2.1 - Radicals

A radical is the root of a number – for our purposes, the square root of a number.

Square Root sign $\rightarrow \underbrace{\sqrt{n}}_{\textbf{Radical}} \leftarrow$ Radicand

Examples: $\sqrt{3}, \sqrt{5}, \ldots$

Properties of Radicals

$\sqrt{4} \times \sqrt{9} = 6$ $\qquad \sqrt{36} = \sqrt{4 \times 9} = 6$

Therefore, $\boxed{\sqrt{ab} = \sqrt{a} \times \sqrt{b} \quad , a \geq 0, b \geq 0}$

$\dfrac{\sqrt{36}}{\sqrt{4}} = 3$ $\qquad \sqrt{36 \div 4} = \sqrt{9} = 3$

Therefore, $\boxed{\sqrt{\dfrac{a}{b}} = \dfrac{\sqrt{a}}{\sqrt{b}} \quad , a \geq 0, b > 0}$

Simplifying Radicals

When working with the simplification of radicals you must remember some basic information about **perfect square** numbers.

Perfect Squares		Radicals (Square Roots)
$1^2 =$		$\sqrt{1} =$
$2^2 =$		$\sqrt{4} =$
$3^2 =$		$\sqrt{9} =$
$4^2 =$		$\sqrt{16} =$
$5^2 =$		$\sqrt{25} =$
$6^2 =$		$\sqrt{36} =$
$7^2 =$		$\sqrt{49} =$
$8^2 =$		$\sqrt{64} =$
$9^2 =$		$\sqrt{81} =$
$10^2 =$		$\sqrt{100} =$
$11^2 =$		$\sqrt{121} =$
$12^2 =$		$\sqrt{144} =$
$13^2 =$		$\sqrt{169} =$
$14^2 =$		$\sqrt{196} =$
$15^2 =$		$\sqrt{225} =$

Recall: **Simplify** means to find another expression with the same value. It does not mean to find a decimal approximation.

The Simplified Radical

A radical is in its simplest form when:

1) The radicand has no perfect square factors other than 1 $\sqrt{20}=\sqrt{4\times 5}=2\sqrt{5}$

2) The radicand does not contain a fraction $\sqrt{\dfrac{1}{9}}=\dfrac{1}{3}$

3) No radical appears in the denominator of a fraction $\dfrac{1}{\sqrt{5}}=\dfrac{1}{\sqrt{5}}\times\dfrac{\sqrt{5}}{\sqrt{5}}=\dfrac{\sqrt{5}}{5}$.

To Simplify (or reduce) a Radical:

We can convert from an entire radical to a mixed radical as long as the radicand is not a perfect square nor is prime (has factors of only itself and 1).

$$\underbrace{\sqrt{75}}_{\textbf{Entire Radical}} \longrightarrow \underbrace{5\sqrt{3}}_{\textbf{Mixed Radical}}$$

1. Find the **largest perfect square** which will divide evenly into the number under your radical sign. This means that when you divide, you get no remainders, no decimals, no fractions.

   Reduce: $\sqrt{75}$ the largest perfect square that divides evenly into 75 is 25

   * If the number under your radical cannot be divided evenly by any of the perfect squares, your radical is already in simplest form and cannot be reduced further.

2. Write the number appearing under your radical as the product (multiplication) of the perfect square and your answer from dividing.

$$\sqrt{75}=\sqrt{25\cdot 3}$$

3. Give each number in the product its own radical sign.

$$\sqrt{75} = \sqrt{25} \cdot \sqrt{3}$$

4. Reduce the "perfect" radical which you have now created.

Therefore, $\sqrt{75} = 5\sqrt{3}$

Practice: Simplify a) $\sqrt{24}$ b) $\sqrt{48}$

Ex. 2 Reduce: $3\sqrt{50}$

Don't let the number in front of the radical distract you!

It is simply "along for the ride" and will be multiplied times our final answer.

The largest perfect square dividing evenly into 50 is 25.

$$3\sqrt{50} = 3\sqrt{25 \cdot 2} = 3\sqrt{25} \cdot \sqrt{2}$$

Reduce the "perfect" radical and multiply times the 3 (who is "along for the ride")

$$3\sqrt{50} = 3 \cdot 5\sqrt{2} = 15\sqrt{2}$$

Practice: Simplify $5\sqrt{72}$

Multiplication & Division of Radicals

When multiplying radicals:

- multiply the numbers OUTSIDE (O) the radicals AND then
- multiply the numbers INSIDE (I) the radicals.

$$O_1\sqrt{I_1} \cdot O_2\sqrt{I_2} = O_1 O_2 \sqrt{I_1} \cdot \sqrt{I_2}$$

Ex. $2\sqrt{3} \cdot 4\sqrt{5} = 2 \cdot 4 \cdot \sqrt{3} \cdot \sqrt{5} = 8\sqrt{15}$ * make sure to simplify radical if necessary

When dividing radicals:

- divide the numbers OUTSIDE (O) the radicals AND then
- divide the numbers INSIDE (I) the radicals.

$$\frac{O_1\sqrt{I_1}}{O_2\sqrt{I_2}} = \frac{O_1}{O_2} \cdot \frac{\sqrt{I_1}}{\sqrt{I_2}}$$

Ex. $\dfrac{4\sqrt{15}}{2\sqrt{3}} = \dfrac{4}{2} \cdot \dfrac{\sqrt{15}}{\sqrt{3}} = 2\sqrt{5}$ * make sure to simplify radical if necessary

Practice: Simplify

a) $2\sqrt{18} \cdot 3\sqrt{8}$

b) $\dfrac{-12\sqrt{24}}{3\sqrt{2}}$

c) $\sqrt{\dfrac{5}{9}}$

d) $\dfrac{6-\sqrt{45}}{3}$

e) $\sqrt{2}(\sqrt{3}-5\sqrt{2})$

f) $(3\sqrt{2}+4\sqrt{5})(4\sqrt{2}-3\sqrt{5})$

<u>Addition & Subtraction of Radicals</u>

Steps:

1. Simplify each radical by extracting all perfect square factors
2. Combine **like-radicals** by addition and subtraction

Ex. a) $3\sqrt{2}-5\sqrt{3}+6\sqrt{2}+2\sqrt{3}-\sqrt{5}$

b) $4\sqrt{3}+2\sqrt{20}-\sqrt{12}+6\sqrt{45}$

Rationalizing the Denominator

Method: multiply the numerator and denominator by the same radical

Goal: produce a perfect square in the denominator

Ex. 2 Simplify each fully

a) $\dfrac{3}{\sqrt{7}}$

b) $\dfrac{5}{\sqrt{12}}$

c) $\dfrac{1}{3\sqrt{2}}$

Homework: Complete Radicals Worksheet

NOW
IT'S YOUR TURN!

1. Simplify.

a) $\sqrt{12}$ b) $\sqrt{20}$ c) $\sqrt{45}$

d) $\sqrt{50}$ e) $\sqrt{24}$ f) $\sqrt{63}$

g) $\sqrt{200}$ h) $\sqrt{32}$ i) $\sqrt{44}$

j) $\sqrt{60}$ k) $\sqrt{18}$ l) $\sqrt{54}$

m) $\sqrt{128}$ n) $\sqrt{90}$ o) $\sqrt{125}$

2. Simplify.

a) $\dfrac{\sqrt{14}}{\sqrt{7}}$ b) $\dfrac{\sqrt{10}}{\sqrt{2}}$ c) $\dfrac{\sqrt{60}}{\sqrt{3}}$ d) $\dfrac{\sqrt{40}}{\sqrt{5}}$

e) $\dfrac{\sqrt{33}}{\sqrt{3}}$ f) $\dfrac{\sqrt{7}}{\sqrt{4}}$ g) $\dfrac{\sqrt{20}}{\sqrt{9}}$ h) $\dfrac{3\sqrt{8}}{\sqrt{2}}$

i) $\dfrac{27\sqrt{15}}{3\sqrt{5}}$ j) $\dfrac{12\sqrt{75}}{4\sqrt{3}}$ k) $\dfrac{4\sqrt{2}}{\sqrt{8}}$ l) $\dfrac{2\sqrt{2}}{\sqrt{18}}$

3. Simplify.

a) $\sqrt{2} \times \sqrt{10}$

b) $\sqrt{3} \times \sqrt{6}$

c) $\sqrt{15} \times \sqrt{5}$

d) $\sqrt{7} \times \sqrt{11}$

e) $4\sqrt{3} \times \sqrt{7}$

f) $3\sqrt{6} \times 3\sqrt{6}$

g) $2\sqrt{2} \times 3\sqrt{6}$

h) $2\sqrt{5} \times 3\sqrt{10}$

i) $3\sqrt{3} \times 4\sqrt{15}$

j) $4\sqrt{7} \times 2\sqrt{14}$

k) $\sqrt{6} \times \sqrt{3} \times \sqrt{2}$

l) $2\sqrt{7} \times 3\sqrt{1} \times \sqrt{7}$

4. Simplify.

a) $\dfrac{10 + 15\sqrt{5}}{5}$ b) $\dfrac{21 - 7\sqrt{6}}{7}$ c) $\dfrac{6 + \sqrt{8}}{2}$

d) $\dfrac{12 - \sqrt{27}}{3}$ e) $\dfrac{-10 - \sqrt{50}}{5}$ f) $\dfrac{-12 + \sqrt{48}}{4}$

ANSWERS:

Section 2.1, pp. 106–109

1. a) $2\sqrt{3}$ **b)** $2\sqrt{5}$ **c)** $3\sqrt{5}$ **d)** $5\sqrt{2}$ **e)** $2\sqrt{6}$ **f)** $3\sqrt{7}$ **g)** $10\sqrt{2}$ **h)** $4\sqrt{2}$ **i)** $2\sqrt{11}$ **j)** $2\sqrt{15}$ **k)** $3\sqrt{2}$ **l)** $3\sqrt{6}$ **m)** $8\sqrt{2}$ **n)** $3\sqrt{10}$ **o)** $5\sqrt{5}$ **2. a)** $\sqrt{2}$ **b)** $\sqrt{5}$ **c)** $2\sqrt{5}$ **d)** $2\sqrt{2}$ **e)** $\sqrt{11}$ **f)** $\dfrac{\sqrt{7}}{2}$ **g)** $\dfrac{2\sqrt{5}}{3}$ **h)** 6 **i)** $9\sqrt{3}$ **j)** 15 **k)** 2 **l)** $\dfrac{2}{3}$ **3. a)** $2\sqrt{5}$ **b)** $3\sqrt{2}$ **c)** $5\sqrt{3}$ **d)** $\sqrt{77}$ **e)** $4\sqrt{21}$ **f)** 54 **g)** $12\sqrt{3}$ **h)** $30\sqrt{2}$ **i)** $36\sqrt{5}$ **j)** $56\sqrt{2}$ **k)** 6 **l)** 42 **4. a)** $2 + 3\sqrt{5}$ **b)** $3 - \sqrt{6}$ **c)** $3 + \sqrt{2}$ **d)** $4 - \sqrt{3}$ **e)** $-2 - \sqrt{2}$ **f)** $-3 + \sqrt{3}$ **5. a)** $3i$ **b)** $5i$ **c)** $9i$

1. Simplify.

a) $2\sqrt{5} + 3\sqrt{5} + 6\sqrt{5}$

b) $4\sqrt{3} + 2\sqrt{3} - \sqrt{3}$

c) $6\sqrt{2} - \sqrt{2} + 7\sqrt{2} - 3\sqrt{2}$

d) $5\sqrt{7} + 3\sqrt{7} - 2\sqrt{7}$

e) $8\sqrt{10} - 2\sqrt{10} - 7\sqrt{10}$

f) $\sqrt{2} - 3\sqrt{2} - 9\sqrt{2} + 11\sqrt{2}$

g) $\sqrt{5} + \sqrt{5} + \sqrt{5} + \sqrt{5}$

2. Simplify.

a) $5\sqrt{3} + 2\sqrt{6} + 3\sqrt{3}$

b) $8\sqrt{5} - 3\sqrt{7} + 7\sqrt{7} - 4\sqrt{5}$

c) $2\sqrt{2} + 3\sqrt{10} + 5\sqrt{2} - 4\sqrt{10}$

d) $7\sqrt{6} - 4\sqrt{13} - \sqrt{13} + \sqrt{6}$

e) $9\sqrt{11} - \sqrt{11} + 6\sqrt{14} - 3\sqrt{14} - 2\sqrt{11}$

f) $12\sqrt{7} + 9 - 3\sqrt{7} + 4$

g) $8 + 7\sqrt{11} - 9 - 9\sqrt{11}$

3. Simplify.

a) $\sqrt{12} + \sqrt{27}$

b) $\sqrt{20} + \sqrt{45}$

c) $\sqrt{18} - \sqrt{8}$

d) $\sqrt{50} + \sqrt{98} - \sqrt{2}$

e) $\sqrt{75} + \sqrt{48} + \sqrt{27}$

f) $\sqrt{54} + \sqrt{24} + \sqrt{72} - \sqrt{32}$

g) $\sqrt{28} - \sqrt{27} + \sqrt{63} + \sqrt{300}$

4. Simplify.

a) $8\sqrt{7} + 2\sqrt{28}$

b) $3\sqrt{50} - 2\sqrt{32}$

c) $5\sqrt{27} + 4\sqrt{48}$

d) $3\sqrt{8} + \sqrt{18} + 3\sqrt{2}$

e) $\sqrt{5} + 2\sqrt{45} - 3\sqrt{20}$

f) $4\sqrt{3} + 3\sqrt{20} - 2\sqrt{12} + \sqrt{45}$

g) $3\sqrt{48} - 4\sqrt{8} + 4\sqrt{27} - 2\sqrt{72}$

5. Expand and simplify.

a) $\sqrt{2}\left(\sqrt{10}+4\right)$

b) $\sqrt{3}\left(\sqrt{6}-1\right)$

c) $\sqrt{6}\left(\sqrt{2}+\sqrt{6}\right)$

d) $2\sqrt{2}\left(3\sqrt{6}-\sqrt{3}\right)$

e) $\sqrt{2}\left(\sqrt{3}+4\right)$

f) $3\sqrt{2}\left(2\sqrt{6}+\sqrt{10}\right)$

g) $\left(\sqrt{5}+\sqrt{6}\right)\left(\sqrt{5}+3\sqrt{6}\right)$

h) $\left(2\sqrt{3}-1\right)\left(3\sqrt{3}+2\right)$

i) $\left(4\sqrt{7}-3\sqrt{2}\right)\left(2\sqrt{7}+5\sqrt{2}\right)$

j) $\left(3\sqrt{3}+1\right)^2$

k) $\left(2\sqrt{2}-\sqrt{5}\right)^2$

l) $\left(2+\sqrt{3}\right)\left(2-\sqrt{3}\right)$

m) $\left(\sqrt{6}-\sqrt{2}\right)\left(\sqrt{6}+\sqrt{2}\right)$

n) $\left(2\sqrt{7}+3\sqrt{5}\right)\left(2\sqrt{7}-3\sqrt{5}\right)$

6. Simplify.

a) $\dfrac{1}{\sqrt{3}}$ b) $\dfrac{2}{\sqrt{5}}$ c) $\dfrac{2}{\sqrt{7}}$

d) $\dfrac{\sqrt{1}}{\sqrt{2}}$ e) $\dfrac{5\sqrt{5}}{2\sqrt{3}}$ f) $\dfrac{2\sqrt{2}}{\sqrt{18}}$

g) $\dfrac{4\sqrt{2}}{\sqrt{8}}$ h) $\dfrac{3\sqrt{5}}{\sqrt{3}}$ i) $\dfrac{4\sqrt{7}}{2\sqrt{14}}$

j) $\dfrac{3\sqrt{6}}{4\sqrt{10}}$ k) $\dfrac{7\sqrt{11}}{2\sqrt{3}}$ l) $\dfrac{2\sqrt{5}}{5\sqrt{2}}$

ANSWERS:

Section 2.4, pp. 139–142

1. a) $11\sqrt{5}$ **b)** $5\sqrt{3}$ **c)** $9\sqrt{2}$ **d)** $6\sqrt{7}$ **e)** $-\sqrt{10}$ **f)** 0 **g)** $4\sqrt{5}$
2. a) $8\sqrt{3}+2\sqrt{6}$ **b)** $4\sqrt{5}+4\sqrt{7}$ **c)** $7\sqrt{2}-\sqrt{10}$ **d)** $8\sqrt{6}-5\sqrt{13}$
e) $6\sqrt{11}+3\sqrt{14}$ **f)** $13+9\sqrt{7}$ **g)** $-1-2\sqrt{11}$ **3. a)** $5\sqrt{3}$ **b)** $5\sqrt{5}$

c) $\sqrt{2}$ **d)** $11\sqrt{2}$ **e)** $12\sqrt{3}$ **f)** $5\sqrt{6}+2\sqrt{2}$ **g)** $5\sqrt{7}+7\sqrt{3}$ **4. a)** $12\sqrt{7}$
b) $7\sqrt{2}$ **c)** $31\sqrt{3}$ **d)** $12\sqrt{2}$ **e)** $\sqrt{5}$ **f)** $9\sqrt{5}$ **g)** $24\sqrt{3}-20\sqrt{2}$
5. a) $2\sqrt{5}+4\sqrt{2}$ **b)** $3\sqrt{2}-\sqrt{3}$ **c)** $2\sqrt{3}+6$ **d)** $12\sqrt{3}-2\sqrt{6}$
e) $\sqrt{6}+4\sqrt{2}$ **f)** $12\sqrt{3}+6\sqrt{5}$ **g)** $23+4\sqrt{30}$ **h)** $16+\sqrt{3}$
i) $26+14\sqrt{14}$ **j)** $28+6\sqrt{3}$ **k)** $13-4\sqrt{10}$ **l)** 1 **m)** 4 **n)** -17
6. a) $\frac{\sqrt{3}}{3}$ **b)** $\frac{2\sqrt{5}}{5}$ **c)** $\frac{2\sqrt{7}}{7}$ **d)** $\frac{\sqrt{2}}{2}$ **e)** $\frac{5\sqrt{15}}{6}$ **f)** $\frac{2}{3}$ **g)** 2 **h)** $\sqrt{15}$ **i)** $\sqrt{2}$
j) $\frac{3\sqrt{15}}{20}$ **k)** $\frac{7\sqrt{33}}{6}$ **l)** $\frac{\sqrt{10}}{5}$ **7. a)** $\frac{2-\sqrt{2}}{2}$ **b)** $\frac{3+3\sqrt{5}}{4}$