For questions 1 to 3, choose the best answer.

1. Which of the following is not a derivative rule? Justify your answer with an example.

A $\dfrac{d}{dx}[f(x) + g(x)] = \dfrac{d}{dx}f(x) + \dfrac{d}{dx}g(x)$

B $\dfrac{d}{dx}f[g(x)] = \dfrac{d}{dx}f(x)\dfrac{d}{dx}g(x)$

C $\dfrac{d}{dx}\dfrac{f(x)}{g(x)} = f(x)\dfrac{d}{dx}[g(x)]^{-1} + [g(x)]^{-1}\dfrac{d}{dx}f(x)$

D $\dfrac{d}{dx}cf(x) = c\dfrac{d}{dx}f(x)$

2. Which statement is always true for an object moving along a vertical straight line? Explain why each of the other statements is not true.

A The object is speeding up when $v(t)a(t)$ is negative.

B The object is slowing down when $v(t)a(t)$ is positive.

C The object is moving upward when $v(t)$ is positive.

D The object is at rest when the acceleration is zero.

3. Which of the following are incorrect derivatives for $y = \dfrac{-4x}{x^2 + 1}$? Justify your answers.

A $y' = \dfrac{-4}{2x}$

B $y' = \dfrac{(x^2 + 1)(-4) - 4x(2x)}{(x^2 + 1)^2}$

C $y' = -4(x^2 + 1)^{-1} + 8x^2(x^2 + 1)^{-2}$

D $y' = \dfrac{(x^2 + 1)(-4) + 4x(2x)}{(x^2 + 1)^2}$

4. Determine $f''(3)$ for the function $f(x) = (5x^2 - 3x)^2$.

5. Describe two different methods that can be used to differentiate each of the following. Differentiate each function using the methods you described.

a) $y = (3x^6)^{\frac{1}{3}}$ **b)** $y = (x^2 - 4)(2x + 1)$

6. Differentiate each function.

a) $y = -5x^3 + \dfrac{4}{x^5} + 1.7\pi$

b) $g(x) = (8x^2 - 3x)^3$

c) $m(x) = \sqrt{9 - 2x}\left(x^2 + \dfrac{2}{x^3}\right)$

d) $f(x) = \dfrac{3x - 2}{\sqrt{1 - x^2}}$

7. Mia shoots an arrow upward with an initial vertical velocity of 11 m/s from a platform that is 2 m high. The height, h, in metres, of the arrow after t seconds is modelled by the equation $h(t) = -4.9t^2 + 11t + 2$, $t \geq 0$.

a) Determine the velocity and acceleration of the arrow after 3 s.

b) When is the arrow moving upward? When is it moving downward? Justify your answer.

c) When is the arrow momentarily at rest?

d) What is the height of the arrow for the time found in part c)? What is the significance of this value?

e) When does the arrow hit the ground? With what velocity does it hit the ground?

8. Determine the equation of the tangent to the curve $y = \dfrac{-x}{(3x + 2)^3}$ at the point where $x = -1$.

9. Determine the coordinates of the point on the graph of $f(x) = \sqrt{2x + 1}$ where the tangent line is perpendicular to the line $3x + y + 4 = 0$.

10. The graph below shows the position function of a vehicle.
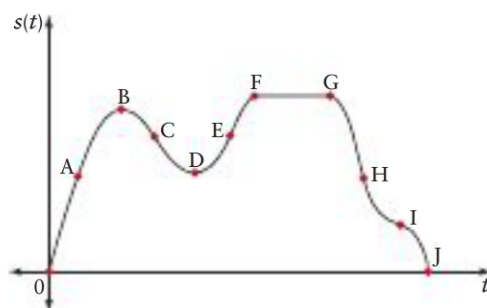
a) Is the vehicle going faster at A or at E? Is it going faster at C or at H?

b) What is the velocity of the vehicle at B and at D?

c) What happens between F and G?

d) Is the vehicle speeding up or slowing down at C and I?

e) What happens at J?

f) State whether the acceleration is positive, negative, or zero over each interval.
  i) 0 to A   **ii)** B to C
  iii) D to E   **iv)** F to G
  v) I to J

11. The student council normally sells 1500 school T-shirts for $12 each. This year they plan to decrease the price of the T-shirts. Based on student feedback, they know that for every $0.50 decrease in price, 20 more T-shirts will be sold.

a) Determine the demand, or price, function.

b) Determine the marginal revenue from the sales of 1800 T-shirts.

c) The cost of producing x T-shirts is $C(x) = -0.0005x^2 + 7.5x + 200$. Determine the marginal cost of producing 1800 T-shirts.

d) Determine the actual cost of producing the 1801st T-shirt.

e) Determine the profit and marginal profit from the sale of 1800 T-shirts.

12. Suppose the function $V(t) = \dfrac{100\,000 + 5t}{1 + 0.02t}$ represents the value, in dollars, of a new motorboat t years after it is purchased.

a) What is the rate of change of the value of the motorboat at 1, 3, and 6 years?

b) What was the initial value of the motorboat?

c) Do the values in part a) support the purchase of a new motorboat or a used one? Explain your reasoning.

13. The cost, C, in dollars, of manufacturing x MP3 players per day is represented by the function $C(x) = 0.01x^2 + 42x + 300$, $0 \le x \le 300$. The demand function is $p(x) = 130 - 0.4x$.

a) Determine the marginal cost at a production level of 250 players.

b) Determine the actual cost of producing the 251st player.

c) Compare and describe your results from parts a) and b).

d) Determine the revenue function and the profit function.

e) Determine the marginal revenue and marginal profit for the sale of 250 players.

f) Interpret the meaning of the values in part e) for this situation.

14. The value of an antique solid wood dining set t years after it is purchased is modelled by the function $V(t) = \dfrac{(5500 + 6t^3)}{\sqrt{0.002t^2 + 1}}$, where V is in dollars, and $t \ge 0$.

a) What was the price of the dining set when it was purchased?

b) Determine the rate of change of the value of the dining set after t years.

c) Is the value of the dining set increasing or decreasing? Justify your answer.

d) What is the dining set worth after 3 years and after 10 years?

e) Compare $V'(3)$ and $V'(10)$. Interpret the meaning of these values for this situation.

f) **Use Technology** When will the dining set be worth about $10\,500? What is the rate of change of the value of the dining set at this time?