* Graeco, Latin Square Design

B γ	A β	D δ	C α
A δ	B α	C γ	D β
D α	C δ	B β	A γ
C β	D γ	A α	B δ

	1	....	p	total
1				R_1
⋮		$x_{ij(ks)}$		R_2 R_i
⋮				⋮
p				R_p
total	C_1	C_2 --- C_j	C_p	G

L_k = latin letter total

g_s = Greake " "

$i, j, k, s = 1, 2, \dots, p$

[Modle] each observation $x_{ij(ks)} = \mu + \overset{r}{K_i} + \overset{c}{M_j} + \overset{L}{N_k} + \overset{G}{O_s} + \underset{\sim N(0,\sigma^2)}{\epsilon_{ij(ks)}}$.

μ : Genral mean

k_i = the effect of i^{th} row with $\sum_i k_i = 0$

M_j = the effect of j^{th} column with $\sum_j M_j = 0$

N_k = " " " k^{th} latin letter treatments with $\sum_k N_k = 0$

O_s = " " " s^{th} Graec " " " $\sum_{s=1} O_s = 0$

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$\boxed{\text{Hypthisth}}$

1) For rows:

to test $H_{01}: K_i = 0 \quad \forall\ i = 1, 2, \dots, p$

against: $H_{11}: K_i \neq 0$ for some i

2) For columns:

$H_{02}: M_j = 0 \quad \forall\ j = 1, \dots, p$

v.s $H_{12}: M_j \neq 0$ for some j

3) For latin treatments:

test: $H_{03}: N_K = 0 \quad \forall\ K = 1, \dots, p$

v.s $H_{13}: N_K \neq 0$ for Same K

4) For Greac treatments

test $H_{04}: O_s = 0 \quad \forall\ s = 1, \dots, p$

v.s $H_{14}: O_s \neq 0$ for some s

<u>Calculation</u>

1) $C = \frac{G^2}{p^2}$

2) $SSR = \sum_{i=1}^{p} \frac{R_i^2}{p} - C$

3) $SSC = \sum_{j=1}^{p} \frac{C_j^2}{p} - C$

4) $SSL = \sum_{K=1}^{p} \frac{L_K^2}{p} - C$

5) $SSg = \sum_{s=1}^{p} \frac{g_s^2}{p} - C$

6) $SST = \sum_{i=1}^{p} \sum_{j=1}^{p} x^2_{ij(ks)} - C$

7) $SSE = SST - SSR - SSC - SSL - SSg$

ANOVA table

S.O.V	d.o.f	S.S	M.S.	F_{cal}	F_{tab} Test
R	$P-1$	SSR	MSR	$F_R = MSR/MSE$	$F_\alpha(P-1,(P-1)(P-3))$
C	$P-1$	SSC	MSC	$F_C = MSC/MSE$	$F_\alpha(P-1,(P-1)(P-3))$
L	$P-1$	SSL	MSL	$F_L = MSL/MSE$	↓
g	$P-1$	SSg	MSg	$F_g = MSg/MSE$	
E	$(P-1)(P-3)$	SSE	MSE		
Total	P^2-1	SST			

if $F_{cal} \leq F_{tab}$ accept $H_{0\ell}$; $\ell = 1,2,3,4$

$\quad\quad >$ reject $H_{0\ell}$ and accept $H_{1\ell}$

Ex: Aanlyse the following experment Example (1) Pag 5 Lec 10

Sub a sinutable no. say "36"

A 2 α	B -8 β	C -4 γ	D 7 δ	-3
B -7 δ	C -3 α	D 4 β	A 8 γ	2
C -4 γ	D 4 δ	A 1 α	B -2 β	-1
D 0 β	A 2 γ	B 0 δ	C 6 α	8
-9	-5	1	19	6

$P = 4$ & $L_1 = \sum A = 13$ & $L_2 = \sum B = -17$

$L_3 = \sum C = -5$ & $L_4 = \sum D = 15$ & $\sum_{i=1}^{4} L_i = 6$

$g_1 = \sum \alpha = 6$ & $g_2 = \sum \beta = -6$

$g_3 = \sum \gamma = 2$ & $g_4 = \sum \delta = 4$ & $\sum_{i=1}^{4} g_i = 6$

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$$C = \frac{G^2}{p^2} = \frac{36}{16} = 2.25$$

$$SSR = \frac{\sum_1^p R_i^2}{p} - C = 17.25$$

$$SSC = \frac{\sum_1^p C_j^2}{p} - C = 114.75$$

$$SSL = \frac{\sum_1^p L_k^2}{p} - C = 174.75$$

$$SSg = \frac{\sum \theta_s^2}{p} - C = 20.75$$

$$SST = \sum\sum x_{ij}^2 - C = 345.75$$

$$SSE = SST - SSR - SSC - SSL - SSg = \cancel{345.75}\ 18.25$$

S.o.V	d.o.f	SS.	M.S.	F
R	3	17.25	5.75	0.95
C	3	114.75	38.25	6.29
L	3	174.75	58.25	9.58
g	3	20.75	6.91	1.14
E	3	~~345.75~~ 18.25	6.08	
Total	15	345.75		

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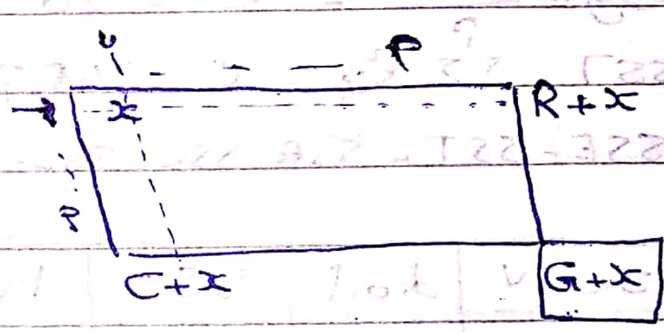
(EX) in Greaco. latin Square of order $p\times p$ the observation $x_{11(12)}$ is missing. find a linear estimator of this observation which minimize the experimental error.

(note: $P\times P \Rightarrow 15\times 15$ نفس الحل والعوض بـ 15 عالآخر)

let $x_{11(12)} = x$

$L_1 = \Sigma A = L_1(A) = L + x$

$g_2 = \Sigma \beta = g_2(\beta) = g + x$



in this case we have

$SSE = SST - SSR - SSC - SSL - SSg$

$$= x^2 - \frac{(x+R)^2}{p} - \frac{1}{p}(x+C)^2 - \frac{1}{p}(x+L)^2 - \frac{1}{p}(x+g)^2 + \frac{3}{p^2}(G+x)^2 + k \quad (1)$$

where k = Sum terms that does not contain x

For minimum SSE we have $\frac{\partial SSE}{\partial x} = 0$

$$(1) \Rightarrow 2x - \frac{2}{p}(x+R) - \frac{2}{p}(x+C) - \frac{2}{p}(x+L) - \frac{2}{p}(x+g) + \frac{6}{p^2}(x+G) = 0$$

$$\xrightarrow[\times p^2]{\div 2} x(p^2 - 4p + 3) = p(R + C + L + g) - 3G$$

$$\boxed{\hat{x} = \frac{p(R+G+L+g) - 3G}{(p-1)(p-3)}}$$