

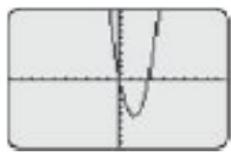
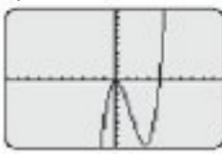
Chapter 1 Practice Test, pages 66–67

1. C 2. C; there is a cusp at $x = 2$. 3. a) 35 b) 141

c) 7 d) $-\frac{9}{5}$ e) 14 f) 0 4. C

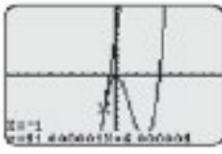
5. a) $\frac{dy}{dx} = 3x^2 - 8x$

b)



c) $y = 11x + 6$

d)



6. a) -3 b) 1 c) -2 d) -2 e) 1 f) ∞ 7. a) domain: $\{x \mid x \in \mathbb{R}, x \neq 4\}$; range: $\{y \mid y \in \mathbb{R}, y \neq 3\}$ b) i) 3 from above ii) 3 from below iii) $+\infty$ iv) $-\infty$ v) 9 vi) 1 c) The graph is not continuous. Answers may vary: As x approaches 4 from the left, the graph becomes large and negative. As x approaches 4 from the right, the graph becomes large and positive. Therefore, there is an infinite discontinuity at $x = 4$.

8. a) $V(x) = 4x - \frac{1}{4}x^3$ b) $0.0625 \text{ m}^3/\text{m}$ c) $-2.75 \text{ m}^3/\text{m}$

9. a) i) 0.2 m ii) 0.6 m iii) 1 m b) $\frac{dA}{dr} = 2\pi r \text{ m}^2/\text{m}$

c) i) $1.3 \text{ m}^2/\text{m}$ ii) $3.8 \text{ m}^2/\text{m}$ iii) $6.3 \text{ m}^2/\text{m}$ 10. a) C b) A c) D d) B

CHAPTER 2 DERIVATIVES

Prerequisite Skills, pages 70–71

1. a) polynomial b) sinusoidal c) polynomial d) root e) exponential f) rational g) logarithmic h) polynomial

2. a) $-\frac{1}{2}$ b) $\frac{1}{5}$ c) $-\frac{3}{2}$ d) undefined e) -1 f) 0 3. a) $x^{\frac{1}{2}}$

b) $x^{\frac{1}{3}}$ c) $x^{\frac{3}{4}}$ d) $x^{\frac{2}{5}}$ 4. a) x^{-1} b) $-2x^{-4}$ c) $x^{-\frac{1}{2}}$ d) $x^{-\frac{2}{3}}$

5. a) $(x^3 - 1)(5x + 2)^{-1}$ b) $(3x^4)(5x + 6)^{-\frac{1}{2}}$

c) $(9 - x^2)^3(2x + 1)^{-4}$ d) $(x + 3)^2(1 - 7x^2)^{-\frac{1}{3}}$

6. a) $\frac{1}{x^6}$ b) $2 - \frac{1}{x} + \frac{3}{x^2}$ c) $\frac{1}{x^3}$ d) $x^{\frac{1}{2}} - \frac{1}{x^{\frac{1}{2}}}$ e) c^9

f) $\frac{(4x - 3)^2}{(x^2 + 3)^{\frac{3}{2}}}$ 7. a) i) increasing: $(-\infty, -1), (2.5, \infty)$;

decreasing: $(-1, 2.5)$ ii) positive: $(-3, 2), (3, \infty)$; negative: $(-\infty, -3), (2, 3)$ iii) zero slope: $x = -1, x = 2.5$; positive slope: $(-\infty, -1), (2.5, \infty)$; negative slope: $(-1, 2.5)$ b) i) decreasing: $(-\infty, -2.5), (1.5, 4.5)$; increasing: $(-2.5, 1.5), (4.5, \infty)$ ii) positive: $(-\infty, -4), (-0.5, 4), (5, \infty)$; negative: $(-4, -0.5), (4, 5)$ iii) zero slope: $x = -2.5, x = 1.5, x = 4.5$; positive slope: $(-2.5, 1.5), (4.5, \infty)$; negative slope: $(-\infty, -2.5), (1.5, 4.5)$

8. a) 2, 6 b) $-3, 7$ c) $\frac{4}{5}, 2$ d) $\frac{2}{3}, \frac{3}{2}$ e) $\frac{-5 \pm \sqrt{41}}{2}$

f) $\frac{-13 \pm \sqrt{217}}{4}$ g) $\frac{9 \pm \sqrt{33}}{8}$ h) $\frac{7 \pm 3\sqrt{5}}{2}$

9. a) $-4, -1, 2$ b) $-1, -0.5, 2$ c) $-4, -\frac{1}{3}, 3$

d) $-3, -\frac{1}{5}, 1$ e) $-2, \frac{1}{3}, 1$ f) $-3, -1, 2, 4$

10. a) $15x^2 - 14x + 20$ b) $-175x^4 - 30x^2 + 126x$

c) $42x^6 + 75x^4 - 48x^3 - 60x$

11. a) $(x^3 - 1)^4(2x + 7)^3(38x^3 + 105x^2 - 8)$

b) $(x^3 + 4)^{-2}(-12x^3 + 15x^2 + 24)$

c) $2\sqrt{x}(x - 1)(x^2 + x + 1)$ d) $\frac{(x + 1)^2}{x^2}$ 12. a) 23 b) -5 c) 3

13. a) $f \circ g(x) \left(\frac{1}{x-2} \right)^3 + 1$ b) $g \circ f(x) \frac{1}{\sqrt{1-x^2}-2}$

c) $b[f(x)]\sqrt{-x^6-2x^3}$ d) $g[f(x)]\frac{1}{x^3-1}$

14. a) $h(x) = f[g(x)]$ if $f(x) = x^2$ and $g(x) = 2x - 3$

b) $h(x) = f[g(x)]$ if $f(x) = \sqrt{x}$ and $g(x) = 2 + 4x$

c) $h(x) = f[g(x)]$ if $f(x) = \frac{1}{x}$ and $g(x) = 3x^2 - 7x$

d) $h(x) = f[g(x)]$ if $f(x) = \frac{1}{x^2}$ and $g(x) = x^3 - 4$

2.1 Derivative of a Polynomial Function, pages 83–86

1. A, B, E, G, and H 2. a) $\frac{dy}{dx} = 1$ b) $\frac{dy}{dx} = \frac{1}{2}x$ c) $\frac{dy}{dx} = 5x^4$

d) $\frac{dy}{dx} = -12x^3$ e) $\frac{dy}{dx} = 4.5x^2$ f) $\frac{dy}{dx} = \frac{3}{5\sqrt[5]{x^2}}$ g) $\frac{dy}{dx} = \frac{-5}{x^2}$

h) $\frac{dy}{dx} = \frac{2}{\sqrt{x^3}}$ 3. a) 0 b) 90 c) $\frac{3}{16}$ d) -34.3

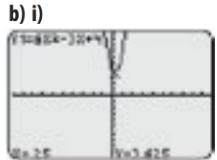
e) $\frac{3\pi}{2}$ f) $\frac{1}{12}$ 4. a) $f'(x) = 4x + 3x^2$; sum rule, power rule, constant multiple rule b) $\frac{dy}{dx} = 4x^4 - 3$; difference rule, power rule, constant multiple rule c) $h'(t) = -4.4t^3$; sum rule, power rule, constant rule, constant multiple rule

d) $V'(r) = 4\pi r^2$; power rule, constant multiple rule

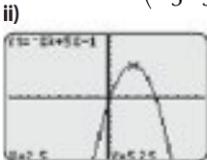
e) $p'(a) = \frac{1}{3}a^4 - \frac{1}{\sqrt{a}}$; difference rule, power rule, constant multiple rule

f) $k'(s) = \frac{2}{s^3} + 28s^3$; sum rule, power rule, constant multiple rule

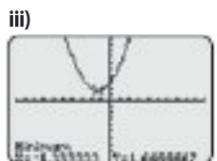
5. a) i) $(0.25, 3.625)$ ii) $(2.5, 5.25)$ iii) $\left(-\frac{4}{3}, \frac{5}{3}\right) = (1.\dot{3}, 1.\dot{6})$



local minimum point



local maximum point



local minimum point

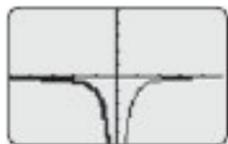
6. a) $f(x) = 5x^2 - 3x$; $f'(x) = 10x - 3$
 b) $g(x) = 6x^2 + 5x - 4$; $g'(x) = 12x + 5$
 c) $p(x) = \frac{1}{4}x^5 - x^3 + \frac{1}{2}$; $p'(x) = \frac{5}{4}x^4 - 3x^2$
 d) $f(x) = 25x^2 + 20x + 4$; $f'(x) = 50x + 20$

7. Answers may vary. a) Use a graphing calculator to graph the curve, and then draw the tangent to the curve and determine the equation of the tangent to the curve at the given x -value. b) The derivative is equal to the slope of the tangent at the given point. The equation formed can then be solved to find the x -value of the tangent point. The x -value is then substituted into the equation of the function to find the y -value of the tangent point.

8. a) Answers may vary: The rules in this chapter require expansion and solving by the sum, power, and constant multiple rules. This question needs to be in a more expanded form to solve via these methods.

b) $f(x)$ would need to be expanded and simplified to make solving simpler. c) $f(x) = 4x^3 + 8x^2 - 11x + 3$; $f'(x) = 12x^2 + 16x - 11$

9. Answers may vary: The function could be plotted on a graphing calculator and the slope of a tangent line found. The slope of the tangent line will match the derivative of the function at a given point. Simple derivative rules can be used to prove this algebraically, while numerical evidence can be taken from the graphing procedure.

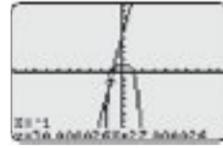


10. $f'(t)$ is the rate of water entering the first barrel, $g'(t)$ is the rate of water entering the second barrel, $f'(t) + g'(t)$ is the sum of the rates of water entering the two barrels, and, according to the sum and difference rule, $(f+g)'(t)$ is also the sum of the rates of the water entering the two barrels. 11. -49 m/s b) 17.5 s c) 171.5 m/s

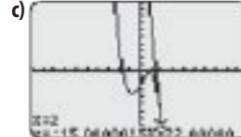
12. Earth $= -39.2$ m/s; Venus $= -35.6$ m/s; Mars $= -14.8$ m/s; Saturn $= -42.0$ m/s; Neptune $= -44.8$ m/s

13. a) $\frac{dy}{dx} = -24x^3 + 6x^2 = 30$ b) $y = 30x + 27$

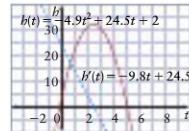
c) Answers may vary; plotting both curves would allow verification of tangent and intercept.



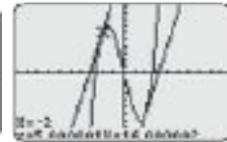
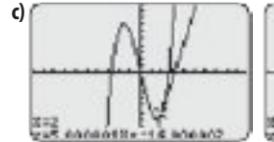
14. a) -15 b) $y = -15x + 22$



15. a) 31.4 m b) 14.7 m/s; 4.9 m/s; -14.7 m/s; -24.5 m/s
 c) The arrow is at the initial launch height of 2 m off the ground. d) $t = 5.08$ s e) -25.28 m/s f) Answers may vary; plot the $b(t)$ and $b'(t)$ functions from 0 to about 5.08 s and read the values off the plots.



16. a) $(2, -6), (-2, 6)$ b) $y = 5x - 16$; $y = 5x + 16$



17. a) $x = \frac{4}{3}, 0$ b) $y = \frac{16}{3}x - \frac{32}{9}$; $y = \frac{16}{3}x - \frac{128}{27}$; $y = 0$



19. a) $\$4850$; $\$3250$

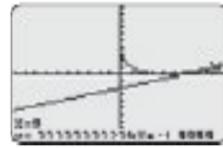
b) $C'(1000) = 1.3$; $C'(3000) = 0.9$; provides marginal cost of production per unit c) never; $C'(x) = 1.5 - 0.0002x$; lowest value of $C'(x) = 1.0$ within range of x

d) $R'(x) = 3.25$ is the rate of change of revenue per yogourt bar produced. e) $P(x) = 0.0001x^2 + 1.75x - 3450$

f) Positive: $x \geq 1788$; negative: $x \leq 1788$; this provides the break-even point. 20. a) -5 b) $y = 0.2x + 4.6$

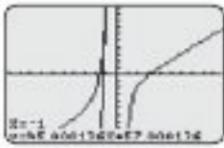
21. a) $-\frac{29}{2}$ b) $y = \frac{2}{29}x - \frac{60}{29}$ 22. a) $y = \frac{1}{3}x - 2$

b)



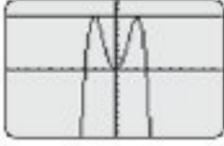
23. a) $y = 45x + 57$

b)



24. a) $y = 0; y = 16$

b)



25. a) Setting the derivative equal to -5 results in an equation that has no solution: $-5 = 18x^2 + 4x$

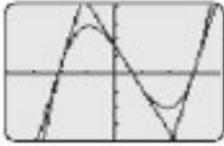
b) Answers may vary; plot the first derivative to see that the slope is never equal to -5 .



27. a) $a = 2; b = -\frac{5}{2}$ 28. $y = 6x - 11; y = -2x - 3$

29. a) $y = 25x + 52; y = -12.5x + 8.25; y = 25x - 73$

b)



31. D 32. E

2.2 The Product Rule, pages 93–96

1. a) $f'(x) = 4x + 7$ b) $b'(x) = -20x + 11$ c) $b'(x) = -6x - 5$
- d) $g'(x) = -12x + 11$ 2. a) $f'(x) = 80x - 14$
- b) $b'(t) = -4t + 7$ c) $p'(x) = -4x + 21$
- d) $g'(x) = 12x^2 - 10x + 8$ e) $f'(x) = -3x^2 + 2x + 5$
- f) $b'(t) = 4t(3t^2 + 1)$ 3. a) $M'(u) = -12u^2 - 16u + 1$
- b) $g'(x) = -2x + 13$ c) $p'(n) = -15n^2 - 2n + 15$
- d) $A'(r) = 12r^2 + 4r - 12$ e) $b'(k) = 0.4k - 4.4$
4. a) $f(x) = 5x^2 + 7$; $g(x) = 21 - 3x$
- b) $f(x) = -4x^3 + 8x$; $g(x) = 2x^2 - 4x$
- c) $f(x) = 2x^3 - x$; $g(x) = 0.5x^2 + x$
- d) $f(x) = -\frac{3}{4}x^4 + 6x$; $g(x) = 7x - \frac{2}{3}x^2$ 5. a) 54 b) 60
- c) -11 d) 215 e) -116 f) -274 6. a) $y = -240x - 739$
- b) $y = -36x + 65$ c) $y = -4x - 24$
- d) $y = -1491x + 3648$ 7. a) $\left(-\frac{9}{8}, \frac{225}{16}\right)$
- b) $(1.57, -87.021)$ c) $(1.53, -3.42); (-1.20, 8.70)$
- d) $(0.43, -3.38); (-0.77, 0.76)$

8. a) $Y(t) = (10t + 120)(280 + 15t)$ b) 5200; rate of

change of apple production at $t = 2$ years

c) $Y'(6) = 6400$; represents the rate of change of apple production at $t = 6$ years.

9. a) $\frac{dy}{dx} = 15x^2 + 18x - 1$ b) $\frac{dy}{dx} = \frac{3}{x^4} - 6x^2 - \frac{1}{x^2} + 2x$

c) $\frac{dy}{dx} = -24x^5 - 32x^3 - 30x^2 + 5x^4 + 6x$

d) $\frac{dy}{dx} = 16x^3 - 10x^{\frac{3}{2}} + 1$ e) $\frac{dy}{dx} = 36x^3 - 18x^2 - 10x + 2$

10. a) Expand and differentiate; use the product rule.

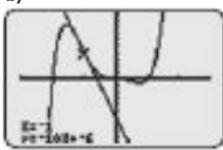
$R'(n) = 14.50 - 4n$ b) -1.5 ; tells the manager that he is losing revenue at a price of \$8.50 c) $n = 3.625$; determines the price for maximum revenue: \$8.31.

d) Maximum revenue is \$552.78. Notice that revenue deteriorates after $n = 3.625$. e) When the derivative is zero, the revenue is maximized. This occurs at $n = 3.625$.

11. a) $R(x) = (30 + 2.5x)(550 - 5x)$ b) $R'(x) = 1225 - 25x$

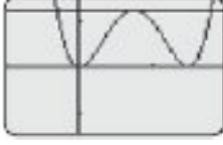
c) 1150; represents the rate of change of revenue at a \$7.50 increase d) $x = 49$ e) The owner could increase the price from \$30 to \$152.50 per haircut to maximize revenue. 12. a) $y = -10x - 6$

b)



13. a) $\left(\frac{3}{2}, 0\right); (0, 0); \left(\frac{3}{4}, \frac{81}{64}\right)$

b)



14. a) 90 L of gas b) 3.33 L/h c) -6.66 L/h

15. a) 3600 b) 1575 fish per year c) 1071.7 fish per year d) 3.45 years; 1813.6 fish per year.

16. a) $R(n) = (1.75 + 0.25n)(150 - 10n)$

b) $R'(0.25) = 18.75$; $R'(1) = 15$; $R'(3) = 5$; $R'(4) = 0$; $R'(5) = -5$; $R'(6) = -10$ c) $n = 4$; at this point the revenue is not changing, so profit is maximized.

d) $P'(1) = 22.50$; $P'(3) = 12.50$; $P'(4) = 7.50$

e) $P'(5) = 2.50$; $P'(6) = -2.50$ f) The profit numbers have a rate of change that is 7.5 greater than that of the revenue numbers.

17. a) i) $\frac{dy}{dx} = (2x - 3)(x^2 - 3x) + (2x - 3)(x^2 - 3x)$

ii) $\frac{dy}{dx} = (6x^2 + 1)(2x^3 + x) + (6x^2 + 1)(2x^3 + x)$

iii) $\frac{dy}{dx} = (-4x^3 + 10x)(-x^4 + 5x^2) + (-4x^3 + 10x)(-x^4 + 5x^2)$

b) $\frac{d}{dx}[f(x)]^2 = 2f(x)\frac{d}{dx}[f(x)]$ c)-d) Answers may vary

18. a) $f'gb + fg'b + fgb'$

b) $f'(x) = 2x(3x^4 - 2)(5x + 1) + (x^2 + 4)(12x^2)(5x + 1) + (x^2 + 4)(x^2 - 2)(5)$ c) Expand and differentiate.

$f'(A) = 105x^6 + 18x^5 + 300x^4 + 48x^3 - 30x^2 - 4x - 40$

19. a) $\frac{d}{dx}[f(x)]^3 = 3f(x)^2 \frac{d}{dx}[f(x)]$

c) i) $\frac{dy}{dx} = 384x^5 - 240x^4 + 48x^3 - 3x^2$

ii) $\frac{dy}{dx} = 9x^8 + 21x^6 + 15x^4 + 3x^2$

iii) $\frac{dy}{dx} = -96x^{11} + 120x^9 - 48x^7 + 6x^5$

20. a) $h'(x) = 3x^2f(x) + x^3f'(x)$

b) $p'(x) = g(x)(4x^3 - 6x) + (x^4 - 3x^2)g'(x)$

c) $q'(x) = (-12x^3 - 16x + 5)f(x) + (-3x^4 - 8x^2 + 5x + 6)f'(x)$

d) $r'(x) = f'(x)(2x^3 + 5x^2)^2 + 4f(x)(2x^3 + 5x^2)(3x^2 + 5x)$

21. a) $\frac{d}{dx}[f(x)]^n = n[f(x)]^{n-1} \left(\frac{d}{dx}[f(x)] \right)$

b) $\frac{dy}{dx} = 4(2x^3 + x^2)^3(6x^2 + 2x);$

$\frac{dy}{dx} = 5(2x^3 + x^2)^4(6x^2 + 2x); \frac{dy}{dx} = 6(2x^3 + x^2)^5(6x^2 + 2x)$

22. C 23. E

2.3 Velocity, Acceleration, and Second Derivatives, pages 106–110

1. a) $\frac{d^2y}{dx^2} = 12x$ b) $s''(t) = -12t^2 + 30t - 4$

c) $h''(x) = 5x^4 - 4x^3$ d) $f''(x) = \frac{3}{2}x - 4$

e) $g''(x) = 20x^3 + 36x^2 - 12x$ f) $h''(t) = -9.8$

2. a) 174 b) 72 c) -234 d) -48 e) 98 f) 2100

3. a) $s'(t) = 7 - 24t^2$; $s''(t) = -48t$

b) $s'(t) = -7 - 20t$; $s''(t) = -20$

c) $s'(t) = -9t^2 - 10t - 3$; $s''(t) = -18t - 10$

d) $s'(t) = -t - \frac{1}{4}$; $s''(t) = -1$

4. a) 1 m/s; 6 m/s² b) -4.6 m/s; -9.8 m/s²

c) -95 m/s; -86 m/s² d) 32 m/s; 48 m/s²

5. a) ①: $v(t)$; ②: $s(t)$; ③: $a(t)$ b) ①: $v(t)$; ②: $a(t)$; ③: $s(t)$

6. a)

Interval	$v(t)$	$a(t)$	$v(t) \times a(t)$	Motion of Object	Description of Slope of $s(t)$
[0, 3)	+	-	-	forward slowing	+ decreasing
(3, 6]	-	-	+	reverse accelerating	- decreasing

b)

Interval	$v(t)$	$a(t)$	$v(t) \times a(t)$	Motion of Object	Description of Slope of $s(t)$
[0, 1)	-	+	-	reverse slowing	- increasing
(1, 2)	+	+	+	forward accelerating	+ increasing
(2, 3)	+	-	-	forward slowing	+ decreasing
(3, ∞)	-	-	+	reverse accelerating	- decreasing

7. Answers may vary 8. a) i) increasing ii) +

b) i) increasing ii) + c) i) decreasing ii) - d) i) decreasing

ii) - e) i) constant ii) zero 9. a) zero b) zero c) A

d) $v(t)=0$; $a(t)=0$ e) A speeding up; B slowing down;

D speeding up f) bus returns to origin; slows to a stop

10. a) + b) - c) + d) - e) + ii) - iii) +

iv) 0 v) + vi) 0 b) i) all positive acceleration ii) all

negative acceleration iii) opposite accelerations

12. a) -19.6 m/s b) -29.4 m/s c) 4.04 s d) -39.6 m/s

13. a) 5.1 m/s; -9.8 m/s² b) $t = 3.5$ s c) 63.9 m

d) 7.13 s e) -35.4 m/s 14. a) + b) decreasing c) -

15. a) $s(t) = 48t$ b) $v(t) = 48$; $a(t) = 0$ c) $s(t) = 50 - 4.9t^2$

d) $v(t) = -9.8t$; $a(t) = -9.8$ e) -31.3 m/s f) 1.36 s

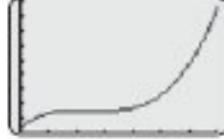
g) $v_{\text{tot}}(t) = \sqrt{2304 + 96.04t^2}$ h) $a_{\text{tot}}(t) = -9.8$

i) -9.8 m/s² 16. a) $v(1) = 12$; $a(1) = -18$;

$v(4) = 12$; $a(4) = 18$ b) $t = 2$; $t = 3$; $s(2) = 38$; $s(3) = 37$

c) positive: $(-\infty, 2)$; $(3, \infty)$; negative: $(2, 3)$ d) 205 m

e)



17. a) $v(t) = gt + v_0$; $a(t) = g$

b) $h(t) = -4.9t^2 + 17.5t + 4$; $v(t) = -9.8t + 17.5$;

$a(t) = -9.8$ c) $h(t) = -4.9t^2 + 30t + 2$; $v(t) = -9.8t + 30$;

$a(t) = -9.8$ 18. a) $s(t) = -6t^2 + 24t$; $v(t) = -12t + 24$;

$a(t) = -12$ b) 2 s 19. E 20. D

2.4 The Chain Rule, pages 117–119

1. a) $f'(x) = 24x^2$ b) $g'(x) = 64x^3$ c) $p'(x) = \frac{3x}{\sqrt{x^2}}$

d) $f'(x) = 12x^2$ e) $q'(x) = \frac{8}{3x^{\frac{1}{3}}}$

2.

$f(x) = g[h(x)]$	$g(x)$	$h(x)$	$h'(x)$	$g'[h(x)]$	$f'(x) = g'[h(x)]h'(x)$
a) $(6x - 1)^2$	x^2	$6x - 1$	6	$2h(x)$	$12(6x - 1)$
b) $(x^2 + 3)^3$	x^3	$x^2 + 3$	$2x$	$3[h(x)]^2$	$6x(x^2 + 3)^2$
c) $(2 - x^3)^4$	x^4	$2 - x^3$	$3x^2$	$4[h(x)]^3$	$12x^2(x^3 - 2)^3$
d) $(-3x + 4)^{-1}$	x^{-1}	$-3x + 4$	-3	$[-h(x)]^{-2}$	$\frac{3}{(-3x + 4)^2}$
e) $(7 + x^2)^{-2}$	x^{-2}	$7 + x^2$	$2x$	$-2[h(x)]^{-3}$	$\frac{-4x}{(x^2 + 7)^3}$
f) $\sqrt{x^4 - 3x^2}$	$x^{\frac{1}{2}}$	$x^4 - 3x^2$	$4x^3 - 6x$	$\frac{1}{2}[h(x)]^{-\frac{1}{2}}$	$\frac{2x^2 - 3}{\sqrt{x^2 - 3}}$

3. a) $y' = 8(4x + 1)$ b) $y' = 18x(3x^2 - 2)^2$
 c) $y' = \frac{-3(3x^2 - 1)}{(x^3 - x)^4}$ d) $y' = \frac{-2(8x + 3)}{(4x^2 + 3x)^3}$

4. a) $y = (2x - 3x^5)^2$; $\frac{dy}{dx} = \frac{2 - 15x^4}{2\sqrt{2x - 3x^5}}$

b) $y = (-x^3 + 9)^{\frac{1}{2}}$; $\frac{dy}{dx} = \frac{-3x^2}{2\sqrt{-x^3 + 9}}$

c) $y = (x - x^4)^{\frac{1}{3}}$; $\frac{dy}{dx} = \frac{1 - 4x^3}{3(x - x^4)^{\frac{2}{3}}}$

d) $y = \sqrt[5]{2 + 3x^2 - x^3}$; $\frac{dy}{dx} = \frac{6x - 3x^2}{5(2 + 3x^2 - x^3)^{\frac{4}{5}}}$

5. a) $y = (-x^3 + 1)^{-2}$; $\frac{dy}{dx} = \frac{6x^2}{(-x^3 + 1)^3}$

b) $y = (3x^2 - 2)^{-1}$; $\frac{dy}{dx} = \frac{6x}{(3x^2 - 2)^2}$

c) $y = (x^2 + 4x)^{\frac{1}{2}}$; $\frac{dy}{dx} = \frac{x + 2}{(x^2 + 4x)^{\frac{3}{2}}}$

d) $y = (x - 7x^2)^{-\frac{1}{3}}$; $\frac{dy}{dx} = \frac{1 - 14x}{3(x - 7x^2)^{\frac{4}{3}}}$

6. a) $f'(x) = 10x$ b) Answers may vary. c) No.

7. a) 56 b) $-\frac{2}{27}$ c) $\frac{4}{5}\sqrt{5}$ d) 0 8. a) $\frac{7}{4}$

b) $-\frac{9}{26}\sqrt{13}$ c) $-\frac{7}{4}$ d) $-\frac{5}{16}$ 9. $y = 729x - 2916$

10. $y = \frac{13}{80}x + \frac{33}{40}$ 11. $\frac{7}{3}$

12. $(-1, 0), \left(-\frac{1}{\sqrt{2}}, \frac{1}{16}\right), (0, 0), \left(\frac{1}{\sqrt{2}}, \frac{1}{16}\right), (1, 0)$

13. a) $N'(t) = \frac{1800t}{(16 + 3t^2)^{\frac{3}{2}}}$ represents the rate at which the customers are being served b) $N(4) = 75$; $N'(4) \doteq 14.06$; after 4 h, 75 customers are served at an instantaneous rate of change of $\frac{225}{16}$ customers per hour c) $t \doteq 7$ h represents the time when 103 customers are served

d) $N'(7) = \frac{12600}{26569}\sqrt{163} \doteq 6.05$; the customers are being served at a slower rate at 7 h.

14. $P'(2) = -12.01$; $P'(4) = -11.56$; $P'(7) = -10.92$

15. $V'(3) = 1617$; this represents the rate of change of the volume of the cube with respect to x .

16. $y = (4x - x^3)(3x^2 + 2)^{-2}$; $\frac{dy}{dx} = \frac{-42x^2 + 8 + 3x^4}{(3x^2 + 2)^3}$

18. $y = \frac{35}{3}x - \frac{26}{3}$; $y = \frac{35}{3}x + \frac{26}{3}$ 19. -12

20. $\frac{d^2y}{dx^2} = \frac{1}{(2x + 1)^2}$ 21. Answers may vary.

a) $f(x) = x^2$; $g(x) = 2x$ b) $f(x) = g(x) = x^2$

22. a) $\frac{dy}{dx} = -\frac{2(x + 1)}{x^2(x + 2)^2}$ b) $\frac{dy}{dx} = -\frac{2(x + 1)}{x^2(x + 2)^2}$

c) $\frac{dy}{dx} = -\frac{2(x^2 + 1)}{x(x^2 + 2)\sqrt{x^2(x^2 + 2)}}$ d) $\frac{dy}{dx} = -\frac{2(x^2 + 1)}{x^3\sqrt{2x^2 + 1}}$

23. $\frac{dy}{dx} = f'(g[h(x)])(g'[h(x)][h'(x)])$ 24. D 25. E

2.5 Derivatives of Quotients, pages 124–126

1. a) $q(x) = (3x + 5)^{-1}$; $x \neq -\frac{5}{3}$ b) $f(x) = -2(x - 4)^{-1}$; $x \neq 4$

c) $g(x) = 6(7x^2 + 1)^{-1}$; no restrictions

d) $r(x) = -2(x^3 - 27)^{-1}$; $x \neq 3$

2. a) $q'(x) = -(3x + 5)^{-2}(3)$ b) $f'(x) = 2(x - 4)^{-2}(1)$

c) $g'(x) = -6(7x^2 + 1)^{-2}(14x)$ d) $r'(x) = 2(x^3 - 27)^{-2}(3x^2)$

3. a) $q(x) = 3x(x + 1)^{-1}$; $x \neq -1$

b) $f(x) = -x(2x + 3)^{-1}$; $x \neq -\frac{3}{2}$

c) $g(x) = x^2(5x - 4)^{-1}$; $x \neq \frac{4}{5}$ d) $r(x) = 8x^2(x^2 - 9)^{-1}$; $x \neq \pm 3$

4. a) $q'(x) = 3(x + 1)^{-1} + 3x(-1)(x + 1)^{-2}(1)$

b) $f'(x) = -1(2x + 3)^{-1} + (-x)(-1)(2x + 3)^{-2}(2)$

c) $g'(x) = 2x(5x - 4)^{-1} + x^2(-1)(5x - 4)^{-2}(5)$

d) $r'(x) = 16x(x^2 - 9)^{-1} + 8x^2(-1)(x^2 - 9)^{-2}(2x)$

5. a) $\frac{dy}{dx} = \frac{2x^2 - 12x - 5}{(2x^2 + 5)^2}$ b) $\frac{dy}{dx} = \frac{8x^3 + 3x^2 + 8}{(x^3 - 2)^2}$

c) $\frac{dy}{dx} = 3$ d) $\frac{dy}{dx} = \frac{x^3(2x^2 - 3x + 4)}{(x^2 - x + 1)^2}$

6. a) $\frac{4}{25}$ b) $-\frac{5}{4}$ c) $-\frac{17}{32}$ d) 0 e) $\frac{7}{5}$

7. a) Express as a product and Product Rule or Simplify

and Sum Rule: $q'(x) = \frac{5x^2 - 4x + 18}{x^4}$

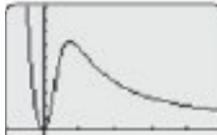
b) Answers may vary

8. a) $(-1, 1), (-3, -9)$ 9. $y = -20x - 16$

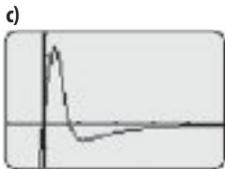
10. a) $\frac{3}{5}$ m/s b) 2 s 11. a) $C'(1) \doteq 7.9$; $C'(3) \doteq 17.4$;

$C'(5) \doteq 10.4$; $C'(8) \doteq -1.4$; C' is the number of new clients in the fund per week.

b)



The slope of the line indicates if $C'(w)$ is positive, zero, or negative



e) Answers may vary.

12. a) 1805.3 b) 206.55 c) $N'(1) \approx 69.5; N'(6) \approx 328.3$

d) Never. The number of customers will continue to increase. 13. a) \$1767.77

b) $V'(t) = \frac{0.4(3t^2 + 12517t + 87508)}{(2t + 8)^3}$ c) always increasing

d) $V'(2) = 1083.05; V'(22) = 388.65$; these values represent the rate of increase in value of the painting. It gains value more quickly 2 years after purchase than 22 years after purchase.

14. a) $f^{(n)}(x) = \frac{(-1)^n n! a^n}{(ax + b)^{n+1}}$, $x \neq -\frac{b}{a}$ b) $f^{(4)}(x) = \frac{384}{(2x - 3)^5}$

15. a) $(0, -1)$ b) $\left(-\frac{2}{\sqrt{3}}, -\frac{1}{2}\right); \left(\frac{2}{\sqrt{3}}, -\frac{1}{2}\right)$



local minimum and points of inflection



local extrema

16. a) $\frac{dy}{dx} = \frac{x^4 - 2x^3 + 2x - 1}{2x^2(x^2 - x + 1)}$, $x \neq 0$

b) $\frac{dy}{dx} = \frac{x^3 - 3x^2 + 3x - 2}{2x^2(x - 1)^2}$ x $\neq 0, 1$ 17. D 18. D

Extension: The Quotient Rule, page 129

1–3. See Section 2.5, Q4, Q5 and Q6 answers

4. a) i), ii) $\frac{dy}{dx} = \frac{5(2x + 3)}{(x^2 + 3x)^6}$ b) The power of a function rule is more efficient because the numerator is one.

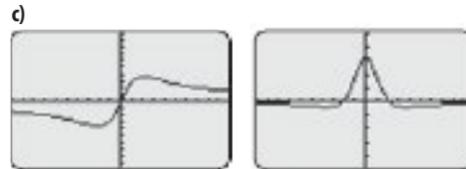
c) Answers may vary 5. $\frac{28}{25}$ 6. $(0, 0); (-5, -5)$ 7. No.

8. a) $n'(1) = 150; n'(5) = \frac{750}{169}$ b) No 9. $-\frac{92}{961}$; the rate of change of antibiotic concentration at 3 h

10. a) $c'(1) = \frac{42}{121}; c'(4) = -\frac{138}{1681}; c'(7) = -\frac{534}{11449}$

b) zero: $t = \frac{3}{\sqrt{2}}$; positive $t \in \left(0, \frac{3}{\sqrt{2}}\right)$;

negative: $t \in \left(\frac{3}{\sqrt{2}}, \infty\right)$



d) The concentration increases to a peak value at $t = \frac{3}{\sqrt{2}}$.

After this point, the concentration begins to decrease

e) $\frac{480}{68921}$ represents the rate of dissipation of the cleaner at 4 days

2.6 Rate of Change Problems, pages 137–141

1. a) $R(x) = \frac{575x}{\sqrt{x}} - 3x$ b) $R'(x) = \frac{575}{2\sqrt{x}} - 3$ c) \$17.33

2. a) $P(x) = 0.002x^2 - 153x - 575\sqrt{x} - 2000$

b) $P'(x) = 0.004x - 153 + \frac{575}{2\sqrt{x}}$ d) -\$138.14

3. a) $R(x) = 17.5x$ b) $R'(x) = 17.5$

c) $P(x) = 0.001x^3 - 0.025x^2 + 13.5x$

d) $P'(x) = 0.003x^2 - 0.05x + 13.5$

e) $R'(300) = \$17.50, P'(300) = \268.50

4. a) 0.41 g/m b) $f'(5) = \frac{1}{3}$ g/m; $f'(8) = \frac{1}{\sqrt{15}}$ g/m; the

density of the wire decreases as the distance increases.

5. a) $p(x) = 86 - 0.2x$ b) $R(x) = 86x - 0.2x^2$

c) $R'(x) = 86 - 0.4x$ d) $x = 215$; revenue is maximized at this point e) $p(215) = \$43.00$ 6. a) 12.75 kg/m

b) 11.75 kg/m 7. a) i) Increasing: The slope of the graph is positive ii) The rate of growth is positive during this period; the slope of the graph is positive. b) i, ii, vi, iii, iv; the intervals are ordered from the least steep to the interval with the steepest slope. c) The rate of inflation is higher after 1975. You can conclude the economy was doing well d) Decreasing; the slope of the graph decreases slightly.

8. a) \$2.80 b) \$1.70 c) \$4.50; the gross income, or revenue, will be the number of containers sold times the price. Thus, the price is the revenue divided by the number of containers sold. 9. a) $C'(5) = 45$;

$[C(5.001) - C(5)] = 0.045$ b) \$1.64 c) -0.32; the

change in cost of producing an additional unit

10. a) 12 500 b) $P'(3) = 313.25; P'(8) = 272$

c) in 14 years and in 26.66 years d) in 10 years

e) Increasing until year 20.66; At this point the population begins decreasing 11. a) \$1920; rate of change of total cost with respect to x b) \$1918.98 c) The cost of the 751st hot tub is less than the marginal cost at 750

d) $R(x) = 9200x$ e) \$7280 12. $-\frac{10}{27}$ A/Ω 13. 7.2°F/min

14. Negative. The pupil becomes smaller as it is exposed to more light. 15. a) -\$15 per item; -\$0.60 per item

b) Rate of change of revenue decreases with sales of commodity c) $x = 3$ d) \$1500

16. a) $-8.9375c$ **b)** $r = 0$; The rate of change of airflow is zero; $c = 1$. **17. a)** $p(x) = 5.75 - 0.002x$ **b)** \$1767.50; \$4.35 **c)** \$3.15 **d)** \$3.1495 **e)** \$203.75; \$1.20 **f)** \$5.03; \$0.60; the profit is much lower than revenue due to the cost of producing the mocha lattes. **18. a)** 0.206 g/s

b) No; the function is $M'(t) = \frac{13.86}{(t + 2.2)^2}$, which is always positive. **19. a)** $R(x) = \frac{650x}{\sqrt{x}} - 4.5x$ **b)** \$10.03

c) $P(x) = \frac{650x}{\sqrt{x}} - 129.5x$ **d)** $-\$114.97$ **20.** $\frac{15(2c - 45b)}{abc}$

21. a) Answers will vary. **b)** i) $\frac{du}{dM} = \frac{1181.25}{(M + 150)^2}$

ii) 0.000 82 m/s² **22. D** **23. C**

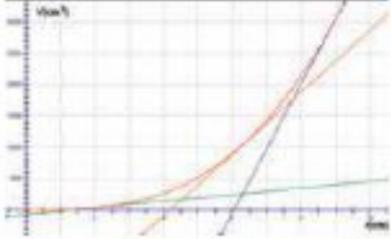
Chapter 2 Review, pages 142–143

1. Rules used may vary. **a)** $b'(t) = 3t^2 - 4t - \frac{2}{t^3}$

b) $p'(n) = -5n^4 + 15n^2 + \frac{2}{3\sqrt[3]{n}}$ **c)** $p'(r) = 6r^5 + \frac{1}{5\sqrt[5]{r^3}} + 1$

2. a) $V'(1.5) = 9\pi$; $V'(6) = 144\pi$; $V'(9) = 324\pi$

b)

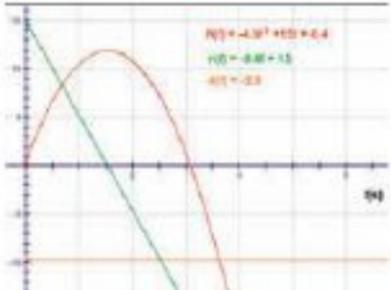


c) $y_{1,5} = 28.27x - 28.27$; $y_6 = 452.39x - 1809.56$;
 $y_9 = 1017.88x - 6107.26$

3. a) $f'(x) = 20x - 49$ **b)** $h'(t) = \frac{72x^{\frac{8}{3}} - 60x^{\frac{5}{3}} + 16x - 5}{3x^{\frac{2}{3}}}$

c) $g'(x) = -27x^5 + 84x^6 - 8$ **d)** $p'(n) = 99n^2 + 12n - 55$

4. a) $y = 3$ **b)** $y = 21x - 40$ **5. 34** **6. a)** 10.8 m **b)** 5.2 m/s; -24.2 m/s **c)** 3.088 s **d)** -15.26 m/s **e)** i) 1.53 s
ii) 11.88 m **iii)** 0 m/s; at $t = 1.53$, the velocity function passes through zero.



7. a) 3.67 bushes per year **b)** 10.85 years

c) 3.68 bushes per year

8. a) $\frac{7}{4}$ **b)** $\frac{1}{3\sqrt{2}}$ **c)** $\frac{1}{4}$ **9. a)** $\frac{16}{9}$ **b)** $\frac{31}{50}$ **c)** 1 **d)** $\frac{18}{25}$

10. a) $q'(x) = \frac{140x^2 + 21 - 48x}{(4x^2 - 3)^4}$

b) $\frac{dy}{dx} = \frac{12x^2(5x - 4)}{(3x - 2)^2}$ **c)** $m'(x) = -\frac{2(x - 2)(-23 + 5x)}{(3 + 5x)^5}$

d) $\frac{dy}{dx} = \frac{2(x^2 - 3)(7x^2 + 10x + 3)}{(4x + 5)^2}$

e) $\frac{dy}{dx} = \frac{-3(2\sqrt{x} + 7)^2(13x^3 - 25x^2 - 1 + 49x^2 - 98x^2)}{(x^3 - 3x^2 + 1)^8\sqrt{x}}$

11. $y = -216x - 459$ **12. a)** $p(x) = 42 - 0.15x$

b) \$3.00 **c)** \$3.30 **d)** -\$6.30

13. a) $V(t) = (4.85 - 0.01t^2)(15 + 0.11t)$

b) $V'(t) = -0.3t - 0.0033t^2 + 0.5335$ The rate of change in voltage over time **c)** -0.0797 V/s **d)** -0.04 A/s **e)** 0.11 Ω/s **f)** No. The product rule dictates that $V'(2) = I(2)R'(2) + I'(2)R(2)$

Chapter 2 Practice Test, pages 144–145

1. B; **2. C;** **3. A** and **B** are incorrect.

4. 2178 **5.** Methods may vary. **a)** $\frac{dy}{dx} = 2 - \frac{1}{3^3 x}$

b) $\frac{dy}{dx} = 6x^2 + 2x - 8$ **6. a)** $\frac{dy}{dx} = \frac{-5(3x^8 + 4)}{x^6}$

b) $g(x) = 3x^2(8x - 3)^2(16x - 3)$

c) $m'(x) = \frac{5x^6 - 10x - 18x^5 + 54}{\sqrt{9 - 2x(x^4)}}$

d) $f'(x) = \frac{3 - 2x}{(1 - x^2)^{\frac{3}{2}}}$

7. a) -18.4 m/s; -9.8 m/s²

b) The arrow moves upward until 1.12 s and then stops and moves downward. **c)** at $t = 1.12$ s **d)** 8.17 m; at this point the arrow has zero velocity and is at its maximum height **e)** 2.41 s; -12.62 m/s **8. $y = -8x - 9$** **9. (4, 3)**

10. a) Faster at A and H **b)** 0 **c)** The vehicle is stopped **d)** Slowing down **e)** Vehicle returns to starting position; **f)** i) - ii) - iii) + iv) 0 **v)** - **11. a)** $p(x) = 49.5 - 0.025x$

b) -\$40.50 **c)** \$5.70 **d)** \$5.70 **e)** -\$3980; -\$46.20

12. a) -\$1917.53; -\$1775.54; -\$1590.40

b) \$100 000.00 **c)** Used; The new boat depreciates in value rapidly after purchase and less so at later times

13. a) \$47 **b)** \$47.01 **c)** The cost of producing the 251st player only slightly exceeds the marginal cost of production at the 250 production level.

- d) $R(x) = 130x - 0.4x^2$; $P(x) = -0.41x^2 + 88x - 300$
e) $-\$70$; $-\$117$ f) The production level is too high; profit is not being maximized.

14. a) \$5500 b) $V'(t) = \frac{1000(3t^3 + 2250t - 1375)}{(5t^2 + 2500)^{\frac{3}{2}}}$

- c) Increasing; $V'(t)$ is always positive.
d) \$5611.72; \$10 498.45 e) \$127.49; \$1468.20; the dining set is gaining value at a faster rate with time.
f) 10.0 years; \$1468.20 per year

CHAPTER 3 CURVE SKETCHING

Prerequisite Skills, pages 148–149

1. a) $(x+1)(x^2+x+1)$ b) $(z+2)(z^2-2z-2)$
c) $(t+5)(t+4)(t-3)$ d) $(b+4)(b+3)(b+1)$
e) $(n+1)(n-1)(3n-1)$ f) $(2p-1)(p-1)(p-3)$
g) $(k+1)(4k+3)(k-1)$ h) $(3w+5)(w-3)(2w-1)$
2. a) $x = 3$; $x = 4$ b) $x = -\frac{3}{2}$; $x = \frac{3}{2}$ c) $v = 0$; $v = 2$
d) $a = -7$; $a = 5$ e) $t = \frac{98 \pm 3\sqrt{931}}{49}$
f) $x = -5$; $x = -2$; $x = 1$ g) $x = -2$; $x = 7$
3. a) $x > 5$ b) $-5 < x < 0$ c) $x > 4$ d) $-7 < x < 2$
e) $-2 < x < 1$ or $x > 3$ f) $-1 < x < 0$ or $x > 1$; $x \neq \pm 1$
4. a) $x = 3$ b) $x = -4$, $x = 7$ c) $x = -3$, $x = -2$, $x = -1$
d) $x = \pm 3$ 5. a) \mathbb{R} ; \mathbb{R} b) \mathbb{R} ; $\{y | y \in \mathbb{R}, y \geq -9\}$ c) \mathbb{R} ; \mathbb{R}
d) $\{x | x \in \mathbb{R}, x \neq -1\}$; $\{y | y \in \mathbb{R}, y \neq 0\}$
e) $\{y \in \mathbb{R} | y \neq 4\}$; \mathbb{R} f) $\{y \in \mathbb{R}, y < -\frac{1}{3} \text{ or } y > 0\}$;
 $\left\{ y \mid y \in \mathbb{R}, -\frac{1}{3} < y \leq 0 \right\}$ g) $\{x \in \mathbb{R} | y \in \mathbb{R} | -0.5 \leq y \leq 0.5\}$;
 $\{y \mid y \in \mathbb{R}, y > 0\}$
6. a) none b) none c) none d) $x = -1$, $y = 0$
e) no asymptotes f) $x = -3$, $x = 3$, $y = 0$ g) $y = 0$
7. a) increasing: $-\infty < x < 0$, $2 < x < \infty$;
decreasing: $0 < x < 2$ b) increasing: $-2 < x < 0$,
 $2 < x < \infty$; decreasing: $-\infty < x < -2$, $0 < x < 2$
8. a) $f'(x) = 10x - 7$ b) $\frac{dy}{dx} = 3x^2 - 4x + 4$
c) $f'(x) = \frac{1}{x^2}$ d) $\frac{dy}{dx} = \frac{20x}{(x^2 + 1)^2}$
9. $V(x) = 4x^3 - 200x^2 + 2400x$, $0 < x < 20$
10. $SA(r) = 2\pi r^2 + \frac{2000}{r}$ 11. a) odd b) even c) even
d) neither 12. a) odd b) neither c) even d) neither
e) odd f) even

3.1 Increasing and Decreasing Functions, pages 156–158

1. a) $x = 3$ b) $x = 1$, $x = -9$ c) $x = -2$, $x = 2$
d) $x = 6$, $x = 0$ e) $x = -1 \pm \sqrt{5}$ f) $x = -4$, $x = 2$, $x = 5$
g) $x = -3$, $x = -2$, $x = 2$ h) $x = -1$, $x = 0$, $x = 1$
2. a) increasing: $x < 3$; decreasing: $x > 3$

- b) increasing: $x > 1$ and $x < -9$; decreasing: $-9 < x < 1$
c) increasing: $x < -2$ and $x > 2$; decreasing: $-2 < x < 2$
d) increasing: $x > 6$; decreasing: $x < 0$, $0 < x < 6$

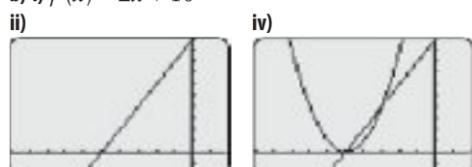
- e) increasing: $x < -1 - \sqrt{5}$, $x > -1 + \sqrt{5}$;
decreasing: $-1 - \sqrt{5} < x < -1 + \sqrt{5}$
f) increasing: $-4 < x < 2$, $x > 5$; decreasing: $x < -4$,
 $2 < x < 5$ g) increasing: $-3 < x < -2$, $x > 2$;
decreasing: $x < -3$, $-2 < x < 2$ h) increasing: $x < -1$,
 $0 < x < 1$, $x > 1$ decreasing: $-1 < x < 0$

3. a) i) $f'(x) = 6$



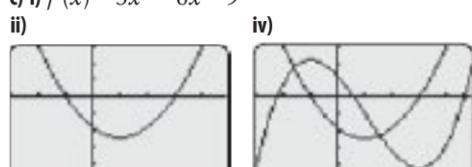
- iii) always increasing

- b) i) $f'(x) = 2x + 10$



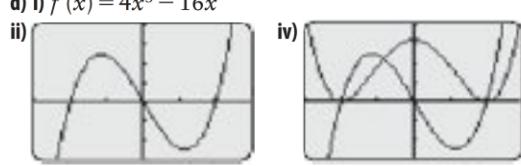
- iii) increasing: $x > -5$; decreasing: $x < -5$

- c) i) $f'(x) = 3x^2 - 6x - 9$

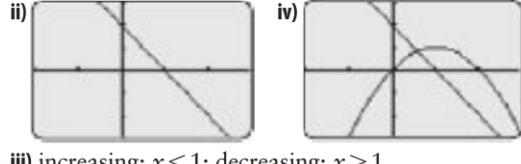


- iii) increasing: $x < -1$, $x > 3$; decreasing: $-1 < x < 3$

- d) i) $f'(x) = 4x^3 - 16x$



- iii) increasing: $-2 < x < 0$, $x > 2$; decreasing: $x < -2$, $0 < x < 2$ e) i) $f'(x) = 2 - 2x$



- iii) increasing: $x < 1$; decreasing: $x > 1$

- f) i) $f'(x) = 3x^2 + 2x - 1$

