1

$y = 2u^3 + 3u^2$

$u = x + \sqrt{x}$

find $\frac{dy}{dx}$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

$$= (6u^2 + 6u) \cdot \left(1 + \frac{1}{2\sqrt{x}}\right)$$

$$= 6\left[(x+\sqrt{x})^2 + (x+\sqrt{x})\right] \cdot \left(1 + \frac{1}{2\sqrt{x}}\right)$$

$$= \boxed{6\left[(x+\sqrt{x})(x+\sqrt{x}+1)\right] \cdot \left(1 + \frac{1}{2\sqrt{x}}\right)}$$

2

$y = 5u^2 + 3u - 1$

$u = \frac{18}{x^2+5}$

$$\frac{dy}{du} = \frac{dy}{du} \cdot \frac{du}{dx}$$

$$= (10u + 3)\left(\frac{-18(2x)}{(x^2+5)^2}\right)$$

$$= \frac{-36x\,(10u+3)}{(x^2+5)^2}$$

$$= \frac{-36x\left[10\left(\frac{18}{x^2+5}\right) + 3\right]}{(x^2+5)^2}$$

$$= \frac{-36x\left[180 + 3(x^2+5)\right]}{(x^2+5)^3}$$

$$= \frac{-36x(180 + 3x^2 + 15)}{(x^2+5)^3}$$

$$= \frac{-36x(195 + 3x^2)}{(x^2+5)^3}$$

$$= \boxed{\frac{-7020x - 108x^3}{(x^2+5)^3}} \text{ or } \boxed{\frac{-108x(65 + x^2)}{(x^2+5)^3}}$$

$\frac{1}{2}+\frac{1}{2}$

(3) $g(x) = \sqrt{x}\,(x^3 - x)$

$g(x) = x^{\frac{1}{2}}(x^3 - x)$

$g'(x) = \frac{1}{2}x^{-\frac{1}{2}}(x^3 - x) + x^{\frac{1}{2}}(3x^2 - 1)$

$= x^{-\frac{1}{2}}\left[\frac{1}{2}(x^3 - x) + x(3x^2 - 1)\right]$

$= x^{-\frac{1}{2}}\left[\frac{1}{2}x^3 - \frac{1}{2}x + 3x^3 - x\right]$

$= x^{-\frac{1}{2}}\left[\frac{1}{2}x^3 - \frac{1}{2}x + \frac{6x^3}{2} - \frac{2x}{2}\right]$

$= \frac{7x^3 - 3x}{2x^{\frac{1}{2}}}$

$= \frac{x(7x^2 - 3)}{2x^{1/2}}$

$= \boxed{\frac{\sqrt{x}(7x^2 - 3)}{2}}$

(4) $h(x) = \frac{(2x^4 - x^3)^2}{(3x^2 + x - 2)^4}$

$h'(x) = \frac{2(2x^4 - x^3)(8x^3 - 3x^2)(3x^2 + x - 2)^4 - (2x^4 - x^3)^2(4)(3x^2 + x - 2)^3(6x + 1)}{(3x^2 + x - 2)^8}$

$= \frac{2(2x^4 - x^3)\cancel{(3x^2 + x - 2)^3}\left[(8x^3 - 3x^2)(3x^2 + x - 2) - (2x^4 - x^3)(2)(6x + 1)\right]}{(3x^2 + x - 2)^{8\,5}}$

$= \frac{2(2x^4 - x^3)\left[\cancel{24x^5} + 8x^4 - 16x^3 - 9x^4 - 3x^3 + 6x^2 - 2(\cancel{12x^5} + \cancel{2x^4} - \cancel{6x^4} - \cancel{x^3})\right]}{(3x^2 + x - 2)^5}$

$= \frac{2(2x^4 - x^3)(-7x^4 - 17x^3 + 6x^2)}{(3x^2 + x - 2)^5}$

$= \frac{2x^2(2x^4 - x^3)(7x^2 - 17x + 6)}{(3x^2 + x - 2)^5}$

$\begin{matrix} 1 & -2 \\ 7 & -3 \end{matrix}$

$= \boxed{\frac{2x^2(2x^4 - x^3)(x - 2)(7x - 3)}{(3x^2 + x - 2)^5}}$

5 Find the equation of the tangent to the curve

$y = \dfrac{1}{4\sqrt{x^2-2x+1}}$ at $x=8$. We must simplify the derivative first.

1

$$y = \frac{1}{4(x^2-2x+1)^{\frac{1}{2}}}$$

$$y' = \frac{-\left[\boxed{0} + 4\left(\frac{1}{2}\right)(x^2-2x+1)^{-\frac{1}{2}}(2x-2)\right]}{\left(4(x^2-2x+1)^{1/2}\right)^2}$$

$$= \frac{-\left[2(x^2-2x+1)^{-1/2}(\overset{2(x-1)}{2x-2})\right]}{16(x^2-2x+1)}$$

$$= \frac{-\cancel{4}(x^2-2x+1)^{-3/2}[(x-1)]}{4\cancel{16}(x^2-2x+1)^{1}}$$

$-\frac{1}{2}-1$

$$= \frac{-x+1}{4(x^2-2x+1)^{3/2}}$$

$$= \frac{-x+1}{4\left(\sqrt{(x-1)(x-1)}\right)^3} \text{ or } \frac{-x+1}{4(x-1)^3}$$

$$= \frac{-(x-1)}{4[(x-1)(x-1)]^{3/2}}$$

$$= \frac{-\cancel{(x-1)}}{4\cancel{(x-1)}^3}$$

$$= -\frac{1}{4(x-1)^2}$$

2

$$\therefore y'\Big|_{x=8} = \frac{-1}{4(8-1)^2}$$

$$= -\frac{1}{4(7)^2}$$

$$= \frac{-1}{4(49)}$$

$$= \boxed{\frac{-1}{196}}$$

and

3 $y = \dfrac{1}{4\sqrt{(x-1)(x-1)}}$

when $x=8$ $y = \dfrac{1}{4\sqrt{7\cdot 7}}$

$$= \frac{1}{4\cdot 7}$$

$$= \boxed{\frac{1}{28}}$$

4 The equation of the tangent at $x=8$ is:

$$\therefore y = \frac{-1}{196}(x-8) + \frac{1}{28}$$

$$y = \frac{-x}{196} + \frac{8}{196} + \frac{1}{28}$$

$$\boxed{y = -\frac{1}{196}x + \frac{15}{196}}$$