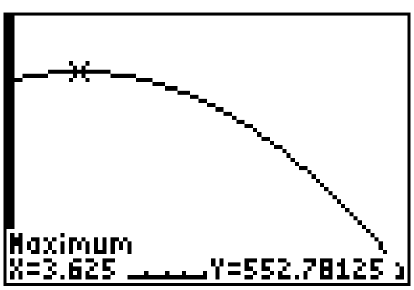
c) $R'(n) = 0$

$14.5 - 4n = 0$

$n = 3.625$

This tells the manager that the maximum price for a ticket is 6.50 + 0.50(3.625) = \$8.31.

d)



The maximum revenue is \$552.78. This occurs for n = 3.625, which is the value of n found in part c). As n grows larger than 3.625, the revenue decreases.

e) The maximum value of the function is found at $R'(n) = 0$. $R(n)$ is maximized at n = 3.625.

Chapter 2 Section 2 **Question 11 Page 95**

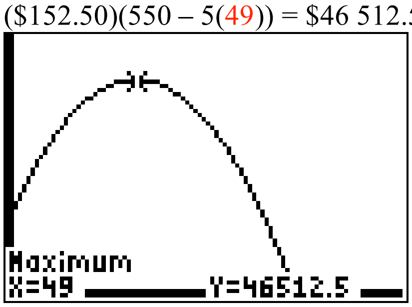
a) For x increases in \$2.50 increments the cost of a visit will be $(30+2.50x)$ dollars.
The number of clients for x increases will be $(550-5x)$.
$R(x)=(30+2.50x)(550-5x)$

b) $$R'(x)=\left[\frac{d}{dx}\left(30+2.5x\right)\right](550-5x)+(30+2.5x)\left[\frac{d}{dn}\left(550-5x\right)\right]$$
$$=(2.5)(550-5x)+(30+2.5x)(-5)$$
$$=1225-25x$$

c) $$R'(3)=1225-25(3)$$
$$=1150$$
This represents the rate of change of revenue at a \$7.50 increase.

d) $$R'(x)=0$$
$$1225-25x=0$$
$$x=49$$

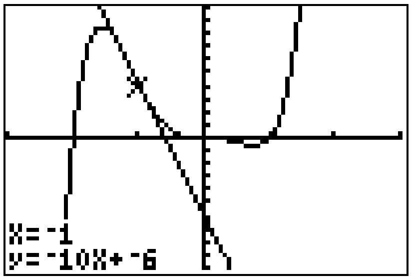
e) The owner could maximize revenue by making 49 increases of \$2.50 increments. A visit to the hair salon would then cost 30 + 49(\$2.50) = \$152.50 resulting in a maximum revenue of (\$152.50)(550 – 5(49)) = \$46 512.50.



Chapter 2 Section 2 **Question 12 Page 95**

a)
$$f'(x)=\left[\frac{d}{dx}\left(2x^2\right)\right](x^2+2x)(x-1)+2x^2\left[\frac{d}{dx}\left(x^2+2x\right)\right](x-1)+2x^2(x^2+2x)\left[\frac{d}{dx}\left(x-1\right)\right]$$
$$=4x(x^2+2x)(x-1)+2x^2(2x+2)(x-1)+2x^2(x^2+2x)(1)$$
$$=(4x^4+4x^3-8x^2)+(4x^4-4x^2)+(2x^4+4x^3)$$
$$=10x^4+8x^3-12x^2$$
$f'(-1)=10(-1)^4+8(-1)^3-12(-1)^2$ so the slope at $x=-1$ is $m=-10$.
The y-coordinate of the point on the graph at $x=-1$ is $f(-1)=4$.
If the tangent to the graph is $y = mx + b$ then use the point (–1, 4) and m = –10 to find b.
$4=-10(-1)+b$
$b=-6$
The equation of the tangent line is y = –10x – 6.

b)



Chapter 2 Section 2 **Question 13 Page 95**

a)

$$f'(x) = 2\left[\frac{d}{dx}\left(3x - 2x^2\right)\right](3x - 2x^2)$$
$$= 2(3 - 4x)(3x - 2x^2)$$
$$= 16x^3 - 36x^2 + 18x$$

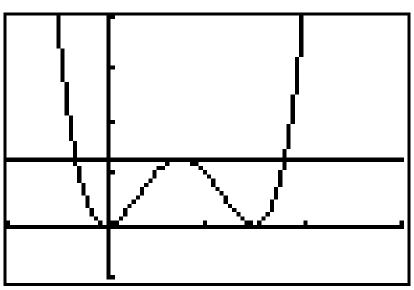
If the tangent line is parallel to the x-axis, then its slope is $m = 0$.
To find the points on the graph where the slope of its tangent is 0, set $f'(x) = 0$ and solve for x.

$$16x^3 - 36x^2 + 18x = 0$$
$$2x(8x^2 - 18x + 9) = 0$$
$$2x(4x - 3)(2x - 3) = 0$$

The solutions are $x = 0$, $x = \frac{3}{2}$, $x = \frac{3}{4}$.

Therefore, the points on the graph corresponding to tangents with slope parallel to the x-axis are (0, 0), $\left(\frac{3}{2}, 0\right)$ and $\left(\frac{3}{4}, \frac{81}{64}\right)$.

b)



Chapter 2 Section 2 **Question 14 Page 95**

a) At the start, $t = 0$ so the amount of gas is $V(0) = 90$ L.

b) $$V'(t) = 90\left(2\left[\frac{d}{dt}\left(1-\frac{t}{18}\right)\right]\left(1-\frac{t}{18}\right)\right)$$
$$= 180\left(-\frac{1}{18}\right)\left(1-\frac{t}{18}\right)$$
$$= -10 + \frac{5}{9}t$$

After 12 h, the rate of gas leakage is $V'(12) = -10 + \frac{5}{9}(12)$, which is –3.33.

So the tank is leaking 3.33 L/h after 12 h.

c) Set $V(t) = 40$ and find t.

$$40 = 90\left(1-\frac{t}{18}\right)^2$$
$$\frac{4}{9} = \left(1-\frac{t}{18}\right)^2$$
$$\frac{2}{3} = 1-\frac{t}{18}$$
$$t = \frac{20}{3}$$

Substitute $t = \frac{20}{3}$ into the derivative function to find $V'(t)$.

$$V'\left(\frac{20}{3}\right) = -10 + \frac{5}{9}\left(\frac{20}{3}\right)$$
$$= -6.30$$

The gas is leaking at 6.30 L/h when there is 40 L in the tank.

Chapter 2 Section 2 **Question 15 Page 95**

a) For t = 0, the population of fish is $p(0) = 15(30)(8)$, which is 3600 fish.

b) $p'(t) = 15(2t)(t+8) + 15(t^2 + 30)(1)$

$p'(t) = 45t^2 + 240t + 450$

$p'(3) = 45(9) + 720 + 450$

$= 1575$

In 3 years, the rate of change of the fish population is 1575 fish per year.

c) Set $p(t) = 5000$ and solve for t.

$5000 = 15(t^2 + 30)(t+8)$

$5000 = 15(t^3 + 30t + 8t^2 + 240)$

$5000 = 15t^3 + 120t^2 + 450t + 3600$

$0 = 3t^3 + 24t^2 + 90t - 280$

Use CAS or a graphing calculator to find the values of t.
t = 1.908 years

$p'(1.908) = 45(1.908)^2 + 240(1.908) + 450$

$= 1071.74$

The rate of change of the fish is 1071.7 fish per year.

d) Doubling the number of fish gives 2 × 3600 = 7200 fish. Set $p(t) = 7200$ and solve for t.

$7200 = 15t^3 + 120t^2 + 450t + 3600$

$0 = 15t^3 + 120t^2 + 450t - 3600$

$0 = 3t^3 + 24t^2 + 90t - 720$

Use CAS or a graphing calculator to find the values of t.
t = 3.45 years

$p'(3.45) = 45(3.45)^2 + 240(3.45) + 450$

$= 1813.61$

The population will double 3.45 years from now and the rate of change at this time is 1813.6 fish/year.

Chapter 2 Section 2 **Question 16 Page 95**

a) After n increases the cost will be $(1.75 + 0.25n)$ dollars and the number of sales will be $(150 - 10n)$.
$R(n) = (1.75 + 0.25n)(150 - 10n)$

b) Compare $R'(n)$ for n = 1, 3, 4, 5, and 6 increases.

$$\begin{aligned} R'(n) &= (0.25)(150 - 10n) + (1.75 + 0,25n)(-10) \\ &= 20 - 5n. \end{aligned}$$

$R'(1) = 15,\ R'(3) = 5,\ R'(4) = 0,\ R'(5) = -5,\ R'(6) = -10$

c)
$$\begin{aligned} R'(n) &= 0 \\ 20 - 5n &= 0 \\ n &= 4 \end{aligned}$$

This means that the profit is maximized after 4 increases of \$0.25.

d) The profit function is the revenue function minus the cost function.
The cost function is the cost to make one smoothie multiplied by the number of smoothies.

$$\begin{aligned} P(n) &= R(n) - C(n) \\ &= R(n) - 0.75(150 - 10n) \\ P'(n) &= R'(n) - 0.75(-10) \\ &= R'(n) + 7.5 \end{aligned}$$

$P'(1) = 22.50,\ P'(3) = 12.50,\ P'(4) = 7.50,\ P'(5) = 2.50,\ P'(6) = -2.50$

e) The maximum profit occurs when $P'(n) = 0$.

$$\begin{aligned} 0 &= R'(n) + 7.5 \\ 0 &= (20 - 5n) + 7.5 \\ n &= 5.5 \end{aligned}$$

When n = 3.85, the cost of a smoothie will be \$1.75 + \$0.25(5.5) = \$3.13. This is the price for maximum profit.

f) Answers may vary. For example:

The profit numbers have a rate of change that is 7.5 greater than that of the revenue numbers.

Chapter 2 Section 2 **Question 17 Page 96**

a) i) $\frac{dy}{dx}=(2x-3)(x^2-3x)+(x^2-3x)(2x-3)$

ii) $\frac{dy}{dx}=(6x^2+1)(2x^3+x)+(2x^3+x)(6x^2+1)$

iii) $\frac{dy}{dx}=(-4x^3+10x)(-x^4+5x^2)+(-x^4+5x^2)(-4x^3+10x)$

Both terms in the answers are the same.

b) $\frac{d}{dx}[f(x)]^2=2f(x)\frac{d}{dx}[f(x)]$

c) $\frac{d}{dx}[f(x)g(x)]=f(x)\frac{d}{dx}[g(x)]+g(x)\frac{d}{dx}[f(x)]$

Set $f(x)=g(x)$.

$\frac{d}{dx}[f(x)f(x)]=f(x)\frac{d}{dx}[f(x)]+f(x)\frac{d}{dx}[f(x)]$

$\frac{d}{dx}[f(x)]^2=2f(x)\frac{d}{dx}[f(x)]$

d) i) $\frac{dy}{dx}=2(x^2-3x)(2x-3)$

ii) $\frac{dy}{dx}=2(2x^3+x)(6x^2+1)$

iii) $\frac{dy}{dx}=2(-x^4+5x^2)(-4x^3+10x)$

Since $\frac{d}{dx}[f(x)]^2=2f(x)\frac{d}{dx}[f(x)]$, the answers are the same as in part a).

Chapter 2 Section 2 **Question 18 Page 96**

a) $[fgh]' = f'gh + f(gh)'$
$= f'gh + f(g'h + fgh')$
$= f'gh + fg'h + fgh'$

b) $f'(x) = (2x)(3x^4 - 2)(5x + 1) + (x^2 + 4)(12x^3)(5x + 1) + (x^2 + 4)(3x^4 - 2)(5)$

c) Expand and differentiate using the sum rule and power rule.

$$f(x) = 15x^7 + 3x^6 + 60x^5 + 12x^4 - 10x^3 - 2x^2 - 40x - 8$$
$$f'(x) = 105x^6 + 18x^5 + 300x^4 + 48x^3 - 30x^2 - 4x - 40$$

d) Parts b) and c) simplify to the same polynomial.

$$\begin{aligned} f'(x) &= 2x(3x^4 - 2)(5x + 1) + (x^2 + 4)(12x^3)(5x + 1) + (x^2 + 4)(3x^4 - 2)(5) \\ &= 2x(15x^5 + 3x^4 - 10x - 2) + 12x^3(5x^3 + x^2 + 20x + 4) + 5(3x^6 - 2x^2 + 12x^4 - 8) \\ &= (30x^6 + 6x^5 - 20x^2 - 4x) + (60x^6 + 12x^5 + 240x^4 + 48x^3) + (15x^6 + 60x^4 - 10x^2 - 40) \\ &= 105x^6 + 18x^5 + 300x^4 + 48x^3 - 30x^2 - 4x - 40 \end{aligned}$$

Chapter 2 Section 2 **Question 19 Page 96**

a) $\dfrac{d}{dx}\left[f(x)\right]^3 = 3f(x)^2 \dfrac{d}{dx}[f(x)]$

b) Using $[fgh]' = f'gh + fg'h + fgh'$ let $g = h = f$.
$[fff]' = f'ff + ff'f + fff'$
$[f^3]' = f'f^2 + f^2f' + f^2f'$
$\dfrac{d}{dx}\left[f(x)\right]^3 = 3f(x)^2 \dfrac{d}{dx}[f(x)]$

c) **i)**
$$\begin{aligned} \frac{dy}{dx} &= 3(4x^2 - x)^2(8x - 1) \\ &= 384x^5 - 240x^4 + 48x^3 - 3x^2 \end{aligned}$$

ii)
$$\begin{aligned} \frac{dy}{dx} &= 3(x^3 + x)^2(3x^2 + 1) \\ &= 9x^8 + 21x^6 + 15x^4 + 3x^2 \end{aligned}$$

iii)
$$\begin{aligned} \frac{dy}{dx} &= 3(-2x^4 + x^2)^2(-8x^3 + 2x) \\ &= -96x^{11} + 120x^9 - 48x^7 + 6x^5 \end{aligned}$$

Chapter 2 Section 2 **Question 20 Page 96**

a) $h'(x) = 3x^2 f(x) + x^3 f'(x)$

b) $p'(x) = g(x)(4x^3 - 6x) + (x^4 - 3x^2)g'(x)$

c) $q'(x) = (-12x^3 - 16x + 5)f(x) + (-3x^4 - 8x^2 + 5x + 6)f'(x)$

d) $r'(x) = 4f(x)(2x^3 + 5x^2)(3x^2 + 5x) + (2x^3 + 5x^2)^2 f'(x)$

Chapter 2 Section 2 **Question 21 Page 96**

a) $\frac{d}{dx}[f(x)]^n = n[f(x)]^{n-1}\left(\frac{d}{dx}[f(x)]\right)$

b) $n = 4: \ \frac{dy}{dx} = 4(2x^3 + x^2)^3(6x^2 + 2x)$

$n = 5: \ \frac{dy}{dx} = 5(2x^3 + x^2)^4(6x^2 + 2x)$

$n = 6: \ \frac{dy}{dx} = 6(2x^3 + x^2)^5(6x^2 + 2x)$

Chapter 2 Section 2 **Question 22 Page 96**

C

Chapter 2 Section 2 **Question 23 Page 96**

E

Chapter 2 Section 3 **Velocity, Acceleration, and Second Derivatives**

Chapter 2 Section 3 **Question 1 Page 106**

a) $\frac{dy}{dx} = 6x^2, \frac{d^2y}{dx^2} = 12x$

b) $s'(t) = -4t^3 + 15t^2 - 4t + 1, s''(t) = -12t^2 + 30t - 4$

c) $h'(x) = x^5 - x^4, h''(x) = 5x^4 - 4x^3$

d) $f'(x) = \frac{3}{4}x^2 - 4x, f''(x) = \frac{3}{2}x - 4$

e) $g'(x) = 5x^4 + 12x^3 - 6x^2, g''(x) = 20x^3 + 36x^2 - 12x$

f) $h'(t) = -9.8t + 25, h''(t) = -9.8$

Chapter 2 Section 3 **Question 2 Page 106**

a) $f'(x) = 8x^3 - 9x^2 + 12x, f''(x) = 24x^2 - 18x + 12$

$$\begin{aligned} f''(3) &= 24(3)^2 - 18(3) + 12 \\ &= 174 \end{aligned}$$

b) $f'(x) = 12x^2 - 5, f''(x) = 24x$

$$\begin{aligned} f''(3) &= 24(3) \\ &= 72 \end{aligned}$$

c) $f'(x) = -2x^4 - 3x^2, f''(x) = -8x^3 - 6x$

$$\begin{aligned} f''(3) &= -8(3)^3 - 6(3) \\ &= -234 \end{aligned}$$

d)
$$\begin{aligned} f'(x) &= 6x(1-x) + (3x^2+2)(-1) \\ &= 6x - 6x^2 - 3x^2 - 2 \\ &= -9x^2 + 6x - 2 \end{aligned}$$

$$\begin{aligned} f''(x) &= -18x + 6 \\ f''(3) &= -18(3) + 6 \\ &= -48 \end{aligned}$$

e)
$$\begin{aligned} f'(x) &= (6x-5)(2x) + 6(x^2+4) \\ f'(x) &= 12x^2 - 10x + 6x^2 + 24 \\ &= 18x^2 - 10x + 24 \end{aligned}$$

$$\begin{aligned} f''(x) &= 36x - 10 \\ f''(3) &= 36(3) - 10 \\ &= 98 \end{aligned}$$

f) $f'(x) = 20x^4 - 2x^3 - 6x, f''(x) = 80x^3 - 6x^2 - 6$

$$\begin{aligned} f''(3) &= 80(3)^3 - 6(3)^2 - 6 \\ &= 2100 \end{aligned}$$

Chapter 2 Section 3 **Question 3 Page 107**

a) $s(t) = 5 + 7t - 8t^3$

$$v(t) = s'(t) = 7 - 24t^2 \qquad a(t) = s''(t) = -48t$$

b) $s(t) = (2t+3)(4-5t)$

$$= 8t + 12 - 10t^2 - 15t$$
$$= -10t^2 - 7t + 12$$

$$v(t) = s'(t) = -20t - 7 \qquad a(t) = s''(t) = -20$$

c) $s(t) = -(t+2)(3t^2 - t + 5)$

$$= -(3t^3 - t^2 + 5t + 6t^2 - 2t + 10)$$
$$= -3t^3 - 5t^2 - 3t - 10$$

$$v(t) = s'(t) = -9t^2 - 10t - 3 \qquad a(t) = s''(t) = -18t - 10$$

d) $s(t) = \dfrac{-2t^4 - t^3 + 8t^2}{4t^2}$

$$= -\frac{1}{2}t^2 - \frac{1}{4}t + 2$$

$$v(t) = s'(t) = -t - \frac{1}{4} \qquad a(t) = s''(t) = -1$$

Chapter 2 Section 3 **Question 4 Page 107**

a) $v(t) = 3t^2 - 6t + 1$ $\qquad a(t) = 6t - 6$

$$v(2) = 3(2)^2 - 6(2) + 1 = 1 \qquad a(2) = 6(2) - 6 = 6$$

The velocity 1 m/s. The acceleration is 6 m/s^2.

b) $v(t) = -9.8t + 15$ $\qquad a(t) = -9.8$

$$v(2) = -9.8(2) + 15 = -4.6 \qquad a(2) = -9.8$$

The velocity is –4.6 m/s. The acceleration is –9.8 m/s^2.

c) $s(t) = t(3t+5)(1-2t)$
$= (3t^2+5t)(1-2t)$
$= -6t^3 - 7t^2 + 5t$

$v(t) = -18t^2 - 14t + 5$

$v(2) = -18(2)^2 - 14(2) + 5$
$= -95$
The velocity is –95 m/s.

$a(t) = -36t - 14$

$a(2) = -36(2) - 14$
$= -86$
The acceleration is –86 m/s^2.

d) $s(t) = (t^2-2)(t^2+2)$
$= t^4 - 4$

$v(t) = 4t^3$

$v(2) = 4(2)^3$
$= 32$
The velocity is 32 m/s.

$a(t) = 12t^2$

$a(2) = 12(2)^2$
$= 48$
The acceleration is 48 m/s^2.

Chapter 2 Section 3 **Question 5 Page 107**

Explanations may vary. For example:

a) $s(t)$ is curve (2); $v(t)$ is line (1); $a(t)$ is line (3) **.**
Curve (2) represents the position function since it is a quadratic with the highest exponent. Curve (1) represents the velocity since its exponent is one less so it is a sloping line. Curve (3) is the acceleration since its exponent is one less than the velocity so it is just a constant function, which is the flat line.

b) $s(t)$ is curve (3); $v(t)$ is curve (1); $a(t)$ is line (2) **.**
Curve (3) represents the position function since it is a cubic with the highest exponent. Curve (1) represents the velocity since its exponent is one less so it is a quadratic function. Curve (3) is the acceleration since its exponent is one less than the velocity so it is a line.

Chapter 2 Section 3 **Question 6 Page 107**

a)

Interval	***v*(*t*)**	***a*(*t*)**	***v*(*t*) × *a*(*t*)**	**Motion of Object**	**Description of slope of slope of *s*(*t*)**
[0, 3)	+	–	–	forward slowing	+decreasing
(3, 6]	–	–	+	reverse accelerating	–decreasing

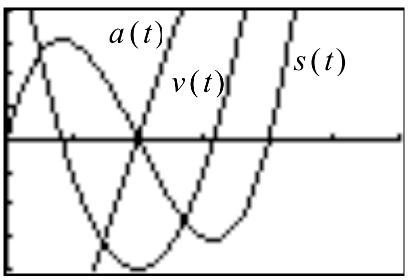
b)

Interval	***v*(*t*)**	***a*(*t*)**	***v*(*t*) × *a*(*t*)**	**Motion of Object**	**Description of slope of slope of *s*(*t*)**
[0, 1)	–	+	–	reverse slowing	–increasing
(1, 2)	+	+	+	forward accelerating	+increasing
(2, 3)	+	–	–	forward slowing	+decreasing
(3, infinity]	–	–	+	reverse accelerating	– decreasing

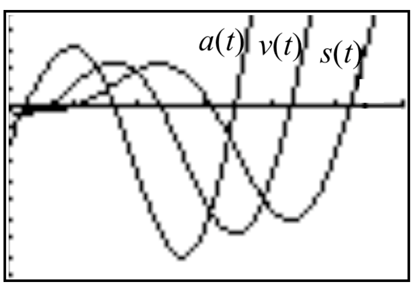
Chapter 2 Section 3 **Question 7 Page 107**

Answers may vary. For example:

a)



b)



Chapter 2 Section 3 **Question 7 Page 108**

Explanations may vary. For example:

a) i) $v(t)$ is increasing because the slope of the tangents are increasing.

ii) $a(t)$ is positive because the velocity is increasing.

b) i) $v(t)$ is increasing because the slope of the tangents are increasing.

ii) $a(t)$ is positive because the velocity is increasing.

c) i) $v(t)$ is decreasing because the slope of the tangents are decreasing.

ii) $a(t)$ is negative because the velocity is decreasing.

d) i) $v(t)$ is decreasing because the slope of the tangents are decreasing.

ii) $a(t)$ is negative because the velocity is decreasing.

e) i) $v(t)$ is constant because the slope of the tangents do not change.

ii) $a(t)$ is zero because the velocity is constant.

Chapter 2 Section 3 Question 9 Page 108

a) The initial velocity is zero.

b) The buses velocity at C and at F is zero.

c) The bus is going faster at A since the velocity is the slope of $s(t)$ and the slope is greater at A.

d) The bus is stopped since the velocity and acceleration of the bus are zero.

e) At A, the bus is speeding up. At B, the bus is slowing down. At D, the bus is speeding up.

f) At J, the bus returns to the starting point and slows to a stop.

Chapter 2 Section 3 Question 10 Page 108

a) The acceleration is positive.

b) The acceleration is negative.

c) The acceleration is positive.

d) The acceleration is negative.

e) The acceleration is positive.

Chapter 2 Section 3 Question 11 Page 109

a) i) The acceleration is positive.

ii) The acceleration is negative.

iii) The acceleration is positive.

iv) The acceleration is zero.

v) The acceleration is positive.

vi) At A the acceleration is positive and at D the acceleration is zero.

b) Justifications may vary. For example:

i) In all 3 intervals, the acceleration is positive as the velocity is increasing.

ii) In both intervals, the acceleration is negative as the velocity is decreasing.

iii) Both intervals have negative accelerations; The acceleration in the interval from B to C becomes more negative, while the acceleration in the interval from C to D becomes less negative.

Chapter 2 Section 3 **Question 12 Page 109**

a) $$\begin{aligned}\text{Average velocity} &= \frac{\Delta s}{\Delta t}\\ &= \frac{s(3)-s(1)}{3-1}\\ &= \frac{[80-4.9(3)^2]-[80-4.9(1)^2]}{3-1}\\ &= -19.6\end{aligned}$$

The average velocity is –19.6 m/s.

b) The velocity of the bottle at 3 s is $v(3)$.

$$\begin{aligned}v(t) &= s'(t)\\ &= -9.8t\end{aligned} \qquad \begin{aligned}v(3) &= -9.8(3)\\ &= -29.4\end{aligned}$$

The velocity of the bottle is –29.4 m/s.

c) The bottle will hit the ground when $s(t)=0$.

$$0 = 80-4.9t^2$$

$$t = \sqrt{\frac{80}{4.9}}$$

$$t \doteq 4.04$$

The time is 4.04 s.

d) The impact velocity is $v(4.04)$.

$$\begin{aligned}v(4.04) &= -9.8(4.04)\\ &= -39.6\end{aligned}$$

The impact velocity is –39.6 m/s.

Chapter 2 Section 3 **Question 13 Page 109**

a) $v(t) = h'(t)$
$= -9.8t + 34.5$

$a(t) = h''(t)$
$= -9.8$

$v(3) = -9.8(3) + 34.5$
$= 5.1$

$a(3) = -9.8$

The velocity of the rocket at 3 s is 5.1 m/s and the acceleration is –9.8 m/s^2.

b) When the rocket reaches its maximum height, $v(t) = 0$.
$0 = -9.8t + 34.5$
$t = 3.5$
The time is 3.5 s.

c) The starburst display occurs when t = 3.5 s so find $h(3.5)$ to get the height.
$h(3.5) = -4.9(3.5)^2 + 34.5(3.5) + 3.2$
$= 63.9$
The height is 63.9 m.

d) When the rocket is at ground level, $h(t) = 0$.
$0 = -4.9t^2 + 34.5t + 3.2$
$$t = \frac{-34.5 \pm \sqrt{34.5^2 - 4(-4.9)(3.2)}}{2(-4.9)}$$
$t \doteq 7.13,\ t \doteq -0.09$
Time cannot be negative, so it would take 7.13 s for the rocket to return to the ground.

e) $v(7.13) = -9.8(7.13) + 34.5$
$= -35.4$
The velocity is –35.4 m/s.

Chapter 2 Section 3 **Question 14 Page 109**

Justifications may vary. For example:

a) The velocity is positive because the truck is moving forward.

b) The velocity is decreasing because the truck is slowing down.

c) The acceleration is negative because the velocity is decreasing.

Chapter 2 Section 3 **Question 15 Page 109**

a) If the horizontal displacement is $s(t)$, then $s(t) = 48t$ where s is in kilometres and t is in hours.

b) $v(t) = s'(t)$ $\quad$ $a(t) = s''(t)$

$\quad = 48$ $\quad\quad = 0$

c) Since the acceleration due to gravity is –9.8 m/s^2, $s''(t) = -9.8$. This means that $s'(t)$ should be $-9.8t$ and $s(t)$ should be $-4.9t^2$. But, when $t = 0$ the height needs to be $s(0) = 50$ m. This works for $s(0) = -4.9(0)^2 + 50$.

Therefore, the vertical displacement can be represented by $s(t) = 50 - 4.9t^2$, where s is the vertical height in metres and t is the time in seconds.

d) $v(t) = s'(t)$ $\quad$ $a(t) = s''(t)$

$\quad = -9.8t$ $\quad\quad = -9.8$

e) When the prey hits the ground $s(t) = 0$ in the vertical displacement function.

$$0 = 50 - 4.9t^2$$
$$t = \sqrt{\frac{50}{4.9}}$$
$$t = 3.19$$

The vertical velocity when it hits the ground is $v(3.19)$.

$$\begin{aligned} v(3.19) &= -9.8(3.19) \\ &\approx -31.3 \end{aligned}$$

The velocity is about –31.3 m/s.

f) First, convert 48 km/h into metres per second.

The horizontal speed is $\dfrac{48 \times 1000}{1 \times 60 \times 60}$ or 13.33 m/s.

The vertical speed is $9.8t$.

When $9.8t = 13.3$, $t = 1.36$ s.

The vertical speed is greater than the horizontal speed when t is greater than 1.36 s.

g) Let V_T be the total velocity, V_V be the vertical velocity and V_H be the horizontal velocity.

$$\begin{aligned} V_T &= \sqrt{{V_V}^2 + {V_H}^2} \\ &= \sqrt{(9.8)^2 t^2 + (48)^2} \\ &= \sqrt{2304 + 96.04t^2} \end{aligned}$$

h) Since the eagle and prey travel horizontally at a constant velocity, the horizontal acceleration is zero. Thus, the total acceleration is $a_{\text{tot}}(t) = -9.8$.

i) Since the acceleration due to gravity is constant, the prey's acceleration is –9.8 m/s^2.

Chapter 2 Section 3 **Question 16 Page 110**

a) Find $v(t)$ and $a(t)$ for $t = 1$ s and $t = 4$ s.

$v(t) = 6t^2 - 30t + 36$ $a(t) = 12t - 30$

$v(1) = 12$ m/s; $v(4) = 12$ m/s $a(1) = -18$ m/s^2; $a(4) = 18$ m/s^2

b) The object is stopped when $v(t) = 0$.

$0 = 6t^2 - 30t + 36$

$0 = t^2 - 5t + 6$

$0 = (t-2)(t-3)$

$t = 2$ or $t = 3$

The object is at rest at $t = 2$ s and $t = 3$ s.
The object's position at these times is $s(2) = 38$ m and $s(3) = 37$ m.

c) The object is moving in a positive direction for $v(t) > 0$ so when t belongs to the intervals $(-\infty, 2)$ and $(3, \infty)$.

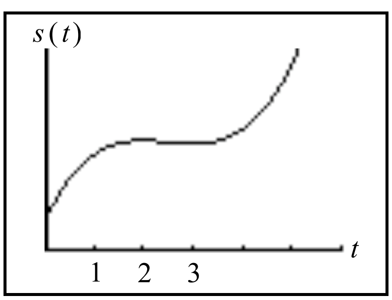
The object is moving in a negative direction when $v(t) < 0$ so when t belongs to the interval (2, 3).

d) Since the object travels in a negative direction from $t = 2$ s to $t = 3$ s, split up the intervals by the direction the object is moving and take the absolute values of the distances.

$$\begin{aligned}\text{Distance travelled} &= |s(2) - s(0)| + |s(3) - s(2)| + |s(7) - s(3)| \\ &= |28| + |-1| + |176| \\ &= 205\end{aligned}$$

The distance is 205 m.

e) Sketches may vary. For example:



Chapter 2 Section 3 **Question 17 Page 110**

a) $v(t) = h'(t)$ $a(t) = h''(t)$

$= gt + v_0$ $= g$

b) Set v_0 = 17.4 m/s, s_0 = 4 m, and g = –9.8 m/s^2.

$$h(t) = -4.9t^2 + 17.5t + 4$$
$$v(t) = -9.8t + 17.5$$
$$a(t) = -9.8$$

c) Find v_0 using the velocity function.

$$v(t) = -9.8t + v_0$$
$$10.4 = -9.8(2) + v_0$$
$$v_0 = 30$$

The original velocity is 30 m/s.
Find s_0 using the position function.

$$h(t) = -4.9t^2 + v_0 t + s_0$$
$$42.4 = -4.9(2)^2 + 30(2) + s_0$$
$$s_0 = 2$$

The original position is 2 m.
The functions are $h(t) = -4.9t^2 + 30t + 2$, $v(t) = -9.8t + 30$, and $a(t) = -9.8$.

Chapter 2 Section 3 **Question 18 Page 110**

a) $v(t) = h'(t) = gt + v_0$ $\qquad$ $a(t) = h''(t) = g$

The initial velocity is given in kilometres per hour so convert to metre per second.

$$v_0 = \frac{86.4}{1} \times \frac{1000}{60 \times 60} = 24$$

The initial velocity is 24 m/s.

Set v_0 = 24 m/s, s_0 = 0 m, and g = –12 m/s^2.

$$s(t) = -6t^2 + 24t$$
$$v(t) = -12t + 24$$
$$a(t) = -12$$

b) The truck is stopped when $v(t) = 0$ so use this to find t.

$$0 = -12t + 24$$
$$t = 2$$

So the truck will take 2 s to stop.

Chapter 2 Section 3 **Question 19 Page 110**

E

Chapter 2 Section 3 **Question 20 Page 110**

D

Chapter 2 Section 4 **The Chain Rule**

Chapter 2 Section 4 **Question 1 Page 117**

i) a) $f'(x) = 3(2x)^2(2)$
$= 24x^2$

b) $g'(x) = 2(-4x^2)(-8x)$
$= 64x^3$

c) $p'(x) = \frac{1}{2}(9x^2)^{-\frac{1}{2}}(18x)$
$= \frac{9x}{\sqrt{9x^2}}$
$= \frac{3x}{\sqrt{x^2}}$

d) $f'(x) = \frac{3}{4}(-16x^2)^{-\frac{1}{4}}(-32x)$
$= \frac{-96x}{4(-16x^2)^{\frac{1}{4}}}$
$= \frac{-24x}{(-16x^2)^{\frac{1}{4}}}$

e) $q'(x) = \frac{2}{3}(8x)^{-\frac{1}{3}}(8)$
$= \frac{16}{3(8)^{\frac{1}{3}}(x)^{\frac{1}{3}}}$
$= \frac{8}{3x^{\frac{1}{3}}}$

ii) a) $f(x) = (2x)^3$
$= 8x^3$
$f'(x) = 24x^2$

b) $g(x) = (-4x^2)^2$
$= 16x^4$
$g'(x) = 64x^3$

c) $p(x) = \sqrt{9x^2}$

$= 3\sqrt{x^2}$

$p'(x) = 3\left(\frac{1}{2}(x^2)^{-\frac{1}{2}}(2x)\right)$

$= \frac{3x}{\sqrt{x^2}}$

d) $f(x) = (-16x^2)^{\frac{3}{4}}$

$= (-1)^{\frac{3}{4}}16^{\frac{3}{4}}x^{\frac{3}{2}}$

$= (-1)^{\frac{3}{4}}8x^{\frac{3}{2}}$

$f'(x) = \frac{3(-1)^{\frac{3}{4}}8x^{\frac{1}{2}}}{2}$

$= \frac{24(-1)^{\frac{3}{4}}x^{\frac{1}{2}}}{(2^4)^{\frac{1}{4}}} \cdot \frac{(-1)^{\frac{1}{4}}x^{\frac{1}{2}}}{(-1)^{\frac{1}{4}}x^{\frac{1}{2}}}$

$= \frac{-24x}{(-16x^2)^{\frac{1}{4}}}$

e) $q(x) = (8)^{\frac{2}{3}}(x^{\frac{2}{3}})$

$= 4x^{\frac{2}{3}}$

$q'(x) = 4\left(\frac{2}{3}\right)x^{-\frac{1}{3}}$

$= \frac{8}{3x^{\frac{1}{3}}}$

Chapter 2 Section 4 **Question 2 Page 117**

$f(x) = g[h(x)]$	$g(x)$	$h(x)$	$h'(x)$	$g'[h(x)]$	$f'(x) = g'[h(x)]\,h'(x)$
a) $(6x-1)^2$	x^2	$6x-1$	6	$2h(x)$	$12(6x-1)$
b) $(x^2+3)^3$	x^3	x^2+3	$2x$	$3[h(x)]^2$	$6x(x^2+3)^2$
c) $(2-x^3)^4$	x^4	$2-x^3$	$-3x^2$	$4[h(x)]^3$	$(12x^2)(x^3-2)^3$
d) $(-3x+4)^{-1}$	$x-1$	$-3x+4$	-3	$-[h(x)]^{-2}$	$\dfrac{3}{(3x-4)^2}$
e) $(7+x^2)^{-2}$	$x-2$	$7+x^2$	$2x$	$-2[h(x)]^{-3}$	$-\dfrac{4x}{(x^2+7)^3}$
f) $(x^4-3x^2)^{\frac{1}{2}}$	$x^{\frac{1}{2}}$	x^4-3x^2	$4x^3-6x$	$\dfrac{1}{2}(x^4-3x^2)^{-\frac{1}{2}}$	$\dfrac{2x^3-3x}{(x^4-3x^2)^{\frac{1}{2}}}$

Chapter 2 Section 4 **Question 3 Page 118**

a)

$$\begin{aligned} y' &= 2(4x+1)(4) \\ &= 8(4x+1) \end{aligned}$$

b)

$$\begin{aligned} y' &= 3(3x^2-2)^2(6x) \\ &= 18x(9x^4-12x^2+4) \\ &= 162x^5-216x^3+72x \end{aligned}$$

c)

$$\begin{aligned} y' &= -3(x^3-x)^{-4}(3x^2-1) \\ &= \frac{-3(3x^2-1)}{(x^3-x)^4} \\ &= \frac{-3(3x^2-1)}{x^4(x^2-1)^4} \end{aligned}$$

d)

$$\begin{aligned} y' &= -2(4x^2+3x)^{-3}(8x+3) \\ &= \frac{-2(8x+3)}{(4x^2+3x)^3} \\ &= \frac{-2(8x+3)}{x^3(4x+3)^3} \end{aligned}$$

Chapter 2 Section 4 **Question 4 Page 118**

a) $y = (2x - 3x^5)^{\frac{1}{2}}$

$$\frac{dy}{dx} = \frac{1}{2}(2x - 3x^5)^{-\frac{1}{2}}(2 - 15x^4)$$
$$= \frac{2 - 15x^4}{2(2x - 3x^5)^{\frac{1}{2}}}$$

b) $y = (-x^3 + 9)^{\frac{1}{2}}$

$$\frac{dy}{dx} = \frac{1}{2}(-x^3 + 9)^{-\frac{1}{2}}(-3x^2)$$
$$= \frac{-3x^2}{2(-x^3 + 9)^{\frac{1}{2}}}$$

c) $y = (x - x^4)^{\frac{1}{3}}$

$$\frac{dy}{dx} = \frac{1}{3}(x - x^4)^{-\frac{2}{3}}(1 - 4x^3)$$
$$= \frac{1 - 4x^3}{3(x - x^4)^{\frac{2}{3}}}$$

d) $y = (2 + 3x^2 - x^3)^{\frac{1}{5}}$

$$\frac{dy}{dx} = \frac{1}{5}(2 + 3x^2 - x^3)^{-\frac{4}{5}}(6x - 3x^2)$$
$$= \frac{6x - 3x^2}{5(2 + 3x^2 - x^3)^{\frac{4}{5}}}$$

Chapter 2 Section 4 **Question 5 Page 118**

a) $y = (-x^3 + 1)^{-2}$

$$\begin{aligned}\frac{dy}{dx} &= -2(-x^3 + 1)^{-3}(-3x^2)\\ &= \frac{6x^2}{(-x^3 + 1)^3}\end{aligned}$$

b) $y = (3x^2 - 2)^{-1}$

$$\begin{aligned}\frac{dy}{dx} &= -(3x^2 - 2)^{-2}(6x)\\ &= -\frac{6x}{(3x^2 - 2)^2}\end{aligned}$$

c) $y = (x^2 + 4x)^{-\frac{1}{2}}$

$$\begin{aligned}\frac{dy}{dx} &= -\frac{1}{2}(x^2 + 4x)^{-\frac{3}{2}}(2x + 4)\\ &= -\frac{x + 2}{(x^2 + 4x)^{\frac{3}{2}}}\end{aligned}$$

d) $y = (x - 7x^2)^{-\frac{1}{3}}$

$$\begin{aligned}\frac{dy}{dx} &= -\frac{1}{3}(x - 7x^2)^{-\frac{4}{3}}(1 - 14x)\\ &= -\frac{1 - 14x}{3(x - 7x^2)^{\frac{4}{3}}}\end{aligned}$$

Chapter 2 Section 4 **Question 6 Page 118**

a) Use the chain rule.

$$\begin{aligned} f'(x) &= \frac{1}{2}(25x^4)^{-\frac{1}{2}}(100x^3) \\ &= \frac{50x^3}{5x^2} \\ &= 10x \end{aligned}$$

Alternately, simplify the function first and then differentiate.

$$f(x) = 5x^2$$
$$f'(x) = 10x$$

b) Answers may vary. For example:

I prefer the second method because simplifying the function first makes differentiating much easier.

c) No, the function cannot be simplified because I cannot take the square root of $25x^4 - 3$.

Chapter 2 Section 4 **Question 7 Page 118**

a) $f'(x) = 2(4x^2 - x + 1)(8x - 1)$

$f'(1) = 56$

b) $f'(x) = -2(3 - x + x^2)^{-3}(-1 + 2x)$

$f'(1) = \dfrac{-2}{27}$

c) $f(x) = (4x^2 + 1)^{\frac{1}{2}}$

$f'(x) = \dfrac{1}{2}(4x^2 + 1)^{-\frac{1}{2}}(8x)$

$f'(1) = \dfrac{4}{\sqrt{5}}$

d) $f(x) = 5(2x - x^2)^{-\frac{1}{3}}$

$f'(x) = 5\left(-\dfrac{1}{3}(2x - x^2)^{-\frac{4}{3}}(2 - 2x)\right)$

$f'(1) = 0$

Chapter 2 Section 4 **Question 8 Page 118**

Use the chain rule in Leibniz notation: $\frac{dy}{dx}=\frac{dy}{du}\frac{du}{dx}$

a) $$\frac{dy}{dx}=(2u+3)\left(\frac{1}{2}x^{-\frac{1}{2}}\right)$$
$$=(2\sqrt{x}+3)\left(\frac{1}{2\sqrt{x}}\right)$$
$$=1+\frac{3}{2\sqrt{x}}$$
$$\left.\frac{dy}{dx}\right|_{x=4}=1+\frac{3}{2\sqrt{4}}$$
$$=\frac{7}{4}$$

b) $$\frac{dy}{dx}=\left(\frac{1}{2}u^{-\frac{1}{2}}\right)(4x+3)$$
$$=\left(\frac{1}{2}(2x^2+3x+4)^{-\frac{1}{2}}\right)(4x+3)$$
$$=\frac{4x+3}{2\sqrt{2x^2+3x+4}}$$
$$\left.\frac{dy}{dx}\right|_{x=-3}=\frac{4(-3)+3}{2\sqrt{2(-3)^2+3(-3)+4}}$$
$$=-\frac{9}{2\sqrt{13}} \text{ or } -\frac{9}{26}\sqrt{13}$$

c) $$\frac{dy}{dx}=(-2u^{-3})(3x^2-5)$$
$$=-2(x^3-5x)^{-3}(3x^2-5)$$
$$=\frac{-2(3x^2-5)}{(x^3-5x)^{-3}}$$
$$\left.\frac{dy}{dx}\right|_{x=-2}=\frac{-2(3(-2)^2-5)}{((-2)^3-5(-2))^{-3}}$$
$$=-\frac{7}{4}$$

d) $\dfrac{dy}{dx} = (2 - 3u^2)(-x^{-2})$

$= (2 - 3(x^{-1})^2)(-x^{-2})$

$= \dfrac{3}{x^4} - \dfrac{2}{x^2}$

$\left.\dfrac{dy}{dx}\right|_{x=2} = \dfrac{3}{(2)^4} - \dfrac{2}{(2)^2}$

$= -\dfrac{5}{16}$

Chapter 2 Section 4 Question 9 Page 118

The slope m of the tangent will be $\left.\dfrac{dy}{dx}\right|_{x=3}$

$\dfrac{dy}{dx} = 3(x^3 - 4x^2)^2(3x^2 - 8x)$

$\left.\dfrac{dy}{dx}\right|_{x=3} = 3((3)^3 - 4(3)^2)^2(3(3)^2 - 8(3))$

$= 729$

The y-coordinate at $x = 3$ is $((3)^3 - 4(3)^2)^3 = -729$.
Use $m = 729$ and the point $(3, -729)$ to find b in the equation $y = mx + b$.

$-729 = 729(3) + b$
$b = -2916$

The equation of the tangent is $y = 729x - 2916$.

Chapter 2 Section 4 Question 10 Page 118

The slope m of the tangent will be $\left.\dfrac{dy}{dx}\right|_{x=2}$

$\dfrac{dy}{dx} = -\dfrac{1}{5}(5x^3 - 2x^2)^{-\frac{6}{5}}(15x^2 - 4x)$

$\left.\dfrac{dy}{dx}\right|_{x=2} = -\dfrac{1}{5}(5(2)^3 - 2(2)^2)^{-\frac{6}{5}}(15(2)^2 - 4(2))$

$= -\dfrac{13}{80}$

The y-coordinate at $x = 2$ is $(5(2)^3 - 2(2)^2)^{-\frac{1}{5}} = \dfrac{1}{2}$.

Use $m = -\dfrac{13}{80}$ and the point $\left(2, \dfrac{1}{2}\right)$ to find b in the equation $y = mx + b$.

$$\frac{1}{2} = -\frac{13}{80}(2) + b$$

$$b = \frac{33}{40}$$

The equation of the tangent is $y = -\frac{13}{80}x + \frac{33}{40}$.

Chapter 2 Section 4 **Question 11 Page 118**

$$v(t) = s'(t)$$
$$= \frac{1}{3}(t^5 - 750t^2)^{\frac{-2}{3}}(5t^4 - 1500t)$$

Substitute $t = 5$ s into the velocity function and factor the numbers using powers of 5 to find $v(5)$.

$$v(5) = \frac{1}{3}((5)^5 - 750(5)^2)^{-\frac{2}{3}}(5(5)^4 - 1500(5))$$
$$= \frac{1}{3}(-15625)^{\frac{-2}{3}}(-4375)$$
$$= \frac{1}{3}(-5^6)^{\frac{-2}{3}}(7)(-5^4)$$
$$= -\frac{1}{3}(-5^{-4})(-5^4)(7)$$
$$= -\frac{7}{3}$$

The velocity is $-\frac{7}{3}$ m/s.

Chapter 2 Section 4 **Question 12 Page 118**

Expand y and find the derivative.

$$y = (x^4(x^2 - 1)^2)$$
$$= x^4(x^4 - 2x^2 + 1)$$
$$= x^8 - 2x^6 + x^4$$

$$\frac{dy}{dx} = 8x^7 - 12x^5 + 4x^3$$

The points on the curve where the tangent line is horizontal are the points where $\frac{dy}{dx} = 0$.

$0 = 8x^7 - 12x^5 + 4x^3$

$0 = 4x^3(2x^4 - 3x^2 + 1)$

$0 = 4x^3(2x^2 - 1)(x^2 - 1)$

$x = 0,\ x = 1,\ x = -1,\ x = \frac{1}{\sqrt{2}},\ x = -\frac{1}{\sqrt{2}}$

Substituting the x-values into y gives the points (–1, 0), $\left(-\frac{1}{\sqrt{2}}, \frac{1}{16}\right)$, (0, 0), $\left(\frac{1}{\sqrt{2}}, \frac{1}{16}\right)$, (1, 0).

Chapter 2 Section 4 **Question 13 Page 118**

a)

$$N'(t) = 600\left(\frac{1}{2}\right)(16 + 3t^2)^{-\frac{3}{2}}(6t)$$
$$= \frac{1800t}{(16 + 3t^2)^{\frac{3}{2}}}$$

$N'(t)$ represents the rate at which the customers are being served.

b) $N(4) = 150 - \frac{600}{\sqrt{16 + 3(4)^2}}$

$$= 75$$

$$N'(4) = \frac{1800(4)}{(16 + 3(4)^2)^{\frac{3}{2}}}$$
$$\doteq \frac{225}{16} \text{ or } 14.06$$

After 4 h, 75 customers have been served at an instantaneous rate of change of 14.06 customers per hour.

c) $103 = 150 - \frac{600}{\sqrt{16 + 3t^2}}$

$$16 + 3t^2 = \left(\frac{600}{47}\right)^2$$
$$t \doteq \pm 7$$

This means that 103 customers have been served at time $t \doteq 7$ h.

d) $N'(7) = \dfrac{1800(7)}{(16+3(7)^2)^{\frac{3}{2}}}$

$= \dfrac{12600}{26569}\sqrt{163}$

$\doteq 6.05$

At $t = 7$ h, there are about 6.05 customers served per hour. At $t = 4$ h from before, there were 14.06 customers served per hour, which means that the customers are being served at a slower rate at 7 h.

Chapter 2 Section 4 **Question 14 Page 118**

$$P'(t) = -1250(1+0.01t)^{-2}(0.01)$$

$$= \frac{-12.5}{(1+0.01t)^2}$$

$$P'(2) = \frac{-12.5}{(1+0.01(2))^2}$$

$$= -12.01$$

The rate is –12.01 people/year.

$$P'(4) = \frac{-12.5}{(1+0.01(4))^2}$$

$$= -11.56$$

The rate is –11.56 people/year.

$$P'(7) = \frac{-12.5}{(1+0.01(7))^2}$$

$$= -10.92$$

The rate is –10.92 people/year.

Chapter 2 Section 4 **Question 15 Page 119**

$$\frac{dV}{dx} = \frac{dV}{ds}\cdot\frac{ds}{dx}$$

$$= (3s^2)(6x-7)$$

$$= 3(3x^2-7x+1)^2(6x-7)$$

$$\left.\frac{dV}{dx}\right|_{x=3} = 3\left(3(3)^2-7(3)+1\right)^2(6(3)-7)$$

$$= 1617$$

This value represents the rate of change of the volume of the cube with respect to x, when $x = 3$ m.

Chapter 2 Section 4 **Question 16 Page 119**

$$y=(4x-x^3)(3x^2+2)^{-2}$$

$$\begin{aligned}\frac{dy}{dx}&=(4x-x^3)(-2)(3x^2+2)^{-3}(6x)+(4-3x^2)(3x^2+2)^{-2}\\&=\frac{(-48x^2+12x^4)+(4-3x^2)(3x^2+2)}{(3x^2+2)^3}\\&=\frac{-48x^2+12x^4+6x^2-9x^4+8}{(3x^2+2)^3}\\&=\frac{3x^4-42x^2+8}{(3x^2+2)^3}\end{aligned}$$

Chapter 2 Section 4 **Question 17 Page 119**

Solutions to the Achievement Checks are shown in the Teacher's Resource.

Chapter 2 Section 4 **Question 18 Page 119**

The slope of the tangent at x is $\frac{dy}{dx}$.

$$\begin{aligned}\frac{dy}{dx}&=(3x^2)(8x^2+1)^{\frac{1}{2}}+x^3\left(\frac{1}{2}(8x^2+1)^{\frac{-1}{2}}(16x)\right)\\&=3x^2\sqrt{8x^2+1}+\frac{8x^4}{\sqrt{8x^2+1}}\end{aligned}$$

Since all the exponents of x in the derivative are even, the slope at $x=-1$ and $x=1$ will be the same.

$$\begin{aligned}\left.\frac{dy}{dx}\right|_{x=1}&=3(1)^2\sqrt{8(1)^2+1}+\frac{8(1)^4}{\sqrt{8(1)^2+1}}\\&=\frac{35}{3}\end{aligned}$$

The slopes at both $x=-1$ and $x=1$ are $m=\frac{35}{3}$.

When $x=-1$, the $y=3$ so use m and the point $(-1, 3)$ to find b in the equation for the tangent.

$$3=\frac{35}{3}(-1)+b$$

$$b=\frac{26}{3}$$

When $x = 1$, the $y = 3$ so use m and the point (1, 3) to find b in the equation for the second tangent.

$$3 = \frac{35}{3}(1) + b$$

$$b = -\frac{26}{3}$$

Therefore, the equations of the tangents are $y = \frac{35}{3}x + \frac{26}{3}$ and $y = \frac{35}{3}x - \frac{26}{3}$.

The tangents are related because they are parallel.
This is true because the curve is odd, so the tangents at $x = a$ and $x = -a$ will have the same slope for all values of x in the domain.

Chapter 2 Section 4 **Question 19 Page 119**

$$f'(x) = g'\left[h(x)\right] \times h'(x)$$

$$\begin{aligned} f'(2) &= g'\left[h(2)\right] \times h'(2) \\ &= g'(-6) \times 4 \\ &= -3 \times 4 \\ &= -12 \end{aligned}$$

Chapter 2 Section 4 **Question 20 Page 119**

$$\begin{aligned} \frac{dy}{dx} &= \frac{1}{2}(2x+1)^{-\frac{1}{2}}(2) \\ &= (2x+1)^{-\frac{1}{2}} \\ \frac{d^2y}{dx^2} &= -\frac{1}{2}(2x+1)^{-\frac{3}{2}}(2) \\ &= -\frac{1}{(2x+1)^{\frac{3}{2}}} \end{aligned}$$

Chapter 2 Section 4 **Question 21 Page 119**

Answers may vary. For example:

a) Choose $f(x) = x^2$ and $g(x) = 2x$.

$$\begin{aligned} g \circ f(x) &= 2x^2 \\ f \circ g(x) &= (2x)^2 \\ &= 4x^2 \end{aligned}$$

$\frac{d}{dx} f \circ g(x) = 8x$ and $\frac{d}{dx} g \circ f(x) = 4x$ so they are not the same.

b) Choose $f(x)=x^2$ and $g(x)=x^2$.

$$f\circ g(x)=(x^2)^2 \qquad g\circ f(x)=(x^2)^2$$
$$=x^4 \qquad\qquad =x^4$$

Both compositions have the same derivative of $4x^3$.

Chapter 2 Section 4 **Question 22 Page 119**

a)
$$\begin{aligned} y&=f\left[g\left[h(x)\right]\right]\\ &=\left(\frac{1}{\sqrt{x^2+2x}}\right)^2\\ &=(x^2+2x)^{-1} \end{aligned}$$

$$\begin{aligned} \frac{dy}{dx}&=-(x^2+2x)^{-2}(2x+2)\\ &=\frac{-(2x+2)}{(x^2+2x)^2}\\ &=\frac{-2(x+1)}{x^2(x+2)^2} \end{aligned}$$

b)
$$\begin{aligned} y&=f\left[g\left[h(x)\right]\right]\\ &=(x^2+2x)^{-1}\\ \frac{dy}{dx}&=\frac{-2(x+1)}{x^2(x+2)^2} \end{aligned}$$

c)
$$\begin{aligned} y&=g\left[h\left[f(x)\right]\right]\\ &=\frac{1}{\sqrt{x^4+2x^2}}\\ &=(x^4+2x^2)^{-\frac{1}{2}} \end{aligned}$$

$$\begin{aligned} \frac{dy}{dx}&=-\frac{1}{2}(x^4+2x^2)^{-\frac{3}{2}}(4x^3+4x)\\ &=\frac{-2(x^3+x)}{(x^2(x^2+2))^{\frac{3}{2}}}\\ &=\frac{-2x(x^2+1)}{x^2(x^2+2)\sqrt{x^2(x^2+2)}}\\ &=\frac{-2(x^2+1)}{x(x^2+2)\sqrt{x^2(x^2+2)}} \end{aligned}$$

d) $y = g\left[h\left[f(x)\right]\right]$

$$\begin{aligned}
&= \sqrt{\left(\frac{1}{x^2}\right)^2 + 2\left(\frac{1}{x^2}\right)} \\
&= (x^{-4} + 2x^{-2})^{\frac{1}{2}}
\end{aligned}$$

$$\begin{aligned}
\frac{dy}{dx} &= \frac{1}{2}(x^{-4} + 2x^{-2})^{-\frac{1}{2}}(-4x^{-5} - 4x^{-3}) \\
&= \frac{-2(x^{-5} + x^{-3})}{\left(\frac{1}{x^4} + \frac{2}{x^2}\right)^{\frac{1}{2}}} \\
&= \frac{-2x^{-5}(1 + x^2)}{\sqrt{\frac{1}{x^4} + \frac{2}{x^2}}} \\
&= \frac{-2(1 + x^2)}{x^5\sqrt{\frac{1 + 2x^2}{x^4}}}
\end{aligned}$$

Chapter 2 Section 4 **Question 23 Page 119**

$$\begin{aligned}
\frac{dy}{dx} &= f'(g \circ h(x)) \times (g \circ h(x))' \\
&= f'(g[h(x)])(g'[h(x)])[h'(x)]
\end{aligned}$$

Chapter 2 Section 4 **Question 24 Page 119**

D

Chapter 2 Section 4 **Question 25 Page 119**

E

Chapter 2 Section 5 **Derivatives of Quotients**

Chapter 2 Section 5 **Question 1 Page 124**

a) $q(x)=(3x+5)^{-1}$; domain: $x \neq \frac{-5}{3}$

b) $f(x)=-2(x-4)^{-1}$; domain: $x \neq 4$

c) $g(x)=6(7x^2+1)^{-1}$; domain: no restrictions

d) $r(x)=-2(x^3-27)^{-1}$; domain: $x \neq 3$

Chapter 2 Section 5 **Question 2 Page 124**

a) $q'(x)=-(3x+5)^{-2}(3)$

b) $f'(x)=2(x-4)^{-2}$

c) $g'(x)=-6(7x^2+1)^{-2}(14x)$

d) $r'(x)=2(x^3-27)^{-2}(3x^2)$

Chapter 2 Section 5 **Question 3 Page 124**

a) $q(x)=3x(x+1)^{-1}$; domain: $x \neq -1$

b) $f(x)=-x(2x+3)^{-1}$; domain: $x \neq -\frac{3}{2}$

c) $g(x)=x^2(5x-4)^{-1}$; domain: $x \neq \frac{4}{5}$

d) $r(x)=8x^2(x^2-9)^{-1}$; domain: $x \neq \pm 3$

Chapter 2 Section 5 **Question 4 Page 124**

a) $q'(x)=3(x+1)^{-1}+3x(-1)(x+1)^{-2}$

b) $f'(x)=(-1)(2x+3)^{-1}-x(-1)(2x+3)^{-2}(2)$

c) $g'(x)=2x(5x-4)^{-1}+x^2(-1)(5x-4)^{-2}(5)$

d) $r'(x)=16x(x^2-9)^{-1}+8x^2(-1)(x^2-9)^{-2}(2x)$

Chapter 2 Section 5 **Question 5 Page 125**

a) $\frac{dy}{dx} = (-x+3)(-1)(2x^2+5)^{-2}(4x) + (2x^2+5)^{-1}(-1)$

$= (2x^2+5)^{-2}[-4x(-x+3) - (2x^2+5)]$

$= \frac{2x^2-12x-5}{(2x^2+5)^2}$

b) $\frac{dy}{dx} = (4x+1)(-1)(x^3-2)^{-2}(3x^2) + (x^3-2)^{-1}(4)$

$= (x^3-2)^{-2}[-3x^2(4x+1) + 4(x^3-2)]$

$= -\frac{8x^3+3x^2+8}{(x^3-2)^2}$

c) $\frac{dy}{dx} = (9x^2-1)(-1)(1+3x)^{-2}(3) + (1+3x)^{-1}(18x)$

$= (1+3x)^{-2}[-3(9x^2-1) + 18x(1+3x)]$

$= 3(1+3x)^{-2}(1+3x)^2$

$= 3$

d) $\frac{dy}{dx} = x^4(-1)(x^2-x+1)^{-2}(2x-1) + (x^2-x+1)^{-1}(4x^3)$

$= (x^2-x+1)^{-2}[-x^4(2x-1) + 4x^3(x^2-x+1)]$

$= x^3(x^2-x+1)^{-2}[-x(2x-1) + 4(x^2-x+1)]$

$= \frac{x^3(2x^2-3x+4)}{(x^2-x+1)^2}$

Chapter 2 Section 5 **Question 6 Page 125**

a) $\frac{dy}{dx} = x^2(-1)(6x+2)^{-2}(6) + (6x+2)^{-1}(2x)$

$\left.\frac{dy}{dx}\right|_{x=-2} = (-2)^2(-1)(6(-2)+2)^{-2}(6) + (6(-2)+2)^{-1}2(-2)$

$= \frac{4}{25}$

The slope of the tangent at $x = 2$ is $\frac{4}{25}$.

b) $\dfrac{dy}{dx} = x^{\frac{1}{2}}(-1)(3x^2-1)^{-2}(6x) + (3x^2-1)^{-1}\left(\dfrac{1}{2}x^{-\frac{1}{2}}\right)$

$$\left.\frac{dy}{dx}\right|_{x=1} = 1^{\frac{1}{2}}(-1)(3(1)^2-1)^{-2}6(1) + (3(1)^2-1)^{-1}\left(\frac{1}{2}(1)^{-\frac{1}{2}}\right)$$
$$= -\frac{5}{4}$$

The slope of the tangent at $x = 1$ is $-\dfrac{5}{4}$.

c) $\dfrac{dy}{dx} = 4(x^2-1)^{-1} + (4x+1)(-1)(x^2-1)^{-2}(2x)$

$$\left.\frac{dy}{dx}\right|_{x=-3} = 4((-3)^2-1)^{-1} + (4(-3)+1)(-1)((-3)^2-1)^{-2}2(-3)$$
$$= -\frac{17}{32}$$

The slope of the tangent at $x = 1$ is $-\dfrac{17}{32}$.

d) $\dfrac{dy}{dx} = 2(x^2-x+1)^{-1} + 2x(-1)(x^2-x+1)^{-2}(2x-1)$

$$\left.\frac{dy}{dx}\right|_{x=-1} = 2((-1)^2-(-1)+1)^{-1} + 2(-1)(-1)((-1)^2-(-1)+1)^{-2}(2(-1)-1)$$
$$= 0$$

The slope of the tangent at $x = 1$ is 0.

e) $\dfrac{dy}{dx} = (3x^2)(x^2+x-1)^{-1} + (x^3-3)(-1)(x^2+x-1)^{-2}(2x+1)$

$$\left.\frac{dy}{dx}\right|_{x=2} = 3(2)^2(2^2+2-1)^{-1} + (2^3-3)(-1)(2^2+2-1)^{-2}(2(2)+1)$$
$$= \frac{7}{5}$$

The slope of the tangent at $x = 2$ is $\dfrac{7}{5}$.

Chapter 2 Section 5 **Question 7 Page 125**

Explanations may vary. For example:

a) Express the function as a product and use the product rule.

$$q'(x) = (-4x^3 + 5x^2 - 2x + 6)(-3x^{-4}) + x^{-3}(-12x^2 + 10x - 2)$$
$$q'(x) = x^{-4}[-3(-4x^3 + 5x^2 - 2x + 6) + x(-12x^2 + 10x - 2)]$$
$$q'(x) = \frac{-5x^2 + 4x - 18}{x^4}$$

Simplify the function and use the sum rule.

$$q(x) = -4 + 5x^{-1} - 2x^{-2} + 6x^{-3}$$
$$q'(x) = -5x^{-2} + 4x^{-3} - 18x^{-4}$$

I prefer simplifying the function first and using the sum rule because it is easier.

b) No, it is not possible to simplify and use the sum rule for the given function.

Chapter 2 Section 5 **Question 8 Page 125**

$$\begin{aligned} y' &= x^2(-1)(x+2)^{-2} + (x+2)^{-1}(2x) \\ &= (x+2)^{-2}[-x^2(x+2)^{-2} + 2x(x+2)] \\ &= \frac{x^2 + 4x}{(x+2)^2} \end{aligned}$$

The slope of the tangent is 3, so find x when $y' = -3$.

$$\begin{aligned} -3 &= \frac{x^2 + 4x}{(x+2)^2} \\ -3(x^2 + 4x + 4) &= x^2 + 4x \\ 4x^2 + 16x + 12 &= 0 \\ x^2 + 4x + 3 &= 0 \\ (x+1)(x+3) &= 0 \\ x = -1 \text{ or } x &= -3 \end{aligned}$$

The points on the curve where the slope of the tangents are –3 are (–1, 1) and (–3, –9).

Chapter 2 Section 5 **Question 9 Page 125**

$$\begin{aligned} y &= \frac{(x^3 - 1)^2}{(x+2)^2} \\ &= (x^3 - 1)^2(x+2)^{-2} \end{aligned}$$

$$y' = (x^3 - 1)^2(-2)(x+2)^{-3} + (2)(x^3 - 1)(3x^2)(x+2)^{-2}$$

The slope of the tangent at $x = -1$ is $y'(-1)$.

$$\begin{aligned} y'(-1) &= ((-1)^3 - 1)^2(-2)(-1+2)^{-3} + (2)((-1)^3 - 1)(3(-1)^2)(-1+2)^{-2} \\ &= -20 \end{aligned}$$

When $x = -1$, $y = 4$ so use this point and $m = -20$ to find b in the equation of the tangent, $y = mx + b$.
$4 = -20(-1) + b$ so $b = -16$.

The equation of the tangent at $x = -1$ is $y = -20x - 16$.

Chapter 2 Section 5 **Question 10 Page 125**

a)
$$\begin{aligned} v(t) &= s'(t) \\ &= 5t(-1)(t^2+4)^{-2}(2t) + (t^2+4)^{-1}(5) \\ &= (t^2+4)^{-2}[5t(-1)(2t) + (t^2+4)(5)] \\ &= \frac{-5t^2+20}{(t^2+4)^2} \end{aligned}$$

The hamster is moving at $v(1) = \frac{3}{5}$ m/s after 1 s.

b) First, find the time when the hamster stops, which is when $v(t) = 0$.

$$0 = \frac{-5t^2+20}{(t^2+4)^2}$$
$$0 = -5t^2 + 20$$
$$t = \pm 2$$

Since time cannot be negative, $t = 2$ s.
If $0 \le t < 2$, then $v(t) > 0$ and if $t > 2$, then $v(t) < 0$.
Therefore, the hamster changes direction when $t = 2$ s.

Chapter 2 Section 5 **Question 11 Page 125**

a) $C'(w) = 800w^2(-1)(200 + w^3)^{-2}(3w^2) + (200 + w^3)^{-1}(1600w)$

When $w = 1$, $C'(1) = 800(1)^2(-1)(200 + 1^3)^{-2}3(1)^2 + (200 + 1^3)^{-1}1600(1)$
$\doteq 7.9$

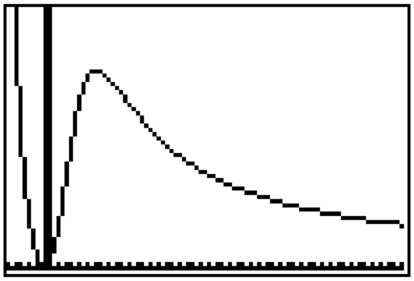
When $w = 3$, $C'(3) = 800(3)^2(-1)(200 + 3^3)^{-2}3(3)^2 + (200 + 3^3)^{-1}1600(3)$
$\doteq 17.4$

When $w = 5$, $C'(5) = 800(5)^2(-1)(200 + 5^3)^{-2}3(5)^2 + (200 + 5^3)^{-1}1600(5)$
$\doteq 10.4$

When $w = 8$, $C'(8) = 800(8)^2(-1)(200 + 8^3)^{-2}3(8)^2 + (200 + 8^3)^{-1}1600(8)$
$\doteq -1.4$

$C'(w)$ is the number of new clients in the fund per week.

b)



The slope of the curve indicates if $C'(w)$ is positive, negative, or zero.

c)

d)

$$C'(w) = 800w^2(-1)(200+w^3)^{-2}(3w^2) + (200+w^3)^{-1}(1600w)$$
$$= (200+w^3)^{-2}[800w^2(-1)(3w^2) + (200+w^3)(1600w)]$$
$$= \frac{-800w^4 + 320000w}{(200+w^3)^2}$$

Find $C'(w) = 0$.

$$-800w^4 + 320000w = 0$$
$$-w^4 + 400w = 0$$
$$-w(w^3 - 400) = 0$$
$$w = 0,\ w = \sqrt[3]{400} \text{ or } 2\sqrt[3]{50}$$

$C'(w) = 0$ when $w = 0$, and $w = 2\sqrt[3]{50}$. $C'(w)$ is positive for $0 < w < 2\sqrt[3]{50}$ and is negative for $w > 2\sqrt[3]{50}$.

e) Answers may vary. For example:

The number of new clients per week increases from $w = 0$ to $w = 2\sqrt[3]{50}$ weeks, then declines for $w > 2\sqrt[3]{50}$.

Chapter 2 Section 5 **Question 12 Page 125**

a) $N(8) = \dfrac{500(8)^2}{\sqrt{280+8^2}} + 10(8)$

$= 1805.3$

The predicted number of new customers after 8 weeks is 1805.3 customers.

b) $N(1)=\dfrac{500(1)^2}{\sqrt{280+1^2}}+10(1)$

$\doteq 39.8$

$N(6)=\dfrac{500(6)^2}{\sqrt{280+6^2}}+10(6)$

$\doteq 1072.6$

$$\begin{aligned}\text{Average}&=\frac{N(6)-N(1)}{6-1}\\&=\frac{1072.6-39.8}{5}\\&\doteq 206.55\end{aligned}$$

The predicted average number of new customers between weeks 1 and 6 is 206.55 customers.

c) $N'(x)=500x^2\left(-\dfrac{1}{2}\right)(280+x^2)^{-\frac{3}{2}}(2x)+(280+x^2)^{-\frac{1}{2}}(1000x)+10$

$N'(1)=500(1)^2\left(-\dfrac{1}{2}\right)(280+1^2)^{-\frac{3}{2}}2(1)+(280+1^2)^{-\frac{1}{2}}1000(1)+10$

$\doteq 69.5$

$N'(6)=500(6)^2\left(-\dfrac{1}{2}\right)(280+6^2)^{-\frac{3}{2}}2(6)+(280+6^2)^{-\frac{1}{2}}1000(6)+10$

$\doteq 328.3$

The rate of change of the predicted number of new customers is 69.5 customers/week at week 1 and is 328.3 customers/week at week 6.

d) The rate of change, $N'(x)$, can never be zero, so the number of customers will always increase.

Chapter 2 Section 5 **Question 13 Page 126**

a) $V(0)=\dfrac{2500}{\sqrt{2}}$

$\doteq 1767.77$

The purchase price of the painting was \$1767.77.

b) Find $V'(t)$.

$$\begin{aligned}V'(t) &= 0.2(1+t)(0.5t+2)^{-0.5} + (2500+0.2t)(0.5t+2)^{-0.5} + (2500+0.2t)(1+t)(-0.5)(0.5t+2)^{-1.5}(0.5)\\
&= (0.5t+2)^{-1.5}[0.2(1+t)(0.5t+2) + (2500+0.2t)(0.5t+2) + (2500+0.2t)(1+t)(-0.5)(0.5)]\\
&= (0.5t+2)^{-1.5}[(0.1t^2+0.5t+0.4) + (0.1t^2+1250.4t+5000) + (-0.05t^2-625.05t-625)]\\
&= \frac{0.15t^2+625.85t+4375.4}{(0.5t+2)^{1.5}}\\
&= \frac{0.6t^2+2503.4t+17501.6}{4(0.5t+2)^{1.5}} \text{ which can be transformed to}\\
&= \frac{0.4(3t^2+12517t+87508)}{(2t+8)^{1.5}}\end{aligned}$$

c) The value of the painting will always increase since $V'(t) > 0$ for all values of $t > 0$.

d) $$\begin{aligned}V'(2) &= \frac{0.4(3(2)^2+12517(2)+87508)}{(2(2)+8)^{1.5}}\\ &= 1083.05\end{aligned} \qquad \begin{aligned}V'(22) &= \frac{0.4(3(22)^2+12517(22)+87508)}{(2(22)+8)^{1.5}}\\ &= 388.65\end{aligned}$$

These values represent the increase in value of the painting after 2 years and after 22 years. The painting gains value more quickly 2 years after purchase than 22 years after purchase.

Chapter 2 Section 5 **Question 14 Page 126**

a) $$\begin{aligned}f'(x) &= -(ax+b)^{-2}(a)\\
f''(x) &= (-1)(-2)(ax+b)^{-3}(a)(a)\\
&= 2(ax+b)^{-3}a^2\\
f'''(x) &= (-1)(-2)(-3)(ax+b)^{-3}(a)(a)(a)\\
&= -6(ax+b)^{-4}a^3\end{aligned}$$

By extrapolation:

$$\begin{aligned}f^{(n)}(x) &= (-1)^n(n!)(ax+b)^{-(n+1)}(a^n)\\
&= \frac{(-1)^n(n!)(a^n)}{(ax+b)^{n+1}}\end{aligned}$$

b) $$\begin{aligned}f^{(4)}(x) &= (-1)^4(4!)(2x-3)^{-5}(2^4)\\
&= \frac{384}{(2x-3)^5}\end{aligned}$$

Chapter 2 Section 5 **Question 15 Page 126**

a) Find $p'(x)$ and set it to zero to find the value of x.

$$p'(x) = (x^2 - 4)(-1)(x^2 + 4)^{-2}(2x) + 2x(x^2 + 4)^{-1}$$
$$0 = (x^2 + 4)^{-2}[(x^2 - 4)(-1)(2x) + 2x(x^2 + 4)]$$
$$0 = \frac{16x}{(x^2 + 4)^2}$$
$$0 = 16x$$
$$x = 0$$

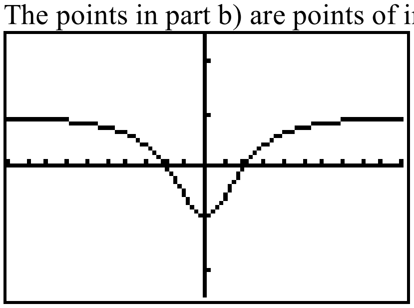
When $x = 0$, $y = 1$ so the point is (0, –1).

b) Find the points where $p''(x) = 0$.

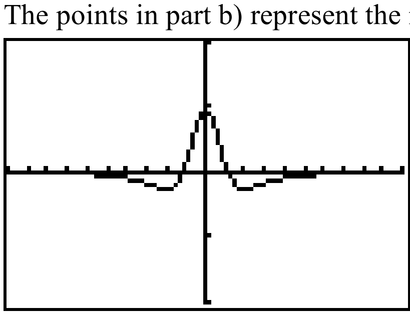
$$p''(x) = 16x(-2)(x^2 + 4)^{-3}(2x) + 16(x^2 + 4)^{-2}$$
$$0 = (x^2 + 4)^{-3}[16x(-2)(2x) + 16(x^2 + 4)]$$
$$0 = \frac{64 - 48x^2}{(x^2 + 4)^3}$$
$$0 = 64 - 48x^2$$
$$x = \pm\frac{2}{\sqrt{3}}$$

Substituting the x-values into $p(x)$ gives the points $\left(\frac{2}{\sqrt{3}}, -\frac{1}{2}\right)$ and $\left(-\frac{2}{\sqrt{3}}, -\frac{1}{2}\right)$.

c) The point in part a) represents the point on the curve with zero slope, which is the minimum of $p(x)$. The points in part b) are points of inflection.



d) The points in part b) represent the maximum and minimum points on the graph of $p'(x)$.



Chapter 2 Section 5 **Question 16 Page 126**

a) $$y = \frac{\left(\frac{1}{x} + x\right)}{\sqrt{\left(\frac{1}{x} + x\right) - 1}}$$

$$= (x^{-1} + x)(x^{-1} + x - 1)^{-0.5}$$

$$\frac{dy}{dx} = (x^{-1} + x)(-0.5)(x^{-1} + x - 1)^{-1.5}(-x^{-2} + 1) + (-x^{-2} + 1)(x^{-1} + x - 1)^{-0.5}$$

$$= (x^{-1} + x - 1)^{-1.5}[(x^{-1} + x)(-0.5)(-x^{-2} + 1) + (-x^{-2} + 1)(x^{-1} + x - 1)]$$

$$= (x^{-1} + x - 1)^{-1.5}[(0.5x^{-3} - 0.5x^{-1} + 0.5x^{-1} - 0.5x) + (-x^{-3} - x^{-1} + x^{-2} + x^{-1} + x - 1)]$$

$$= \frac{-0.5x^{-3} + x^{-2} + 0.5x - 1}{(x^{-1} + x - 1)^{1.5}}$$

$$= \frac{-0.5 + x + 0.5x^4 - x^3}{x^3(x^{-1}(1 + x^2 - x))(x^{-1}(1 + x^2 - x))^{0.5}}$$

$$= \frac{x^4 - 2x^3 + 2x - 1}{2x^2(x^2 - x + 1)\sqrt{\frac{x^2 - x + 1}{x}}}$$

domain: $x \neq 0$

b) $$y = x^{-1}(x-1)^{\frac{1}{2}} + x(x-1)^{-\frac{1}{2}}$$

$$\frac{dy}{dx} = (x-1)^{\frac{1}{2}}(-1)x^{-2} + x^{-1}\left(\frac{1}{2}\right)(x-1)^{-\frac{1}{2}} + x\left(-\frac{1}{2}\right)(x-1)^{-\frac{3}{2}} + (x-1)^{-\frac{1}{2}}(1)$$

$$= x^{-2}(x-1)^{-\frac{3}{2}}[(x-1)^2(-1) + x\left(\frac{1}{2}\right)(x-1) + x^3\left(-\frac{1}{2}\right) + x^2(x-1)]$$

$$= x^{-2}(x-1)^{-\frac{3}{2}}[(-x^2 + 2x - 1) + \left(\frac{1}{2}x^2 - \frac{1}{2}x\right) + \left(-\frac{1}{2}x^3\right) + (x^3 - x^2)]$$

$$= \frac{x^3 - 3x^2 + 3x - 2}{2x^2(x-1)^{\frac{3}{2}}}$$

domain: $x \neq 0,\ x \neq 1$

Chapter 2 Section 5 **Question 17 Page 126**

D

Chapter 2 Section 5 **Question 18 Page 126**

D

Chapter 2 Section 5 Extension **Question 1 Page 129**

a) $$q'(x) = \frac{(x+1)(3) - 3x(1)}{(x+1)^2}$$
$$= \frac{3}{(x+1)^2}$$

b) $$f'(x) = \frac{(2x+3)(-1) - (-x)(2)}{(2x+3)^2}$$
$$= \frac{-3}{(2x+3)^2}$$

c) $$g'(x) = \frac{(5x-4)(2x) - (x^2)(5)}{(5x-4)^2}$$
$$= \frac{x(5x-8)}{(5x-4)^2}$$

d) $$r'(x) = \frac{(x^2-9)(16x) - (8x^2)(2x)}{(x^2-9)^2}$$
$$= \frac{-144x}{(x^2-9)^2}$$

Chapter 2 Section 5 Extension **Question 2 Page 129**

a) $$y' = \frac{(2x^2+5)(-1) - (-x+3)(4x)}{(2x^2+5)^2}$$
$$= \frac{2x^2 - 12x - 5}{(2x^2+5)^2}$$

b) $$y' = \frac{(x^3-2)(4) - (4x+1)(3x^2)}{(x^3-2)^2}$$
$$= -\frac{(8x^3 + 3x^2 + 8)}{(x^3-2)^2}$$

c) $$y' = \frac{(1+3x)(18x) - (9x^2-1)(3)}{(1+3x)^2}$$
$$= \frac{27x^2 + 18x + 3}{(1+3x)^2}$$
$$= \frac{3(1+3x)^2}{(1+3x)^2}$$
$$= 3$$

d) $y' = \dfrac{(x^2 - x + 1)(4x^3) - x^4(2x - 1)}{(x^2 - x + 1)^2}$

$= \dfrac{2x^5 - 3x^4 + 4x^3}{(x^2 - x + 1)^2}$

$= \dfrac{x^3(2x^2 - 3x + 4)}{(x^2 - x + 1)^2}$

Chapter 2 Section 5 Extension Question 3 Page 129

a) $y' = \dfrac{(6x + 2)(2x) - x^2(6)}{(6x + 2)^2}$

$y'(-2) = \dfrac{(6(-2) + 2)(2(-2)) - (-2)^2(6)}{(6(-2) + 2)^2}$

$= \dfrac{4}{25}$

The slope of the tangent at $x = -2$ is $\dfrac{4}{25}$.

b) $y' = \dfrac{(3x^2 - 1)(0.5x^{-0.5}) - x^{0.5}(6x)}{(3x^2 - 1)^2}$

$y'(1) = \dfrac{(3(1)^2 - 1)(0.5(1)^{-0.5}) - (1)^{0.5}(6(1))}{(3(1)^2 - 1)^2}$

$= -\dfrac{5}{4}$

The slope of the tangent at $x = 1$ is $-\dfrac{5}{4}$.

c) $y' = \dfrac{(x^2 - 1)(4) - (4x + 1)(2x)}{(x^2 - 1)^2}$

$y'(-3) = \dfrac{((-3)^2 - 1)(4) - (4(-3) + 1)(2(-3))}{((-3)^2 - 1)^2}$

$= -\dfrac{17}{32}$

The slope of the tangent at $x = -3$ is $-\dfrac{17}{32}$.

d) $y' = \dfrac{(x^2 - x + 1)(2) - 2x(2x - 1)}{(x^2 - x + 1)^2}$

$y'(-1) = \dfrac{((-1)^2 - (-1) + 1)(2) - 2(-1)(2(-1) - 1)}{((-1)^2 - (-1) + 1)^2}$

$= 0$

The slope of the tangent at $x = -1$ is 0.

e)

$$y' = \frac{(x^2+x-1)(3x^2)-(x^3-3)(2x+1)}{(x^2+x-1)^2}$$

$$\begin{aligned} y'(2) &= \frac{((2)^2+(2)-1)(3(2)^2)-((2)^3-3)(2(2)+1)}{((2)^2+(2)-1)^2} \\ &= \frac{7}{5} \end{aligned}$$

The slope of the tangent at $x = 2$ is $\frac{7}{5}$.

Chapter 2 Section 5 Extension Question 4 Page 129

a) i)

$$\begin{aligned} \frac{dy}{dx} &= \frac{(x^2+3x)^5(0)-(1)(5(x^2+3x)^4(2x+3))}{(x^2+3x)^{10}} \\ &= \frac{-5(2x+3)}{(x^2+3x)^6} \end{aligned}$$

ii)

$$\begin{aligned} y &= (x^2+3x)^{-5} \\ \frac{dy}{dx} &= -5(x^2+3x)^{-6}(2x+3) \\ &= \frac{-5(2x+3)}{(x^2+3x)^6} \end{aligned}$$

b) The power of a function rule is more efficient because the numerator is 1.

c) Answers may vary. For example:

I prefer the power of a functional rule because it is quicker.

Chapter 2 Section 5 Extension Question 5 Page 129

$$\begin{aligned} \frac{dy}{dx} &= \frac{dy}{du} \cdot \frac{du}{dx} \\ &= \left(\frac{(u^2+1)(3u^2)-u^3(2u)}{(u^2+1)^2} \right)(3-2x) \\ &= \frac{u^2(u^2+3)}{(u^2+1)^2}(3-2x) \end{aligned}$$

When $x = 2$, $u = 3(2) - (2)^2$ or 2.

$$\begin{aligned} \left.\frac{dy}{dx}\right|_{x=2} &= \frac{(2)^2((2)^2+3)}{((2)^2+1)^2}(3-2(2)) \\ &= -\frac{28}{25} \end{aligned}$$

Chapter 2 Section 5 Extension Question 6 Page 129

$$y' = \frac{(2x+5)(2x)-(x^2)(2)}{(2x+5)^2}$$
$$= \frac{2x^2+10x}{(2x+5)^2}$$

Solve $y'(x) = 0$.

$$0 = \frac{2x^2+10x}{(2x+5)^2}$$
$$0 = 2x(x+5)$$
$$x = 0, x = -5$$

If $x = 0$, then $y = 0$ and if $x = –5$, then $y = –5$.
The points on the curve with horizontal tangent lines are the points (0, 0) and (–5, –5).

Chapter 2 Section 5 Extension Question 7 Page 129

No, the statement is not true.
$f'(x) = 75x^4 - 27x^2$ and $g'(x) = 6x$

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{3x^2(75x^4-27x^2)-(15x^5-9x^3)(6x)}{(3x^2)^2}$$
$$= \frac{135x^6-27x^4}{9x^4}$$
$$= 15x^2 - 3$$
$$\frac{f'(x)}{g'(x)} = \frac{75x^4-27x^2}{6x}$$
$$= \frac{25}{2}x^3 - \frac{9}{2}x$$
$$\therefore \frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] \neq \frac{f'(x)}{g'(x)}$$

Chapter 2 Section 5 Extension Question 8 Page 129

a) $$n'(w) = \frac{(1+w^2)(600w)-300w^2(2w)}{(1+w^2)}$$
$$= \frac{600w}{(1+w^2)}$$

$$n'(1) = \frac{600(1)}{(1+1^2)}$$
$$= 150$$
$$n'(5) = \frac{600(5)}{(1+5^2)}$$
$$= \frac{750}{169}$$

b) No. The number of sales does not decrease since $n'(w)$ is not less than 0 for $0 \le w \le 10$.

Chapter 2 Section 5 Extension Question 9 Page 129

$$C'(t) = \frac{(3t^2+4)(4) - 4t(6t)}{(3t^2+4)^2}$$
$$C'(3) = \frac{(3(3)^2+4)(4) - 4(3)(6(3))}{(3(3)^2+4)^2}$$
$$= -\frac{92}{961}$$
$C'(3)$ represents the rate of change of concentration of the antibiotic in the blood at 3 h.

Chapter 2 Section 5 Extension Question 10 Page 129

a) $$c'(t) = \frac{(2t^2+9)(6) - 6t(4t)}{(2t^2+9)^2}$$
$$= \frac{54-12t^2}{(2t^2+9)^2}$$

$$c'(1) = \frac{54-12(1)^2}{(2(1)^2+9)^2}$$
$$= \frac{42}{121}$$

The rate is $\frac{42}{121}$ g/L per day.

$$c'(4) = \frac{54-12(4)^2}{(2(4)^2+9)^2}$$
$$= -\frac{138}{1681}$$
The rate is $-\frac{138}{1681}$ g/L per day.

$$c'(7) = \frac{54 - 12(7)^2}{(2(7)^2 + 9)^2}$$
$$= -\frac{534}{11\,449}$$

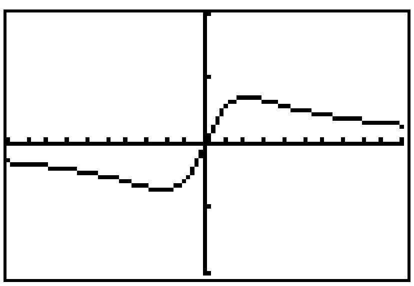
The rate is $-\frac{534}{11\,449}$ g/L per day.

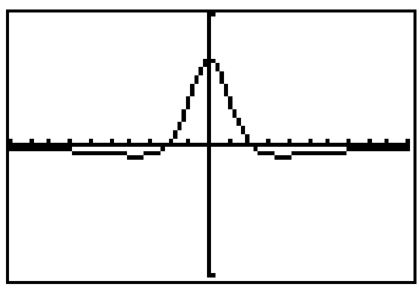
b) Find t when $c'(t) = 0$.

$$0 = \frac{54 - 12t^2}{(2t^2 + 9)^2}$$
$$0 = 54 - 12t^2$$
$$t = \frac{3}{\sqrt{2}}$$

The rate is positive for $0 \le t < \frac{3}{\sqrt{2}}$ days and negative for $t > \frac{3}{\sqrt{2}}$ days.

c)





d) The concentration of the cleaner rises to a maximum at $t = \frac{3}{\sqrt{2}}$ days and then starts to decrease.

e) $$c''(t) = \frac{(2t^2 + 9)^2(-24t) - (54 - 12t^2)(2)(2t^2 + 9)(4t)}{(2t^2 + 9)^4}$$
$$c''(4) = \frac{(2(4)^2 + 9)^2(-24(4)) - (54 - 12(4)^2)(2)(2(4)^2 + 9)(4(4))}{(2(4)^2 + 9)^4}$$
$$= \frac{480}{68\,921}$$

$c''(4)$ represents the rate of dissipation of the cleaner at 4 days.

Chapter 2 Section 6 **Rate of Change Problems**

Chapter 2 Section 6 **Question 1 Page 137**

a) $R(x) = xp(x)$
$= 575\sqrt{x} - 3x$

b) $R'(x) = 575(0.5)x^{-0.5} - 3$
$= \dfrac{575}{2\sqrt{x}} - 3$

c) $R'(200) = \dfrac{575}{2\sqrt{200}} - 3$
$= 17.33$

When 200 DVD's are sold, the marginal revenue is \$17.33 per DVD.

Chapter 2 Section 6 **Question 2 Page 137**

a) $P(x) = R(x) - C(x)$
$= (575\sqrt{x} - 3x) - (2000 + 150x - 0.002x^2)$
$= 0.002x^2 - 153x + 575\sqrt{x} - 2000$

b) $P'(x) = 0.004x + \dfrac{575}{2\sqrt{x}} - 153$

c) $P'(500) = 0.004(500) + \dfrac{575}{2\sqrt{500}} - 153$
$= -138.14$

When 500 DVD's are sold, the marginal profit is –\$138.14 per DVD.

Chapter 2 Section 6 **Question 3 Page 138**

a) $R(x) = xp(x)$
$= 17.5x$

b) $R'(x) = 17.5$

c) $P(x) = R(x) - C(x)$
$= (17.5x) - (-0.001x^3 + 0.025x^2 + 4x)$
$= 0.001x^3 - 0.025x^2 + 13.5x$

d) $P'(x) = 0.003x^2 - 0.05x + 13.5$

e) $R'(300) = 17.50$

$$P'(300) = 0.003(300)^2 - 0.05(300) + 13.5$$
$$= 268.50$$

When 300 large pizza combos are sold, the marginal revenue is \$17.50 per combo and the marginal profit is \$268.50 per combo.

Chapter 2 Section 6 **Question 4 Page 138**

a) $$\text{Average linear density} = \frac{f(8) - f(1)}{8 - 1}$$
$$= \frac{\sqrt{2(8)-1} - \sqrt{2(1)-1}}{8-1}$$
$$= 0.41$$

The average linear density is 0.41 g/m.

b) $$f'(x) = 0.5(2x-1)^{-0.5}(2)$$
$$= \frac{1}{\sqrt{2x-1}}$$
$$f'(5) = \frac{1}{3},\ f'(8) = \frac{1}{\sqrt{15}}$$

The linear mass density of the wire is $\frac{1}{3}$ g/m at x = 5 m and $\frac{1}{\sqrt{15}}$ g/m at x = 8 m.

The density of the wire at x = 8 m is less than the density at x = 5 m, so the density of the wire decreases as the distance increases.

Chapter 2 Section 6 **Question 5 Page 138**

a) Let n represent the number of price decreases, and x represent the number of cans sold.

$$x = 270 + 6n$$
$$p(x) = 32 - 1.2n$$
$$= 32 - 1.2\left(\frac{x-270}{6}\right)$$
$$= 86 - 0.2x$$

b) $$R(x) = xp(x)$$
$$= 86x - 0.2x^2$$

c) $R'(x) = 86 - 0.4x$

d) Solve $R'(x) = 0$.

$$0 = 86 - 0.4x$$
$$x = 215$$

When 215 cans per month are sold, revenue is at a maximum.

e) Solve for $p(215)$.

$$p(215) = 86 - 0.2(215)$$
$$= 43$$

The price in this case is \$43.00. This is the price of at which the revenue will be a maximum.

Chapter 2 Section 6 **Question 6 Page 138**

a)
$$\text{Average linear density} = \frac{f(3) - f(1)}{3 - 1}$$
$$= \frac{(3-0.5)^3 + 5(3) - [(1-0.5)^3 + 5(1)]}{2}$$
$$= 12.75$$

The average linear density is 12.75 kg/m.

b)
$$f'(x) = 3(x - 0.5)^2 + 5$$
$$f'(2) = 3(2 - 0.5)^2 + 5$$
$$= 11.75$$

The linear mass density of the rod at $x = 2$ m is 11.75 kg/m.

Chapter 2 Section 6 **Question 7 Page 138**

a) **i)** The CPI is increasing since the slope of the graph is positive.

ii) The rate of growth during this period is positive since the slope of the graph is positive.

b) i, ii, vi, v, iii, iv
I ordered the intervals from the least steep to the interval with the steepest slope.
A to B: 1951 – 1961; B to C: 1961 – 1971; C to D: 1971 – 1981; D to E: 1981 – 1983;
E to F: 1983 – 1991; F to G: 1991 – 2007

c) The rate of inflation is higher after 1975 because the slope of the graph is greater after 1975. I can conclude that the economy was doing well.

d) The rate of inflation has been decreasing since the slope of the graph begins to decrease at this time.

Chapter 2 Section 6 **Question 8 Page 139**

a) Find $C'(4000)$.

$$C'(x) = 0.0002x + 2$$
$$C'(4000) = 2.80$$

The marginal cost of producing 4000 containers of yogurt is \$2.80 per container.

b) Find $P'(4000)$.

$$\begin{aligned} P(x) &= R(x) - C(x) \\ &= 4.5x - (0.0001x^2 + 2x + 3200) \\ P'(x) &= 2.5 - 0.0002x \end{aligned}$$

$$P'(4000) = 1.70$$

The marginal profit from producing 4000 containers of yogurt is \$1.70 per container.

c) Since the revenue for selling x containers of yogourt is $R(x) = 4.5x$, the price of one container is \$4.50.

Chapter 2 Section 6 **Question 9 Page 139**

a) Determine $C'(5)$.

$$\begin{aligned} C'(5) &= 5 + 40 \\ &= 45 \end{aligned}$$

The marginal cost of producing 5000 kitchen utensils is \$45 per utensil.

$$\begin{aligned} C(5.001) - C(5.000) &= 0.5(5.001)^2 + 40(5.001) + 8000 - (0.5(5.000)^2 + 40(5.000) + 8000) \\ &= 0.045 \end{aligned}$$

The cost of producing the 5001st item is \$0.045.

b)
$$\begin{aligned} \text{Average cost} &= \frac{C(5)}{5000} \\ &= \frac{0.5(5)^2 + 200 + 8000}{5000} \\ &= 1.64 \end{aligned}$$

The average cost of producing 5000 items is \$1.64 per item.

c) Let $A(x)$ represent the average cost, so find $A'(5)$.

$$\begin{aligned} A(x) &= \frac{C(x)}{1000x} \\ &= \frac{0.5x^2 + 40x + 8000}{1000x} \\ &= \frac{x}{2000} + \frac{1}{25} + \frac{8}{x} \\ A'(x) &= \frac{1}{2000} - \frac{8}{x^2} \end{aligned}$$

$$A'(5) = -0.32$$

The rate of change of the average cost of producing 5000 items is –\$0.32 per item.
This is the change in cost of producing an additional item.

Chapter 2 Section 6 **Question 10 Page 139**

a) $P(0) = 12\,500$; The present population is 12 500 people.

b) $P'(x) = 320 - 0.75x^2$

$P'(3) = 313.25;\ P'(8) = 272$

The rate of change of the population in 3 years is 313.25 people/year and 272 people/year in 8 years.

c) Find x when $P(x) = 16\,294$.

$$16\,294 = 12\,500 + 320x - 0.25x^3$$
$$0 = 0.25x^3 - 320x + 3794$$

Using CAS or a graphing calculator, the positive solutions are $x = 14,\ x = (\sqrt{133} - 7)$ or 26.66.

The population will reach 16 294 after 14 and 26.66 years.

d) Set $P'(x) = 245$.

$$245 = 320 - 0.75x^2$$
$$x = \pm 10$$

The rate of growth of the population will be 245 people/year after 10 years.

e) First find when $P'(x) = 0$.

$$0 = 320 - 0.75x^2$$
$$x = 20.66$$

If $0 \le x < 20.66$ years, then $P'(x) > 0$ and if $x > 20.66$ years, then $P'(x) < 0$.

The population will increase up to 20.66 years and then will start to decrease.

Chapter 2 Section 6 **Question 11 Page 139**

a) Find $C'(750)$.

$$C'(x) = 3450 - 2.04x$$
$$C'(750) = 1920$$

The marginal cost at a production level of 750 hot tubs is \$1920 per hot tub. The marginal cost shows that the rate of change in cost of producing x items reduces for greater values of x.

b) $C(751) - C(750) = 3450(751) - 1.02(751^2) - (3450(750) - 1.02(750^2))$

$= 1918.98$

The cost of producing the 751st tub is \$1918.98.

c) The cost of the 751st hot tub is less than the marginal cost at 750 tubs.

d) $R(x) = 9200x$

e) Find $P'(750)$.

$$\begin{aligned} P(x) &= R(x) - C(x) \\ &= 9200x - (3450x - 1.02x^2) \\ &= 5750x + 1.02x^2 \\ P'(x) &= 5750 + 2.04x \\ P'(750) &= 7280 \end{aligned}$$

The rate of change of profit for the sale of 750 hot tubs is \$7280 per hot tub.

Chapter 2 Section 6 Question 12 Page 139

Find $I'(18)$.

$$\begin{aligned} I'(R) &= -\frac{120}{R^2} \\ I'(18) &= -\frac{120}{18^2} \\ &= -\frac{10}{27} \end{aligned}$$

The rate of change of the current when the resistance is 18 Ω is $-\frac{10}{27}$ A/Ω.

Chapter 2 Section 6 Question 13 Page 139

$$\begin{aligned} F &= 1.8(10 + 4t) + 32 \\ F &= 7.2t + 50 \\ F'(t) &= 7.2 \end{aligned}$$

The rate of change of the temperature of the bar is 7.2ºF/min for any t, so at $t = 4$ min too.

Chapter 2 Section 6 Question 14 Page 139

$$\begin{aligned} f'(x) &= \frac{(3x^{-0.5} + 18)(155(-0.5)x^{-1.5}) - (155x^{-0.5} + 85)(3)(-0.5)x^{-1.5}}{\left(3x^{-0.5} + 18\right)^2} \\ &= \frac{-3(155)(0.5)x^{-2} - 9(155)x^{-1.5} + 3(155)(0.5)x^{-2} + 85(3)(0.5)x^{-1.5}}{\left(3x^{-0.5} + 18\right)^2} \\ &= \frac{-1267.5x^{-1.5}}{\left(3x^{-0.5} + 18\right)^2} \\ &= -\frac{2535}{2x^{1.5}\left(3x^{-0.5} + 18\right)^2} \end{aligned}$$

This expression is always negative in value. The pupil gets smaller as it is exposed to more light.

Chapter 2 Section 6 **Question 15 Page 139**

a) Find $R'(1000)$ and $R'(5000)$.

$$R'(x) = \frac{(x^2+15)(15-2x)-(15x-x^2)(2x)}{(x^2+15)^2}$$
$$= \frac{-15x^2-30x+225}{(x^2+15)^2}$$

$R'(1000) = -0.000\,015$

$R'(5000) = -0.000\,000\,6$

The rate of change for the sale of 1000 items is \$–0.015 per item and is \$–0.0006 for 5000 items.

b) The rate of change of the revenue decreases with the increase in sales.

c) Find x for $R'(x) = 0$.

$$0 = \frac{-15x^2-30x+225}{(x^2+15)^2}$$

$0 = -15x^2 - 30x + 225$

$0 = (x-3)(x+5)$

$x = 3$ or $x = -5$

Since x cannot be negative, 3 items must be sold to obtain a \$0 rate of change in revenue.

d) Find $R(3)$.

$$R(3) = \frac{15(3)-3^2}{3^2+15}$$
$$= 1.5$$

The revenue for 3 items sold is \$1500.

The number of items that need to be sold in order to maximize revenue is 3 items.

Chapter 2 Section 6 **Question 16 Page 140**

a) Find $V'(2.75)$.

$V'(r) = 5cr - 3cr^2$

$V'(2.75) = -8.9375c$

The rate of change of velocity of air when $r = 2.75$ cm is $8.9375c$.

b) Find r for $V'(r) = 0$.

$$0 = 5cr - 3cr^2$$
$$0 = cr(5 - 3r)$$
$$r = 0 \text{ or } r = \frac{5}{3}$$

Since the radius of the windpipe is 2.5 cm when there is no cough, it cannot have a radius of $\frac{5}{3}$ cm so $r = 0$. This means that the rate of change of airflow is zero when the windpipe is closed.

Chapter 2 Section 6 Question 17 Page 140

a) Let n represent the number of price decreases, and x represent the number of lattes sold.

$$x = 500 + 125n$$
$$\begin{aligned} p(x) &= 4.75 - 0.25n \\ &= 4.75 - 0.25\left(\frac{x - 500}{125}\right) \\ &= 5.75 - 0.002x \end{aligned}$$

b)
$$\begin{aligned} R(x) &= xp(x) \\ &= 5.75x - 0.002x^2 \\ R'(x) &= 5.75 - 0.004x \\ R(350) &= 5.75(350) - 0.002(350)^2 \\ &= 1767.50 \\ R'(350) &= 5.75 - 0.004(350) \\ &= 4.35 \end{aligned}$$

The revenue and marginal revenue from the monthly sales of 350 mocha lattes is \$1767.50 and \$4.35 per latte, respectively.

c)
$$\begin{aligned} C'(x) &= -0.001x + 3.5 \\ C'(350) &= -0.001(350) + 3.5 \\ &= 3.15 \end{aligned}$$

The marginal cost of producing 350 lattes is \$3.15 per latte.

d)
$$\begin{aligned} C(351) - C(350) &= -0.0005(351^2 - 350^2) + 3.5(1) \\ &= -0.0005(701) + 3.5 \\ &= 3.1495 \end{aligned}$$

The actual cost of producing the 351st latte is \$3.1495.

e)

$$\begin{aligned} P(x) &= R(x) - C(x) \\ &= (5.75x - 0.002x^2) - (-0.0005x^2 + 3.5x + 400) \\ &= -0.0015x^2 + 2.25x - 400 \end{aligned}$$

$$P(350) = 203.75$$

$$\begin{aligned} P'(x) &= -0.003x + 2.25 \\ P'(350) &= -0.003(350) + 2.25 \\ &= 1.20 \end{aligned}$$

The profit and marginal profit from selling 350 lattes is \$203.75 and \$1.20 per latte respectively.

f) $$\begin{aligned} \text{Average revenue} &= \frac{R(360)}{360} \\ &= \frac{5.75(360) - 0.002(360)^2}{360} \\ &= 5.03 \end{aligned}$$

$$\begin{aligned} \text{Average profit} &= \frac{P(360)}{360} \\ &= \frac{-0.0015(360)^2 + 2.25(360) - 400}{360} \\ &= 0.60 \end{aligned}$$

The profit is much lower at \$0.60 than the revenue of \$5.03 due to the cost of producing the mocha lattes.

Chapter 2 Section 6 **Question 18 Page 140**

a) Determine $M'(6)$.

$$\begin{aligned} M'(t) &= \frac{(t + 2.2)(6.3) - 6.3t(1)}{(t + 2.2)^2} \\ &= \frac{13.86}{(t + 2.2)^2} \\ M'(6) &= \frac{13.86}{(6 + 2.2)^2} \\ &= 0.206 \end{aligned}$$

The rate of change of the mass after 6 s is 0.206 g/s.

b) No. The numerator and denominator of $M'(t)$ are always positive, so $M'(t)$ is always positive.

Chapter 2 Section 6 **Question 19 Page 140**

a) $R(x) = xp(x)$
$= 650\sqrt{x} - 4.5x$

b) $R'(x) = (0.5)650x^{-0.5} - 4.5$
$= \frac{325}{\sqrt{x}} - 4.5$
$R'(500) = \frac{325}{\sqrt{500}} - 4.5$
$= 10.03$

The marginal revenue for the sale of 500 PDAs is \$10.03 per PDA.

c) $P(x) = R(x) - C(x)$
$= (650\sqrt{x} - 4.5x) - 125x$
$= 650\sqrt{x} - 129.5x$

d) $P'(x) = (0.5)650x^{-0.5} - 129.5$
$= \frac{325}{\sqrt{x}} - 129.5$
$P'(500) = \frac{325}{\sqrt{500}} - 129.5$
$= -114.97$

The marginal profit for the sale of 500 PDAs is –\$114.97 per PDA.

Chapter 2 Section 6 **Question 20 Page 140**

$$\frac{dr}{dm} = \frac{2m}{a}\left(\frac{1}{b} - \frac{m}{c}\right) + \frac{m^2}{a}\left(-\frac{1}{c}\right)$$
$$= \frac{2m}{ab} - \frac{3m^2}{ac}$$
$$\left.\frac{dr}{dm}\right|_{m=15} = \frac{2(15)}{ab} - \frac{3(15)^2}{ac}$$
$$= \frac{15(2c - 45b)}{abc}$$

Chapter 2 Section 6 **Question 21 Page 141**

a) $$\frac{du}{dM}=\frac{(M+m)(V(1+c)+vc)-[MV(1+c)+v(cM-m)(1)]}{(M+m)^2}$$

$$=\frac{MV(1+c)+Mvc+mV(1+c)+mvc-MV(1+c)-vcM+vm}{(M+m)^2}$$

$$=\frac{V(1+c)m+cvm+vm}{(M+m)^2} \text{ as required}$$

b) i) Find $\frac{du}{dM}$ using $m=150$ g, $c=0.575$, $v=40$ m/s, and $V=-35$ m/s.

$$\frac{du}{dM}=\frac{-35(1+0.575)(150)+(150)(40)(0.575)+40(150)}{(M+150)^2}$$

$$=\frac{1181.25}{(M+150)^2}$$

ii) Find $\frac{du}{dM}$, where M = 1050 g.

$$\left.\frac{du}{dM}\right|_{M=1050}=\frac{1181.25}{(1050+150)^2}$$

$$=0.000\,82$$

The rate of change of the velocity of the ball is 0.000 82 m/s^2.

Chapter 2 Section 6 **Question 22 Page 141**

D

Chapter 2 Section 6 **Question 23 Page 141**

C

Chapter 2 Review

Chapter 2 Review **Question 1 Page 142**

a) $h'(t) = 3t^2 - 4t - \dfrac{2}{t^3}$

Use the sum rule and the power rule.

b) $p(n) = -n^5 + 5n^3 + n^{\frac{2}{3}}$

$p'(n) = -5n^4 + 15n^2 + \dfrac{2}{3\sqrt[3]{n}}$

Use the sum rule and the power rule.

c) $p(r) = r^6 - \dfrac{2}{5}r^{-\frac{1}{2}} + r - 1$

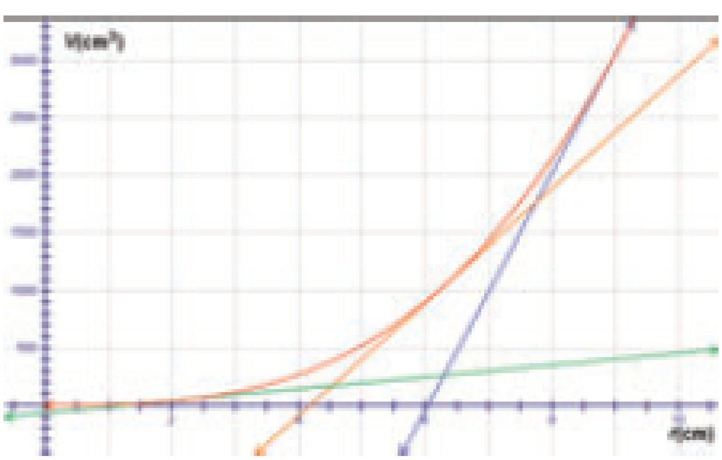
$p'(r) = 6r^5 + \dfrac{1}{5\sqrt{r^3}} + 1$

Use the sum rule, the power rule, and the constant rule.

Chapter 2 Review **Question 2 Page 142**

a)
$$\begin{aligned} V'(r) &= 4\pi r^2 \\ V'(1.5) &= 4\pi(1.5)^2 \\ &= 9\pi \\ V'(6) &= 4\pi(6)^2 \\ &= 144\pi \\ V'(9) &= 4\pi(9)^2 \\ &= 324\pi \end{aligned}$$

b)



c) If $r = 1.5$, then $V(1.5) = 4.5\pi$. Use $(1.5, 4.5\pi)$ and $m = 9\pi$ to find b.

$4.5\pi = 9\pi(1.5) + b$

$b = -9\pi$

So $y_{1.5} = 9\pi x - 9\pi$ or $y_{1.5} = 28.27x - 28.27$

If $r = 6$, then $V(6) = 288\pi$. Use $(6, 288\pi)$ and $m = 144\pi$ to find b.

$288\pi = 144\pi(6) + b$

$b = -576\pi$

So $y_6 = 144\pi x - 576\pi$ or $y_6 = 452.39x - 1809.56$

If $r = 9$, then $V(9) = 972\pi$. Use $(9, 972\pi)$ and $m = 324\pi$ to find b.

$972\pi = 324\pi(9) + b$

$b = -1944\pi$

So $y_9 = 324\pi x - 1944\pi$ or $y_9 = 1017.88x - 6107.26$

Chapter 2 Review **Question 3 Page 142**

a)
$$\begin{aligned} f(x) &= (5x+3)(2x-11) \\ &= 10x^2 - 49x - 33 \\ f'(x) &= 20x - 49 \end{aligned}$$

b)
$$\begin{aligned} h(t) &= 8t^3 - 10t^2 + 4t^{\frac{4}{3}} - 5t^{\frac{1}{3}} \\ h'(t) &= 24t^2 - 20t + \frac{16}{3}t^{\frac{1}{3}} - \frac{5}{3}t^{-\frac{2}{3}} \\ &= \frac{72t^{\frac{8}{3}} - 60t^{\frac{5}{3}} + 16t - 5}{3t^{\frac{2}{3}}} \end{aligned}$$

c)
$$\begin{aligned} g(x) &= -4.5x^6 + 3 + 12x^7 - 8x \\ g'(x) &= 84x^6 - 27x^5 - 8 \end{aligned}$$

d)
$$\begin{aligned} p(n) &= -55n + 33n^3 - 10 + 6n^2 \\ p'(n) &= 99n^2 + 12n - 55 \end{aligned}$$

Chapter 2 Review **Question 4 Page 142**

a)
$$\begin{aligned} y' &= (6x-3)(-2x) + (6)(-x^2+2) \\ &= -18x^2 + 6x + 12 \\ y'(1) &= -18(1)^2 + 6(1) + 12 \\ &= 0 \end{aligned}$$

When $x = 1$, $y = 3$ so use the point $(1, 3)$ and $m = 0$ in $y = mx + b$ to find b.

$3 = 0(1) + b$ so $b = 3$.

The equation of the tangent to y at $x = 1$ is $y = 3$.

b) $y' = (-3x+8)(3x^2) + (-3)(x^3 - 7)$

$= -12x^3 + 24x^2 + 21$

$y'(2) = -12(2)^3 + 24(2)^2 + 21$

$= 21$

When $x = 2$, $y = 2$ so use the point (2, 2) and $m = 21$ in $y = mx + b$ to find b.

$2 = 21(2) + b$ so $b = -40$.

The equation of the tangent to y at $x = 2$ is $y = 21x - 40$.

Chapter 2 Review **Question 5 Page 142**

$f'(x) = (-2x)(3x+1) + 3(4 - x^2)$

$= -9x^2 - 2x + 12$

$f''(x) = -18x - 2$

$f''(-2) = 34$

Chapter 2 Review **Question 6 Page 142**

a) $h(2) = -4.9(2)^2 + 15(2) + 0.4$

$= 10.8$

The height of the missile after 2 s is 10.8 m.

b) Find $h'(1)$ and $h'(4)$.

$h'(t) = -9.8t + 15$

$h'(1) = 5.2$

$h'(4) = -24.2$

The rate of change of the height of the missile at 1 s is 5.2 m/s and –24.2 m/s at 4 s.

c) When the missile hits the ground $h(t) = 0$.

$0 = -4.9t^2 + 15t + 0.4 = 0$

$$t = \frac{-15 \pm \sqrt{15^2 - 4(-4.9)(0.4)}}{2(-4.9)}$$

$t = 3.088$ or $t = -0.27$

Since time cannot be negative, the toy missile returns to the ground when $t = 3.088$ s.

d) Find $h'(3.088)$.

$h'(3.088) = -9.8(3.088) + 15$

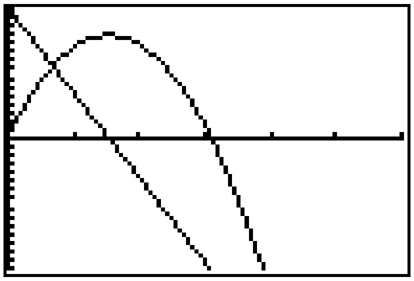
$= -15.26$

The missile was travelling –15.26 m/s when it hit the ground.

e) i) The missile reach it's maximum height when $t = 1.53$ s.

ii) The maximum height of the missile is 11.88 m.

iii) The velocity of the missile when it reaches its maximum height is 0 m/s. I can tell since the graph of the velocity is zero at this point.



Chapter 2 Review **Question 7 Page 142**

a) Find $p'(5)$.

$$p'(t)=\frac{1}{3}(16t+50t^3)^{-\frac{2}{3}}(16+150t^2)$$

$$p'(5)=\frac{(16(5)+50(5)^3)^{-\frac{2}{3}}(16+150(5)^2)}{3}$$
$$\doteq 3.67$$

The rate is 3.67 bushes/year.

b) Set $p(t)=40$ and solve for t.

$$40=(16t+50t^3)^{\frac{1}{3}}$$
$$40^3=16t+50t^3$$
$$0=50t^3+16t-64000$$

Solving by using CAS or a graphing calculator, t = 10.85 years.

c) Solve for $p'(10.85)$.

$$p'(10.85)=\frac{(16(10.85)+50(10.85)^3)^{-\frac{2}{3}}(16+150(10.85)^2)}{3}$$
$$=3.68$$

The rate is 3.68 bushes/year.

Chapter 2 Review **Question 8 Page 142**

a) $$\frac{dy}{dx}=\frac{dy}{du}\cdot\frac{du}{dx}$$
$$=(2u+3)\left(\frac{1}{2}(x-1)^{-\frac{1}{2}}\right)$$

When $x = 5$, $u = 2$.

$$\left.\frac{dy}{dx}\right|_{x=5} = (2(2)+3)\left(\frac{1}{2}(5-1)^{-\frac{1}{2}}\right)$$
$$= \frac{7}{4}$$

b) $$\frac{dy}{dx} = \frac{dy}{du}\cdot\frac{du}{dx}$$
$$= \frac{\sqrt{2}}{2}(u)^{-\frac{1}{2}}(-1)$$

When $x = -3$, $u = 9$.

$$\left.\frac{dy}{dx}\right|_{x=-3} = -\frac{\sqrt{2}}{2}(9)^{-\frac{1}{2}}$$
$$= -\frac{1}{3\sqrt{2}}$$

c) $$\frac{dy}{dx} = \frac{dy}{du}\cdot\frac{du}{dx}$$
$$= (8(1-u)-8u)(-x^{-2})$$

When $x = 4$, $u = \frac{1}{4}$.

$$\left.\frac{dy}{dx}\right|_{x=4} = (8(1-0.25)-8(0.25))(-(4)^{-2})$$
$$= -\frac{1}{4}$$

Chapter 2 Review **Question 9 Page 142**

a) $$y' = 2x^2(-1)(x+1)^{-2} + 4x(x+1)^{-1}$$
$$y'(2) = 2(2)^2(-1)(2+1)^{-2} + 4(2)(2+1)^{-1}$$
$$= \frac{16}{9}$$

The slope of the tangent at $x = 2$ is $\frac{16}{9}$.

b) $y' = (3x)^{0.5}(-1)(x^2-4)^{-2}(2x) + 3.5(3x)^{-0.5}(x^2-4)^{-1}$

$$\begin{aligned} y'(3) &= (3(3))^{0.5}(-1)(3^2-4)^{-2}(2(3)) + 1.5(3(3))^{-0.5}(3^2-4)^{-1} \\ &= -\frac{31}{50} \end{aligned}$$

The slope of the tangent at $x = 3$ is $-\frac{31}{50}$.

c) $y' = (5x+3)(-1)(x^3+1)^{-2}(3x^2) + 5(x^3+1)^{-1}$

$$\begin{aligned} y'(-2) &= (5(-2)+3)(-1)((-2)^3+1)^{-2}(3(-2)^2) + 5((-2)^3+1)^{-1} \\ &= 1 \end{aligned}$$

The slope of the tangent at $x = -2$ is 1.

d) $y'(x) = (-4x+2)(-1)(3x^2-7x-1)^{-2}(6x-7) - 4(3x^2-7x-1)^{-1}$

$$\begin{aligned} y'(1) &= (-4(1)+2)(-1)(3(1)^2-7(1)-1)^{-2}(6(1)-7) - 4(3(1)^2-7(1)-1)^{-1} \\ &= \frac{18}{25} \end{aligned}$$

The slope of the tangent at $x = 1$ is $\frac{18}{25}$.

Chapter 2 Review **Question 10 Page 143**

a)
$$\begin{aligned} q'(x) &= \frac{(4x^2-3)^3(-7) - (-7x+2)(3)(4x^2-3)^2(8x)}{(4x^2-3)^6} \\ &= \frac{(4x^2-3)(-7) - (-7x+2)(24x)}{(4x^2-3)^4} \\ &= \frac{140x^2-48x+21}{(4x^2-3)^4} \end{aligned}$$

b)
$$\begin{aligned} \frac{dy}{dx} &= \frac{(3x-2)^{\frac{1}{2}}(24x^2) - (8x^3)\left(\frac{1}{2}\right)(3x-2)^{-\frac{1}{2}}(3)}{(3x-2)} \\ &= \frac{(3x-2)(24x^2) - 12x^3}{(3x-2)^{\frac{3}{2}}} \\ &= \frac{12x^2(5x-4)}{(3x-2)^{\frac{3}{2}}} \end{aligned}$$

c) $m'(x) = \dfrac{(3+5x)^4(2(-x+2))(-1) - (-x+2)^2(4)(3+5x)^3(5)}{(3+5x)^8}$

$= \dfrac{(3+5x)(2x-4) - 20(-x+2)^2}{(3+5x)^5}$

$= \dfrac{-2(x-2)(5x-23)}{(3+5x)^5}$

d) $\dfrac{dy}{dx} = \dfrac{(4x+5)^{\frac{1}{2}}(2(x^2-3)(2x)) - (x^2-3)^2\left(\frac{1}{2}\right)(4x+5)^{-\frac{1}{2}}(4)}{(4x+5)}$

$= \dfrac{(4x+5)(4x)(x^2-3) - 2(x^2-3)^2}{(4x+5)^{\frac{3}{2}}}$

$= \dfrac{2(x^2-3)(7x^2+10x+3)}{(4x+5)^{\frac{3}{2}}}$

e) $\dfrac{dy}{dx} = \dfrac{(x^3-3x^2+1)^7(3)(2x^{\frac{1}{2}}+7)^2(x^{-\frac{1}{2}}) - (2x^{\frac{1}{2}}+7)^3(7)(x^3-3x^2+1)^6(3x^2-6x)}{(x^3-3x^2+1)^{14}}$

$= \dfrac{(2x^{\frac{1}{2}}+7)^2[(x^3-3x^2+1)(3)(x^{-\frac{1}{2}}) - 7(2x^{\frac{1}{2}}+7)^3(3x^2-6x)]}{(x^3-3x^2+1)^8}$

$= \dfrac{3(2x^{\frac{1}{2}}+7)^2[x^3-3x^2+1-14x^3+28x^2-49x^{\frac{5}{2}}+98x^{\frac{3}{2}}]}{x^{\frac{1}{2}}(x^3-3x^2+1)^8}$

$= \dfrac{3(2x^{\frac{1}{2}}+7)^2[-13x^3+25x^2+1-49x^{\frac{5}{2}}+98x^{\frac{3}{2}}]}{x^{\frac{1}{2}}(x^3-3x^2+1)^8}$

Chapter 2 Review **Question 11 Page 143**

$y' = \dfrac{3(x^2-1)^2(2x)(4x+7) - (x^2-1)^3(3)(4x+7)^2(4)}{(4x+7)^6}$

$y'(-2) = \dfrac{3((-2)^2-1)^2(2(-2))(4(-2)+7) - ((-2)^2-1)^3(3)(4(-2)+7)^2(4)}{(4(-2)+7)^6}$

$= -216$

When $x = -2$, $y = -27$ so use the point (–2, –27) and $m = -216$ to find b in $y = mx + b$.
$-27 = -216(-2) + b$ so $b = -459$.

The equation of the tangent to the curve at $x = -2$ is $y = -216x - 459$.

Chapter 2 Review **Question 12 Page 143**

a) Let x be the number of CD's sold and n be the number of price decreases.

$$x = 120 + 5n$$

$$n = \frac{x-120}{5}$$

$$\begin{aligned} p(x) &= 24 - 0.75n \\ &= 24 - 0.75\left(\frac{x-120}{5}\right) \\ &= 42 - 0.15x \end{aligned}$$

b)

$$\begin{aligned} R(x) &= xp(x) \\ &= 42x - 0.15x^2 \\ R'(x) &= 42 - 0.3x \\ R'(150) &= -3.00 \end{aligned}$$

The marginal revenue from the weekly sales of 150 music CDs is –\$3.00 per CD.

c) Find $C'(150)$.

$$\begin{aligned} C'(x) &= -0.006x + 4.2 \\ C'(150) &= -0.006(150) + 4.2 \\ &= 3.30 \end{aligned}$$

The marginal cost of producing 150 CDs is \$3.30 per CD.

d)

$$\begin{aligned} P(x) &= R(x) - C(x) \\ &= (42x - 0.15x^2) - (-0.003x^2 + 4.2x + 3000) \\ &= -0.147x^2 + 37.8x - 3000 \\ P'(x) &= -0.294x + 37.8 \\ P'(150) &= -6.30 \end{aligned}$$

The marginal profit from the sales of 150 CDs is –\$6.30 per CD.

Chapter 2 Review **Question 13 Page 143**

a)

$$\begin{aligned} V(t) &= I(t)R(t) \\ &= (4.85 - 0.01t^2)(15 + 0.11t) \end{aligned}$$

b) Find $V'(t)$.

$$\begin{aligned} V'(t) &= (4.85 - 0.01t^2)(0.11) + (15 + 0.11t)(-0.02t) \\ &= -0.0033t^2 - 0.3t + 0.5335 \end{aligned}$$

This expression represents the change in voltage over time.

c) Find $V'(2)$.

$$\begin{aligned} V'(2) &= -0.0033(2)^2 - 0.3(2) + 0.5335 \\ &= -0.0797 \end{aligned}$$

The rate of change of voltage after 2 s is –0.0797 V/s.

d) Find $I'(2)$.

$$I'(t) = -0.02t$$

$$I'(2) = -0.04$$

The rate of change of current after 2 s is 0.04 A/s.

e) Find $R'(2)$.

$$R'(t) = 0.11$$

$$R'(2) = 0.11$$

The rate of change of resistance after 2 s is 0.11 Ω/s.

f) No. By the product rule: $V'(2) = I'(2)R(2) + I(2)R'(2) \neq I'(2)R'(2)$.

Chapter 2 Practice Test

Chapter 2 Practice Test Question 1 Page 144

B; Justifications may vary. For example:

$$\frac{d}{dx}\left(x^2+2x\right)^3 = 3(x^2+2x)^2(2x+2)$$
$$\frac{d}{dx}\left(x^3\right)\frac{d}{dx}\left(x^2+2x\right) = 3x^2(2x+2)$$
$$\frac{d}{dx}\left(x^2+2x\right)^3 \neq \frac{d}{dx}\left(x^3\right)\frac{d}{dx}\left(x^2+2x\right)$$

Chapter 2 Practice Test Question 2 Page 144

C; Answer may vary. For example:

In case A, if the velocity is positive and the acceleration is negative, then the product $a(t)v(t)$ is negative. However, an object with negative acceleration is slowing down so A is incorrect.

In case B, if the velocity is positive and the acceleration is positive, then the product $a(t)v(t)$ is positive. However, an object with positive acceleration is speeding up so B is incorrect.

In case D, an object with an acceleration of zero can be travelling at a constant speed so it does not have to be at rest. D is incorrect.

Chapter 2 Practice Test Question 3 Page 144

A and B are incorrect derivatives. Justifications may vary. For example:

In case A, the derivative of y is expressed as the derivative of the numerator divided by the derivative of the denominator, which is incorrect when taking the derivative of a quotient.

In case B, the quotient rule has been applied but the sign in front of the $4x(2x)$ should be a +.

Cases C and D are correct derivatives because one uses the quotient rule properly and the other uses the product rule.

Chapter 2 Practice Test Question 4 Page 144

$$\begin{aligned} f'(x) &= 2(5x^2-3x)(10x-3) \\ f''(x) &= 2[(5x^2-3x)(10)+(10x-3)(10x-3)] \\ f''(3) &= 2[360+27^2] \\ &= 2178 \end{aligned}$$

Chapter 2 Practice Test **Question 5 Page 144**

a) Use the power of a function rule.

$$\begin{aligned}\frac{dy}{dx} &= \frac{1}{3}(3x^6)^{-\frac{2}{3}}(18x^5) \\ &= \frac{6x^5}{(3x^6)^{\frac{2}{3}}} \\ &= 2\cdot 3^{\frac{1}{3}}x\end{aligned}$$

Alternatively, evaluate the equation and use the power rule.

$$y = 3^{\frac{1}{3}}x^2$$

$$\frac{dy}{dx} = 2\cdot 3^{\frac{1}{3}}x$$

b) Evaluate and use the sum rule.

$$y = 2x^3 - 8x + x^2 - 4$$

$$\frac{dy}{dx} = 6x^2 + 2x - 8$$

Alternatively, use the product rule.

$$\begin{aligned}\frac{dy}{dx} &= (x^2-4)(2) + 2x(2x+1) \\ &= 6x^2 + 2x - 8\end{aligned}$$

Chapter 2 Practice Test **Question 6 Page 144**

a) $$\begin{aligned}\frac{dy}{dx} &= -15x^2 - \frac{20}{x^6} \\ &= \frac{-5(3x^8+4)}{x^6}\end{aligned}$$

b) $$\begin{aligned}g'(x) &= 3(8x^2-3x)^2(16x-3) \\ &= 3x^2(8x-3)^2(16x-3)\end{aligned}$$

c) $m'(x) = \dfrac{1}{2}(9-2x)^{-\frac{1}{2}}(-2)\left(x^2 + \dfrac{2}{x^3}\right) + (9-2x)^{\frac{1}{2}}\left(2x - \dfrac{6}{x^4}\right)$

$$= \frac{-x^4\left(x^2 + \dfrac{2}{x^3} + (9-2x)(2x^5-6)\right)}{x^4\sqrt{9-2x}}$$

$$= \frac{-x^6 + 2x + 18x^5 - 54 - 4x^6 - 12x}{x^4\sqrt{9-2x}}$$

$$= -\frac{(5x^6 - 18x^5 - 10x + 54)}{x^4\sqrt{9-2x}}$$

d) $f'(x) = \dfrac{(1-x^2)^{\frac{1}{2}}(3) - (3x-2)\left(\dfrac{1}{2}\right)(1-x^2)^{-\frac{1}{2}}(-2x)}{(1-x^2)}$

$$= \frac{(1-x^2)(3) + x(3x-2)}{(1-x)^{\frac{3}{2}}}$$

$$= \frac{3-2x}{(1-x)^{\frac{3}{2}}}$$

Chapter 2 Practice Test **Question 7 Page 144**

a) Determine $v(t)$ and $a(t)$ after 3 s.
$v(t) = -9.8t + 11$ and $a(t) = -9.8$
$v(3) = -18.4$ m/s and $a(3) = -9.8$ m/s^2

b) The arrow moves upward when $v(t) > 0$ and downwards when $v(t) < 0$ until it hits the ground.
The arrow changes direction when $v(t) = 0$, so when $t = 1.12$ s.
upward: $0 \le t < 1.12$; downward: $t > 1.12$

c) The arrow is at rest when $v(t) = 0$, when $t = 1.12$ s.

d) The height at $t = 1.12$ s is the maximum height.
$h(1.12) = -4.9(1.12)^2 + 11(1.12) + 2$
$\doteq 8.17$
The maximum height is about 8.17 m.

e) The arrow hits the ground when $h(t) = 0$.

$$0 = -4.9t^2 + 11t + 2$$
$$t = \frac{-11 \pm \sqrt{11^2 - 4(-4.9)(2)}}{2(-4.9)}$$
$$= \frac{11 \pm 12.66}{9.8}$$

Since time cannot be negative, $t = 2.41$ s.

$$v(2.41) = -9.8(2.41) + 11$$
$$= -12.62$$

The arrow hits the ground at a velocity of 12.62 m/s.

Chapter 2 Practice Test **Question 8 Page 144**

$$y' = \frac{(3x+2)^3(-1) + x(3)(3x+2)^2(3)}{(3x+2)^6}$$
$$y'(-1) = \frac{(-1)^3(-1) + (-1)(3)(-1)^2(3)}{(-1)^6}$$
$$= -8$$

When $x = -1$, $y = -1$ so use $(-1, -1)$ and $m = -8$ to find b in $y = mx + b$.

$-1 = -8(-1) + b$ so $b = -9$.

The equation of the tangent to the curve at $x = -1$ is $y = -8x - 9$.

Chapter 2 Practice Test **Question 9 Page 144**

The slope of the line is –3, so the slope of the tangent must be $-\left(\frac{1}{-3}\right) = \frac{1}{3}$.

The slope of the tangent is also given by $f'(x)$ so set $f'(x) = \frac{1}{3}$.

$$f'(x) = \frac{1}{2}(2x+1)^{-\frac{1}{2}}(2)$$
$$\frac{1}{3} = \frac{1}{\sqrt{2x+1}}$$
$$3 = \sqrt{2x+1}$$
$$x = 4$$

When $x = 4$, $f(x) = 3$ so the point is (4, 3).

Chapter 2 Practice Test **Question 10 Page 145**

a) The vehicle is going faster at A; The vehicle is going faster at H.

b) The velocity at B and D is zero.

c) Between F and G the vehicle is stopped.

d) At C and I the vehicle is slowing down.

e) At J the vehicle has returned to its starting position.

f) **i)** The acceleration from 0-A is negative.

ii) The acceleration from B-C is negative.

iii) The acceleration from D-E is positive.

iv) The acceleration from F-G is zero.

v) The acceleration from I-J is negative.

Chapter 2 Practice Test **Question 11 Page 145**

a) Determine the demand function.
Let $p(x)$ be the price of one T-shirt, x the number of sales, and n the number of decreases in price.

$$x = 1500 + 20n$$
$$n = \frac{x - 1500}{20}$$
$$p = 12 - 0.50n$$
$$p = 12 - 0.5\left(\frac{x - 1500}{20}\right)$$
$$p(x) = 49.5 - 0.025x$$

b)
$$\begin{aligned} R(x) &= xp(x) \\ &= 49.5x - 0.025x^2 \\ R'(x) &= 49.5 - 0.05x \\ R'(1800) &= 49.5 - 0.05(1800) \\ &= -40.5 \end{aligned}$$
The marginal revenue is $–40.5.

c) Find $C'(1800)$
$$\begin{aligned} C'(x) &= -0.001x + 7.5 \\ C'(1800) &= -0.001(1800) + 7.5 \\ &= 5.70 \end{aligned}$$
The marginal cost is $5.70.

d)
$$\begin{aligned} C(1801) - C(1800) &= -0.0005(1801^2 - 1800^2) + 7.5(1) \\ &= -0.0005(3601) + 7.5 \\ &= 5.70 \end{aligned}$$
The cost of the 18th T-shirt is $5.70.

e) $\begin{aligned} P(x) &= R(x) - C(x) \\ &= 49.5x - 0.025x^2 + 0.0005x^2 - 7.5x - 200 \\ &= -0.0245x^2 + 42x - 200 \end{aligned}$

$P(1800) = -3980$

$P'(x) = -0.049x + 42$

$P'(1800) = -46.20$

The profit for 1800 t-shirts is –\$3980.00 and the marginal profit is –\$46.20.

Chapter 2 Practice Test **Question 12 Page 145**

a) $V'(t) = \dfrac{(1+0.02t)(5) - (100\,000 + 5t)(0.02)}{(1+0.02t)^2}$

$V'(1) = \dfrac{5.1 - 100\,005(0.02)}{(1.02)^2}$

$= -1917.53$

$V'(3) = \dfrac{5.3 - 100\,015(0.02)}{(1.06)^2}$

$= -1775.54$

$V'(6) = \dfrac{5.6 - 100\,030(0.02)}{(1.12)^2}$

$= -1590.40$

The depreciation after 1 year is –\$1917.53, 3 years is –\$1175.54, and 6 years –\$1590.40.

b) $v(0) = \$100\ 000$

c) It is more economical to purchase a used boat, since a new one depreciates much faster in the years just after the purchase was made and more slowly later on.

Chapter 2 Practice Test **Question 13 Page 145**

a) $C'(x) = 0.02x + 42$

$C'(250) = 0.02(250) + 42$

$= 47$

The marginal cost is \$47.

b) $C(251) - C(250) = 0.01(251^2 - 250^2) + 42$

$= 47.01$

The cost is \$47.01.

c) The cost of producing the 251st MP3 player only slightly exceeds the marginal cost of production at the production level of 250 MP3 players.

d) $R(x) = xp(x)$

$$= 130x - 0.4x^2$$

$$P(x) = R(x) - C(x)$$

$$= 130x - 0.4x^2 - 0.01x^2 - 42x - 300$$

$$= -0.41x^2 + 88x - 300$$

e) Determine the marginal revenue and marginal profit for $x = 250$ players.

$$R'(x) = 130 - 0.8x$$

$$R'(250) = 130 - 0.8(250)$$

$$= -70.00$$

$$P'(x) = -0.82x + 88$$

$$P'(250) = -0.82(250) + 88$$

$$= -117.00$$

The marginal revenue is –\$70.00 and the marginal profit is –\$117.00.

f) The reason that the marginal profit and marginal are negative is that the production level is too high. It costs more per unit as production is increased.
The profit would be maximized at

$x = \dfrac{88}{0.82}$ or 107.32 units.

Chapter 2 Practice Test **Question 14 Page 145**

a) $V(0) = \$5500$

b) Find $V'(t)$.

$$V'(t) = \frac{(0.002t^2+1)^{\frac{1}{2}}(18t^2) - (5500+6t^3)\left(\frac{1}{2}\right)(0.002t^2+1)^{-\frac{1}{2}}(0.004t)}{(0.002t^2+1)}$$

$$= \frac{(0.002t^2+1)(18t^2) - (0.002t)(5500+6t^3)}{(0.002t^2+1)^{\frac{3}{2}}} \times \frac{125000}{125000}$$

$$= \frac{1000(4.5t^4 + 2250t^2 - 1375t - 1.5t^4)}{(5t^2+2500)^{\frac{3}{2}}}$$

$$= \frac{1000t(3t^3 + 2250t - 1375)}{(5t^2+2500)^{\frac{3}{2}}}$$

c) The value of the dining set is increasing in value with time as $V'(t) > 0$ for $t > 0.611$ years.

d) Find $V(3)$ and $V(10)$.

$$V(3) = \frac{(5500+6(27))}{\sqrt{1.018}}$$
$$= 5611.72$$
$$V(10) = \frac{(5500+6(1000))}{\sqrt{1.2}}$$
$$= 10\,498.45$$

The value after 3 years is \$5611.72 and 10 years is \$10 498.45.

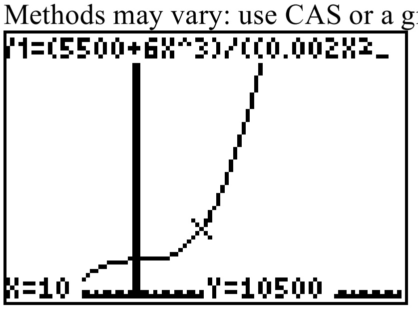
e) Compare $V'(3)$ and $V'(10)$.

$$V'(3) = \frac{3000(3(27)+2250(3)-1375)}{(5(9)+2500)^{\frac{3}{2}}}$$
$$= 127.49$$

$$V'(10) = \frac{10000(3(1000)+22500-1375)}{(500+2500)^{\frac{3}{2}}}$$
$$= 1468.20$$

The dining room set is appreciating in value much faster as time goes by.

f) Methods may vary: use CAS or a graphing calculator.



From parts d) and e), $V(10) = \$10\,500$ and $V'(10) = \$1468.20$ per year.