QMM – Instructor Shahla Asadi

Assignment 2 – Module 2

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Case 1: Back Savers Backpack Company

a. Clearly define the decision variables

Decision variables represent the unknown quantities that management needs to determine to make an optimal decision. In this case, management must decide how many units of each backpack model to produce per week.
Let:
x = number of **Collegiate backpacks** to produce per week
y = number of **Mini backpacks** to produce per week

b. What is the objective function?

The primary goal, as stated by management, is to maximize total weekly profit. The profit contribution from each product is linear and known.

- Each Collegiate contributes $32 of profit.
- Each Mini contributes $24 of profit.

Therefore, the objective function is:
Maximize $Z = 32x + 24y$

c. What are the constraints?
Constraints are the limitations or restrictions that prevent the company from producing unlimited backpacks. They must be based on the available resources (nylon fabric, labor) and market realities (sales forecasts).

1. **Nylon Fabric Constraint:** The total nylon used by both products cannot exceed the weekly shipment of 5,000 square feet.
    - Each Collegiate use 3 sq ft.
    - Each Mini use 2 sq ft.
    - Constraint: $3x + 2y \leq 5000$

2. **Market Demand Constraint for Collegiate and mini models:** Sales forecasts indicate an upper limit on how many Collegiate and Mini models can be sold.
    - Constraint: $x \leq 1000$ (Collegiate Demand)
    - Constraint: $y \leq 1200$ (Mini Demand)

3. **Labor Constraint:** The total labor minutes used in production cannot exceed the total labor minutes available per week.
    - Total Labor Available: 35 laborers × 40 hours (per week) × 60 minutes (per hour) = 84,000 minutes.
    - Each Collegiate requires 45 minutes of labor.
    - Each Mini requires 40 minutes of labor.
    - Constraint: $45x + 40y \leq 84{,}000$

4. **Non-negativity Constraints:** Since production quantities cannot be negative, it is not applicable to produce a negative quantity of backpacks.
    - Constraints: $x \geq 0$ and $y \geq 0$

d. Write down the full mathematical formulation for this LP problem.

This formulation captures the complete linear programming problem that Back Savers needs to solve to determine its optimal weekly production mix.

Maximize: $Z = 32x + 24y$

Subject to:

- $3x + 2y \leq 5000$ (Nylon fabric available)
- $x \leq 1000$ (Collegiate demand limit)
- $y \leq 1200$ (Mini demand limit)
- $45x + 40y \leq 84{,}000$ (Labor constraint)
- $x \geq 0, y \geq 0$ (non-negativity)

Case 2: Weigelt Corporation

Table 1. Product Data

Size	Profit per Unit	Storage Space per Unit (sq ft)	Sales Forecast (units/day)
Large	$420	20	900
Medium	$360	15	1200
Small	$300	12	750

Table 2. Plant Data

Plant	Excess Production Capacity (units/day)	Storage Space Available (sq ft/day)
1	750	13000
2	900	12000
3	450	5000

a. Define the decision variables.

Table 3: Decision Variables

Plant	Large	Medium	Small
Plant 1	x_1	x_4	x_7
Plant 2	x_2	x_5	x_8
Plant 3	x_3	x_6	x_9

- x_1, x_2, x_3 = number of large units produced per day at Plant 1, Plant 2, Plant 3, respectively.
- x_4, x_5, x_6 = number of medium units produced per day at Plant 1, Plant 2, Plant 3, respectively.
- x_7, x_8, x_9 = number of small units produced per day at Plant 1, Plant 2, Plant 3, respectively.

b. Formulate a linear programming model for this problem.

This model includes a standard objective function and resource constraints, but also a unique "same percentage of capacity" constraint to avoid layoffs.

1. Objective Function:

The goal is to maximize total profit. The profit per unit is fixed for each size.

- **Maximize $Z = 420(x_1 + x_2 + x_3) + 360(x_4 + x_5 + x_6) + 300(x_7 + x_8 + x_9)$**

2. Resource Constraints:

- Production Capacity Constraints (for each plant): The total number of units produced at a plant cannot exceed its available excess capacity.
  - Plant 1: $x_1 + x_4 + x_7 \leq 750$
  - Plant 2: $x_2 + x_5 + x_8 \leq 900$
  - Plant 3: $x_3 + x_6 + x_9 \leq 450$

- Storage Space Constraints (for each plant): The total storage space used by a plant's production cannot exceed its available space.
  - Plant 1: $20x_1 + 15x_4 + 12x_7 \leq 13{,}000$
  - Plant 2: $20x_2 + 15x_5 + 12x_8 \leq 12{,}000$
  - Plant 3: $20x_3 + 15x_6 + 12x_9 \leq 5{,}000$
- Sales Forecast Constraints: The total production of each size across all plants cannot exceed its market demand.
  - Large: $x_1 + x_2 + x_3 \leq 900$
  - Medium: $x_4 + x_5 + x_6 \leq 1{,}200$
  - Small: $x_7 + x_8 + x_9 \leq 750$

3. Same Percentage of Capacity Constraint:

   This is the key managerial constraint. To avoid layoffs, each plant must use the same percentage of its excess capacity. This means the ratio of production to capacity must be identical for all three plants.

   - $(x_1 + x_4 + x_7) / 750 = (x_2 + x_5 + x_8) / 900 = (x_3 + x_6 + x_9) / 450$

   To incorporate this into an LP model, we can express this relationship with linear equations. We can set the ratio for Plant 1 equal to Plant 2, and the ratio for Plant 1 equal to Plant 3. This gives us two equations that enforce the same percentage across all plants.

   - Plant 1 % = Plant 2 %: $(x_1 + x_4 + x_7) / 750 = (x_2 + x_5 + x_8) / 900$

     This can be rewritten by cross-multiplying to eliminate the fractions:

     $900(x_1 + x_4 + x_7) - 750(x_2 + x_5 + x_8) = 0$

   - Plant 1 % = Plant 3 %: $(x_1 + x_4 + x_7) / 750 = (x_3 + x_6 + x_9) / 450$

     Similarly, cross-multiply to get a linear equation:

     $450(x_1 + x_4 + x_7) - 750(x_3 + x_6 + x_9) = 0$

4. Non-negativity Constraints:

   All decision variables must be non-negative.

   - Constraints: $x_1,\ldots,x_9 \geq 0$

Full Linear Programming Model:

Maximize Total Profit:

Maximize $Z = 420(x_1 + x_2 + x_3) + 360(x_4 + x_5 + x_6) + 300(x_7 + x_8 + x_9)$

Subject to:

$x_1 + x_4 + x_7 \leq 750$ (Plant 1 production capacity)

$x_2 + x_5 + x_8 \leq 900$ (Plant 2 production capacity)

$x_3 + x_6 + x_9 \leq 450$ (Plant 3 production capacity)

$20x_1 + 15x_4 + 12x_7 \leq 13{,}000$ (Plant 1 storage space)

$20x_2 + 15x_5 + 12x_8 \leq 12{,}000$ (Plant 2 storage space)

$20x_3 + 15x_6 + 12x_9 \leq 5{,}000$ (Plant 3 storage space)

$x_1 + x_2 + x_3 \leq 900$ (Large sales forecast)

$x_4 + x_5 + x_6 \leq 1{,}200$ (Medium sales forecast)

$x_7 + x_8 + x_9 \leq 750$ (Small sales forecast)

Same Percentage of Capacity:

$(x_1 + x_4 + x_7) / 750 = (x_2 + x_5 + x_8) / 900 = (x_3 + x_6 + x_9) / 450$

$x_1,\ldots,x_9 \geq 0$ (non-negativity)

Table 4: Constraint Summary – Weigelt Corporation

Constraint Type	Plant / Product	Constraint Formula	Notes
Assembly Capacity	Plant 1	$x_1 + x_4 + x_7 \leq 750$	Production capacity limit
	Plant 2	$x_2 + x_5 + x_8 \leq 900$	Production capacity limit
	Plant 3	$x_3 + x_6 + x_9 \leq 450$	Production capacity limit
Storage Space	Plant 1	$20x_1 + 15x_4 + 12x_7 \leq 13{,}000$	Storage space limit
	Plant 2	$20x_2 + 15x_5 + 12x_8 \leq 12{,}000$	Storage space limit
	Plant 3	$20x_3 + 15x_6 + 12x_9 \leq 5{,}000$	Storage space limit
Sales Forecast	Large	$x_1 + x_2 + x_3 \leq 900$	Market demand limit
	Medium	$x_4 + x_5 + x_6 \leq 1{,}200$	Market demand limit
	Small	$x_7 + x_8 + x_9 \leq 750$	Market demand limit
Equal Capacity Use	All Plants	$(x_1 + x_4 + x_7)/750 = (x_2 + x_5 + x_8)/900 = (x_3 + x_6 + x_9)/450$	Ensures same % of capacity used
Non-Negativity	All Variables	$x_1, \ldots, x_9 \geq 0$	Cannot produce negative quantities