Chapter 2 REVIEW

2.1 Derivative of a Polynomial Function

1. Differentiate each function. State the derivative rules used.

a) $h(t) = t^3 - 2t^2 + \frac{1}{t^2}$

b) $p(n) = -n^5 + 5n^3 + \sqrt[3]{n^2}$

c) $p(r) = r^6 - \frac{2}{5\sqrt{r}} + r - 1$

2. Air is being pumped into a spherical balloon. The volume of the balloon is $V = \frac{4}{3}\pi r^3$, where the radius, r, is in centimetres.

a) Determine the instantaneous rate of change of the volume of the balloon when its radius is 1.5 cm, 6 cm, and 9 cm.

b) Sketch a graph of the curve and the tangents corresponding to each radius in part a).

c) State the equations of the tangent lines.

2.2 The Product Rule

3. Differentiate using the power rule.

a) $f(x) = (5x + 3)(2x - 11)$

b) $h(t) = (2t^2 + \sqrt[3]{t})(4t - 5)$

c) $g(x) = (-1.5x^6 + 1)(3 - 8x)$

d) $p(n) = (11n + 2)(-5 + 3n^2)$

4. Determine the equation of the tangent to the graph of each curve at the point that corresponds to each value of x.

a) $y = (6x - 3)(-x^2 + 2)$, $x = 1$

b) $y = (-3x + 8)(x^3 - 7)$, $x = 2$

2.3 Velocity, Acceleration, and Second Derivatives

5. Determine $f''(-2)$ for $f(x) = (4 - x^2)(3x + 1)$.

6. A toy missile is shot into the air. Its height, in metres, after t seconds is given by the function $h(t) = -4.9t^2 + 15t + 0.4$, $t \geq 0$.

a) Determine the height of the missile after 2 s.

b) Determine the rate of change of the height of the missile at 1 s and at 4 s.

c) How long does it take the missile to return to the ground?

d) How fast was the missile travelling when it hit the ground? Explain your reasoning.

e) **Use Technology** Graph $h(t)$ and $v(t)$.

i) When does the toy missile reach its maximum height?

ii) What is the maximum height of the toy missile?

iii) What is the velocity of the missile when it reaches its maximum height? How can you tell this from the graph of the velocity function?

2.4 The Chain Rule

7. The population of a certain type of berry bush in a conservation area is represented by the function $p(t) = \sqrt[3]{16t + 50t^3}$, where p is the number of berry bushes and t is time, in years.

a) Determine the rate of change of the number of berry bushes after 5 years.

b) When will there be 40 berry bushes?

c) What is the rate of change of the berry bush population at the time found in part b)?

8. Apply the chain rule, in Leibniz notation, to determine $\frac{dy}{dx}$ at the indicated value of x.

a) $y = u^2 + 3u$, $u = \sqrt{x - 1}$, $x = 5$

b) $y = \sqrt{2u}$, $u = 6 - x$, $x = -3$

c) $y = 8u(1 - u)$, $u = \frac{1}{x}$, $x = 4$

2.5 Derivatives of Quotients

9. Determine the slope of the tangent to each.

a) $y = \frac{2x^2}{x + 1}$ at $x = 2$

b) $y = \frac{\sqrt{3x}}{x^2 - 4}$ at $x = 3$

c) $y = \frac{5x + 3}{x^3 + 1}$ at $x = -2$

d) $y = \frac{-4x + 2}{3x^2 - 7x - 1}$ at $x = 1$

10. Differentiate each function.

a) $q(x)=\dfrac{-7x+2}{(4x^2-3)^3}$ **b)** $y=\dfrac{8x^3}{\sqrt{3x-2}}$

c) $m(x)=\dfrac{(-x+2)^2}{(3+5x)^4}$ **d)** $y=\dfrac{(x^2-3)^2}{\sqrt{4x+5}}$

e) $y=\dfrac{(2\sqrt{x}+7)^3}{(x^3-3x^2+1)^7}$

11. Determine the equation of the tangent to the curve $y=\left(\dfrac{x^2-1}{4x+7}\right)^3$ at the point where $x=-2$.

2.6 Rate of Change Problems

12. A music store sells an average of 120 music CDs per week at \$24 each. A market survey indicates that for each \$0.75 decrease in price, 5 additional CDs will be sold per week.

a) Determine the demand, or price, function.

b) Determine the marginal revenue from the weekly sales of 150 music CDs.

c) The cost of producing x music CDs is $C(x)=-0.003x^2+4.2x+3000$. Determine the marginal cost of producing 150 CDs.

d) Determine the marginal profit from the weekly sales of 150 music CDs.

13. The voltage across a resistor in an electrical circuit is $V=IR$, where $I=4.85-0.01t^2$ is the current through the resistor, in amperes, $R=15.0+0.11t$ is the resistance, in ohms, and t is time, in seconds.

a) Write an equation for V in terms of t.

b) Determine $V'(t)$ and interpret its meaning for this situation.

c) Determine the rate of change of voltage after 2 s.

d) What is the rate of change of current after 2 s?

e) What is the rate of change of resistance after 2 s?

f) Is the product of the values in parts d) and e) equal to the value in part b)? Give reasons why or why not.

CHAPTER

PROBLEM WRAP-UP

The owners of Mooses, Gooses, and Juices hired a research firm to perform a market survey on their products. They discovered that the yearly demand for their **Brain Boost BlueBerry** frozen smoothie, also know as the **B**^{4}, is represented by the function $p(x)=\dfrac{45\,000-x}{10\,000}$, where p is the price, in dollars, and x is the number of **B**^{4}s ordered each year.

a) Graph the demand function.

b) Would you use a graph or an equation to determine the quantity of B^{4}s ordered when the price is \$0.50 and \$3.00? Explain your choice and determine each quantity.

c) Would you use a graph or an equation to determine the quantity of B^{4}s ordered when the price is \$2.75 and \$3.90? Explain your choice and determine each quantity.

d) Determine the marginal revenue when 20 000 **B**^{4}s are made each year. Explain the significance of this value.

e) Research shows that the cost, in dollars, of producing x number of **B**^{4}s is modelled by the function $C(x)=10\,000+0.75x$. Compare the profit and marginal profit when 15 000 **B**^{4}s are sold each year, versus 30 000 **B**^{4}s. Explain the meaning of the marginal profit for these two quantities.