Minia University
Faculty of Computers & Information
Final 1st Term Exam 2023/2024
Course: Probability and Statistics

Time: 2 hours
Date: 4 January 2024
Code: MA203
Class: Second year FCI

Answer the following questions

$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{1}{3}}{\frac{1}{3}}$

Question # 1: Choose the correct answer with the explanation:

1. If A and B are two events such that $P(B) = \frac{1}{3}$ and $P(A \cap B) = \frac{1}{3}$ what will be $P(A|B)$

   $= \frac{\frac{1}{3}}{\frac{1}{3}} = 1$

   a) 1/4   b) 1/2   c) 1/3   d) None of them

2. In meteorology, the intensity of UV-B radiation is classified as follows: weak, moderate, strong, very strong, extreme. What is the best measure for this data?

   a) Mean   b) Median   c) Mode   d) b and c

3. Which measure of central tendency would best depict the following data: 10, 200, 200, 300, 325, 350 and 400?

   a) Mean   b) Median   c) Mode   d) Range

4. A code consists of a digit chosen from 0 to 6 followed by a letter of the alphabet. What is the probability the code is 6Z?

   a) $\frac{1}{260}$ b) $\frac{1}{156}$ c) $\frac{2}{182}$ d) None of them

5. If $A \subset B$ then $p(A^c|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(B) + P(A^c)}{P(B)} - \frac{P(A \cap B)}{P(B)} = \frac{P(B), P(A^c)(\cap A)}{P(B)}$

   a) 1   b) $1 - p(A|B)$ c) $p(B|A)$ d) 0

6. The value of k when the probability mass function is given by,

   $P(x) = \begin{cases} \frac{2x}{k}, & x = 1,2,3,4 \\ 0, & o.w \end{cases}$ is $P(x) = \sum_{x=1}^{4} F(x) = 1$ $A \subset B$ $P(A \cap B) = A$

   a) 10   b) 20   c) $\frac{1}{10}$ d) $\frac{1}{20}$

7. If the average of a series of values is 10 and their variance is 9, then the coefficient of variation is: $\frac{\sqrt{(x)}}{}$

   a) 40%   b) 20%   c) 90%   d) 30%

8. If $\sum x_i = 20$, and $\bar{x} = 4$, then the rank of the Median is $P(B) \cdot P(A)$ $P(A)$ $P(A|B \cap B)$

   a) 4   b) 5   c) 3   d) 3,4

9. If the variance of x $v(x) = 3$, then $v(\frac{x}{3} - 5) =$ $P(A^c) = 1 - P(A)$

   a) $\frac{1}{3}$ b) 3   c) 27 $P(A) = P(A^c) + 1$ d) $\frac{1}{57}$

10. In Exton School, 60% of the boys play baseball and 24% of the boys play baseball and football. What percent of those that play baseball also play football?

   a) 24%   b) 60%   c) 80%   d) 40%

$A^c = 1 - P(A)$

$P(B) + 1 - P(A) \cdot P(x)$

Question # 2:

If the events $B_1, B_2, ..., B_k$ constitute a partition of the sample space S such that $P(B_i) \geq 0$ for $i = 1, 2, ..., k$, then for any event A of S, prove that:

$$P(A) = \sum_{i=1}^{k} P(B_i) \, P(A|B_i)$$

2. A company producing electric relays has three manufacturing plants producing 50%, 30%, and 20%, respectively, of its product. Suppose that the probabilities that a relay manufactured by these plants is defective are 0.02, 0.05, and 0.01, respectively.

   a) If a relay is selected at random from the output of the company, what is the probability that it is defective?

   b) If a relay selected at random is found to be defective, what is the probability that it was manufactured by plant 2?

Question # 3:

1. When Farid plays chess against his favorite computer program, he wins with probability 0.60.

   a) What is the probability that Farid wins 6 games, if he plays 10 games?

   b) What is the probability that Farid wins 2 games at most, if he plays 10 games?

2. Fares makes mistakes in class according to Poisson process with an average rate of 1.2 mistakes per class.

   a) What is the probability that Fares makes 2 mistakes during one class?

   b) What is the probability that Fares makes one mistake at least during one class?

Question # 4:

A company manufactures an electronic device to be used in a very wide temperature range. The company knows that increased temperature shortens the lifetime of the device, and a study is therefore performed in which the lifetime is determined as a function of temperature. The following data is found:

Temperature (Celsius) X	10	20	30	40	50	60	70	80	90
Lifetime in hours Y	420	365	285	220	176	117	69	34	5

   a) Find the best line equation (simple linear regression equation) so that Y may be predicted from X.

   b) Predict the lifetime of the device when the temperature is 100°.

   c) Find the coefficient of correlation r.

Best wishes and wish Good Luck. Dr. Hossam Mahmoud Radwan

1. IF A and B are two events such that $P(B) = \frac{1}{3}$ and $P(A \cap B) = \frac{1}{3}$ What will be $P(A/B)$.

(a) $\frac{1}{4}$ (b) $\frac{1}{2}$ (c) $\frac{1}{3}$ (d) ✓ None of them

$$P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{1/3}{1/3} = \frac{3}{3} = 1$$

② In meteorology, The intensity of UV-B radiation is classified as Follows : weak, moderate, strong, verystrong, extreme what's the best measure of this data.   mode and median

③ which measure of central tendency would best depict The following data : $log 200, 200, 300, 325, 350, 400$

~~mean~~ ~~for 200 + 200 + 300 + 325 + 350 + 400~~ ~~= 255~~ ✗
                        $\frac{}{7}$

~~Median~~   ~~= 4~~   ~~D   300~~ ✗

~~Mode = 200~~

        The best depict ⟹   mode ✻

④ A code consists of a digit chosen From 0 to 6 Followed by The letter of The alphabet what's The Probability The code is b7 ?

$$P(b) = \frac{1}{7} \quad , \quad P(7) = \frac{1}{26}$$

$$P(b7) = P(6) P(7) = \frac{1}{7} \times \frac{1}{26} = \frac{1}{182} \quad ✻ \quad ©$$

⑤ IF $A \subset B$ then $P(A^c/B) =$

$$P(A^c/B) = \frac{P(A^c \cap B)}{P(B)} = \frac{P(B) - P(A \cap B)}{P(B)} = 1 - \frac{P(A \cap B)}{P(B)}$$

$$= 1 - P(A/B) \quad \text{✳ (b)}$$

⑥ The value of k when the probability mass function

is given by $P(x) = \begin{cases} \frac{2x}{k} & x = 1, 2, 3, 4 \\ 0 & o.w \end{cases}$

$P(x=1) = \frac{2}{k}$

$P(x=2) = \frac{4}{k}$, $P(x=3) = \frac{6}{k}$, $P(x=4) = \frac{8}{k}$

∵ Probability mass function

∴ $\sum P(x) = 1$

$\frac{2}{k} + \frac{4}{k} + \frac{6}{k} + \frac{8}{k} = 1 \quad \rightarrow \quad \frac{20}{k} = 1 \quad \rightarrow k = 20$ ✳
ⓑ

⑦ IF the average of a series of values is 10

and their variance is 9 then the coefficient of

variation is : ~

$\bar{x} = 10$ & variance $= 9$

coefficient of variation $= \frac{s}{\bar{x}} \times 100 = \frac{\sqrt{variance}}{\bar{x}} \times 100$

$= \frac{\sqrt{9}}{10} \times 100$

$= 30\%$ ✳

⑧ IF $\sum x_i = 20$ and $\bar{x} = 4$ then the rank of the

median is : ~

$\bar{x} = \frac{\sum x_i}{n} \quad \Rightarrow n = \frac{\sum x_i}{\bar{x}} = \frac{20}{4} = 5$ فيكون

median $= \frac{n+1}{2} = \frac{5+1}{2} = \frac{6}{2} = 3$ ✳ ⓒ

⑨ If the variance of X V(x) = 3 Then

$$V\left(\frac{X}{3} - 5\right) =$$

$$V(ax \pm b) = a^2 V(x)$$

من خواص المتباين

$$V\left(\frac{X}{3} - 5\right) = \left(\frac{1}{3}\right)^2 V(x) \qquad = \frac{1}{9} \times 3 = \frac{1}{3} \ast \text{ⓐ}$$

⑩ In Exton school, 60% of the boys plays baseball and 24% of the boys plays football what percent of those play baseball also play football ?
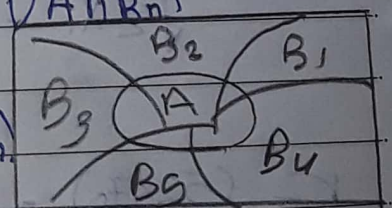
$$P(\text{Football} / \text{baseball}) = \frac{P(F \cap b)}{P(b)}$$

$$= \frac{24\%}{60\%} = 40\% \ast \text{ⓐ}$$

⑪ If the events $B_1, B_2, \dots, B_k$, $P(B_i) \neq 0$

$$P(A = \sum_{i=1}^{k} P(B_i) \, P(A/B_i)$$

$$A = (A \cap B_1) \cup (A \cap B_2) \cup (A \cap B_3) \cup \dots \cup (A \cap B_n)$$

$$P(A) = P(A \cap B_1) + P(A \cap B_2) + \dots + P(A \cap B_n)$$

∵ $P(A \cap B) = P(A/B) \cdot P(B)$

$$P(A) = P(A/B_1) \cdot P(B_1) + \dots + P(A/B_n) \cdot P(B_n)$$

∴ $$P(A) = \sum_{i=1}^{k} P(A/B_i) \cdot P(B_i)$$

② A Company Producing electric relays has Three manufacturing Plants Producing 50%, 30% and 20%, respectively, of its product. suppose That The Probabilities That a relay manufactured by these Plants is defective 0.02, 0.05 and 0.01, respectively

ⓐ If a relay is selected at random from The out Put of The Company, what is the Probability That it's defective?

ⓑ If a relay selected at random is found to be defective, what is the Probability that it was manufactured by Plant 2?

$P(B_1) = 50\%$ $P(A/B_1) = 0.02$
$P(B_2) = 30\%$ $P(A/B_2) = 0.05$
$P(B_3) = 20\%$ $P(A/B_3) = 0.01$

$P(A) = 50\% (0.02) + 30\% (0.05) + 20\% (0.01)$
$= (0.5)(0.02) + (0.3)(0.05) + (0.2)(0.01)$
$= 0.027$

$P(B_2/A) = \dfrac{P(B_2 \cap A)}{P(A)}$

$= \dfrac{P(A/B_2) \cdot P(B_2)}{P(A)}$

$= \dfrac{(0.05)(0.3)}{0.027}$

$= \dfrac{5}{9} = 0.555$

التاريخ   الموضوع

when Farid plays chess against his
favorite computer program, he wins with
probability 0.60

@ What's the probability that Farid wins
6 games, if he plays 10 games?

ⓑ what's the probability that Farid wins
2 games at most, if he plays 10 games?

$P = 0.6$ $, n = 10$

$q = 1 - P = 1 - 0.6 = 0.4$

$P(X=2) = \dfrac{10?}{6?\,(10-6)} (0.6)^6 (0.4)^{10-6}$

$= 0.25$ ❋

$P(X \le 2) = \left[P(x=0) + P(x=1) + P(x=2) \right]$

$= 1.048 \times 10^{-4} + 1.57 \times 10^{-3} + 0.01$

$= 0.0122$ ❋

Farse makes mistakes in class according
to Poisson process with an average rate
of 1.2 mistakes per class.

ⓐ what's the probability that Farse makes
2 mistake during one class?

ⓑ what's the probability that Farse makes
one mistake at least during one class?

$\lambda = 1.2$

$P(x) = \dfrac{\lambda^x\, e^{-\lambda}}{x!}$ $\overset{x=2}{=} \dfrac{(1.2)^2 e^{-1.2}}{2?} = 0.216$ ❋

$P(X \ge 1) = 1 - P(X < 1) = 1 - P(X=0) = 1 - \dfrac{(1.2)^0 e^{-1.2}}{0?}$

$= 0.698$

__Qu__

X	y	x²	y²	xy
10	420	100	176400	4200
20	365	400	133225	7300
30	285	900	81225	8550
40	220	1600	48400	8800
50	176	2500	30976	8800
60	117	3600	13689	7020
70	69	4900	4761	4830
80	34	6400	1156	2720
90	5	8100	25	450
Σx	Σy	Σx^2	Σy^2	Σxy
=450	1691	28500	489857	52670

$$n = 9$$

$$\bar{x} = \frac{\Sigma x}{n} = \frac{450}{9} = 50 \quad \#$$

$$\bar{y} = \frac{\Sigma y}{n} = \frac{1691}{9} = 187.8 \quad \#$$

$$a = \frac{\Sigma x_i y_i - n\bar{x}\bar{y}}{\Sigma x_i^2 - n\bar{x}^2}$$

$$a = \frac{(52670) - (9)(187.8)(50)}{(450)^2 - (9)(50)^2}$$

$$\# \quad a = -3.92 \times 10^{-5} \quad \#$$

$$b = \bar{y} - a\bar{x}$$

$$b = (187.88) - (\overline{})(50) \quad (-3.92 \times 10^{-5})$$

$$b = 187.88 \; \cancel{}$$

$$y = ax + b$$

$$\boxed{y = (-3.92 \times 10^{-5})\, x + 187.88} \rightarrow \text{best line equation}$$
$$= -3.92 \times 10^{-5} X + 187.88$$

at $x = 100 \rightarrow y = ??$

$$y = (-3.92 \times 10^{-5})(100) + 187.88$$

$$y = \overline{} \quad \cancel{} \qquad \text{(b) إجابة}$$

$$y = 187.87 \; \cancel{} \quad \longrightarrow \text{ا}$$

$$r = \frac{\sum x_i y_i - n\bar{x}\bar{y}}{\sqrt{\sum x_i^2 - n\bar{x}} \; \sqrt{\sum y_i^2 - n\bar{y}}}$$

$$r = \frac{(52670) - (9)(50)(187.8)}{\sqrt{[78500]^2 - (9)(50)^2} \; \sqrt{(489857) - (9)(187.8)^2}}$$

$$r = \frac{-31840}{\sqrt{8000} \; \sqrt{172\,V\,37.44}} = -0.99$$