Chapter 1
Introduction to Calculus

Review of Prerequisite Skills, pp. 2–3

1. a. $m = \dfrac{-7-5}{6-2}$
$= -3$

b. $m = \dfrac{4-(-4)}{-1-3}$
$= -2$

c. $m = \dfrac{4-0}{1-0}$
$= 4$

d. $m = \dfrac{4-0}{-1-0}$
$= -4$

e. $m = \dfrac{4-4.41}{-2-(-2.1)}$
$= -4.1$

f. $m = \dfrac{-\frac{1}{4}-\frac{1}{4}}{\frac{7}{4}-\frac{3}{4}}$
$= \dfrac{-\frac{2}{4}}{1}$
$= -\dfrac{1}{2}$

2. a. Substitute the given slope and y-intercept into $y = mx + b$.
$y = 4x - 2$

b. Substitute the given slope and y-intercept into $y = mx + b$.
$y = -2x + 5$

c. The slope of the line is
$m = \dfrac{12-6}{4-(-1)}$
$= \dfrac{6}{5}$

The equation of the line is in the form $y - y_1 = m(x - x_1)$. The point is $(-1, 6)$ and $m = \frac{6}{5}$.
The equation of the line is $y - 6 = \frac{6}{5}(x+1)$ or $y = \frac{6}{5}(x+1) + 6$.

d. $m = \dfrac{8-4}{-6-(-2)}$
$= -1$
$y - 4 = -1(x - (-2))$
$y - 4 = -x - 2$
$x + y - 2 = 0$

e. $x = -3$

f. $y = 5$

3. a. $f(2) = -6 + 5$
$= -1$

b. $f(2) = (8-2)(6-6)$
$= 0$

c. $f(2) = -3(4) + 2(2) - 1$
$= -9$

d. $f(2) = (10+2)^2$
$= 144$

4. a. $f(-10) = \dfrac{-10}{100+4}$
$= -\dfrac{5}{52}$

b. $f(-3) = \dfrac{-3}{9+4}$
$= -\dfrac{3}{13}$

c. $f(0) = \dfrac{0}{0+4}$
$= 0$

d. $f(10) = \dfrac{10}{100+4}$
$= \dfrac{5}{52}$

5. $f(x) = \begin{cases} \sqrt{3-x}, \text{ if } x < 0 \\ \sqrt{3+x}, \text{ if } x \geq 0 \end{cases}$

a. $f(-33) = 6$

b. $f(0) = \sqrt{3}$

c. $f(78) = 9$

d. $f(3) = \sqrt{6}$

6. $s(t) = \begin{cases} \dfrac{1}{t}, \text{ if } -3 < t < 0 \\ 5, \text{ if } t = 0 \\ t^3, \text{ if } t > 0 \end{cases}$

a. $s(-2) = -\dfrac{1}{2}$

b. $s(-1) = -1$

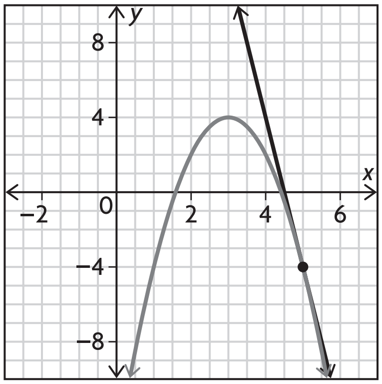
c. $s(0) = 5$
d. $s(1) = 1$
e. $s(100) = 100^3$ or 10^6
7. a. $(x - 6)(x + 2) = x^2 - 4x - 12$
b. $(5 - x)(3 + 4x) = 15 + 17x - 4x^2$
c. $x(5x - 3) - 2x(3x + 2) = 5x^2 - 3x - 6x^2 - 4x$
$= -x^2 - 7x$
d. $(x - 1)(x + 3) - (2x + 5)(x - 2)$
$= x^2 + 2x - 3 - (2x^2 + x - 10)$
$= -x^2 + x + 7$
e. $(a + 2)^3 = (a + 2)(a + 2)(a + 2)$
$= (a^2 + 4a + 4)(a + 2)$
$= a^3 + 6a^2 + 12a + 8$
f. $(9a - 5)^3 = (9a - 5)(9a - 5)(9a - 5)$
$= (81a^2 - 90a + 25)(9a - 5)$
$= 729a^3 - 1215a^2 + 675a - 125$
8. a. $x^3 - x = x(x^2 - 1)$
$= x(x + 1)(x - 1)$
b. $x^2 + x - 6 = (x + 3)(x - 2)$
c. $2x^2 - 7x + 6 = (2x - 3)(x - 2)$
d. $x^3 + 2x^2 + x = x(x^2 + 2x + 1)$
$= x(x + 1)(x + 1)$
e. $27x^3 - 64 = (3x - 4)(9x^2 + 12x + 16)$
f. $2x^3 - x^2 - 7x + 6$
$x = 1$ is a zero, so $x - 1$ is a factor. Synthetic or long division yields
$2x^3 - x^2 - 7x + 6 = (x - 1)(2x^2 + x - 6)$
$= (x - 1)(2x - 3)(x + 2)$
9. a. $\{x \in \mathbf{R} | x \geq -5\}$
b. $\{x \in \mathbf{R}\}$
c. $\{x \in \mathbf{R} | x \neq 1\}$
d. $\{x \in \mathbf{R} | x \neq 0\}$
e. $2x^2 - 5x - 3 = (2x + 1)(x - 3)$
$\left\{x \in \mathbf{R} \middle| x \neq -\frac{1}{2}, 3\right\}$
f. $\{x \in \mathbf{R} | x \neq -5, -2, 1\}$
10. a. $h(0) = 2, h(1) = 22.1$
average rate of change $= \frac{22.1 - 2}{1 - 0}$
$= 20.1$ m/s
b. $h(1) = 22.1, h(2) = 32.4$
average rate of change $= \frac{32.4 - 22.1}{2 - 1}$
$= 10.3$ m/s
11. a. The average rate of change during the second hour is the difference in the volume at $t = 120$ and $t = 60$ (since t is measured in minutes), divided by the difference in time.

$$\frac{V(120) - V(60)}{120 - 60} = \frac{0 - 1200}{60}$$
$$= -20 \text{ L/min}$$

b. To estimate the instantaneous rate of change in volume after exactly 60 minutes, calculate the average rate of change in volume from minute 59 to minute 61.

$$\frac{V(61) - V(59)}{61 - 59} \doteq \frac{1186.56 - 1213.22}{2}$$
$$= -13.33 \text{ L/min}$$

c. The instantaneous rate of change in volume is negative for $0 \leq t \leq 120$ because the volume of water in the hot tub is always decreasing during that time period, a negative change.
12. a., b.



The slope of the tangent line is -8.
c. The instantaneous rate of change in $f(x)$ when $x = 5$ is -8.

1.1 Radical Expressions: Rationalizating Denominators, p. 9

1. a. $2\sqrt{3} + 4$
b. $\sqrt{3} - \sqrt{2}$
c. $2\sqrt{3} + \sqrt{2}$
d. $3\sqrt{3} - \sqrt{2}$
e. $\sqrt{2} + \sqrt{5}$
f. $-\sqrt{5} - 2\sqrt{2}$
2. a. $\frac{\sqrt{3} + \sqrt{5}}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$
$= \frac{\sqrt{6} + \sqrt{10}}{2}$
b. $\frac{2\sqrt{3} - 3\sqrt{2}}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$
$= \frac{2\sqrt{6} - 6}{2}$
$= \sqrt{6} - 3$

c. $\dfrac{4\sqrt{3}+3\sqrt{2}}{2\sqrt{3}} \cdot \dfrac{\sqrt{3}}{\sqrt{3}}$

$= \dfrac{12+3\sqrt{6}}{6}$

$= \dfrac{4+\sqrt{6}}{2}$

d. $\dfrac{3\sqrt{5}-\sqrt{2}}{2\sqrt{2}} \cdot \dfrac{\sqrt{2}}{\sqrt{2}}$

$= \dfrac{3\sqrt{10}-2}{4}$

3. a. $\dfrac{3}{\sqrt{5}-\sqrt{2}} \cdot \dfrac{\sqrt{5}+\sqrt{2}}{\sqrt{5}+\sqrt{2}}$

$= \dfrac{3(\sqrt{5}+\sqrt{2})}{3}$

$= \sqrt{5}+\sqrt{2}$

b. $\dfrac{2\sqrt{5}}{2\sqrt{5}+3\sqrt{2}} \cdot \dfrac{2\sqrt{5}-3\sqrt{2}}{2\sqrt{5}-3\sqrt{2}}$

$= \dfrac{20-6\sqrt{10}}{20-18}$

$= 10-3\sqrt{10}$

c. $\dfrac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}} \cdot \dfrac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}}$

$= \dfrac{3+2\sqrt{6}+2}{3-2}$

$= 5+2\sqrt{6}$

d. $\dfrac{2\sqrt{5}-8}{2\sqrt{5}+3} \cdot \dfrac{2\sqrt{5}-3}{2\sqrt{5}-3}$

$= \dfrac{20-22\sqrt{5}+24}{20-9}$

$= \dfrac{44-22\sqrt{5}}{11}$

$= 4-2\sqrt{5}$

e. $\dfrac{2\sqrt{3}-\sqrt{2}}{5\sqrt{2}+\sqrt{3}} \cdot \dfrac{5\sqrt{2}-\sqrt{3}}{5\sqrt{2}-\sqrt{3}}$

$= \dfrac{10\sqrt{6}-6-10+\sqrt{6}}{50-3}$

$= \dfrac{11\sqrt{6}-16}{47}$

f. $\dfrac{3\sqrt{3}-2\sqrt{2}}{3\sqrt{3}+2\sqrt{2}} \cdot \dfrac{3\sqrt{3}-2\sqrt{2}}{3\sqrt{3}-2\sqrt{2}}$

$= \dfrac{27-12\sqrt{6}+8}{27-8}$

$= \dfrac{35-12\sqrt{6}}{19}$

4. a. $\dfrac{\sqrt{5}-1}{4} \cdot \dfrac{\sqrt{5}+1}{\sqrt{5}+1}$

$= \dfrac{5-1}{4(\sqrt{5}+1)}$

$= \dfrac{1}{\sqrt{5}+1}$

b. $\dfrac{2-3\sqrt{2}}{2} \cdot \dfrac{2+3\sqrt{2}}{2+3\sqrt{2}}$

$= \dfrac{4-18}{2(2+3\sqrt{2})}$

$= \dfrac{-7}{2+3\sqrt{2}}$

c. $\dfrac{\sqrt{5}+2}{2\sqrt{5}-1} \cdot \dfrac{\sqrt{5}-2}{\sqrt{5}-2}$

$= \dfrac{5-4}{10-5\sqrt{5}+2}$

$= \dfrac{1}{12-5\sqrt{5}}$

5. a. $\dfrac{8\sqrt{2}}{\sqrt{20}-\sqrt{18}} \cdot \dfrac{\sqrt{20}+\sqrt{18}}{\sqrt{20}+\sqrt{18}}$

$= \dfrac{8\sqrt{40}+8\sqrt{36}}{20-18}$

$= \dfrac{16\sqrt{10}+48}{2}$

$= 8\sqrt{10}+24$

b. $\dfrac{8\sqrt{2}}{2\sqrt{5}-3\sqrt{2}} \cdot \dfrac{2\sqrt{5}+3\sqrt{2}}{2\sqrt{5}+3\sqrt{2}}$

$= \dfrac{16\sqrt{10}+48}{20-18}$

$= \dfrac{16\sqrt{10}+48}{2}$

$= 8\sqrt{10}+24$

c. The expressions in the two parts are equivalent. The radicals in the denominator of part a. have been simplified in part b.

6. a. $\dfrac{2\sqrt{2}}{2\sqrt{3}-\sqrt{8}} \cdot \dfrac{2\sqrt{3}+\sqrt{8}}{2\sqrt{3}+\sqrt{8}}$

$= \dfrac{4\sqrt{6}+8}{6-8}$

$= -2\sqrt{3}-4$

b. $\dfrac{2\sqrt{6}}{2\sqrt{27}-\sqrt{8}} \cdot \dfrac{2\sqrt{27}+\sqrt{8}}{2\sqrt{27}+\sqrt{8}}$

$= \dfrac{4\sqrt{162}+2\sqrt{48}}{54-8}$

$= \dfrac{36\sqrt{2}+8\sqrt{3}}{46}$

$= \dfrac{18\sqrt{2}+4\sqrt{3}}{23}$

c. $\dfrac{2\sqrt{2}}{\sqrt{16}-\sqrt{12}}$

$= \dfrac{2\sqrt{2}}{4-2\sqrt{3}} \cdot \dfrac{4+2\sqrt{3}}{4+2\sqrt{3}}$

$= \dfrac{8\sqrt{2}+4\sqrt{6}}{16-12}$

$= 2\sqrt{2}+\sqrt{6}$

d. $\dfrac{3\sqrt{2}+2\sqrt{3}}{\sqrt{12}-\sqrt{8}} \cdot \dfrac{\sqrt{12}+\sqrt{8}}{\sqrt{12}+\sqrt{8}}$

$= \dfrac{3\sqrt{24}+12+12+2\sqrt{24}}{12-8}$

$= \dfrac{24+15\sqrt{3}}{4}$

e. $\dfrac{3\sqrt{5}}{4\sqrt{3}-5\sqrt{2}} \cdot \dfrac{4\sqrt{3}+5\sqrt{2}}{4\sqrt{3}+5\sqrt{2}}$

$= \dfrac{12\sqrt{15}+15\sqrt{10}}{48-50}$

$= -\dfrac{12\sqrt{15}+15\sqrt{10}}{2}$

f. $\dfrac{\sqrt{18}+\sqrt{12}}{\sqrt{18}-\sqrt{12}} \cdot \dfrac{\sqrt{18}+\sqrt{12}}{\sqrt{18}+\sqrt{12}}$

$= \dfrac{18+2\sqrt{216}+12}{18-12}$

$= \dfrac{30+12\sqrt{6}}{6}$

$= 5+2\sqrt{6}$

7. a. $\dfrac{\sqrt{a}-2}{a-4} \cdot \dfrac{\sqrt{a}+2}{\sqrt{a}+2}$

$= \dfrac{a-4}{(a-4)(\sqrt{a}-2)}$

$= \dfrac{1}{\sqrt{a}-2}$

b. $\dfrac{\sqrt{x+4}-2}{x} \cdot \dfrac{\sqrt{x+4}+2}{\sqrt{x+4}+2}$

$= \dfrac{x+4-4}{x(\sqrt{x+4}+2)}$

$= \dfrac{x}{x(\sqrt{x+4}+2)}$

$= \dfrac{1}{\sqrt{x+4}-2}$

c. $\dfrac{\sqrt{x+h}-\sqrt{x}}{h} \cdot \dfrac{\sqrt{x+h}+\sqrt{x}}{\sqrt{x+h}+\sqrt{x}}$

$= \dfrac{x+h-x}{h(\sqrt{x+h}+\sqrt{x})}$

$= \dfrac{h}{h(\sqrt{x+h}+\sqrt{x})}$

$= \dfrac{1}{\sqrt{x+h}+\sqrt{x}}$

1.2 The Slope of a Tangent, pp. 18–21

1. a. $m = \dfrac{-8-7}{-3-2}$

$= 3$

b. $m = \dfrac{-\frac{7}{2}-\frac{3}{2}}{\frac{7}{2}-\frac{1}{2}}$

$= \dfrac{-\frac{10}{2}}{\frac{6}{2}}$

$= -\dfrac{5}{3}$

c. $m = \dfrac{-1-(-2.6)}{1.5-6.3}$

$= -\dfrac{1}{3}$

2. a. The slope of the given line is 3, so the slope of a line perpendicular to the given line is $-\frac{1}{3}$.

b. $13x - 7y - 11 = 0$

$-7y = -13x - 11$

$y = \dfrac{13}{7}x + \dfrac{11}{7}$

The slope of the given line is $\frac{13}{7}$, so the slope of a line perpendicular to the given line is $-\frac{7}{13}$.

3. a. $m = \dfrac{-\frac{5}{3} - (-4)}{\frac{5}{3} - (-4)}$

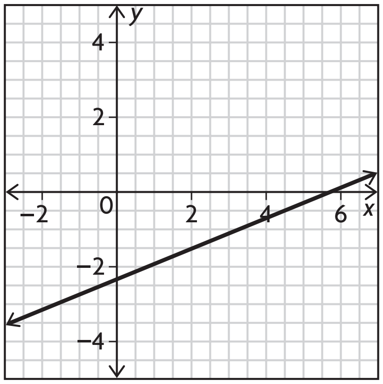
$= \dfrac{\frac{7}{3}}{\frac{17}{3}}$

$= \dfrac{7}{17}$

$y - (-4) = \dfrac{7}{17}(x - (-4))$

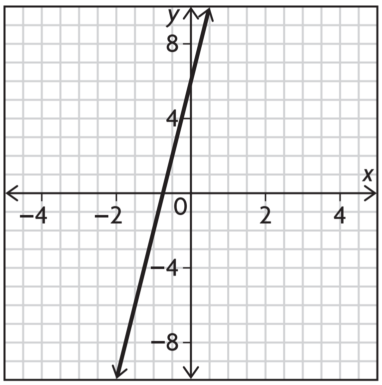
$17y + 68 = 7x + 28$

$7x - 17y - 40 = 0$



b. The slope and y-intercept are given.

$y = 8x + 6$



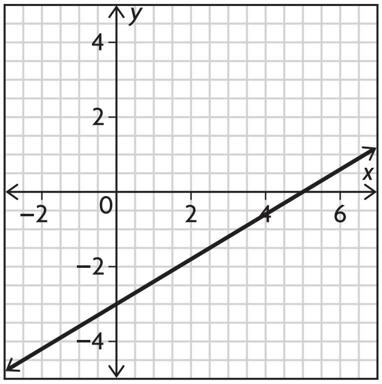
c. $(0, -3)$, $(5, 0)$

$m = \dfrac{0 - (-3)}{5 - 0}$

$= \dfrac{3}{5}$

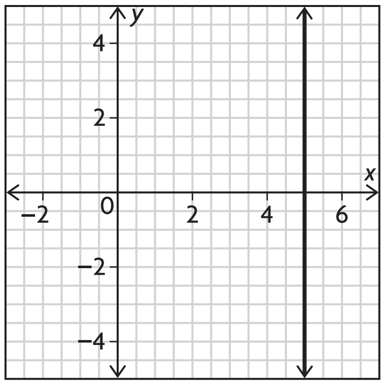
$y - 0 = \dfrac{3}{5}(x - 5)$

$3x - 5y - 15 = 0$



d. The line is a vertical line because both points have the same x-coordinate.

$x = 5$



4. a. $\dfrac{(5 + h)^3 - 125}{h}$

$= \dfrac{(5 + h - 5)((5 + h)^2 + 5(5 + h) + 25)}{h}$

$= \dfrac{h(75 + 15h + h^2)}{h}$

$= 75 + 15h + h^2$

b. $\dfrac{(3 + h)^4 - 81}{h}$

$= \dfrac{((3 + h)^2 - 9)((3 + h)^2 + 9)}{h}$

$= \dfrac{(9 + 6h + h^2 - 9)(9 + 6h + h^2 + 9)}{h}$

$= (6 + h)(18 + 6h + h^2)$

$= 108 + 54h + 12h^2 + h^3$

c. $\dfrac{\frac{1}{1 + h} - 1}{h} = \dfrac{1 - 1 - h}{h(1 + h)} = -\dfrac{1}{1 + h}$

d. $\dfrac{3(1 + h)^2 - 3}{h} = \dfrac{3((1 + h)^2 - 1)}{h}$

$= \dfrac{3(1 + 2h + h^2 - 1)}{h}$

$$= \frac{3(2h + h^2)}{h}$$
$$= 6 + 3h$$

e. $\dfrac{\frac{3}{4+h} - \frac{3}{4}}{h} = \dfrac{\frac{12 - 12 - 3h}{4(4+h)}}{h}$.
$$= \frac{-3}{4(4+h)}$$

f. $\dfrac{\frac{-1}{2+h} + \frac{1}{2}}{h} = \dfrac{\frac{-2+2+h}{2(2+h)}}{h}$
$$= \frac{h}{2h(2+h)}$$
$$= \frac{1}{4+2h}$$

5. a. $\dfrac{\sqrt{16+h} - 4}{h} = \dfrac{16 + h - 16}{h\left(\sqrt{16+h} + 4\right)}$
$$= \frac{1}{\sqrt{16+h} + 4}$$

b. $\dfrac{\sqrt{h^2 + 5h + 4} - 2}{h} = \dfrac{h^2 + 5h + 4 - 4}{h\left(\sqrt{h^2 + 5h + 4} + 2\right)}$.
$$= \frac{h + 5}{\sqrt{h^2 + 5h + 4} + 2}$$

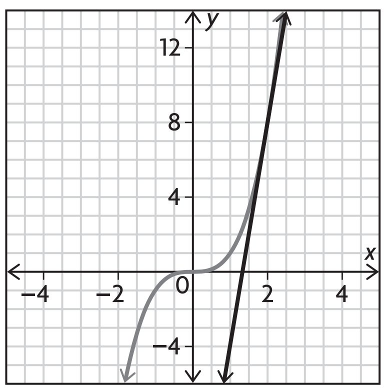
c. $\dfrac{\sqrt{5+h} - \sqrt{5}}{h} = \dfrac{5 + h - 5}{h\left(\sqrt{5+h} + \sqrt{5}\right)}$
$$= \frac{1}{\sqrt{5+h} + \sqrt{5}}$$

6. a. $P(1, 3)$, $Q(1 + h, f(1 + h))$, $f(x) = 3x^2$
$$m = \frac{3(1+h)^2 - 3}{h}$$
$$= 6 + 3h$$
b. $P(1, 3)$, $Q(1 + h, (1 + h)^3 + 2)$
$$m = \frac{(1+h)^3 + 2 - 3}{h}$$
$$= \frac{1 + 3h + 3h^2 + h^3 - 1}{h}$$
$$= 3 + 3h + h^2$$
c. $P(9, 3)$, $Q\left(9 + h, \sqrt{9+h}\right)$
$$m = \frac{\sqrt{9+h} - 3}{h} \cdot \frac{\sqrt{9+h} + 3}{\sqrt{9+h} + 3}$$
$$= \frac{1}{\sqrt{9+h} + 3}$$

7. a.

P	*Q*	Slope of Line *PQ*
(2, 8)	(3, 27)	19
(2, 8)	(2.5, 15.625)	15.25
(2, 8)	(2.1, 9.261)	12.61
(2, 8)	(2.01, 8.120 601)	12.060 1
(2, 8)	(1, 1)	7
(2, 8)	(1.5, 3.375)	9.25
(2, 8)	(1.9, 6.859)	11.41
(2, 8)	(1.99, 7.880 599)	11.940 1

b. 12
c. $(2, 8)$, $((2 + h), (2 + h)^3)$
$$m = \frac{(2+h)^3 - 8}{2 + h - 2}$$
$$= \frac{8 + 12h + 6h^2 + h^3 - 8}{h}$$
$$= 12 + 6h + h^2$$
d. $m = \lim\limits_{h \to 0}(12 + 6h + h^2)$
$$= 12$$
e. They are the same.
f.



8. a. $y = 3x^2$, $(-2, 12)$
$$m = \lim_{h \to 0} \frac{3(-2+h)^2 - 12}{h}$$
$$= \lim_{h \to 0} \frac{12 - 12h + 3h^2 - 12}{h}$$
$$= \lim_{h \to 0}(-12 + 3h)$$
$$= -12$$
b. $y = x^2 - x$ at $x = 3$, $y = 6$.
$$m = \lim_{h \to 0} \frac{(3+h)^2 - (3+h) - 6}{h}$$
$$= \lim_{h \to 0} \frac{9 + 6h + h^2 - 3 - h - 6}{h}$$
$$= \lim_{h \to 0}(5 + h)$$
$$= 5$$

c. $y = x^3$ at $x = -2$, $y = -8$.

$$\begin{aligned} m &= \lim_{h \to 0} \frac{(-2 + h)^3 + 8}{h} \\ &= \lim_{h \to 0} \frac{-8 + 12h - 6h^2 + h^3 + 8}{h} \\ &= \lim_{h \to 0} (12 - 6h + h^2) \\ &= 12 \end{aligned}$$

9. a. $y = \sqrt{x - 2}$; $(3, 1)$

$$\begin{aligned} m &= \lim_{h \to 0} \frac{\sqrt{3 + h - 2} - 1}{h} \\ &= \lim_{h \to 0} \left[\frac{\sqrt{1 + h} - 1}{h} \times \frac{\sqrt{1 + h} + 1}{\sqrt{1 + h} + 1} \right] \\ &= \lim_{h \to 0} \frac{1}{\sqrt{1 + h} + 1} \\ &= \frac{1}{2} \end{aligned}$$

b. $y = \sqrt{x - 5}$ at $x = 9$, $y = 2$

$$\begin{aligned} m &= \lim_{h \to 0} \frac{\sqrt{9 + h - 5} - 2}{h} \\ &= \lim_{h \to 0} \left[\frac{\sqrt{4 + h} - 2}{h} \times \frac{\sqrt{4 + h} + 2}{\sqrt{4 + h} + 2} \right] \\ &= \lim_{h \to 0} \frac{1}{\sqrt{4 + h} + 2} \\ &= \frac{1}{4} \end{aligned}$$

c. $y = \sqrt{5x - 1}$ at $x = 2$, $y = 3$

$$\begin{aligned} m &= \lim_{h \to 0} \frac{\sqrt{10 + 5h - 1} - 3}{h} \\ &= \lim_{h \to 0} \left[\frac{\sqrt{9 + 5h} - 3}{h} \times \frac{\sqrt{9 + 5h} + 3}{\sqrt{9 + 5h} + 3} \right] \\ &= \lim_{h \to 0} \frac{5}{\sqrt{9 + 5h} + 3} \\ &= \frac{5}{6} \end{aligned}$$

10. a. $y = \frac{8}{x}$ at $(2, 4)$

$$\begin{aligned} m &= \lim_{h \to 0} \frac{\frac{8}{2 + h} - 4}{h} \\ &= \lim_{h \to 0} \frac{-4}{2 + h} \\ &= -2 \end{aligned}$$

b. $y = \frac{8}{3 + x}$ at $x = 1$; $y = 2$

$$\begin{aligned} m &= \lim_{h \to 0} \frac{\frac{8}{4 + h} - 2}{h} \\ &= \lim_{h \to 0} \frac{-2}{4 + h} \\ &= -\frac{1}{2} \end{aligned}$$

c. $y = \frac{1}{x + 2}$ at $x = 3$; $y = \frac{1}{5}$

$$\begin{aligned} m &= \lim_{h \to 0} \frac{\frac{1}{5 + h} - \frac{1}{5}}{h} \\ &= \lim_{h \to 0} \frac{-1}{5(5 + h)} \\ &= -\frac{1}{10} \end{aligned}$$

11. a. Let $y = f(x)$.

$$f(2) = (2)^2 - 3(2) = 4 - 6 = -2$$

$$f(2 + h) = (2 + h)^2 - 3(2 + h)$$

Using the limit of the difference quotient, the slope of the tangent at $x = 2$ is

$$\begin{aligned} m &= \lim_{h \to 0} \frac{f(2 + h) - f(2)}{h} \\ &= \lim_{h \to 0} \frac{(2 + h)^2 - 3(2 + h) - (-2)}{h} \\ &= \lim_{h \to 0} \frac{4 + 4h + h^2 - 6 - 3h + 2}{h} \\ &= \lim_{h \to 0} \frac{h^2 + h}{h} \\ &= \lim_{h \to 0} (h + 1) \\ &= 0 + 1 \\ &= 1 \end{aligned}$$

Therefore, the slope of the tangent to $y = f(x) = x^2 - 3x$ at $x = 2$ is 1.

b. $f(-2) = \frac{4}{-2} = -2$

$$f(-2 + h) = \frac{4}{-2 + h}$$

Using the limit of the difference quotient, the slope of the tangent at $x = -2$ is

$$\begin{aligned} m &= \lim_{h \to 0} \frac{f(-2 + h) - f(-2)}{h} \\ &= \lim_{h \to 0} \frac{\frac{4}{-2 + h} - (-2)}{h} \\ &= \lim_{h \to 0} \frac{\frac{4}{-2 + h} + 2}{h} \\ &= \lim_{h \to 0} \left[\frac{4 - 4 + 2h}{-2 + h} \cdot \frac{1}{h} \right] \\ &= \lim_{h \to 0} \left[\frac{2h}{-2 + h} \cdot \frac{1}{h} \right] \end{aligned}$$

$$= \lim_{h\to 0} \frac{2}{-2+h}$$
$$= \frac{2}{-2+0}$$
$$= -1$$

Therefore, the slope of the tangent to $f(x) = \frac{4}{x}$ at $x = -2$ is -1.

c. Let $y = f(x)$.

$$f(1) = 3(1)^3 = 3$$
$$f(1+h) = 3(1+h)^3$$

Using the limit of the difference quotient, the slope of the tangent at $x = 1$ is

$$m = \lim_{h\to 0} \frac{f(1+h) - f(1)}{h}$$
$$= \lim_{h\to 0} \frac{3(1+h)^3 - 3}{h}$$

Using the binomial formula to expand $(1+h)^3$ (or one could simply expand using algebra), the slope m is

$$= \lim_{h\to 0} \frac{3(h^3 + 3h^2 + 3h + 1) - (3)}{h}$$
$$= \lim_{h\to 0} \frac{3h^3 + 9h^2 + 9h + 3 - 3}{h}$$
$$= \lim_{h\to 0} \frac{3h^3 + 9h^2 + 9h}{h}$$
$$= \lim_{h\to 0} (3h^2 + 9h + 9)$$
$$= 3(0) + 9(0) + 9$$
$$= 9$$

Therefore, the slope of the tangent to $y = f(x) = 3x^3$ at $x = 1$ is 9.

d. Let $y = f(x)$.

$$f(16) = \sqrt{16 - 7} = \sqrt{9} = 3$$
$$f(16+h) = \sqrt{16 + h - 7} = \sqrt{h + 9}$$

Using the limit of the difference quotient, the slope of the tangent at $x = 16$ is

$$m = \lim_{h\to 0} \frac{f(16+h) - f(16)}{h}$$
$$= \lim_{h\to 0} \frac{\sqrt{h+9} - 3}{h}$$
$$= \lim_{h\to 0} \frac{\sqrt{h+9} - 3}{h} \cdot \frac{\sqrt{h+9} + 3}{\sqrt{h+9} + 3}$$
$$= \lim_{h\to 0} \frac{(h+9) - 9}{h(\sqrt{h+9} + 3)}$$
$$= \lim_{h\to 0} \frac{h}{h(\sqrt{h+9} + 3)}$$
$$= \lim_{h\to 0} \frac{1}{\sqrt{h+9} + 3}$$
$$= \frac{1}{\sqrt{0+9} + 3}$$
$$= \frac{1}{3+3}$$
$$= \frac{1}{6}$$

Therefore, the slope of the tangent to $y = f(x) = \sqrt{x - 7}$ at $x = 16$ is $\frac{1}{6}$.

e. Let $y = f(x)$.

$$f(3) = \sqrt{25 - (3)^2} = \sqrt{25 - 9} = 4$$
$$f(3+h) = \sqrt{25 - (3+h)^2}$$
$$= \sqrt{25 - (9 + 6h + h^2)}$$
$$= \sqrt{25 - 9 - 6h - h^2}$$
$$= \sqrt{16 - 6h - h^2}$$

Using the limit of the difference quotient, the slope of the tangent at $x = 3$ is

$$m = \lim_{h\to 0} \frac{f(3+h) - f(3)}{h}$$
$$= \lim_{h\to 0} \frac{\sqrt{16 - 6h - h^2} - 4}{h}$$
$$= \lim_{h\to 0} \left[\frac{\sqrt{16 - 6h - h^2} - 4}{h} \times \frac{\sqrt{16 - 6h - h^2} + 4}{\sqrt{16 - 6h - h^2} + 4}\right]$$
$$= \lim_{h\to 0} \frac{16 - 6h - h^2 - 16}{h(\sqrt{16 - 6h - h^2} + 4)}$$
$$= \lim_{h\to 0} \frac{h(-6 - h)}{h(\sqrt{16 - 6h - h^2} + 4)}$$
$$= \lim_{h\to 0} \frac{-6 - h}{\sqrt{16 - 6h - h^2} + 4}$$
$$= \frac{-6 - 0}{\sqrt{16 - 6(0) - (0)^2} + 4}$$
$$= \frac{-6}{\sqrt{16} + 4}$$
$$= \frac{-6}{8}$$
$$= -\frac{3}{4}$$

Therefore, the slope of the tangent to $y = f(x) = \sqrt{25 - x^2}$ at $x = 3$ is $-\frac{3}{4}$.

f. Let $y = f(x)$.

$$f(8) = \frac{4+8}{8-2} = \frac{12}{6} = 2$$
$$f(8+h) = \frac{4 + (8+h)}{(8+h) - 2} = \frac{12+h}{6+h}$$

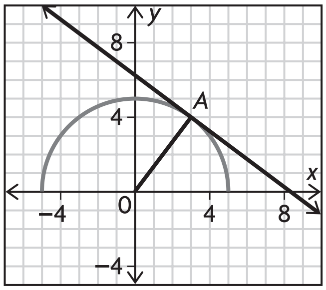
Using the limit of the difference quotient, the slope of the tangent at $x = 8$ is

$$m = \lim_{h\to0} \frac{f(8 + h) - f(8)}{h}$$
$$= \lim_{h\to0} \frac{\frac{12 + h}{6 + h} - 2}{h}$$
$$= \lim_{h\to0} \frac{12 + h - 12 - 2h}{6 + h} \cdot \frac{1}{h}$$
$$= \lim_{h\to0} \frac{-h}{6 + h} \cdot \frac{1}{h}$$
$$= \lim_{h\to0} \frac{-1}{6 + h}$$
$$= \frac{-1}{6 + 0}$$
$$= -\frac{1}{6}$$

Therefore, the slope of the tangent to

$y = f(x) = \frac{4 + x}{x - 2}$ at $x = 8$ is $-\frac{1}{6}$.

12.



$y = \sqrt{25 - x^2} \rightarrow$ Semi-circle centre $(0, 0)$

rad 5, $y \geq 0$

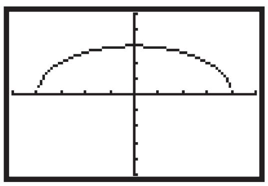
OA is a radius.

The slope of OA is $\frac{4}{3}$.

The slope of tangent is $-\frac{3}{4}$.

13. Take values of x close to the point, then determine $\frac{\Delta y}{\Delta x}$.

14.



Since the tangent is horizontal, the slope is 0.

15. $m = \lim_{h\to0} \frac{(3 + h)^2 - 3(3 + h) + 1 - 1}{h}$

$$= \lim_{h\to0} \frac{9 + 6h + h^2 - 9 - 3h}{h}$$
$$= \lim_{h\to0} \frac{3h + h^2}{h}$$
$$= \lim_{h\to0} (3 + h)$$
$$= 3$$

The slope of the tangent is 3.

$$y - 1 = 3(x - 3)$$
$$3x - y - 8 = 0$$

16. $m = \lim_{h\to0} \frac{(2 + h)^2 - 7(2 + h) + 12 - 2}{h}$

$$= \lim_{h\to0} \frac{4 + 4h + h^2 - 14 - 7h + 10}{h}$$
$$= \lim_{h\to0} \frac{-3h + h^2}{h}$$
$$= \lim_{h\to0} (- 3 + h)$$
$$= -3$$

The slope of the tangent is -3.

When $x = 2$, $y = 2$.

$$y - 2 = -3(x - 2)$$
$$3x + y - 8 = 0$$

17. a. $f(3) = 9 - 12 + 1 = -2$; $(3, -2)$

b. $f(5) = 25 - 20 + 1 = 6$; $(5, 6)$

c. The slope of secant AB is

$$m_{AB} = \frac{6 - (-2)}{5 - 3}$$
$$= \frac{8}{2}$$
$$= 4$$

The equation of the secant is

$$y - y_1 = m_{AB}(x - x_1)$$
$$y + 2 = 4(x - 3)$$
$$y = 4x - 14$$

d. Calculate the slope of the tangent.

$$m = \lim_{h\to0} \frac{f(x + h) - f(x)}{h}$$
$$= \lim_{h\to0} \frac{(x + h)^2 - 4(x + h) + 1 - (x^2 - 4x + 1)}{h}$$
$$= \lim_{h\to0} \frac{x^2 + 2xh + h^2 - 4x - 4h + 1 - x^2 + 4x - 1}{h}$$
$$= \lim_{h\to0} \frac{2xh + h^2 - 4h}{h}$$
$$= \lim_{h\to0} (2x + h - 4)$$
$$= 2x + 0 - 4$$
$$= 2x - 4$$

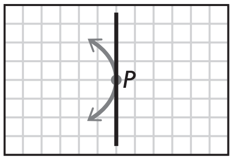
When $x = 3$, the slope is $2(3) - 4 = 2$. So the equation of the tangent at $A(3, -2)$ is

$$y - y_1 = m(x - x_1)$$
$$y + 2 = 2(x - 3)$$
$$y = 2x - 8$$

e. When $x = 5$, the slope of the tangent is $2(5) - 4 = 6$.
So the equation of the tangent at $B(5, 6)$ is

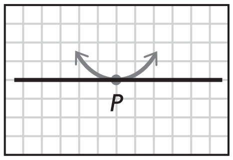
$$y - y_1 = m(x - x_1)$$
$$y - 6 = 6(x - 5)$$
$$y = 6x - 24$$

18. a.



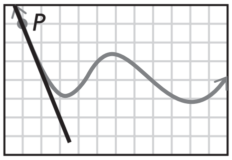
The slope is undefined.

b.



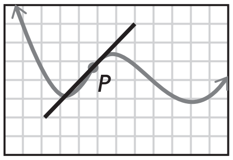
The slope is 0.

c.



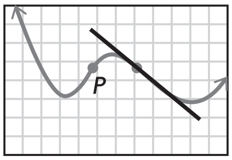
The slope is about –2.5.

d.



The slope is about 1.

e.



The slope is about $-\frac{7}{8}$.

f. There is no tangent at this point.

19. $D(p) = \dfrac{20}{\sqrt{p - 1}}, p > 1$ at $(5, 10)$

$$m = \lim_{h \to 0} \frac{\frac{20}{\sqrt{4 + h}} - 10}{h}$$
$$= 10 \lim_{h \to 0} \frac{2 - \sqrt{4 + h}}{h\sqrt{4 + h}} \times \frac{2 + \sqrt{4 + h}}{2 + \sqrt{4 + h}}$$
$$= 10 \lim_{h \to 0} \frac{4 - 4 - h}{h\sqrt{4 + h}\left(2 + \sqrt{4 + h}\right)}$$
$$= -\frac{10}{8}$$
$$= -\frac{5}{4}$$

20. $C(t) = 100t^2 + 400t + 5000$
Slope at $t = 6$
$C'(t) = 200t + 400$
$C'(6) = 1200 + 400 = 1600$
Increasing at a rate of 1600 papers per month.

21. Point on $f(x) = 3x^2 - 4x$ tangent parallel to $y = 8x$. Therefore, tangent line has slope 8.

$$m = \lim_{h \to 0} \frac{3(h + a)^2 - 4(h + a) - 3(a^2 + 4a)}{h} = 8$$
$$\lim_{h \to 0} \frac{3h^2 + 6ah - 4h}{h} = 8$$
$$6a - 4 = 8$$
$$a = 2$$

The point has coordinates $(2, 4)$.

22. $y = \frac{1}{3}x^3 - 5x - \frac{4}{x}$

$$\frac{1}{3}(a + h)^2 - \frac{1}{3}a^3 = a^2h + ah^2 + \frac{1}{3}h^3$$
$$\lim_{h \to 0}\left(a^2 + ah + \frac{1}{3}h^3\right) = a^2$$
$$5 \lim_{h \to 0} - \frac{(a + h) - (-a)}{h} = -5$$
$$-\frac{4}{a + h} + \frac{4}{a} = -\frac{4a + 4a + 4h}{a(a + h)}$$
$$\lim_{h \to 0} \frac{4}{a(a + h)} = \frac{4}{a^2}$$
$$m = a^2 - 5 + \frac{4}{a^2} = 0$$
$$a^4 - 5a^2 + 4 = 0$$
$$(a^2 - 4)(a^2 - 1) = 0$$
$$a = \pm 2, a = \pm 1$$

Points on the graph for horizontal tangents are: $\left(-2, \frac{28}{3}\right), \left(-1, \frac{26}{3}\right), \left(1, -\frac{26}{3}\right), \left(2, -\frac{28}{3}\right)$.

23. $y = x^2$ and $y = \frac{1}{2} - x^2$

$$x^2 = \frac{1}{2} - x^2$$
$$x^2 = \frac{1}{4}$$
$$x = \frac{1}{2} \text{ or } x = -\frac{1}{2}$$

The points of intersection are $P\left(\frac{1}{2}, \frac{1}{4}\right), Q\left(-\frac{1}{2}, \frac{1}{4}\right)$.
Tangent to $y = x^2$:

$$m = \lim_{h \to 0} \frac{(a + h)^2 - a^2}{h}$$
$$= \lim_{h \to 0} \frac{2ah + h^2}{h}$$
$$= 2a.$$

The slope of the tangent at $a = \frac{1}{2}$ is $1 = m_p$,
at $a = -\frac{1}{2}$ is $-1 = m_q$.
Tangents to $y = \frac{1}{2} - x^2$:

$$m = \lim_{h\to 0}\frac{\left[\frac{1}{2} - (a+h)^2\right] - \left[\frac{1}{2} - a^2\right]}{h}$$
$$= \lim_{h\to 0}\frac{-2ah - h^2}{h}$$
$$= -2a.$$

The slope of the tangents at $a = \frac{1}{2}$ is $-1 = M_p$;
at $a = -\frac{1}{2}$ is $1 = M_q$
$m_pM_p = -1$ and $m_qM_q = -1$
Therefore, the tangents are perpendicular at the points of intersection.

24. $y = -3x^3 - 2x$, $(-1, 5)$

$$m = \lim_{h\to 0}\frac{-3(-1+h)^3 - 2(-1+h) - 5}{h}$$
$$= \lim_{h\to 0}\frac{-3(-1+3h-3h^2+h^3) + 2 - 2h - 5}{h}$$
$$= \lim_{h\to 0}\frac{-3(-1+3h-3h^2+h^3) + 2 - 2h - 5}{h}$$
$$= \lim_{h\to 0}\frac{3 - 9h + 9h^2 - 3h^3 + 2 - 2h - 5}{h}$$
$$= \lim_{h\to 0}\frac{-11h + 9h^2 - 3h^3}{h}$$
$$= \lim_{h\to 0}(-11 + 9h - 3h^2)$$
$$= -11$$

The slope of the tangent is -11.
We want the line that is parallel to the tangent (i.e. has slope -11) and passes through (2, 2). Then,

$$y - 2 = -11(x - 2)$$
$$y = -11x + 24$$

25. a. Let $y = f(x)$.

$$f(a) = 4a^2 + 5a - 2$$
$$f(a+h) = 4(a+h)^2 + 5(a+h) - 2$$
$$= 4(a^2 + 2ah + h^2) + 5a + 5h - 2$$
$$= 4a^2 + 8ah + 4h^2 + 5a + 5h - 2$$

Using the limit of the difference quotient, the slope of the tangent at $x = a$ is

$$m = \lim_{h\to 0}\frac{f(a+h) - f(a)}{h}$$
$$= \lim_{h\to 0}\left[\frac{4a^2 + 8ah + 4h^2 + 5a + 5h - 2}{h} - \frac{(4a^2 + 5a - 2)}{h}\right]$$
$$= \lim_{h\to 0}\left[\frac{4a^2 + 8ah + 4h^2 + 5a + 5h - 2}{h} + \frac{-4a^2 - 5a + 2}{h}\right]$$
$$= \lim_{h\to 0}\frac{8ah + 4h^2 + 5h}{h}$$
$$= \lim_{h\to 0}(8a + 4h + 5)$$
$$= 8a + 4(0) + 5$$
$$= 8a + 5$$

b. To be parallel, the point on the parabola and the line must have the same slope. So, first find the slope of the line. The line $10x - 2y - 18 = 0$ can be rewritten as

$$-2y = 18 - 10x$$
$$y = \frac{18 - 10x}{-2}$$
$$y = -9 + 5x$$
$$y = 5x - 9$$

So, the slope, m, of the line $10x - 2y - 18 = 0$ is 5. To be parallel, the slope at a must equal 5. From part a., the slope of the tangent to the parabola at $x = a$ is $8a + 5$.

$$8a + 5 = 5$$
$$8a = 0$$
$$a = 0$$

Therefore, at the point $(0, -2)$ the tangent line is parallel to the line $10x - 2y - 18 = 0$.

c. To be perpendicular, the point on the parabola and the line must have slopes that are negative reciprocals of each other. That is, their product must equal -1. So, first find the slope of the line. The line $x - 35y + 7 = 0$ can be rewritten as

$$-35y = -x - 7$$
$$y = \frac{-x - 7}{-35}$$
$$y = \frac{1}{35}x + \frac{7}{35}$$

So, the slope, m, of the line $x - 35y + 7 = 0$ is $\frac{1}{35}$. To be perpendicular, the slope at a must equal the negative reciprocal of the slope of the line $x - 35y + 7 = 0$. That is, the slope of a must equal -35. From part a., the slope of the tangent to the parabola at $x = a$ is $8a + 5$.

$$8a + 5 = -35$$
$$8a = -40$$
$$a = -5$$

Therefore, at the point $(-5, 73)$ the tangent line is perpendicular to the line $x - 35y + 7 = 0$.

1.3 Rates of Change, pp. 29–31

1. $v(t) = 0$ when $t = 0$ or $t = 4$.

2. a. $\frac{s(9) - s(2)}{7}$. Slope of the secant between the points $(2, s(2))$ and $(9, s(9))$.

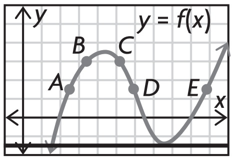
b. $\lim_{h\to 0}\frac{s(6 + h) - s(6)}{h}$. Slope of the tangent at the point $(6, s(6))$.

3. $\lim_{h\to 0}\frac{\sqrt{4 + h} - 2}{h}$. Slope of the tangent to the function with equation $y = \sqrt{x}$ at the point $(4, 2)$.

4. a. A and B

b. greater; the secant line through these two points is steeper than the tangent line at B.

c.



5. Speed is represented only by a number, not a direction.

6. Yes, velocity needs to be described by a number and a direction. Only the speed of the school bus was given, not the direction, so it is not correct to use the word "velocity."

7. $s(t) = 320 - 5t^2, 0 \le t \le 8$

a. Average velocity during the first second:

$\frac{s(1) - s(0)}{1} = 5$ m/s;

third second:

$\frac{s(3) - s(2)}{1} = \frac{45 - 20}{1} = 25$ m/s;

eighth second:

$\frac{s(8) - s(7)}{1} = \frac{320 - 245}{1} = 75$ m/s.

b. Average velocity $3 \le t \le 8$

$\frac{s(8) - s(3)}{8 - 3} = \frac{320 - 45}{5} = \frac{275}{5} = 55$ m/s

c. $s(t) = 320 - 5t^2$

$$v(t) = \lim_{h\to 0}\frac{320 - 5(2 + h)^2 - (320 - 5(2)^2)}{h}$$
$$= 5\lim_{h\to 0}\frac{-4h + h^2}{h}$$
$$= -20$$

Velocity at $t = 2$ is 20 m/s downward.

8. $s(t) = 8t(t + 2), 0 \le t \le 5$

a. i. from $t = 3$ to $t = 4$

Average velocity $\frac{s(4) - s(3)}{1}$

$= 32(6) - 24(5)$
$= 24(8 - 5)$
$= 72$ km/h

ii. from $t = 3$ to $t = 3.1$

$\frac{s(3.1) - s(3)}{0.1}$
$= \frac{126.48 - 120}{0.1}$
$= 64.8$ km/h

iii. $3 \le t \le 3.01$

$\frac{s(3.01) - s(3)}{0.01}$
$= 64.08$ km/h

b. Instantaneous velocity is approximately 64 km/h.

c. At $t = 3$

$s(t) = 8t^2 + 16t$
$v(t) = 16t + 16$
$v(3) = 48 + 16$
$= 64$ km/h

9. a. $N(t) = 20t - t^2$

$\frac{N(3) - N(2)}{1}$
$= \frac{51 - 36}{1}$
$= 15$

15 terms are learned between $t = 2$ and $t = 3$.

b. $\lim_{h\to 0}\frac{20(2 + h) - (2 + h)^2 - 36}{h}$

$$= \lim_{h\to 0}\frac{40 + 20h - 4 - 4h - h^2 - 36}{h}$$
$$= \lim_{h\to 0}\frac{16h - h^2}{h}$$
$$= \lim_{h\to 0}(16 - h)$$
$$= 16$$

At $t = 2$, the student is learning at a rate of 16 terms/h.

10. a. M in mg in 1 mL of blood t hours after the injection.

$M(t) = -\frac{1}{3}t^2 + t; 0 \le t \le 3$

Calculate the instantaneous rate of change when $t = 2$.

$$\lim_{h\to 0}\frac{-\frac{1}{3}(2 + h)^2 + (2 + h) - (-\frac{4}{3} + 2)}{h}$$
$$= \lim_{h\to 0}\frac{-\frac{4}{3} - \frac{4}{3}h - \frac{1}{3}h^2 + 2 + h + \frac{4}{3} - 2}{h}$$
$$= \lim_{h\to 0}\frac{-\frac{1}{3}h - \frac{1}{3}h^2}{h}$$
$$= \lim_{h\to 0}\left(-\frac{1}{3} - \frac{1}{3}h\right)$$
$$= -\frac{1}{3}$$

Rate of change is $-\frac{1}{3}$ mg/h.

b. Amount of medicine in 1 mL of blood is being dissipated throughout the system.

11. $t = \sqrt{\frac{s}{5}}$

Calculate the instantaneous rate of change when $s = 125$.

$$\lim_{h\to 0} \frac{\sqrt{\frac{125+h}{5}} - \sqrt{\frac{125}{5}}}{h}$$

$$= \lim_{h\to 0} \frac{\sqrt{\frac{125+h}{5}} - 5}{h}$$

$$= \lim_{h\to 0} \left[\frac{\sqrt{\frac{125+h}{5}} - 5}{h} \cdot \frac{\sqrt{\frac{125+h}{5}} + 5}{\sqrt{\frac{125+h}{5}} + 5} \right]$$

$$= \lim_{h\to 0} \left[\frac{\frac{125+h}{5} - 25}{h\left(\sqrt{\frac{125+h}{5}} + 5\right)} \right]$$

$$= \lim_{h\to 0} \left[\frac{\frac{125+h-125}{5}}{h\left(\sqrt{\frac{125+h}{5}} + 5\right)} \right]$$

$$= \lim_{h\to 0} \frac{1}{5\left(\sqrt{\frac{125+h}{5}} + 5\right)}$$

$$= \frac{1}{5\left(\sqrt{\frac{125}{5}} + 5\right)}$$

$$= \frac{1}{5(5+5)}$$

$$= \frac{1}{50}$$

At $s = 125$, rate of change of time with respect to height is $\frac{1}{50}$ s/m.

12. $T(h) = \frac{60}{h+2}$

Calculate the instantaneous rate of change when $h = 3$.

$$\lim_{k\to 0} \frac{\frac{60}{(3+k)+2} - \frac{60}{(3+2)}}{k}$$

$$= \lim_{k\to 0} \frac{\frac{60}{5+k} - 12}{k}$$

$$= \lim_{k\to 0} \frac{\frac{60}{5+k} - \frac{60+12k}{5+k}}{k}$$

$$= \lim_{k\to 0} \frac{-12k}{k(5+k)}$$

$$= \lim_{k\to 0} \frac{-12}{(5+k)}$$

$$= -\frac{12}{5}$$

Temperature is decreasing at $\frac{12}{5}$ °C/km.

13. $h = 25t^2 - 100t + 100$

When $h = 0$, $25t^2 - 100t + 100 = 0$

$$t^2 - 4t + 4 = 0$$

$$(t-2)^2 = 0$$

$$t = 2$$

Calculate the instantaneous rate of change when $t = 2$.

$$\lim_{h\to 0} \frac{25(2+h)^2 - 100(2+h) + 100 - 0}{h}$$

$$= \lim_{h\to 0} \frac{100 + 100h + 25h^2 - 200 - 100h + 100}{h}$$

$$= \lim_{h\to 0} \frac{25h^2}{h}$$

$$= \lim_{h\to 0} 25h$$

$$= 0$$

It hit the ground in 2 s at a speed of 0 m/s.

14. Sale of x balls per week:

$P(x) = 160x - x^2$ dollars.

a. $P(40) = 160(40) - (40)^2$

$= 4800$

Profit on the sale of 40 balls is \$4800.

b. Calculate the instantaneous rate of change when $x = 40$.

$$\lim_{h\to 0} \frac{160(40+h) - (40+h)^2 - 4800}{h}$$

$$= \lim_{h\to 0} \frac{6400 + 160h - 1600 - 80h - h^2 - 4800}{h}$$

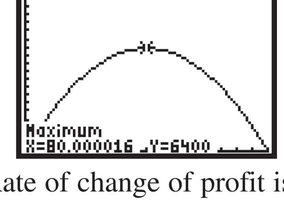
$$= \lim_{h\to 0} \frac{80h - h^2}{h}$$

$$= \lim_{h\to 0} (80 - h)$$

$$= 80$$

Rate of change of profit is \$80 per ball.

c.



Rate of change of profit is positive when the sales level is less than 80.

15. a. $f(x) = -x^2 + 2x + 3$; $(-2, -5)$

$$\lim_{x\to -2} \frac{f(x) - f(-2)}{x + 2}$$
$$= \lim_{x\to -2} \frac{-x^2 + 2x + 3 + 5}{x + 2}$$
$$= \lim_{x\to -2} \frac{-(x^2 - 2x - 8)}{x + 2}$$
$$= -\lim_{x\to -2} \frac{(x - 4)(x + 2)}{x + 2}$$
$$= -\lim_{x\to -2} (x - 4)$$
$$= 6$$

b. $f(x) = \dfrac{x}{x - 1}$, $x = 2$

$$\lim_{x\to 2} \frac{\frac{x}{x-1} - 2}{x - 2}$$
$$= \lim_{x\to 2} \frac{x - 2x + 2}{(x - 1)(x - 2)}$$
$$= \lim_{x\to 2} \frac{-(x - 2)}{(x - 1)(x - 2)}$$
$$= -1$$

c. $f(x) = \sqrt{x + 1}$, $x = 24$

$$= \lim_{x\to 24} \frac{f(x) - f(24)}{x - 24}$$
$$= \lim_{x\to 24} \frac{\sqrt{x + 1} - 5}{x - 24} \cdot \frac{\sqrt{x + 1} + 5}{\sqrt{x + 1} + 5}$$
$$= \lim_{x\to 24} \frac{x - 24}{(x - 24)(\sqrt{x + 1} + 5)}$$
$$= \frac{1}{10}$$

16. $S(x) = 246 + 64x - 8.9x^2 + 0.95x^3$

Calculate the instantaneous rate of change.

$$= \lim_{h\to 0} \frac{S(x + h) - S(x)}{h}$$
$$= \lim_{h\to 0} \frac{246 + 64(x + h) - 8.9(x + h)^2 + 0.95(x + h)^3 - (246 - 64x - 8.9x^2 + 0.95x^3)}{h}$$
$$= \lim_{h\to 0} \frac{246 - 246 + 64(x + h - x) - 8.9(x^2 + 2xh + h^2 - x^2) + 0.95(x^3 + 3x^2h + 3xh^2 + h^3 - x^3)}{h}$$
$$= \lim_{h\to 0} \frac{64h - 8.9(2xh + h^2) + 0.95(3x^2h + 3xh^2 + h^3)}{h}$$
$$= \lim_{h\to 0} [64 - 8.9(2x + h) + 0.95(3x^2 + 3xh + h^2)]$$
$$= 64 - 8.9(2x + 0) + 0.95\,[3x^2 + 3x(0) + (0)^2]$$
$$= 64 - 17.8x + 2.85x^2$$

For the year 2005, $x = 2005 - 1982 = 23$. Hence, the rate at which the average annual salary is changing in 2005 is

$P'(23) = 64 - 17.8(23) + 2.85(23)^2 =$
\$1 162 250/years since 1982

17. $s(t) = 3t^2$

a. The distance travelled from 0 s to 5 s is
$s(5) = 3(5)^2 = 75$ m

b. $s(10) = 3(10)^2 = 300$ m

The rate at which the avalanche is moving from 0 s to 10 s is

$$\frac{\Delta s}{\Delta t} = \frac{300 - 0}{10 - 0}$$
$$= 30 \text{ m/s}$$

c. Calculate the instantaneous rate of change when $t = 10$.

$$\lim_{h\to 0} \frac{3(10 + h)^2 - 300}{h}$$
$$= \lim_{h\to 0} \frac{300 + 60h + 3h^2 - 300}{h}$$
$$= \lim_{h\to 0} \frac{60h + 3h^2}{h}$$
$$= \lim_{h\to 0} (60 + 3h)$$
$$= 60$$

At 10 s the avalanche is moving at 60 m/s.

d. Set $s(t) = 600$:

$$3t^2 = 600$$
$$t^2 = 200$$
$$t = \pm 10\sqrt{2}$$

Since $t \geq 0$, $t = 10\sqrt{2} \doteq 14$ s.

18. The coordinates of the point are $\left(a, \frac{1}{a}\right)$. The slope of the tangent is $-\frac{1}{a^2}$. The equation of the tangent is $y - \frac{1}{a} = -\frac{1}{a^2}(x - a)$ or $y = -\frac{1}{a^2}x + \frac{2}{a}$. The intercepts are $\left(0, \frac{2}{a}\right)$ and $(-2a, 0)$. The tangent line and the axes form a right triangle with legs of length $\frac{2}{a}$ and $2a$. The area of the triangle is $\frac{1}{2}\left(\frac{2}{a}\right)(2a) = 2$.

19. $C(x) = F + V(x)$
$C(x + h) = F + V(x + h)$
Rate of change of cost is

$$\lim_{x\to R}\frac{C(x + h) - C(x)}{h}$$
$$= \lim_{x\to h}\frac{V(x + h) - V(x)}{h} h,$$

which is independent of F (fixed costs).

20. $A(r) = \pi r^2$
Rate of change of area is

$$\lim_{h\to 0}\frac{A(r + h) - A(r)}{h}$$
$$= \lim_{h\to 0}\frac{\pi(r + h)^2 - \pi r^2}{h}$$
$$= \pi\lim_{h\to 0}\frac{(r + h - r)(r + h + r)}{h}$$
$$= 2\pi r$$

$r = 100$ m
Rate is 200π m^2/m.

21. Cube of dimensions x by x by x has volume $V = x^3$. Surface area is $6x^2$.
$V'(x) = 3x^2 = \frac{1}{2}$ surface area.

22. a. The surface area of a sphere is given by $A(r) = 4\pi r^2$.
The question asks for the instantaneous rate of change of the surface when $r = 10$. This is

$$\lim_{h\to 0}\frac{A(10 + h) - A(10)}{h}$$
$$= \lim_{h\to 0}\frac{4\pi(10 + h)^2 - 4\pi(10)^2}{h}$$
$$= \lim_{h\to 0}\frac{4\pi(100 + 20h + h^2) - 4\pi(100)}{h}$$
$$= \lim_{h\to 0}\frac{400\pi + 80\pi h + 4\pi h^2 - 400\pi}{h}$$
$$= \lim_{h\to 0}\frac{80\pi h + 4\pi h^2}{h}$$
$$= \lim_{h\to 0}(80\pi + 4\pi h)$$
$$= 80\pi + 4\pi(0)$$
$$= 80\pi$$

Therefore, the instantaneous rate of change of the surface area of a spherical balloon as it is inflated when the radius reaches 10 cm is 80π cm^2/unit of time.

b. The volume of a sphere is given by $V(r) = \frac{4}{3}\pi r^3$. The question asks for the instantaneous rate of change of the volume when $r = 5$.
Note that the volume is deflating. So, find the rate of the change of the volume when $r = 5$ and then make the answer negative to symbolize a deflating spherical balloon.

$$\lim_{h\to 0}\frac{V(5 + h) - V(5)}{h}$$
$$= \lim_{h\to 0}\frac{\frac{4}{3}\pi(5 + h)^3 - \frac{4}{3}\pi(5)^3}{h}$$

Using the binomial formula to expand $(5 + h)^3$ (or one could simply expand using algebra), the limit is

$$= \lim_{h\to 0}\frac{\frac{4}{3}\pi(h^3 + 15h^2 + 75h + 125) - \frac{4}{3}\pi(5)^3}{h}$$
$$= \lim_{h\to 0}\frac{\frac{4}{3}\pi h^3 + 20\pi h^2 + 100\pi h + \frac{4}{3}\pi(125)}{h}$$
$$\frac{-\frac{4}{3}\pi(125)}{h}$$
$$= \lim_{h\to 0}\frac{\frac{4}{3}\pi h^3 + 20\pi h^2 + 100\pi h}{h}$$
$$= \lim_{h\to 0}\left(\frac{4}{3}\pi h^2 + 20\pi h + 100\pi\right)$$
$$= \frac{4}{3}\pi(0)^2 + 20\pi(0) + 100\pi$$
$$= 100\pi$$

Because the balloon is deflating, the instantaneous rate of change of the volume of the spherical balloon when the radius reaches 5 cm is -100π cm^3/unit of time.

Mid-Chapter Review pp. 32–33

1. a. Corresponding conjugate: $\sqrt{5} + \sqrt{2}$.

$$(\sqrt{5} - \sqrt{2})(\sqrt{5} + \sqrt{2})$$
$$= (\sqrt{25} + \sqrt{10} - \sqrt{10} - \sqrt{4})$$
$$= 5 - 2$$
$$= 3$$

b. Corresponding conjugate: $3\sqrt{5} - 2\sqrt{2}$.

$$(3\sqrt{5} + 2\sqrt{2})(3\sqrt{5} - 2\sqrt{2})$$
$$= (9\sqrt{25} - 6\sqrt{10} + 6\sqrt{10} - 4\sqrt{4})$$
$$= 9(5) - 4(2)$$
$$= 45 - 8$$
$$= 37$$

c. Corresponding conjugate: $9 - 2\sqrt{5}$.

$(9 + 2\sqrt{5})(9 - 2\sqrt{5})$

$= (81 - 18\sqrt{5} + 18\sqrt{5} - 4\sqrt{25})$

$= 81 - 4(5)$

$= 81 - 20$

$= 61$

d. Corresponding conjugate: $3\sqrt{5} + 2\sqrt{10}$.

$(3\sqrt{5} - 2\sqrt{10})(3\sqrt{5} + 2\sqrt{10})$

$= (9\sqrt{25} + 6\sqrt{50} - 6\sqrt{50} - 4\sqrt{100})$

$= 9(5) - 4(10)$

$= 45 - 40$

$= 5$

2. a. $\frac{6 + \sqrt{2}}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$

$= \frac{6\sqrt{3} + \sqrt{6}}{\sqrt{9}}$

$= \frac{6\sqrt{3} + \sqrt{6}}{3}$

b. $\frac{2\sqrt{3} + 4}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$

$= \frac{2\sqrt{9} + 4\sqrt{3}}{\sqrt{9}}$

$= \frac{6 + 4\sqrt{3}}{3}$

c. $\frac{5}{\sqrt{7} - 4} \cdot \frac{\sqrt{7} + 4}{\sqrt{7} + 4}$

$= \frac{5(\sqrt{7} + 4)}{\sqrt{49} + 4\sqrt{7} - 4\sqrt{7} - 16}$

$= \frac{5(\sqrt{7} + 4)}{7 - 16}$

$= -\frac{5(\sqrt{7} + 4)}{9}$

d. $\frac{2\sqrt{3}}{\sqrt{3} - 2} \cdot \frac{\sqrt{3} + 2}{\sqrt{3} + 2}$

$= \frac{2\sqrt{9} + 4\sqrt{3}}{\sqrt{9} + 2\sqrt{3} - 2\sqrt{3} - 4}$

$= \frac{6 + 4\sqrt{3}}{3 - 4}$

$= \frac{6 + 4\sqrt{3}}{-1}$

$= -2(3 + 2\sqrt{3})$

e. $\frac{5\sqrt{3}}{2\sqrt{3} + 4} \cdot \frac{2\sqrt{3} - 4}{2\sqrt{3} - 4}$

$= \frac{10\sqrt{9} - 20\sqrt{3}}{4\sqrt{9} - 8\sqrt{3} + 8\sqrt{3} - 16}$

$= \frac{30 - 20\sqrt{3}}{12 - 16}$

$= \frac{30 - 20\sqrt{3}}{-4}$

$= \frac{10\sqrt{3} - 15}{2}$

f. $\frac{3\sqrt{2}}{2\sqrt{3} - 5} \cdot \frac{2\sqrt{3} + 5}{2\sqrt{3} + 5}$

$= \frac{3\sqrt{2}(2\sqrt{3} + 5)}{4\sqrt{9} + 10\sqrt{3} - 10\sqrt{3} - 25}$

$= \frac{3\sqrt{2}(2\sqrt{3} + 5)}{4(3) - 25}$

$= \frac{3\sqrt{2}(2\sqrt{3} + 5)}{12 - 25}$

$= \frac{3\sqrt{2}(2\sqrt{3} + 5)}{-13}$

$= -\frac{3\sqrt{2}(2\sqrt{3} + 5)}{13}$

3. a. $\frac{\sqrt{2}}{5} \cdot \frac{\sqrt{2}}{\sqrt{2}}$

$= \frac{\sqrt{4}}{5\sqrt{2}}$

$= \frac{2}{5\sqrt{2}}$

b. $\frac{\sqrt{3}}{6 + \sqrt{2}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$

$= \frac{\sqrt{9}}{\sqrt{3}(6 + \sqrt{2})}$

$= \frac{3}{\sqrt{3}(6 + \sqrt{2})}$

c. $\frac{\sqrt{7} - 4}{5} \cdot \frac{\sqrt{7} + 4}{\sqrt{7} + 4}$

$= \frac{\sqrt{49} + 4\sqrt{7} - 4\sqrt{7} - 16}{5(\sqrt{7} + 4)}$

$= \frac{7 - 16}{5(\sqrt{7} + 4)}$

$= -\frac{9}{5(\sqrt{7} + 4)}$

d. $\frac{2\sqrt{3} - 5}{3\sqrt{2}} \cdot \frac{2\sqrt{3} + 5}{2\sqrt{3} + 5}$

$= \frac{4\sqrt{9} + 10\sqrt{3} - 10\sqrt{3} - 25}{3\sqrt{2}(2\sqrt{3} + 5)}$

$= \frac{4(3) - 25}{3\sqrt{2}(2\sqrt{3} + 5)}$

$= \frac{12 - 25}{3\sqrt{2}(2\sqrt{3} + 5)} = -\frac{13}{3\sqrt{2}(2\sqrt{3} + 5)}$

e. $\dfrac{\sqrt{3}-\sqrt{7}}{4}\cdot\dfrac{\sqrt{3}+\sqrt{7}}{\sqrt{3}+\sqrt{7}}$

$$= \frac{\sqrt{9}+\sqrt{21}-\sqrt{21}-\sqrt{49}}{4(\sqrt{3}+\sqrt{7})}$$
$$= \frac{3-7}{4(\sqrt{3}+\sqrt{7})}$$
$$= -\frac{4}{4(\sqrt{3}+\sqrt{7})}$$
$$= -\frac{1}{(\sqrt{3}+\sqrt{7})}$$

f. $\dfrac{2\sqrt{3}+\sqrt{7}}{5}\cdot\dfrac{2\sqrt{3}-\sqrt{7}}{2\sqrt{3}-\sqrt{7}}$

$$= \frac{4\sqrt{9}-2\sqrt{21}+2\sqrt{21}-\sqrt{49}}{5(2\sqrt{3}-\sqrt{7})}$$
$$= \frac{4(3)-7}{5(2\sqrt{3}-\sqrt{7})}$$
$$= \frac{12-7}{5(2\sqrt{3}-\sqrt{7})}$$
$$= \frac{1}{(2\sqrt{3}-\sqrt{7})}$$

4. a. $m = -\dfrac{2}{3};$

$$y-6 = -\frac{2}{3}(x-0)$$
$$y-6 = -\frac{2}{3}x$$
$$\frac{2}{3}x + y - 6 = 0$$

b. $m = \dfrac{11-7}{6-2} = \dfrac{4}{4} = 1$

$$y-7 = 1(x-2)$$
$$y-7 = x-2$$
$$-x+y-5 = 0$$
$$x-y+5 = 0$$

c. $m = 4$

$$y-6 = 4(x-2)$$
$$y-6 = 4x-8$$
$$-4x+y+2 = 0$$
$$4x-y-2 = 0$$

d. $m = \dfrac{1}{5}$

$$y-(-2) = \frac{1}{5}(x-(-1))$$
$$y+2 = \frac{1}{5}x+\frac{1}{5}$$
$$-\frac{1}{5}x + y + \frac{10}{5} - \frac{1}{5} = 0$$
$$-\frac{1}{5}x + y + \frac{9}{5} = 0$$
$$\frac{1}{5}x - y - \frac{9}{5} = 0$$
$$x - 5y - 9 = 0$$

5. The slope of PQ is

$$m = \lim_{h\to 0}\frac{f(1+h)-(-1)}{(1+h)-1}$$
$$= \lim_{h\to 0}\frac{-(1+h)^2+1}{h}$$
$$= \lim_{h\to 0}\frac{-(1+2h+h^2)+1}{h}$$
$$= \lim_{h\to 0}\frac{-1-2h-h^2+1}{h}$$
$$= \lim_{h\to 0}\frac{-2h-h^2}{h}$$
$$= \lim_{h\to 0}(-2-h)$$
$$= -2-(0)$$
$$= -2$$

So, the slope of PQ with $f(x) = -x^2$ is -2.

6. a. Unlisted y-coordinates for Q are found by substituting the x-coordinates into the given function. The slope of the line PQ with the given points is given by the following: Let $P = (x_1, y_1)$ and $Q = (y_1, y_2)$. Then, the slope $= m = \dfrac{y_2-y_1}{x_2-x_1}$.

P	Q	Slope of Line PQ
(−1, 1)	(−2, 6)	−5
(−1, 1)	(−1.5, 3.25)	−4.5
(−1, 1)	(−1.1, 1.41)	−4.1
(−1, 1)	(−1.01, 1.040 1)	−4.01
(−1, 1)	(−1.001, 1.004 001)	−4.001

P	Q	Slope of Line PQ
(−1, 1)	(0, −2)	−3
(−1, 1)	(−0.5, −0.75)	−3.5
(−1, 1)	(−0.9, 0.61)	−3.9
(−1, 1)	(−0.99, 0.9601)	−3.99
(−1, 1)	(−0.999, 0.996 001)	−3.999

b. The slope from the right and from the left appear to approach -4. The slope of the tangent to the graph of $f(x)$ at point P is about -4.

c. With the points $P = (-1, 1)$ and $Q = (-1+h, f(-1+h))$, the slope, m, of PQ is the following:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{[(-1+h)^2 - 2(-1+h) - 2] - (1)}{(-1+h) - (-1)}$$
$$= \frac{1 - 2h + h^2 + 2 - 2h - 2 - 1}{-1 + h + 1}$$
$$= \frac{h^2 - 4h}{h}$$
$$= h - 4$$

d. The slope of the tangent is $\lim_{h\to 0} f(x)$.

In this case, as h goes to zero, $h - 4$ goes to $h - 4 = 0 - 4 = -4$. The slope of the tangent to the graph of $f(x)$ at the point P is -4.

e. The answers are equal.

7. a. $$m = \lim_{h\to 0} \frac{f(-3+h) - f(-3)}{h}$$
$$= \lim_{h\to 0} \frac{[(-3+h)^2 + 3(-3+h) - 5] - [(-3)^2 + 3(-3) - 5]}{h}$$
$$= \lim_{h\to 0} \frac{9 - 6h + h^2 - 9 + 3h - 5 - (9 - 9 - 5)}{h}$$
$$= \lim_{h\to 0} \frac{h^2 - 3h - 5 - (-5)}{h}$$
$$= \lim_{h\to 0} \frac{h^2 - 3h}{h}$$
$$= \lim_{h\to 0} (h - 3)$$
$$= 0 - 3$$
$$= -3$$

b. $y = f(x) = \frac{1}{x}$

$$m = \lim_{h\to 0} \frac{f(\frac{1}{3} + h) - f(\frac{1}{3})}{h}$$
$$= \lim_{h\to 0} \frac{\frac{1}{\frac{1}{3} + h} - \frac{1}{\frac{1}{3}}}{h}$$
$$= \lim_{h\to 0} \frac{\frac{(\frac{1}{3}) - (\frac{1}{3} + h)}{\frac{1}{3}(\frac{1}{3} + h)}}{h}$$
$$= \lim_{h\to 0} \left(\frac{-h}{\frac{1}{9} + \frac{1}{3}h}\right)\frac{1}{h}$$
$$= \lim_{h\to 0} \frac{-1}{\frac{1}{9} + \frac{1}{3}h}$$
$$= \frac{-1}{\frac{1}{9} + \frac{1}{3}(0)}$$
$$= -9$$

c. $y = f(x) = \frac{4}{x - 2}$

$$m = \lim_{h\to 0} \frac{f(6+h) - f(6)}{h}$$
$$= \lim_{h\to 0} \frac{\frac{4}{6 + h - 2} - \frac{4}{6 - 2}}{h}$$
$$= \lim_{h\to 0} \frac{\frac{4}{h + 4} - \frac{4}{4}}{h}$$
$$= \lim_{h\to 0} \frac{\frac{4}{h + 4} - 1}{h}$$
$$= \lim_{h\to 0} \left(\frac{4 - (h + 4)}{h + 4}\right)\frac{1}{h}$$
$$= \lim_{h\to 0} \left(\frac{-h}{h + 4}\right)\frac{1}{h}$$
$$= \lim_{h\to 0} \frac{-1}{h + 4}$$
$$= \frac{-1}{0 + 4}$$
$$= -\frac{1}{4}$$

d. $$m = \lim_{h\to 0} \frac{f(5+h) - f(5)}{h}$$
$$= \lim_{h\to 0} \frac{\sqrt{5 + h + 4} - \sqrt{5 + 4}}{h}$$
$$= \lim_{h\to 0} \frac{\sqrt{9 + h} - \sqrt{9}}{h}$$
$$= \lim_{h\to 0} \frac{\sqrt{9 + h} - 3}{h}$$
$$= \lim_{h\to 0} \frac{\sqrt{9 + h} - 3}{h} \cdot \frac{\sqrt{9 + h} + 3}{\sqrt{9 + h} + 3}$$
$$= \lim_{h\to 0} \frac{9 + h + 3\sqrt{9 + h} - 3\sqrt{9 + h} - 9}{h(\sqrt{9 + h} + 3)}$$
$$= \lim_{h\to 0} \frac{h}{h(\sqrt{9 + h} + 3)}$$
$$= \lim_{h\to 0} \frac{1}{\sqrt{9 + h} + 3}$$

$= \dfrac{1}{\sqrt{9+0}+3}$
$= \dfrac{1}{6}$

8. $s(t) = 6t(t+1) = 6t^2 + 6t$

a. i. average velocity $= \dfrac{s(3) - s(2)}{3 - 2}$
$= [6(3)^2 + 6(3)] - [6(2)^2 + 6(2)]$
$= 6(9) + 18 - (24 + 12)$
$= 54 + 18 - 36$
$= 36$ km/h

ii. average velocity $= \dfrac{s(2.1) - s(2)}{2.1 - 2}$
$= \dfrac{[6(2.1)^2 + 6(2.1)] - [6(2)^2 + 6(2)]}{0.1}$
$= \dfrac{[26.46 + 12.6] - [24 + 12]}{0.1}$
$= \dfrac{39.06 - 36}{0.1}$
$= \dfrac{3.06}{0.1}$
$= 30.6$ km/h

iii. average velocity $= \dfrac{s(2.01) - s(2)}{2.01 - 2}$
$= \dfrac{[6(2.01)^2 + 6(2.01)] - [6(2)^2 + 6(2)]}{0.01}$
$= \dfrac{[24.2406 + 12.06] - [6(2)^2 + 6(2)]}{0.01}$
$= \dfrac{36.3006 - [24 + 12]}{0.01}$
$= \dfrac{36.3006 - 36}{0.01}$
$= \dfrac{0.3006}{0.01}$
$= 30.06$ km/h

b. At the time $t = 2$, the velocity of the car appears to approach 30 km/h.

c. average velocity $= \dfrac{f(2+h) - f(2)}{(2+h) - (2)}$
$= \dfrac{[6(2+h)^2 + 6(2+h)] - [6(2)^2 + 6(2)]}{h}$
$= \dfrac{[6(4 + 4h + h^2) + 12 + 6h] - [24 + 12]}{h}$
$= \dfrac{[24 + 24h + 6h^2 + 12 + 6h] - 36}{h}$
$= \dfrac{6h^2 + 30h + 36 - 36}{h}$
$= \dfrac{6h^2 + 30h}{h}$
$= (6h + 30)$ km/h

d. When $t = 2$, the velocity is the limit as h approaches 0.
velocity $= \lim_{h \to 0} (6h + 30)$
$= 6(0) + 30$
$= 30$
Therefore, when $t = 2$ the velocity is 30 km/h.

9. a. The instantaneous rate of change of $f(x)$ with respect to x at $x = 2$ is given by
$\lim_{h \to 0} \dfrac{f(2+h) - f(2)}{h}$
$= \lim_{h \to 0} \dfrac{[5 - (2+h)^2] - [5 - (2)^2]}{h}$
$= \lim_{h \to 0} \dfrac{5 - (4 + 4h + h^2) - 1}{h}$
$= \lim_{h \to 0} \dfrac{5 - 4 - 4h - h^2 - 1}{h}$
$= \lim_{h \to 0} \dfrac{-h^2 - 4h}{h}$
$= \lim_{h \to 0} (-h - 4)$
$= -(0) - 4$
$= -4$

b. The instantaneous rate of change of $f(x)$ with respect to x at $x = \frac{1}{2}$ is given by
$\lim_{h \to 0} \dfrac{f(\frac{1}{2} + h) - f(\frac{1}{2})}{h}$
$= \lim_{h \to 0} \dfrac{\frac{3}{\frac{1}{2} + h} - \frac{3}{\frac{1}{2}}}{h}$
$= \lim_{h \to 0} \dfrac{\frac{3}{\frac{1}{2} + h} - 6}{h}$
$= \lim_{h \to 0} \dfrac{3 - 6(\frac{1}{2} + h)}{\frac{1}{2} + h} \cdot \dfrac{1}{h}$
$= \lim_{h \to 0} \dfrac{3 - 3 - 6h}{\frac{1}{2} + h} \cdot \dfrac{1}{h}$
$= \lim_{h \to 0} \dfrac{-6h}{\frac{1}{2} + h} \cdot \dfrac{1}{h}$
$= \lim_{h \to 0} \dfrac{-6}{\frac{1}{2} + h}$
$= \dfrac{-6}{\frac{1}{2} + 0}$
$= -12$

10. a. The average rate of change of $V(t)$ with respect to t during the first 20 minutes is given by

$$\frac{f(20) - f(0)}{20 - 0}$$
$$= \frac{[50(30 - 20)^2] - [50(30 - 0)^2]}{20}$$
$$= \frac{5000 - 45\,000}{20}$$
$$= -\frac{40\,000}{20}$$
$$= -2000 \text{ L/min}$$

b. The rate of change of $V(t)$ with respect to t at the time $t = 20$ is given by

$$\lim_{h\to 0}\frac{f(20 + h) - f(20)}{h}$$
$$= \lim_{h\to 0}\frac{[50(30 - (20 + h))^2] - [50(30 - 20)^2]}{h}$$
$$= \lim_{h\to 0}\frac{[50(10 - h)^2] - [50(10)^2]}{h}$$
$$= \lim_{h\to 0}\frac{[50(100 - 20h + h^2)] - [50(100)]}{h}$$
$$= \lim_{h\to 0}\frac{5000 - 1000h + 50h^2 - 5000}{h}$$
$$= \lim_{h\to 0}\frac{50h^2 - 1000h}{h}$$
$$= \lim_{h\to 0} 50h - 1000$$
$$= 50(0) - 1000$$
$$= -1000 \text{ L/min}$$

11. a. Let $y = f(x)$.

$$f(4) = (4)^2 + (4) - 3 = 16 + 1 = 17$$
$$f(4 + h) = (4 + h)^2 + (4 + h) - 3$$
$$= 16 + 8h + h^2 + h + 1$$
$$= h^2 + 9h + 17$$

Using the limit of the difference quotient, the slope of the tangent at $x = 4$ is

$$m = \lim_{h\to 0}\frac{f(4 + h) - f(4)}{h}$$
$$= \lim_{h\to 0}\frac{h^2 + 9h + 17 - (17)}{h}$$
$$= \lim_{h\to 0}\frac{h^2 + 9h}{h}$$
$$= \lim_{h\to 0}(h + 9)$$
$$= 0 + 9$$
$$= 9$$

Therefore, the slope of the tangent to $y = f(x) = x^2 + x - 3$ at $x = 4$ is 9.

So an equation of the tangent at $x = 4$ is given by

$$y - 17 = 9(x - 4)$$
$$y - 17 = 9x - 36$$
$$-9x + y - 17 + 36 = 0$$
$$-9x + y + 19 = 0$$

b. Let $y = f(x)$.

$$f(-2) = 2(-2)^2 - 7 = 2(4) - 7 = 1$$
$$f(-2 + h) = 2(-2 + h)^2 - 7$$
$$= 2(4 - 4h + h^2) - 7$$
$$= 8 - 8h + 2h^2 - 7$$
$$= 2h^2 - 8h + 1$$

Using the limit of the difference quotient, the slope of the tangent at $x = 4$ is

$$m = \lim_{h\to 0}\frac{f(-2 + h) - f(-2)}{h}$$
$$= \lim_{h\to 0}\frac{2h^2 - 8h + 1 - (1)}{h}$$
$$= \lim_{h\to 0}\frac{2h^2 - 8h}{h}$$
$$= \lim_{h\to 0}(2h - 8)$$
$$= 2(0) - 8$$
$$= -8$$

Therefore, the slope of the tangent to $y = f(x) = 2x^2 - 7$ at $x = -2$ is -8.

So an equation of the tangent at $x = -2$ is given by

$$y - 1 = -8(x - (-2))$$
$$y - 1 = -8x - 16$$
$$8x + y - 1 + 16 = 0$$
$$8x + y + 15 = 0$$

c. $f(-1) = 3(-1)^2 + 2(-1) - 5 = 3 - 2 - 5$

$$= -4$$
$$f(-1 + h) = 3(-1 + h)^2 + 2(-1 + h) - 5$$
$$= 3(1 - 2h + h^2) - 2 + 2h - 5$$
$$= 3 - 6h + 3h^2 - 7 + 2h$$
$$= 3h^2 - 4h - 4$$

Using the limit of the difference quotient, the slope of the tangent at $x = 4$ is

$$m = \lim_{h\to 0}\frac{f(-1 + h) - f(-1)}{h}$$
$$= \lim_{h\to 0}\frac{3h^2 - 4h - 4 - (-4)}{h}$$
$$= \lim_{h\to 0}\frac{3h^2 - 4h}{h}$$
$$= \lim_{h\to 0}(3h - 4)$$
$$= 3(0) - 4$$
$$= -4$$

Therefore, the slope of the tangent to
$y = f(x) = 3x^2 + 2x - 5$ at $x = -1$ is -4.
So an equation of the tangent at $x = -4$ is given by

$$\begin{aligned} y - (-4) &= -4(x - (-1)) \\ y + 4 &= -4(x + 1) \\ y + 4 &= -4x - 4 \\ 4x + y + 4 + 4 &= 0 \\ 4x + y + 8 &= 0 \end{aligned}$$

d. $f(1) = 5(1)^2 - 8(1) + 3 = 5 - 8 + 3 = 0$

$$\begin{aligned} f(1 + h) &= 5(1 + h)^2 - 8(1 + h) + 3 \\ &= 5(1 + 2h + h^2) - 8 - 8h + 3 \\ &= 5 + 10h + 5h^2 - 5 - 8h \\ &= 5h^2 + 2h \end{aligned}$$

Using the limit of the difference quotient, the slope of the tangent at $x = 1$ is

$$\begin{aligned} m &= \lim_{h\to 0} \frac{f(1 + h) - f(1)}{h} \\ &= \lim_{h\to 0} \frac{5h^2 + 2h - (0)}{h} \\ &= \lim_{h\to 0} (5h + 2) \\ &= 5(0) + 2 \\ &= 2 \end{aligned}$$

Therefore, the slope of the tangent to
$y = f(x) = 5x^2 - 8x + 3$ at $x = 1$ is 2.
So an equation of the tangent at $x = 1$ is given by

$$\begin{aligned} y - 0 &= 2(x - 1) \\ y &= 2x - 2 \\ -2x + y + 2 &= 0 \end{aligned}$$

12. a. Using the limit of the difference quotient, the slope of the tangent at $x = -5$ is

$$\begin{aligned} m &= \lim_{h\to 0} \frac{f(-5 + h) - f(-5)}{h} \\ &= \lim_{h\to 0} \left(\frac{-5 + h}{-5 + h + 3} - \frac{-5}{-5 + 3}\right) \cdot \frac{1}{h} \\ &= \lim_{h\to 0} \left(\frac{-5 + h}{-2 + h} - \frac{5}{2}\right) \cdot \frac{1}{h} \\ &= \lim_{h\to 0} \left(\frac{-10 + 2h - (-10 + 5h)}{-4 + 2h}\right) \cdot \frac{1}{h} \\ &= \lim_{h\to 0} \left(\frac{-10 + 2h + 10 - 5h}{-4 + 2h}\right) \cdot \frac{1}{h} \\ &= \lim_{h\to 0} \left(\frac{-3h}{-4 + 2h}\right) \cdot \frac{1}{h} \\ &= \lim_{h\to 0} \left(\frac{-3}{-4 + 2h}\right) \\ &= \frac{-3}{-4 + 2(0)} \\ &= \frac{3}{4} \end{aligned}$$

Therefore, the slope of the tangent to
$f(x) = \frac{x}{x + 3}$ at $x = -5$ is $\frac{3}{4}$.
So an equation of the tangent at $x = \frac{3}{4}$ is given by

$$\begin{aligned} y - \frac{5}{2} &= \frac{3}{4}(x - (-5)) \\ y - \frac{5}{2} &= \frac{3}{4}x + \frac{15}{4} \\ -\frac{3}{4}x + y - \frac{10}{4} - \frac{15}{4} &= 0 \\ -\frac{3}{4}x + y - \frac{25}{4} &= 0 \\ -3x + 4y - 25 &= 0 \end{aligned}$$

b. Using the limit of the difference quotient, the slope of the tangent at $x = -1$ is

$$\begin{aligned} m &= \lim_{h\to 0} \frac{f(-1 + h) - f(-1)}{h} \\ &= \lim_{h\to 0} \left(\frac{2(-1 + h) + 5}{5(-1 + h) - 1} - \frac{2(-1) + 5}{5(-1) - 1}\right) \cdot \frac{1}{h} \\ &= \lim_{h\to 0} \left(\frac{-2 + 2h + 5}{-5 + 5h - 1} - \frac{-2 + 5}{-5 - 1}\right) \cdot \frac{1}{h} \\ &= \lim_{h\to 0} \left(\frac{2h + 3}{5h - 6} - \frac{3}{-6}\right) \cdot \frac{1}{h} \\ &= \lim_{h\to 0} \left(\frac{2h + 3}{5h - 6} + \frac{1}{2}\right) \cdot \frac{1}{h} \\ &= \lim_{h\to 0} \left(\frac{4h + 6 + 5h - 6}{10h - 12}\right) \cdot \frac{1}{h} \\ &= \lim_{h\to 0} \left(\frac{9h}{10h - 12}\right) \cdot \frac{1}{h} \\ &= \lim_{h\to 0} \left(\frac{9}{10h - 12}\right) \\ &= \frac{9}{10(0) - 12} \\ &= -\frac{9}{12} \\ &= -\frac{3}{4} \end{aligned}$$

Therefore, the slope of the tangent to
$f(x) = \frac{2x + 5}{5x - 1}$ at $x = -1$ is $-\frac{3}{4}$.

So an equation of the tangent at $x = -\frac{3}{4}$ is given by

$$\begin{aligned} y - \left(-\frac{1}{2}\right) &= -\frac{3}{4}(x - (-1)) \\ y + \frac{1}{2} &= -\frac{3}{4}x - \frac{3}{4} \\ 4y + 2 &= -3x - 3 \\ 3x + 4y + 2 + 3 &= 0 \\ 3x + 4y + 5 &= 0 \end{aligned}$$

1.4 The Limit of a Function, pp. 37–39

1. a. $\frac{27}{99}$

b. π

2. One way to find a limit is to evaluate the function for values of the independent variable that get progressively closer to the given value of the independent variable.

3. a. A right-sided limit is the value that a function gets close to as the values of the independent variable decrease and get close to a given value.

b. A left-sided limit is the value that a function gets close to as the values of the independent variable increase and get close to a given value.

c. A (two-sided) limit is the value that a function gets close to as the values of the independent variable get close to a given value, regardless of whether the values increase or decrease toward the given value.

4. a. -5

b. $3 + 7 = 10$

c. $10^2 = 100$

d. $4 - 3(-2)^2 = -8$

e. 4

f. $2^3 = 8$

5. Even though $f(4) = -1$, the limit is 1, since that is the value that the function approaches from the left and the right of $x = 4$.

6. a. 0

b. 2

c. -1

d. 2

7. a. 2

b. 1

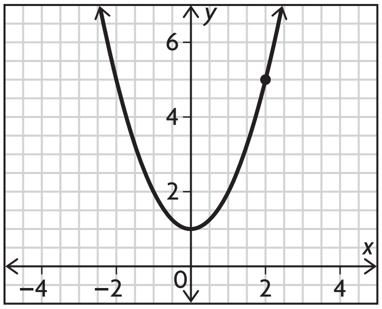
c. does not exist

8. a. $9 - (-1)^2 = 8$

b. $\sqrt{\frac{0 + 20}{0 + 5}} = \sqrt{4}$
$= 2$

c. $\sqrt{5 - 1} = \sqrt{4}$
$= 2$

9. $2^2 + 1 = 5$



10. a. Since 0 is not a value for which the function is undefined, one may substitute 0 in for x to find that

$$\lim_{x\to 0^+} x^4 = \lim_{x\to 0} x^4 = (0)^4 = 0$$

b. Since 2 is not a value for which the function is undefined, one may substitute 2 in for x to find that

$$\lim_{x\to 2^-} (x^2 - 4) = \lim_{x\to 2} (x^2 - 4) = (2)^2 - 4 = 4 - 4 = 0$$

c. Since 3 is not a value for which the function is undefined, one may substitute 3 in for x to find that

$$\lim_{x\to 3^-} (x^2 - 4) = \lim_{x\to 3} (x^2 - 4) = (3)^2 - 4 = 9 - 4 = 5$$

d. Since 1 is not a value for which the function is undefined, one may substitute 1 in for x to find that

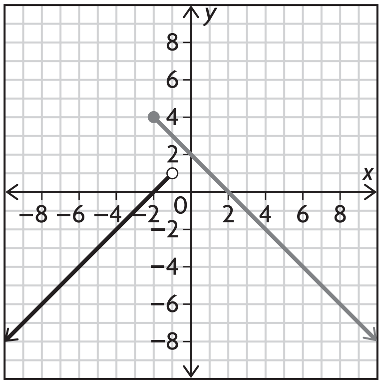
$$\lim_{x\to 1^+} \frac{1}{x - 3} = \lim_{x\to 1} \frac{1}{x - 3} = \frac{1}{1 - 3} = -\frac{1}{2}$$

e. Since 3 is not a value for which the function is undefined, one may substitute 3 in for x to find that

$$\lim_{x\to 3^+} \frac{1}{x + 2} = \lim_{x\to 3} \frac{1}{x + 2} = \frac{1}{3 + 2} = \frac{1}{5}$$

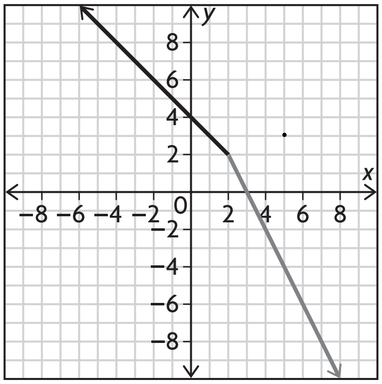
f. If 3 is substituted in the function for x, then the function is undefined because of division by zero. There does not exist a way to divide out the $x - 3$ in the denominator. Also, $\lim_{x\to3^+}\frac{1}{x-3}$ approaches infinity, while $\lim_{x\to3^-}\frac{1}{x-3}$ approaches negative infinity. Therefore, since $\lim_{x\to3^+}\frac{1}{x-3}\neq\lim_{x\to3^-}\frac{1}{x-3}$, $\lim_{x\to3}\frac{1}{x-3}$ does not exist.

11. a.



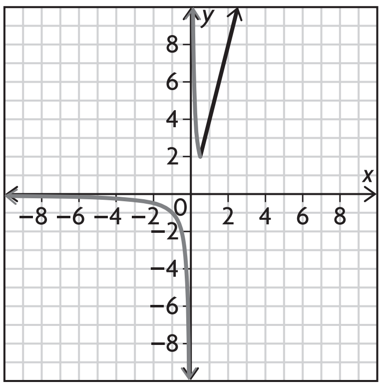
$\lim_{x\to-1^+}f(x)\neq\lim_{x\to-1^-}f(x)$. Therefore, $\lim_{x\to-1}f(x)$ does not exist.

b.



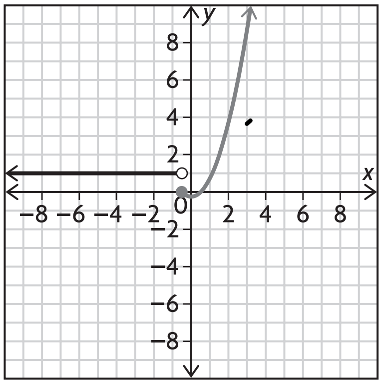
$\lim_{x\to2^+}f(x)=\lim_{x\to2^-}f(x)$. Therefore, $\lim_{x\to2}f(x)$ exists and is equal to 2.

c.



$\lim_{x\to\frac{1}{2}^+}f(x)=\lim_{x\to\frac{1}{2}^-}f(x)$. Therefore, $\lim_{x\to\frac{1}{2}}f(x)$ exists and is equal to 2.

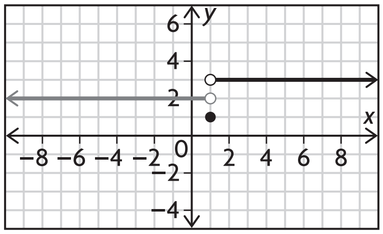
d.



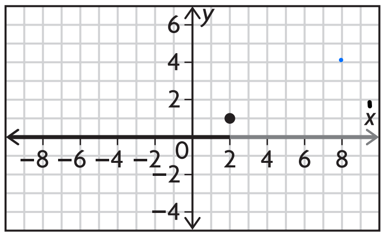
$\lim_{x\to-0.5^+}f(x)\neq\lim_{x\to-0.5^-}f(x)$. Therefore, $\lim_{x\to-0.5}f(x)$ does not exist.

12. Answers may vary. For example:

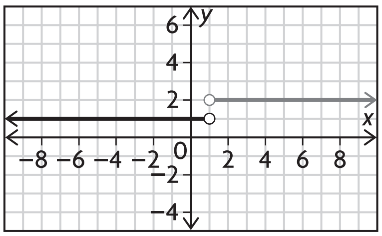
a.



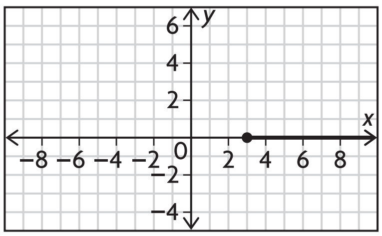
b.



c.



d.



13. $f(x)=mx+b$

$\lim_{x\to1}f(x)=-2 \quad m+b=-2$

$\lim_{x\to-1}f(x)=4 \quad -m+b=4$

$2b=2$

$b=1, m=-3$

14. $f(x) = ax^2 + bx + c, a \neq 0$

$$f(0) = 0 \qquad c = 0$$

$$\lim_{x \to 1} f(x) = 5 \qquad a + b = 5$$

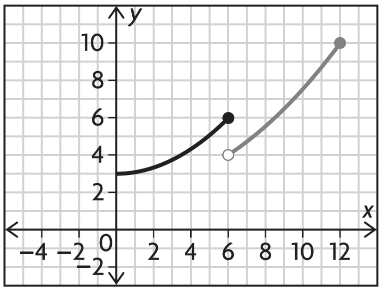
$$\lim_{x \to -2} f(x) = 8 \quad 4a - 2b = 8$$

$$6a = 18$$

$$a = 3, \qquad b = 2$$

Therefore, the values are $a = 3$, $b = 2$, and $c = 0$.

15. a.



b. $\lim_{t \to 6^-} p(t) = 3 + \frac{1}{12}(6)^2$

$$= 3 + \frac{36}{12}$$
$$= 3 + 3$$
$$= 6$$

$$\lim_{t \to 6^+} p(t) = 2 + \frac{1}{18}(6)^2$$
$$= 2 + \frac{36}{18}$$
$$= 2 + 2$$
$$= 4$$

c. Since $p(t)$ is measured in thousands, right before the chemical spill there were 6000 fish in the lake. Right after the chemical spill there were 4000 fish in the lake. So, $6000 - 4000 = 2000$ fish were killed by the spill.

d. The question asks for the time, t, after the chemical spill when there are once again 6000 fish in the lake. Use the second equation to set up an equation that is modelled by

$$6 = 2 + \frac{1}{18}t^2$$

$$4 = \frac{1}{18}t^2$$

$$72 = t^2$$

$$\sqrt{75} = t$$

(The question asks for time so the negative answer is disregarded.)

So, at time $t = \sqrt{72} \doteq 8.49$ years the population has recovered to the level before the spill.

1.5 Properties of Limits, pp. 45–47

1. $\lim_{x \to 2}(3 + x)$ and $\lim_{x \to 2}(x + 3)$ have the same value, but $\lim_{x \to 2} 3 + x$ does not. Since there are no brackets around the expression, the limit only applies to 3, and there is no value for the last term, x.

2. Factor the numerator and denominator. Cancel any common factors. Substitute the given value of x.

3. If the two one-sided limits have the same value, then the value of the limit is equal to the value of the one-sided limits. If the one-sided limits do not have the same value, then the limit does not exist.

4. a. $\frac{3(2)}{2^2 + 2} = 1$

b. $(-1)^4 + (-1)^3 + (-1)^2 = 1$

c. $\left[\sqrt{9} + \frac{1}{\sqrt{9}}\right]^2 = \left(3 + \frac{1}{3}\right)^2$

$$= \frac{100}{9}$$

d. $(2\pi)^3 + \pi^2(2\pi) - 5\pi^3 = 8\pi^3 + 2\pi^3 - 5\pi^3$

$$= 5\pi^3$$

e. $\sqrt{3 + \sqrt{1 + 0}} = \sqrt{3 + 1}$

$$= 2$$

f. $\sqrt{\frac{-3 - 3}{2(-3) + 4}} = \sqrt{\frac{-6}{-2}}$

$$= \sqrt{3}$$

5. a. $\frac{(-2)^3}{-2 - 2} = -2$

b. $\frac{2}{\sqrt{1 + 1}} = \frac{2}{\sqrt{2}}$

$$= \sqrt{2}$$

6. Since substituting $t = 1$ does not make the denominator 0, direct substitution works.

$$\frac{1 - 1 - 5}{6 - 1} = \frac{-5}{5}$$
$$= -1$$

7. a. $\lim_{x \to 2} \frac{4 - x^2}{2 - x} = \lim_{x \to 2} \frac{(2 - x)(2 + x)}{(2 - x)}$

$$= \lim_{x \to 2}(2 + x)$$
$$= 4$$

b. $\lim_{x \to -1} \frac{2x^2 + 5x + 3}{x + 1} = \lim_{x \to -1} \frac{(x + 1)(2x + 3)}{x + 1}$

$$= 5$$

c. $\lim_{x \to 3} \frac{x^3 - 27}{x - 3} = \lim_{x \to 3} \frac{(x - 3)(x^2 + 3x + 9)}{x - 3}$

$$= 9 + 9 + 9$$
$$= 27$$

d. $\lim_{x\to 0}\left[\frac{2-\sqrt{4+x}}{x}\times\frac{2+\sqrt{4+x}}{2+\sqrt{4+x}}\right]$

$= \lim_{x\to 0}\frac{-1}{2+\sqrt{4+x}}$

$= -\frac{1}{4}$

e. $\lim_{x\to 4}\frac{\sqrt{x}-2}{x-4} = \lim_{x\to 4}\frac{\sqrt{x}-2}{(\sqrt{x}-2)(\sqrt{x}+2)}$

$= \frac{1}{4}$

f. $\lim_{x\to 0}\left[\frac{\sqrt{7-x}-\sqrt{7+x}}{x}\times\frac{\sqrt{7-x}+\sqrt{7+x}}{\sqrt{7-x}+\sqrt{7+x}}\right]$

$= \lim_{x\to 0}\frac{7-x-7-x}{x(\sqrt{7-x}+\sqrt{7+x})}$

$= -\frac{1}{\sqrt{7}}$

8. a. $\lim_{x\to 8}\frac{\sqrt[3]{x}-2}{x-8}$

Let $u = \sqrt[3]{x}$. Therefore, $u^3 = x$ as $x\to 8, u\to 2$.

Here, $\lim_{x\to 2}\frac{u-2}{u^3-8} = \lim_{x\to 2}\frac{1}{u^2+2u+4}$

$= \frac{1}{12}$

b. $\lim_{x\to 27}\frac{27-x}{x^{\frac{1}{3}}-3}$

Let $x^{\frac{1}{3}} = u$
$x = u^3$
$x\to 27, u\to 3.$

$= \lim_{x\to 3}\frac{u^3-27}{u-3}$

$= -\lim_{x\to 3}\frac{(u-3)(u^2+3u+9)}{u-3}$

$= -(9+9+9)$

$= -27$

c. $\lim_{x\to 1}\frac{x^{\frac{1}{6}}-1}{x-1}$

$x^{\frac{1}{6}} = u, x = u^6$
$x\to 1, u\to 1$

$= \lim_{x\to 1}\frac{u-1}{u^6-1}$

$= \lim_{x\to 1}\frac{(u-1)}{(u-1)(u^5+u^4+u^3+u^2+u+1)}$

$= \frac{1}{6}$

d. $\lim_{x\to 1}\frac{x^{\frac{1}{6}}-1}{x^{\frac{1}{3}}-1}$

Let $x^{\frac{1}{6}} = u$
$u^6 = x$
$x^{\frac{1}{3}} = u^2$
As $x\to 1, u\to 1$

$= \lim_{x\to 1}\frac{u-1}{u^2-1}$

$= \lim_{x\to 1}\frac{u-1}{(u-1)(u+1)}$

$= \frac{1}{2}$

e. $\lim_{x\to 4}\frac{\sqrt{x}-2}{\sqrt{x^3}-8}$

Let $x^{\frac{1}{2}} = u$
$x^{\frac{3}{2}} = u^3$
$x\to 4, u\to 2$

$= \lim_{x\to 2}\frac{u-2}{u^3-8}$

$= \lim_{x\to 2}\frac{u-2}{(u-2)(u^2+2u+4)}$

$= \frac{1}{12}$

f. $\lim_{x\to 0}\frac{(x+8)^{\frac{1}{3}}-2}{x}$

Let $(x+8)^{\frac{1}{3}} = u$
$x+8 = u^3$
$x = u^3-8$
$x\to 0, u\to 2$

$\lim_{x\to 2}\frac{u-2}{u^3-8}$

$= \frac{1}{12}$

9. a. $\frac{16-16}{64+64} = 0$

b. $\frac{16-16}{16-20+6} = 0$

c. $\lim_{x\to -1}\frac{x^2+x}{x+1} = \lim_{x\to -1}\frac{x(x+1)}{x+1}$

$= -1$

d. $\lim_{x\to 0}\frac{\sqrt{x+1}-1}{x} = \lim_{x\to 0}\frac{\sqrt{x+1}-1}{x+1-1}$

$= \lim_{x\to 0}\frac{\sqrt{x+1}-1}{(\sqrt{x+1}-1)(\sqrt{x+1}+1)}$

$= \frac{1}{2}$

e. $\lim_{h\to 0}\frac{(x+h)^2-x^2}{h} = \lim_{h\to 0}\frac{2xh+h^2}{h}$

$= 2x$

f. $\lim_{x\to 1}\left(\frac{1}{x-1}\right)\left(\frac{1}{x+3}-\frac{2}{3x+5}\right)$

$= \lim_{x\to 1}\left(\frac{1}{x-1}\right)\left(\frac{3x+5-2x-6}{(x+3)(3x+5)}\right)$

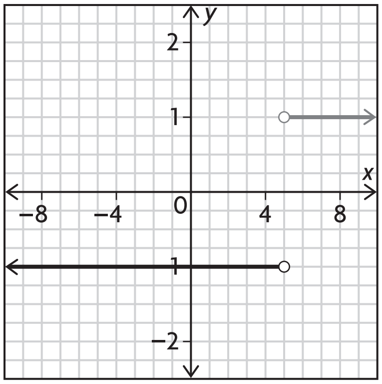
$= \lim_{x\to 1}\frac{1}{(x+3)(3x+5)}$

$= \frac{1}{4(8)}$

$= \frac{1}{32}$

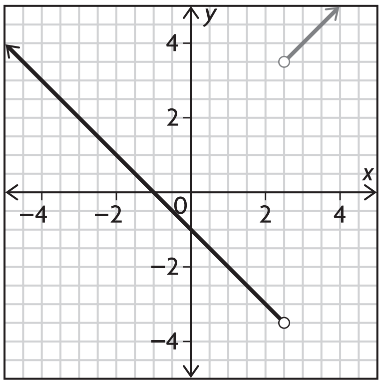
10. a. $\lim\limits_{x \to 5} \dfrac{|x-5|}{x-5}$ does not exist.

$$\lim_{x \to 5^+} \frac{|x-5|}{x-5} = \lim_{x \to 5^+} \frac{x-5}{x-5}$$
$$= 1$$
$$\lim_{x \to 5^-} \frac{|x-5|}{x-5} = \lim_{x \to 5^-} -\left(\frac{x-5}{x-5}\right)$$
$$= -1$$

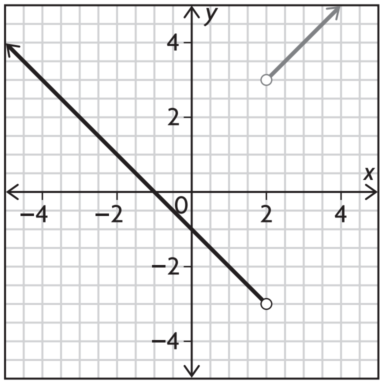


b. $\lim\limits_{x \to \frac{5}{2}} \dfrac{|2x-5|(x+1)}{2x-5}$ does not exist.

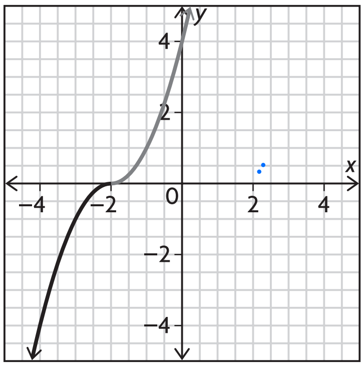
$$|2x-5| = 2x-5, x \geq \frac{5}{2}$$
$$\lim_{x \to \frac{5}{2}^+} \frac{(2x-5)(x+1)}{2x-5} = x+1$$
$$|2x-5| = -(2x-5), x < \frac{5}{2}$$
$$\lim_{x \to \frac{5}{2}^-} \frac{-(2x-5)(x+1)}{2x-5} = -(x+1)$$



c.
$$\lim_{x \to 2} \frac{x^2-x-2}{|x-2|} = \lim_{x \to 2} \frac{(x-2)(x+1)}{|x-2|}$$
$$\lim_{x \to 2^+} \frac{(x-2)(x+1)}{|x-2|} = \lim_{x \to 2^+} \frac{(x-2)(x+1)}{x-2}$$
$$= \lim_{x \to 2^+} x+1$$
$$= 3$$
$$\lim_{x \to 2^-} \frac{(x-2)(x+1)}{|x-2|} = \lim_{x \to 2^-} -\frac{(x-2)(x+1)}{(x-2)}$$
$$= \lim_{x \to 2^-} -(x+1)$$
$$= -3$$



d.
$$|x+2| = x+2 \text{ if } x > -2$$
$$= -(x+2) \text{ if } x < -2$$
$$\lim_{x \to -2^+} \frac{(x+2)(x+2)^2}{x+2} = \lim_{x \to -2^+} (x+2)^2 = 0$$
$$\lim_{x \to -2^-} \frac{(x+2)(x+2)^2}{-(x+2)} = 0$$



11. a.

ΔT	T	V	ΔV
	−40	19.1482	
20			1.6426
	−20	20.7908	
20			1.6426
	0	22.4334	
20			1.6426
	20	24.0760	
20			1.6426
	40	25.7186	
20			1.6426
	60	27.3612	
20			1.6426
	80	29.0038	

ΔV is constant, therefore T and V form a linear relationship.

b. $V = \dfrac{\Delta V}{\Delta T} \cdot T + K$

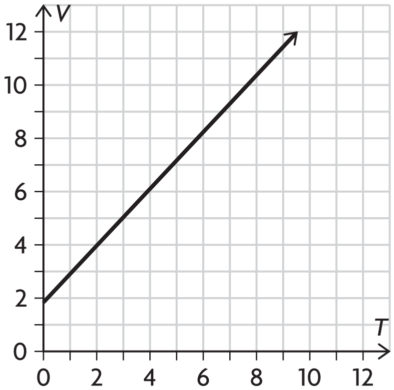
$$\frac{\Delta V}{\Delta T} = \frac{1.6426}{20} = 0.082\,13$$

$V = 0.082\ 13T + K$
$T = 0 \qquad V = 22.4334$
Therefore, $k = 22.4334$ and
$V = 0.082\ 13T + 22.4334$.

c. $T = \dfrac{V - 22.4334}{0.082\ 13}$

d. $\lim\limits_{v \to 0} T = -273.145$

e.



12. $\lim\limits_{x \to 5} \dfrac{x^2 - 4}{f(x)}$

$= \dfrac{\lim\limits_{x \to 5}(x^2 - 4)}{\lim\limits_{x \to 5} f(x)}$

$= \dfrac{21}{3}$

$= 7$

13. $\lim\limits_{x \to 4} f(x) = 3$

a. $\lim\limits_{x \to 4} [f(x)]^3 = 3^3 = 27$

b.

$\lim\limits_{x \to 4} \dfrac{[f(x)]^2 - x^2}{f(x) + x} = \lim\limits_{x \to 4} \dfrac{(f(x) - x)(f(x) + x)}{f(x) + x}$
$= \lim\limits_{x \to 4} (f(x) - x)$
$= 3 - 4$
$= -1$

c. $\lim\limits_{x \to 4} \sqrt{3f(x) - 2x} = \sqrt{3 \times 3 - 2 \times 4}$
$= 1$

14. $\lim\limits_{x \to 0} \dfrac{f(x)}{x} = 1$

a. $\lim\limits_{x \to 0} f(x) = \lim\limits_{x \to 0} \left[\dfrac{f(x)}{x} \times x\right] = 0$

b. $\lim\limits_{x \to 0} \dfrac{f(x)}{g(x)} = \lim\limits_{x \to 0} \left[\dfrac{x}{g(x)} \; \dfrac{f(x)}{x}\right] = 0$

15. $\lim\limits_{x \to 0} \dfrac{f(x)}{x} = 1$ and $\lim\limits_{x \to 0} \dfrac{g(x)}{x} = 2$

a. $\lim\limits_{x \to 0} g(x) = \lim\limits_{x \to 0} x\left(\dfrac{g(x)}{x}\right) = 0 \times 2$
$= 0$

b. $\lim\limits_{x \to 0} \dfrac{f(x)}{g(x)} = \lim\limits_{x \to 0} \dfrac{\frac{f(x)}{x}}{\frac{g(x)}{x}} = \dfrac{1}{2}$

16. $\lim\limits_{x \to 0} \dfrac{\sqrt{x + 1} - \sqrt{2x + 1}}{\sqrt{3x + 4} - \sqrt{2x + 4}}$

$= \lim\limits_{x \to 0} \left[\dfrac{\sqrt{x + 1} - \sqrt{2x + 1}}{\sqrt{x + 1} + \sqrt{2x + 1}}\right.$
$\times \dfrac{\sqrt{x + 1} + \sqrt{2x + 1}}{\sqrt{3x + 4} - \sqrt{2x + 4}}$
$\left.\times \dfrac{\sqrt{3x + 4} + \sqrt{2x + 4}}{\sqrt{3x + 4} + \sqrt{2x + 4}}\right]$

$= \lim\limits_{x \to 0} \left[\dfrac{(x + 1 - 2x - 1)}{(3x + 4 - 2x - 4)} \times \dfrac{\sqrt{3x + 4} + \sqrt{2x + 4}}{\sqrt{x + 1} + \sqrt{2x + 1}}\right]$

$= -\dfrac{2 + 2}{1 + 1}$

$= -2$

17. $\lim\limits_{x \to 1} \dfrac{x^2 + |x - 1| - 1}{|x - 1|}$

$x \to 1^+ \quad |x - 1| = x - 1$

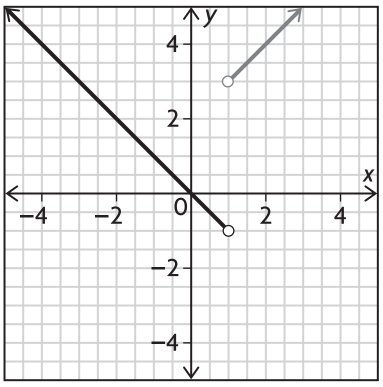
$\dfrac{x^2 + x - 2}{x - 1} = \dfrac{(x + 2)(x - 1)}{x - 1}$

$\lim\limits_{x \to 1^+} \dfrac{x^2 + |x - 1| - 1}{|x - 1|} = 3$

$x \to 1^- \, |x - 1| = -x + 1$

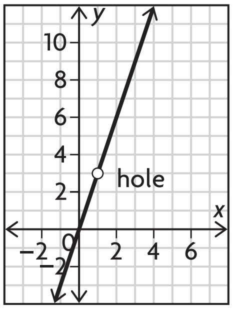
$\lim\limits_{x \to 1^-} \dfrac{x^2 - x}{-x + 1} = \lim\limits_{x \to 1^-} \dfrac{x(x - 1)}{-x + 1}$
$= -1$

Therefore, this limit does not exist.

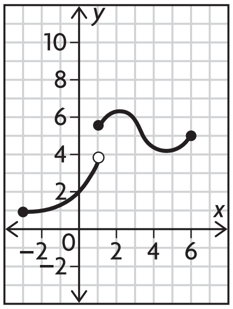


1.6 Continuity, pp. 51–53

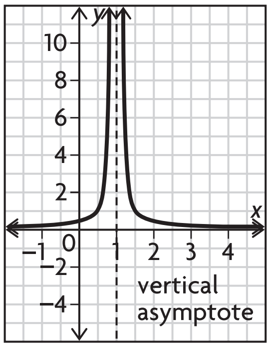
1. Anywhere that you can see breaks or jumps is a place where the function is not continuous.
2. It means that on that domain, you can trace the graph of the function without lifting your pencil.
3. point discontinuity



jump discontinuity

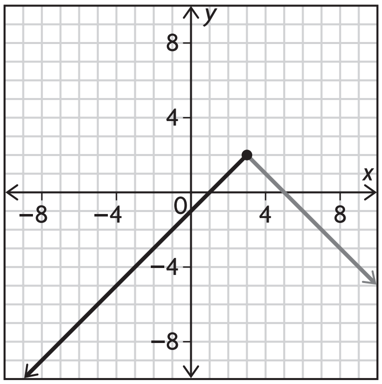


infinite discontinuity

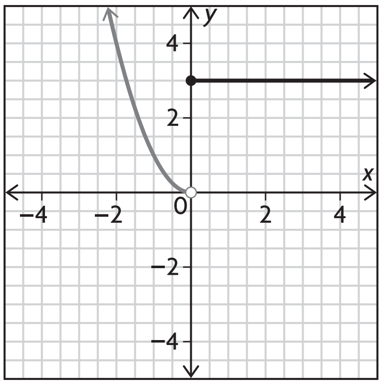


4. a. $x = 3$ makes the denominator 0.
b. $x = 0$ makes the denominator 0.
c. $x = 0$ makes the denominator 0.
d. $x = 3$ and $x = -3$ make the denominator 0.
e. $x^2 + x - 6 = (x + 3)(x - 2)$
$x = -3$ and $x = 2$ make the denominator 0.
f. The function has different one-sided limits at $x = 3$.

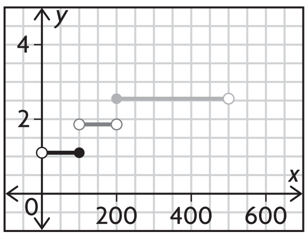
5. a. The function is a polynomial, so the function is continuous for all real numbers.
b. The function is a polynomial, so the function is continuous for all real numbers.
c. $x^2 - 5x = x(x - 5)$
The is continuous for all real numbers except 0 and 5.
d. The is continuous for all real numbers greater than or equal to -2.
e. The is continuous for all real numbers.
f. The is continuous for all real numbers.
6. $g(x)$ is a linear function (a polynomial), and so is continuous everywhere, including $x = 2$.
7.



The function is continuous everywhere.
8.



The function is discontinuous at $x = 0$.
9.



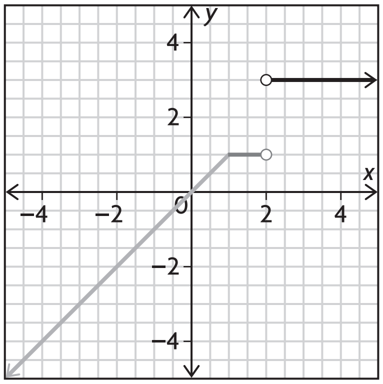
10. $\lim_{x\to 3} f(x) = \lim_{x\to 3}\frac{x^2 - x - 6}{x - 3}$

$= \lim_{x\to 3}\frac{(x-3)(x+2)}{x-3}$

$= 5$

Function is discontinuous at $x = 3$.

11. Discontinuous at $x = 2$



12. $g(x) = \begin{cases} x + 3, \text{ if } x \neq 3 \\ 2 + \sqrt{k}, \text{ if } x = 3 \end{cases}$

$g(x)$ is continuous.

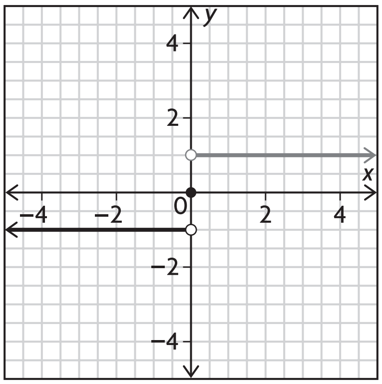
$2 + \sqrt{k} = 6$

$\sqrt{k} = 4,\ k = 16$

13.

$f(x) = \begin{cases} -1, \text{ if } x < 0 \\ 0, \text{ if } x = 0 \\ 1, \text{ if } x > 0 \end{cases}$

a.



b. i. From the graph, $\lim_{x\to 0^-} f(x) = -1$.

ii. From the graph, $\lim_{x\to 0^+} f(x) = 1$.

iii. Since the one-sided limits differ, $\lim_{x\to 0} f(x)$ does not exist.

c. f is not continuous since $\lim_{x\to 0} f(x)$ does not exist.

14. a. From the graph, $f(3) = 2$.

b. From the graph, $\lim_{x\to 3^-} f(x) = 4$.

c. $\lim_{x\to 3^-} f(x) = 4 = \lim_{x\to 3^-} f(x)$

Thus, $\lim_{x\to 3} f(x) = 4$. But, $f(3) = 2$. Hence f is not continuous at $x = 2$ (and also not continuous over $-3 < x < 8$).

15. The function is to be continuous at $x = 1$ and discontinuous at $x = 2$.

$f(x) = \begin{cases} \frac{Ax - B}{x - 2}, \text{ if } x \leq 1 \\ 3x, \text{ if } 1 < x < 2 \\ Bx^2 - A, \text{ if } x \geq 2 \end{cases}$

For $f(x)$ to be continuous at $x = 1$:

$\frac{A(1) - B}{1 - 2} = 3(1)$

$A(1) - B = -3$

$A = B - 3$

For $f(x)$ to be discontinuous at $x = 2$:

$B(2)^2 - A \neq 3(2)$

$4B - A \neq 6$

If $4B - A > 6$, then	if $4B - A < 6$, then
$4B - (B - 3) > 6$	$4B - B + 3 < 6$
$3B + 3 > 6$	$3B + 3 < 6$
$3B > 3$	$3B < 3$
$B > 1$ and	$B < 1$ and
$A > -2$	$A < -2$

This shows that A and B can be any set of real numbers such that

(1) $A = B - 3$

(2) $4B - A \neq 6$ (if $B > 1$, then $A > -2$ if $B < 1$, then $A < -2$)

$A = 1$ and $B = -2$ is not a solution because then the graph would be continuous at $x = 2$.

16. $f(x) = \begin{cases} -x, \text{ if } -3 \leq x \leq -2 \\ ax^2 + b, \text{ if } -2 < x < 0 \\ 6, \text{ if } x = 0 \end{cases}$

at $x = -2$, $4a + b = 2$

at $x = 0$, $b = 6$.

$a = -1$

$f(x) = \begin{cases} -x, \text{ if } -3 \leq x \leq -2 \\ -x^2 + b, \text{ if } -2 < x < 0 \\ 6, \text{ if } x = 0 \end{cases}$

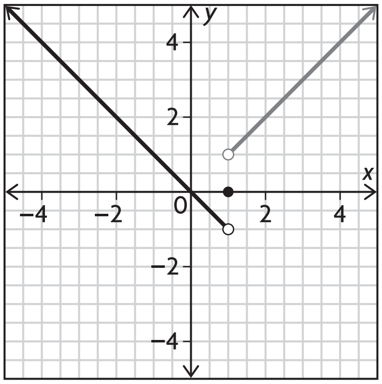
if $a = -1$, $b = 6$. $f(x)$ is continuous.

17. $g(x) = \begin{cases} \frac{x|x - 1|}{x - 1}, \text{ if } x \neq 1 \\ 0, \text{ if } x = 1 \end{cases}$

a. $\left.\begin{array}{l} \lim_{x\to 1^-} g(x) = -1 \\ \lim_{x\to 1^+} g(x) = 1 \end{array}\right\} \lim_{x\to 1} g(x)$

$\lim_{x\to 1} g(x)$ does not exist.

b.



$g(x)$ is discontinuous at $x = 1$.

Review Exercise, pp. 56–59

1. a. $f(-2) = 36, f(3) = 21$

$$m = \frac{21 - 36}{3 - (-2)} = -3$$

b. $f(-1) = 13, f(4) = 48$

$$m = \frac{48 - 13}{4 - (-1)} = 7$$

c. $f(1) = -3$

$$m = \lim_{h \to 0} \frac{5(1 + 2h + h^2) - (-3)}{h} = \lim_{h \to 0} \frac{2h + h^2}{h} = \lim_{h \to 0} 2 + h = 2$$

$$y - (-3) = 2(x - 1)$$
$$2x - y - 5 = 0$$

2. a. $f(x) = \dfrac{3}{x + 1}, P(2, 1)$

$$m = \frac{\frac{3}{3 + h} - 1}{h} = \lim_{h \to 0} -\frac{1}{3 + h} = -\frac{1}{3}$$

b. $g(x) = \sqrt{x + 2}, P(-1, 1)$

$$m = \lim_{h \to 0} \frac{\sqrt{-1 + h + 2} - 1}{h} = \lim_{h \to 0} \left[\frac{\sqrt{h + 1} - 1}{x} \times \frac{\sqrt{h + 1} + 1}{\sqrt{h + 1} + 1}\right] = \lim_{h \to 0} \frac{1}{\sqrt{h + 1} + 1} = \frac{1}{2}$$

c. $h(x) = \dfrac{2}{\sqrt{x + 5}}, P\left(4, \dfrac{2}{3}\right)$

$$m = \lim_{h \to 0} \frac{\frac{2}{\sqrt{4 + h + 5}} - \frac{2}{3}}{h} = 2 \lim_{h \to 0} \left[\frac{3 - \sqrt{9 + h}}{3h\sqrt{9 + h}} \times \frac{3 + \sqrt{9 + h}}{3 + \sqrt{9 + h}}\right] = 2 \lim_{h \to 0} \left[-\frac{1}{3\sqrt{9 + h}(3 + \sqrt{9 + h})}\right] = -\frac{2}{9(6)} = -\frac{1}{27}$$

d. $f(x) = \dfrac{5}{x - 2}, P\left(4, \dfrac{5}{2}\right)$

$$m = \lim_{h \to 0} \frac{\frac{5}{4 + h - 2} - \frac{5}{2}}{h} = \lim_{h \to 0} \frac{10 - 5(2 + h)}{h(2 + h)(2)} = \lim_{h \to 0} -\frac{-5h}{h(2 + h)(2)} = -\frac{5}{4}$$

3. $f(x) = \begin{cases} 4 - x^2, \text{ if } x \le 1 \\ 2x + 1, \text{ if } x > 1 \end{cases}$

a. Slope at $P(-1, 3)$ $f(x) = 4 - x^2$

$$m = \lim_{h \to 0} \frac{4 - (-1 + h)^2 - 3}{h} = \lim_{h \to 0} \frac{4 - 1 + 2h - h^2 - 3}{h} = \lim_{h \to 0} (2 - h) = 2$$

Slope of the graph at $P(-1, 3)$ is 2.

b. Slope at $P(2, 0.5)$

$$f(x) = 2x + 1$$
$$f(2 + h) - f(2) = 2(2 + h) + 1 - 5 = 2h$$

$$m = \lim_{h \to 0} \frac{2h}{h} = 2$$

Slope of the graph at $P(2, 0.5)$ is 2.

4. $s(t) = -5t^2 + 180$

a. $s(0) = 180, s(1) = 175, s(2) = 160$

Average velocity during the first second is

$\dfrac{s(1) - s(0)}{1} = -5$ m/s.

Average velocity during the second second is

$\frac{s(2) - s(1)}{1} = -15$ m/s.

b. At $t = 4$:

$s(4 + h) - s(4)$

$= -5(4 + h)^2 + 180 - (-5(16) + 180)$

$= -80 - 40h - 5h^2 + 180 + 80 - 180$

$\frac{s(4 + h) - s(4)}{h} = \frac{-40h - 5h^2}{h}$

$v(4) = \lim_{h\to 0}(-40 - 5h) = -40$

Velocity is -40 m/s.

c. Time to reach ground is when $s(t) = 0$.

Therefore, $-5t^2 + 180 = 0$

$t^2 = 36$

$t = 6, t > 0.$

Velocity at $t = 6$:

$s(6 + h) = -5(36 + 12h + h^2) + 180$

$= -60h - 5h^2$

$s(6) = 0$

Therefore, $v(6) = \lim_{h\to 0}(-60 - 5h) = -60.$

5. $M(t) = t^2$ mass in grams

a. Growth during $3 \le t \le 3.01$

$M(3.01) = (3.01)^2 = 9.0601$

$M(3) = 3^2$

$= 9$

Grew 0.0601 g during this time interval.

b. Average rate of growth is

$\frac{0.0601}{0.01} = 6.01$ g/min.

c. $s(3 + h) = 9 + 6h + h^2$

$s(3) = 9$

$\frac{s(3 + h) - s(3)}{h} = \frac{6h + h^2}{h}$

Rate of growth is $\lim_{h\to 0}(6 + h) = 6$ g/min.

6. $Q(t) = 10^4(t^2 + 15t + 70)$ tonnes of waste, $0 \le t \le 10$

a. At $t = 0$,

$Q(t) = 70 \times 10^4$

$= 700\,000.$

700 000 t have accumulated up to now.

b. Over the next three years, the average rate of change:

$Q(3) = 10^4(9 + 45 + 70)$

$= 124 \times 10^4$

$Q(0) = 70 \times 10^4$

$\frac{Q(3) - Q(0)}{3} = \frac{54 \times 10^4}{3}$

$= 18 \times 10^4$ t per year.

c. Present rate of change:

$Q(h) = 10^4(h^2 + 15h + 70)$

$Q(0) = 10^4 + 70$

$\lim_{h\to 0}\frac{Q(h) - Q(0)}{h} = \lim_{h\to 0}10^4(h + 15)$

$= 15 \times 10^4$ t per year.

d. $Q(a + h)$

$= 10^4[a^2 + 2ah + h^2 + 15a + 15h + 70]$

$Q(a) = 10^4[a^2 + 15a + 70]$

$\frac{Q(a + h) - Q(a)}{h} = \frac{10^4[2ah + h^2 + 15h]}{h}$

$\lim_{h\to 0}\frac{Q(a + h) - Q(a)}{h} = \lim_{h\to 0}10^4(2a + h + 15)$

$= (2a + 15)10^4$

Now,

$(2a + 15)10^4 = 3 \times 10^5$

$2a + 15 = 30$

$a = 7.5$

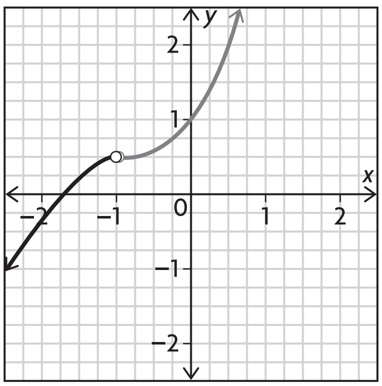
It will take 7.5 years to reach a rate of 3.0×10^5 t per year.

7. a. From the graph, the limit is 10.

b. 7; 0

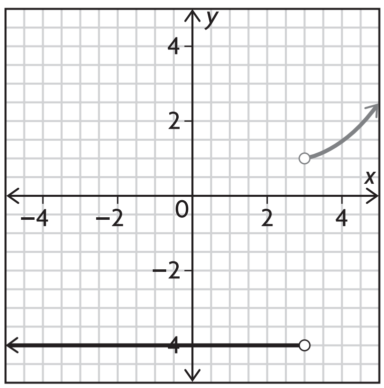
c. $p(t)$ is discontinuous for $t = 3$ and $t = 4$.

8. a. Answers will vary. $\lim_{x\to -1} f(x) = 0.5$, f is discontinuous at $x = -1$

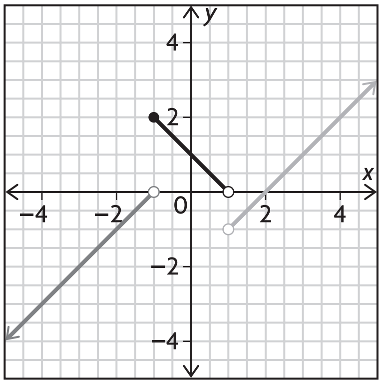


b. $f(x) = -4$ if $x < 3$; f is increasing for $x > 3$

$\lim_{x\to 3^+} f(x) = 1$



9. a.



b. $f(x) = \begin{cases} x + 1, \text{if } x < -1 \\ -x + 1, \text{if } -1 \le x < 1 \\ x - 2, \text{if } x > 1 \end{cases}$

Discontinuous at $x = -1$ and $x = 1$.

c. They do not exist.

10. The function is not continuous at $x = -4$ because the function is not defined at $x = -4$. ($x = -4$ makes the denominator 0.)

11. $f(x) = \dfrac{2x - 2}{x^2 + x - 2}$

$= \dfrac{2(x - 1)}{(x - 1)(x + 2)}$

a. f is discontinuous at $x = 1$ and $x = -2$.

b. $\lim\limits_{x \to 1} f(x) = \lim\limits_{x \to 1} \dfrac{2}{x + 2}$

$= \dfrac{2}{3}$

$\lim\limits_{x \to -2} f(x)$: $= \lim\limits_{x \to -2^+} \dfrac{2}{x + 2} = +\infty$

$\lim\limits_{x \to -2^-} \dfrac{2}{x + 2} = -\infty$

$\lim\limits_{x \to -2} f(x)$ does not exist.

12. a. $f(x) = \dfrac{1}{x^2}$, $\lim\limits_{x \to 0} f(x)$ does not exist.

b. $g(x) = x(x - 5)$, $\lim\limits_{x \to 0} g(x) = 0$

c. $h(x) = \dfrac{x^3 - 27}{x^2 - 9}$,

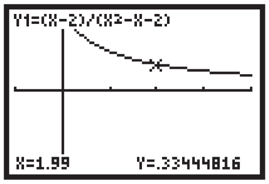
$\lim\limits_{x \to 4} h(x) = \dfrac{37}{7} = 5.2857$

$\lim\limits_{x \to -3} h(x)$ does not exist.

13. a.

x	1.9	1.99	1.999	2.001	2.01	2.1
$\dfrac{x-2}{x^2-x-2}$	0.344 83	0.334 45	0.333 44	0.333 22	0.332 23	0.322 58

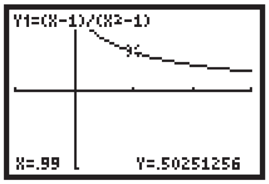
$\dfrac{1}{3}$



b.

x	0.9	0.99	0.999	1.001	1.01	1.1
$\dfrac{x-1}{x^2-1}$	0.526 32	0.502 51	0.500 25	0.499 75	0.497 51	0.476 19

$\dfrac{1}{2}$



14.

x	−0.1	−0.01	−0.001	0.001	0.01	0.1
$\dfrac{\sqrt{x+3}-\sqrt{3}}{x}$	0.291 12	0.288 92	0.2887	0.288 65	0.288 43	0.286 31

$$\lim_{x \to 0} \left[\frac{\sqrt{x+3} - \sqrt{3}}{x} \cdot \frac{\sqrt{x+3} + \sqrt{3}}{\sqrt{x+3} + \sqrt{3}} \right]$$

$$= \lim_{x \to 0} \frac{x + 3 - 3}{x\left(\sqrt{x+3} + \sqrt{3}\right)}$$

$$= \lim_{x \to 0} \frac{x}{x\left(\sqrt{x+3} + \sqrt{3}\right)}$$

$$= \lim_{x \to 0} \frac{1}{\sqrt{x+3} + \sqrt{3}}$$

$$= \frac{1}{2\sqrt{3}}$$

This agrees well with the values in the table.

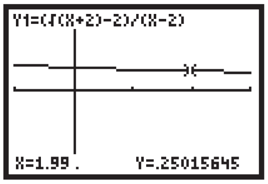
15. a. $f(x) = \dfrac{\sqrt{x+2} - 2}{x - 2}$

x	2.1	2.01	2.001	2.0001
$f(x)$	0.248 46	0.249 84	0.249 98	0.25

$x = 2.0001$

$f(x) \doteq 0.25$

b.



$\lim_{x\to 2} f(x) = 0.25$

c. $\lim_{x\to 2}\left[\frac{\sqrt{x+2}-2}{x-2}\times\frac{\sqrt{x+2}+2}{\sqrt{x+2}+2}\right]$

$= \lim_{x\to 2}\frac{1}{\sqrt{x+2}+2}$

$= \frac{1}{4} = 0.25$

16. a. $\lim_{h\to 0}\frac{(5+h)^2-25}{h}$

$= \lim_{h\to 0}(10+h)$

$= 10$

Slope of the tangent to $y = x^2$ at $x = 5$ is 10.

b. $\lim_{h\to 0}\frac{\sqrt{4+h}-2}{h} = \lim_{h\to 0}\frac{\sqrt{4+h}-2}{4+h-4}$

$= \lim_{h\to 0}\frac{1}{\sqrt{4+h}+2}$

$= \frac{1}{4}$

Slope of the tangent to $y = \sqrt{x}$ at $x = 4$ is $\frac{1}{4}$.

c. $\lim_{h\to 0}\frac{\frac{1}{4+h}-\frac{1}{4}}{h} = \lim_{h\to 0}\frac{4-4-h}{4(4+h)(h)}$

$= \lim_{h\to 0} -\frac{1}{4(4+h)}$

$= -\frac{1}{16}$

Slope of the tangent to $y = \frac{1}{x}$ at $(x = 4)$ is $-\frac{1}{16}$.

17. a. $\lim_{x\to -4}\frac{(x+4)(x+8)}{x+4} = \lim_{x\to -4}(x+8)$

$= (-4) + 8$

$= 4$

b. $\lim_{x\to a}\frac{(x+4a)^2-25a^2}{x-a} = \lim_{x\to a}\frac{(x-a)(x+9a)}{x-a}$

$= 10a$

c. $\lim_{x\to 0}\left[\frac{\sqrt{x+5}-\sqrt{5-x}}{x}\times\frac{\sqrt{x+5}+\sqrt{5-x}}{\sqrt{x+5}+\sqrt{5-x}}\right]$

$= \lim_{x\to 0}\frac{x+5-5+x}{x\left(\sqrt{x+5}+\sqrt{5-x}\right)}$

$= \lim_{x\to 0}\frac{1}{\left(\sqrt{x+5}+\sqrt{5-x}\right)}$

$= \frac{1}{\sqrt{5}}$

d. $\lim_{x\to 2}\frac{(x-2)(x+2)}{(x-2)(x^2+2x+4)}$

$= \lim_{x\to 2}\frac{x+2}{x^2+2x+4}$

$= \frac{(2)+2}{(2)^2+2(2)+4}$

$= \frac{4}{12}$

$= \frac{1}{3}$

e. $\lim_{x\to 4}\left[\frac{4-\sqrt{12+x}}{x-4}\cdot\frac{4+\sqrt{12+x}}{4+\sqrt{12+x}}\right]$

$= \lim_{x\to 4}\frac{16-(12+x)}{(x-4)(4+\sqrt{12+x})}$

$= \lim_{x\to 4}\frac{4-x}{(x-4)(4+\sqrt{12+x})}$

$= \lim_{x\to 4}\frac{-(x-4)}{(x-4)(4+\sqrt{12+x})}$

$= \lim_{x\to 4}\frac{-1}{4+\sqrt{12+x}}$

$= \frac{-1}{4+\sqrt{12+(4)}}$

$= \frac{-1}{4+4}$

$= -\frac{1}{8}$

f. $\lim_{x\to 0}\frac{1}{x}\left(\frac{1}{2+x}-\frac{1}{2}\right)$

$= \lim_{x\to 0}\left[\frac{1}{x}\times -\frac{x}{2(2+x)}\right]$

$= \lim_{x\to 0}\left[-\frac{1}{2(2+x)}\right]$

$= -\frac{1}{4}$

18. a. The function is not defined for $x < 3$, so there is no left-side limit.

b. Even after dividing out common factors from numerator and denominator, there is a factor of $x - 2$ in the denominator; the graph has a vertical asymptote at $x = 2$.

c. $f(x) = \begin{cases} -5, & \text{if } x < 1 \\ 2, & \text{if } x \geq 1 \end{cases}$

$\lim_{x\to 1^-} f(x) = -5 \neq \lim_{x\to 1^+} f(x) = 2$

d. The function has a vertical asymptote at $x = 2$.

e. $\lim_{x\to 0} \frac{|x|}{x}$

$x \to 0^- \; |x| = -x$

$$\lim_{x\to 0^-} \frac{|x|}{x} = -1$$

$$\lim_{x\to 0^+} \frac{|x|}{x} = 1$$

$$\lim_{x\to 0^+} \frac{|x|}{x} \neq \lim_{x\to 0^-} \frac{|x|}{x}$$

f. $f(x) = \begin{cases} 5x^2, \text{ if } x < -1 \\ 2x + 1, \text{ if } x \geq -1 \end{cases}$

$\lim_{x\to -1^+} f(x) = -1$

$\lim_{x\to -1^-} f(x) = 5$

$\lim_{x\to -1^+} f(x) \neq \lim_{x\to -1^-} f(x)$

Therefore, $\lim_{x\to -1} f(x)$ does not exist.

19. a.

$$\begin{aligned} m &= \lim_{h\to 0} \frac{-3(1+h)^2 + 6(1+h) + 4 - (-3+6+4)}{h} \\ &= \lim_{h\to 0} \frac{-3 - 6h - h^2 + 6 + 6h + 4 - 7}{h} \\ &= \lim_{h\to 0} \frac{-h^2}{h} \\ &= \lim_{h\to 0} -h \\ &= 0 \end{aligned}$$

When $x = 1, y = 7$.

The equation of the tangent is $y - 7 = 0(x - 1)$

$y = 7$

b.

$$\begin{aligned} m &= \lim_{h\to 0} \frac{(-2+h)^2 - (-2+h) - 1 - (4+2-1)}{h} \\ &= \lim_{h\to 0} \frac{4 - 4h + h^2 + 2 - h - 1 - 5}{h} \\ &= \lim_{h\to 0} \frac{-5h + h^2}{h} \\ &= \lim_{h\to 0} (-5 + h) \\ &= -5 \end{aligned}$$

When $x = -2, y = 5$.

The equation of the tangent is $y - 5 = -5(x + 2)$

$y = -5x - 5$

c.

$$\begin{aligned} m &= \lim_{h\to 0} \frac{6(-1 + h)^3 - 3 - (-6 - 3)}{h} \\ &= \lim_{h\to 0} \frac{6(-1 + 3h - 3h^2 + h^3) - 3 + 9}{h} \\ &= \lim_{h\to 0} \frac{18h - 18h^2 + 6h^3}{h} \\ &= \lim_{h\to 0} (18 - 18h + 6h^2) \\ &= 18 \end{aligned}$$

When $x = -1, y = -9$.

The equation of the tangent is

$y - (-9) = 18(x - (-1))$

$y = 18x + 9$

d.

$$\begin{aligned} m &= \lim_{h\to 0} \frac{-2(3 + h)^4 - (-162)}{h} \\ &= \lim_{h\to 0} \frac{-2(81 + 108h + 54h^2 + 12h^3 + h^4) + 162}{h} \\ &= \lim_{h\to 0} \frac{-216h - 108h^2 - 24h^3 - 2h^4}{h} \\ &= \lim_{h\to 0} (-216 - 108h - 24h^2 - 2h^3) \\ &= -216 \end{aligned}$$

When $x = 3, y = -162$.

The equation of the tangent is

$y - (-162) = -216(x - 3)$

$y = -216x + 486$

20. $P(t) = 20 + 61t + 3t^2$

a. $P(8) = 20 + 61(8) + 3(8)^2$

$= 700\,000$

b.

$$\begin{aligned} &\lim_{h\to 0} \frac{20 + 61(8 + h) + 3(8 + h)^2 - (20 + 488 + 192)}{h} \\ &= \lim_{h\to 0} \frac{20 + 488 + 61h + 3(64 + 16h + h^2) - 700}{h} \\ &= \lim_{h\to 0} \frac{20 + 488 + 61h + 192 + 48h + 3h^2 - 700}{h} \\ &= \lim_{h\to 0} \frac{109h + 3h^2}{h} \\ &= \lim_{h\to 0} (109 + 3h) \\ &= 109 \end{aligned}$$

The population is changing at the rate of $109\,000$/h.

Chapter 1 Test, p. 60

1. $\lim_{x\to 1} \frac{1}{x - 1}$ does not exist since

$$\lim_{x\to 1^+} \frac{1}{x - 1} = +\infty \neq \lim_{x\to 1^-} \frac{1}{x - 1} = -\infty.$$

2. $f(x) = 5x^2 - 8x$

$f(-2) = 5(4) - 8(-2) = 20 + 16 = 36$

$f(1) = 5 - 8 = -3$

Slope of secant is $\frac{36 + 3}{-2 - 1} = -\frac{39}{3}$

$= -13$

3. a. $\lim_{x\to1} f(x)$ does not exist.

b. $\lim_{x\to2} f(x) = 1$

c. $\lim_{x\to4^-} f(x) = 1$

d. f is discontinuous at $x = 1$ and $x = 2$.

4. a. Average velocity from $t = 2$ to $t = 5$:

$$\frac{s(5) - s(2)}{3} = \frac{(40 - 25) - (16 - 4)}{3} = \frac{15 - 12}{3} = 1$$

Average velocity from $t = 2$ to $t = 5$ is 1 km/h.

b. $s(3 + h) - s(3)$

$= 8(3 + h) - (3 + h)^2 - (24 - 9)$

$= 24 + 8h - 9 - 6h - h^2 - 15$

$= 2h - h^2$

$$v(3) = \lim_{h\to0}\frac{2h - h^2}{h} = 2$$

Velocity at $t = 3$ is 2 km/h.

5. $f(x) = \sqrt{x + 11}$

Average rate of change from $x = 5$ to $x = 5 + h$:

$$\frac{f(5 + h) - f(5)}{h} = \frac{\sqrt{16 + h} - \sqrt{16}}{h}$$

6. $f(x) = \dfrac{x}{x^2 - 15}$

Slope of the tangent at $x = 4$:

$$f(4 + h) = \frac{4 + h}{(4 + h)^2 - 15} = \frac{4 + h}{1 + 8h + h^2}$$

$$f(4) = \frac{4}{1}$$

$$f(4 + h) - f(4) = \frac{4 + h}{1 + 8h + h^2} - 4 = \frac{4 + h - 4 - 32h - 4h^2}{1 + 2h + h^2} = -\frac{31h - 4h^2}{(1 + 2h + h^2)}$$

$$\lim_{h\to0}\frac{f(4 + h) - f(4)}{h} = \lim_{h\to0}\frac{(-31 - 4h)}{1 + 2h + h^2} = -31$$

Slope of the tangent at $x = 4$ is -31.

7. a. $$\lim_{x\to3}\frac{4x^2 - 36}{2x - 6} = \lim_{x\to3}\frac{2(x - 3)(x + 3)}{(x - 3)} = 12$$

b. $$\lim_{x\to2}\frac{2x^2 - x - 6}{3x^2 - 7x + 2} = \lim_{x\to2}\frac{(2x + 3)(x - 2)}{(x - 2)(3x - 1)} = \frac{7}{5}$$

c. $$\lim_{x\to5}\frac{x - 5}{\sqrt{x - 1} - 2} = \lim_{x\to5}\frac{(x - 1) - 4}{\sqrt{x - 1} - 2}$$

$$= \lim_{x\to5}\frac{\left(\sqrt{x - 1} - 2\right)\left(\sqrt{x - 1} + 2\right)}{\sqrt{x - 1} - 2}$$

$= 4$

d. $$\lim_{x\to-1}\frac{x^3 + 1}{x^4 - 1} = \lim_{x\to-1}\frac{(x + 1)(x^2 - x + 1)}{(x - 1)(x + 1)(x^2 + 1)} = \frac{3}{-2(2)} = -\frac{3}{4}$$

e. $$\lim_{x\to3}\left(\frac{1}{x - 3} - \frac{6}{x^2 - 9}\right) = \lim_{x\to3}\frac{(x + 3) - 6}{(x - 3)(x + 3)} = \lim_{x\to3}\frac{1}{x + 3} = \frac{1}{6}$$

f. $$\lim_{x\to0}\frac{(x + 8)^{\frac{1}{3}} - 2}{x} = \lim_{x\to0}\frac{(x + 8)^{\frac{1}{3}} - 2}{(x + 8) - 8}$$

$$= \lim_{x\to0}\frac{(x + 8)^{\frac{1}{3}} - 2}{((x + 8)^{\frac{1}{3}} - 2)((x + 8)^{\frac{2}{3}} + 2(x + 8)^{\frac{1}{3}} + 4)}$$

$$= \frac{1}{4 + 4 + 4} = \frac{1}{12}$$

8. $$f(x) = \begin{cases} ax + 3, \text{ if } x > 5 \\ 8, \text{ if } x = 5 \\ x^2 + bx + a, \text{ if } x < 5 \end{cases}$$

$f(x)$ is continuous.

Therefore, $5a + 3 = 8$ $\qquad a = 1$

$25 + 5b + a = 8$ $\qquad 5b = -18$

$b = -\dfrac{18}{5}$