Minia University - Faculty of Science
Department of Mathematics
Final 2nd Term Exam 2020/2021
Course: Probability and Statistics

Time: 2 hours
Date: June 2021
Code: MATH103

Answer the following questions

Question # 1:

Choose the correct answer with the explanation:

1. If $A \subset B$, Then P(A|B)=
   a) P(B|A) b) P(A) c) P(B) d) None of them
2. If the knowledge that an event A has occurred implies that a second event B cannot occur, the events A and B are said to be
   a) Independent b) Mutually exclusive c) $A \subset B$ d) $B \subset A$
3. If A and B be independent events in a sample space S, then which of the following statement(s) are true
   a) A and B^c are independent b) B and A^c are independent
   c) A^c and B^c are independent d) All of preceding
4. A code consists of a digit chosen from 0 to 6 followed by a letter of the alphabet. What is the probability the code is 6Z?
   a) $\frac{1}{260}$ b) $\frac{1}{156}$ c) $\frac{1}{182}$ d) None of them
5. For a continuous random variable X with a probability density function $f(x) = \begin{cases} k(x^3+1), & 0 \le x \le 1 \\ 0, & o.w. \end{cases}$, then the value of k is
   a) $\frac{1}{2}$ b) $\frac{5}{4}$ c) $\frac{1}{3}$ d) $\frac{4}{5}$
6. The value of k when the probability mass function is given by, $P(x) = \begin{cases} \frac{2x}{k}, & x = 1,2,3,4 \\ 0, & o.w \end{cases}$ is
   a) 10 b) 20 c) $\frac{1}{10}$ d) $\frac{1}{20}$
7. The cumulative distribution function for the random variable X for $f(x) = \begin{cases} \lambda e^{-\lambda x}, & 0 \le x < \infty \\ 0, & o.w. \end{cases}$ is given by
   a) $e^{-\lambda x}$ b) $e^{-\lambda x} - 1$ c) $1 - e^{-\lambda x}$ d) None of them
8. If $f(x) = \begin{cases} 2e^{-2x}, & 0 \le x < \infty \\ 0, & o.w. \end{cases}$, then the value of x such that $p(X > x) = 0.1$ is
   a) 1.15 b) 2.3 c) -2.3 d) None of them

9. If the variance of x $v(x) = \frac{1}{3}$, then $v\left(\frac{x}{3} - 5\right) =$

a) $\frac{1}{3}$ b) 3 c) 27 d) $\frac{1}{27}$

10. $E(a\,x + b)$=

a) $a\,E(x + b)$ b) $aE(x)$ c) $aE(x) + b$ d) None of them

Question # 2:

A company producing electric relays has three manufacturing plants producing 50%, 30%, and 20%, respectively, of its product. Suppose that the probabilities that a relay manufactured by these plants is defective are 0.02, 0.05, and 0.01, respectively.

a) If a relay is selected at random from the output of the company, what is the probability that it is defective?
b) If a relay selected at random is found to be defective, what is the probability that it was manufactured by plant 2?

Question # 3:

1. When Farid plays chess against his favorite computer program, he wins with probability 0.60.
   a) What is the probability that Farid wins 6 games, if he plays 10 games?
   b) What is the probability that Farid wins 2 games at most, if he plays 10 games?
2. Fares makes mistakes in class according to Poisson process with an average rate of 1.2 mistakes per class.
   a) What is the probability that Fares makes 2 mistakes during one class?
   b) What is the probability that Fares makes one mistake at least during one class?

Question # 4:
Consider the following information:

Volume of sales (units)	5	6	7	8	9	10
Total expenses ($)	74	77	82	86	92	95

a) Find the least-squares line for the data.
b) What will be the total expenses when the volume of sales is 7.5 units?

Best wishes and wish Good Luck. Dr. Hossam Radwan

Final / June 2021

Question 1

1) $\because A \subset B \quad \therefore A \cap B = A$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A)}{P(B)}$$

(a) None of them

2) (b) Mutually exclusive

$A \cap B = \phi \qquad P(A \cap B) = 0$

3) (d) All of Preceding

(1) $P(A \cap B^c) = P(A) - P(A \cap B)$

$\sim$ A and B are independent events

$\therefore P(A \cap B^c) = P(A) - P(A)\,P(B)$

$= P(A)[1 - P(B)]$

$= P(A)\,P(B^c)$

$\sim$ A and B^c are independent

(4) $A = \{0, 1, 2, 3, 4, 5, 6\}$

$B = \{A, B, C, \ldots, Z\}$

$P(A \cap B) = P(A)\ P(B)$

$= \frac{1}{7} \times \frac{1}{26} = \frac{1}{182}$

(C) $\frac{1}{182}$

5) $\int_{-\infty}^{\infty} f(x)\, dx = 1$

$\int_{0}^{1} K(x^3 + 1)\, dx = 1$

$\left[K\frac{x^4}{4} + Kx\right]_0^1 = 1$

$\frac{1}{4}K + K = 1$

$\frac{5}{4}K = 1$

$\therefore K = \frac{4}{5}$

(a) $\frac{4}{5}$

(2) $P(B \cap A^c) = P(B) - P(B \cap A)$

$\therefore A, B$ are independent events

$\therefore P(B \cap A^c) = P(B) - P(B)\ P(A)$

$= P(B)[1 - P(A)]$

$= P(B)\ P(A^c)$

$\therefore B$ and A^c are independent

(3) $P(A^c \cap B^c) = P(\overline{A \cup B})$

$= 1 - P(A \cup B)$

$= 1 - [P(A) + P(B) - P(A \cap B)]$

$= 1 - P(A) - P(B) + P(A \cap B)$

$\therefore A, B$ are independent events

$\therefore P(A^c \cap B^c) = 1 - P(A) - P(B) + P(A)\ P(B)$

$= 1 - P(A) - P(B)[1 - P(A)]$

$= [1 - P(A)][1 - P(B)]$

$= P(A^c)\ P(B^c)$

Hence A^c and B^c are independent

8) $F(X) = \int_{-\infty}^{X} f(t)\, dt$

$= \int_{0}^{X} 2e^{-2t}\, dt$

$= -e^{-2t} \Big|_{0}^{X}$

$= -e^{-2X} + 1$

$= 1 - e^{-2X}$

$P(X > x) = 1 - P(X \leq x)$

$= 1 - F(x)$

$= 1 - (1 - e^{-2X})$

$= 1 - 1 + e^{-2X}$

$= e^{-2X}$

$0.1 = e^{-2X}$ نأخذ Ln للطرفين

$-2X = \ln 0.1$

$X = 1.15$

(a) 1.15

6) $\sum_{i=0}^{\infty} P(X_i) = 1$

$\sum_{i=1}^{4} P(X) = 1$

$\frac{2}{K} + \frac{4}{K} + \frac{6}{K} + \frac{8}{K} = 1$

$\frac{20}{K} = 1$

$\therefore K = 20$

(b) 20

7) $F(X) = \int_{-\infty}^{X} f(t)\, dt$

$= \int_{0}^{X} \lambda e^{-\lambda t}\, dt$

$= -e^{-\lambda t} \Big|_{0}^{X}$

$= -e^{-\lambda X} + 1$

$= 1 - e^{-\lambda X}$

(c) $1 - e^{-\lambda X}$

Question 2

$A_1 \rightarrow 0.5 \quad 0.02$

$A_2 \rightarrow 0.3 \quad 0.05$

$A_3 \rightarrow 0.2 \quad 0.01$

a) Let B be the event that the relay is defective, and A_i be the event that the relay is manufactured by Plant i, $(i = 1, 2, 3)$

$$P(B) = \sum_{i=1}^{3} P(B|A_i) P(A_i)$$

$$P(B) = (0.02)(0.5) + (0.05)(0.3) + (0.01)(0.2) = 0.027$$

b) $$P(A_2|B) = \frac{P(A_2 \cap B)}{P(B)} = \frac{P(B|A_2) P(A_2)}{P(B)} = \frac{(0.05)(0.3)}{(0.027)}$$

$$= 0.556$$

9) $V(aX + b) = a^2 V(X)$

$$V\left(\frac{1}{3}X - 5\right) = \frac{1}{9} V(X)$$

$$= \frac{1}{9} \times \frac{1}{3} = \frac{1}{27}$$

(b) $\frac{1}{27}$

10) $$E(aX + b) = \sum (aX + b) P(X)$$

$$= \sum aX P(X) + \sum b P(X)$$

$$= a \sum X P(X) + b \sum P(X)$$

$$= a E(X) + b$$

(c) $a E(X) + b$

Question 4

X	y	xy	x^2
5	74	370	25
6	77	462	36
7	82	574	49
8	86	688	64
9	92	828	81
10	95	950	100
$\Sigma X = 45$	$\Sigma y = 506$	$\Sigma XY = 3872$	$\Sigma X^2 = 355$

$n = 6$

(a) $b = \dfrac{n\Sigma XY - \Sigma X \Sigma y}{n\Sigma x^2 - (\Sigma X)^2} = \dfrac{(6 \times 3872) - (45 \times 506)}{(6 \times 355) - (45)^2} = 4.4$

$\bar{y} = \dfrac{\Sigma y}{n} = \dfrac{506}{6}$ $\quad \bar{X} = \dfrac{\Sigma X}{n} = \dfrac{45}{6}$

$a = \bar{y} - b\bar{X}$

$a = \left(\dfrac{506}{6}\right) - \left(4.4 \times \dfrac{45}{6}\right) = \dfrac{154}{3}$

$\hat{y} = a + bX$

$\hat{y} = \dfrac{154}{3} + 4.4X$

Question 3

1. $n = 10$ $\quad p = 0.60$ $\quad q = 0.40$

a) $P(X=6) = {}^{n}C_{x}\, p^{x}\, q^{n-x}$

$= ({}^{10}C_{6})(0.60)^{6}(0.40)^{4} = 0.25$

b) $P(X \leq 2) = P(X=0) + P(X=1) + P(X=2) = 0.012$

$P(X=0) = ({}^{10}C_{0})(0.60)^{0}(0.40)^{10} =$

$P(X=1) = ({}^{10}C_{1})(0.60)^{1}(0.40)^{9} =$

$P(X=2) = ({}^{10}C_{2})(0.60)^{2}(0.40)^{8} =$

2. $\lambda = 1.2$

a) $P(X=2) = \dfrac{\lambda^{x} e^{-\lambda}}{x!}$

$= \dfrac{(1.2)^{2} e^{-1.2}}{2!} = \dfrac{18}{25 e^{1.2}} = 0.21$

b) $P(X \geq 1) = 1 - P(X < 1)$

$= 1 - P(X=0)$

$= 1 - \dfrac{(1.2)^{0} e^{-1.2}}{0!} = 1 - \dfrac{1}{e^{1.2}} = 0.69$

(b) $X = 7.5$

$$y = \frac{154}{3} + (4.4 \times 7.5) = 84.3$$

The total expenses = 84.3