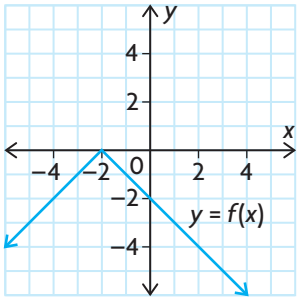
Exercise 2.1

PART A

1. State the domain on which f is differentiable.

a.



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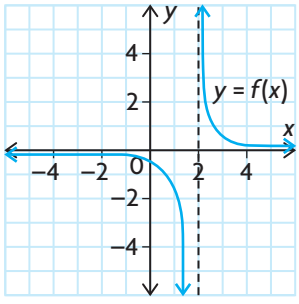

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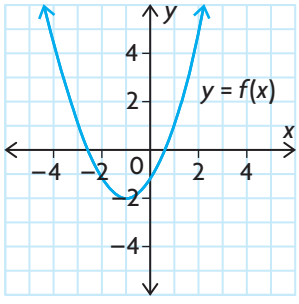

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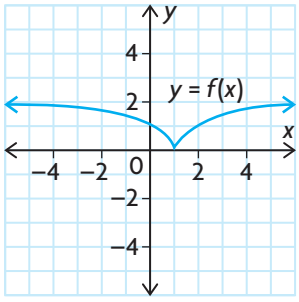

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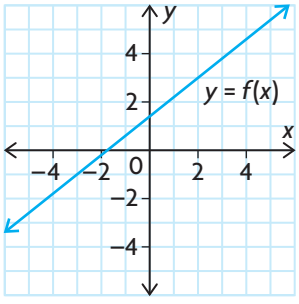

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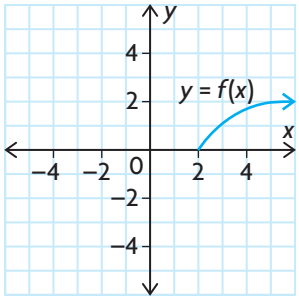

C 2. Explain what the derivative of a function represents.

3. Illustrate two situations in which a function does not have a derivative at $x = 1$.

4. For each function, find $f(a + h)$ and $f(a + h) - f(a)$.

a. $f(x) = 5x - 2$

b. $f(x) = x^2 + 3x - 1$

c. $f(x) = x^3 - 4x + 1$

d. $f(x) = x^2 + x - 6$

e. $f(x) = -7x + 4$

f. $f(x) = 4 - 2x - x^2$

PART B

K

5. For each function, find the value of the derivative $f'(a)$ for the given value of a.

   a. $f(x) = x^2, a = 1$

   b. $f(x) = x^2 + 3x + 1, a = 3$

   c. $f(x) = \sqrt{x + 1}, a = 0$

   d. $f(x) = \frac{5}{x}, a = -1$

6. Use the definition of the derivative to find $f'(x)$ for each function.

   a. $f(x) = -5x - 8$

   b. $f(x) = 2x^2 + 4x$

   c. $f(x) = 6x^3 - 7x$

   d. $f(x) = \sqrt{3x + 2}$

7. In each case, find the derivative $\frac{dy}{dx}$ from first principles.

   a. $y = 6 - 7x$

   b. $y = \frac{x + 1}{x - 1}$

   c. $y = 3x^2$

8. Determine the slope of the tangents to $y = 2x^2 - 4x$ when $x = 0$, $x = 1$, and $x = 2$. Sketch the graph, showing these tangents.

9. a. Sketch the graph of $f(x) = x^3$.

   b. Calculate the slopes of the tangents to $f(x) = x^3$ at points with x-coordinates -2, -1, 0, 1, 2.

   c. Sketch the graph of the derivative function $f'(x)$.

   d. Compare the graphs of $f(x)$ and $f'(x)$.

A

10. An object moves in a straight line with its position at time t seconds given by $s(t) = -t^2 + 8t$, where s is measured in metres. Find the velocity when $t = 0$, $t = 4$, and $t = 6$.

11. Determine an equation of the line that is tangent to the graph of $f(x) = \sqrt{x + 1}$ and parallel to $x - 6y + 4 = 0$.

12. For each function, use the definition of the derivative to determine $\frac{dy}{dx}$, where a, b, c, and m are constants.

    a. $y = c$

    b. $y = x$

    c. $y = mx + b$

    d. $y = ax^2 + bx + c$

13. Does the function $f(x) = x^3$ ever have a negative slope? If so, where? Give reasons for your answer.

14. A football is kicked up into the air. Its height, h, above the ground, in metres, at t seconds can be modelled by $h(t) = 18t - 4.9t^2$.

    a. Determine $h'(2)$.

    b. What does $h'(2)$ represent?

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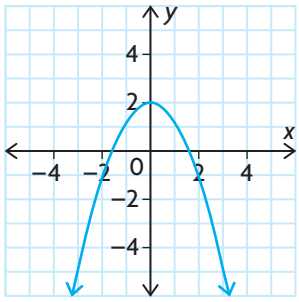
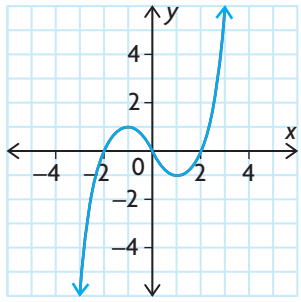
15. Match each function in graphs **a**, **b**, and **c** with its corresponding derivative, graphed in **d**, **e**, and **f**.

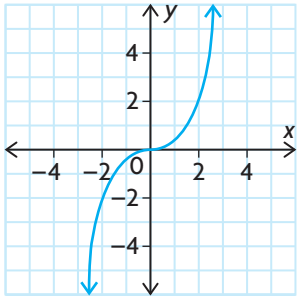
a.



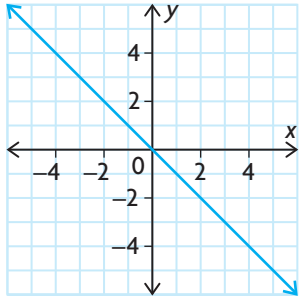
d.



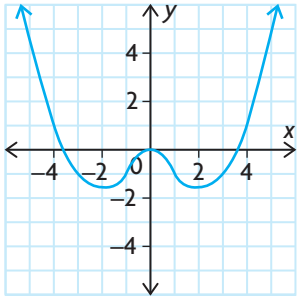
b.



e.



c.



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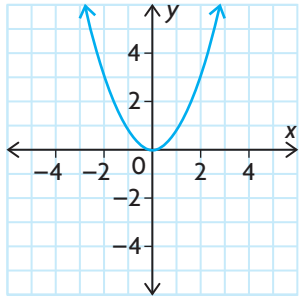

PART C

16. For the function $f(x) = x|x|$, show that $f'(0)$ exists. What is the value?

17. If $f(a) = 0$ and $f'(a) = 6$, find $\lim\limits_{h \to 0} \dfrac{f(a + h)}{2h}$.

18. Give an example of a function that is continuous on $-\infty < x < \infty$ but is not differentiable at $x = 3$.

19. At what point on the graph of $y = x^2 - 4x - 5$ is the tangent parallel to $2x - y = 1$?

20. Determine the equations of both lines that are tangent to the graph of $f(x) = x^2$ and pass through point $(1, -3)$.