Marlics: A finite difference liquid crystal simulation package

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Abstract

In this paper we present Marlics to the world. Marlics is a software written in C++ to solve the Beris-Edwards equation of nematodynamics without flow for both nematic and cholesteric liquid crystals. The program takes as input an descriptive file giving the simulations parameters and initial conditions generating a series of different snapshots. The code is organized in class modules which can be modified by the user base to attend their further needs. Review the abstract after the paper is finished

Keywords: Liquid crystals, Landau-de Gennes, finite differences.

PROGRAM SUMMARY

Program Title: MarLicS

Licensing provisions: GNU General Public License v3.0 (GPL)

Programming language:C++

Supplementary material: Complete instructions about program usage can be found in the user-guide.

Nature of problem: Marlics was developed to simulate liquid crystal devices via solution of the Beris-Edwards system of differential equations equations without flow.

Solution method: The system of equations is solved using finite differences in both time and space. The time integration is performed using an explicit integrator with or without variable time-step.

External routines: The code needs the GSL (Gnu scientific library), an implementation of the CBLas library and an implementation of the OpenMp library(optional).

Running time: From minutes to hours depending on the problem size.

Computer: Single or multi-core processor with shared memory.

RAM: From hundreds of megabytes to gigabytes depending on the problem size.

Restrictions: The code is parallelized using OpenMp, consequently it can only be run in parallel with shared memory processors.

Additional comments: The source code comes with a Mathematica notebook with can be used to aid the user to implement additional interactions, boundary conditions or other situations situation not covered by the current software.

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1. Introduction

Misses an introduction to optical devices or light modulating devices.

Liquid crystals are excellent materials to perform these functions, since it can selectively reflect or transmit incoming light depending on its state, which can be controlled by an external perturbation. In displays, one of the most common application, it is used electric fields to control the liquid crystals state [1]. There are many types o LCDs current available, being the major difference among them given by the distribution of electrodes in the device, and the inclusion of protrusions in the boundaries of the confining surface [2].

The design of new devices requires a great amount of experimentation and empirical knowledge. Performing every trial with a real apparatus would require a unpractical number of prototypes to construct and tested. As an alternative, the researcher can turn to modeling softwares to aid in the discovery process. Before prototyping a device it can be simulated in a package in many forms before being constructed saving time and resources.

There are some commercial available softwares for simulation of liquid crystals devices, for instance LC3D[3] and LCD master[4]. These softwares provides out of the box capabilities for simulation and visualization, however it lacks the possibility for extension by the user. Open sources alternatives have been appearing to fill this gap, Licra [5] is one of this attempt. The licra software is written in C an support a few modes of use, however, its implementation is very monolithic, and if the user wants to change a simulation parameter, or a mode of operation, he/she has to code it directly to the source code and recompile the executable. Recently, another open source liquid crystal simulation software has been released [6]. The program named openQmin, performs the minimization of the liquid crystal energy, searching for its minimum energy state. The program works in many situations and can be fine tuned by the user via a script or a graphical interface. It has many desirable capabilities, and performs very well, but the program is focused

in static problems, and in some situations he dynamic operation of the device is a matter of great importance.

In this paper we present *marlics*, it is a open source code designed to simulate the dynamics of liquid crystal order parameters. The code The code is written in C++ using an independent system of classes, which provide the building blocks for the most common cases, and provide a framework where the user can develop their specific applications. The mode of operation is defined by a script which is passed to the software.

2. Theoretical background

Liquid crystal are anisotropic liquids which have a certain degree of long range ordering. Nematic LCs presents orientational order, but no positional one. Its state can be discribed by a combination of scalar and vectorial fields. The prefered direction can be represented by a vector $\vec{n}$, and due to the phase organization $\vec{n}$ and $-\vec{n}$ are equivalent. In some circunstances, it happens that the liquid crystal presents a second prefered direction, which it is also described by a a vector $\vec{l}$ called co-director. The degree which the system is ordered in in the direction of the director is given by the scalar order paramter S, and by the scalar order parameter P in the co director direction. In this way, when $S = 1$ the system is perfect oriented along $\vec{n}$, while $S = 0$ given an isotropic liquid (no preferred orientation). It is possible to have $S < 0$, in this case the molecules of the liquid crystal is oriented in average perpendicular to the director $\vec{n}$.

The density of energy $f_s(S)$ associated with the scalar order parameter S is given by:

$$f_L(S) = \frac{1}{2}a\,(T - T^*)\,S^2 - \frac{1}{3}S^3 + \frac{1}{4}cS^4 + \frac{1}{2}L(\nabla S)^2 \quad (1)$$

where $\{a, b, c, L$ are thermodynamics constants. The energy associated with variation of $\vec{n}$ in space is given the Frank density of energy:

$$f_f(\vec{n}) = \frac{1}{2}K_{11}(\nabla\cdot\vec{n})^2 + \frac{1}{2}K_{22}(\vec{n}\cdot\nabla\times\vec{n} + q_0)^2 + \frac{1}{2}K_{33}\left|\vec{n}\times\nabla\times\vec{n}\right|^2 + \frac{1}{2}K_{24}\nabla\cdot(\vec{n}\times\nabla\times\vec{n} + \vec{n}\cdot\nabla\vec{n}) \quad (2)$$

being $\{K_{11}, K_{22}, K_{33}, K_{24}\}$ elastic constants with units of force.

The scalar order parameters $\{S, P\}$ and the director and co-director $\{\vec{n}, \vec{l}\}$ can be combined in unique order parameter $\mathbf{Q}$, which is a second rank tensor whose elements are given by:

$$Q_{ij} = \frac{1}{2}S(3n_in_j - 1) + \frac{P}{2}(l_il_j - m_im_j) \quad (3)$$

where $i = 1, 2, 3$ and $j = 1, 2, 3$. This order parameter is symmetric and traceless, therefore only 5 independent elements, for example $\{Q_{11}, Q_{12}, Q_{13}, Q_{22}, Q_{23}\}$ are necessary to fully determine it.

Deviations from the equilibrium value of the scalar order paramter, or spatial variations of the director has associate and energy density given by Landau- de Gennes energy density ($f_{LDG}(\mathbf{Q})$:

$$f_{LDG}(\mathbf{Q}) = \frac{a}{2}(T - T^*)\mathrm{Tr}(\mathbf{Q}^2) + \frac{B}{3}\mathrm{Tr}(\mathbf{Q}^3) + \frac{C}{4}\mathrm{Tr}(\mathbf{Q}^2)^2 + \frac{1}{2}L_1\left(\partial_iQ_{jk}\right)\left(\partial_iQ_{jk}\right) + \frac{1}{2}L_2\left(\partial_iQ_{ji}\right)\left(\partial_kQ_{jk}\right) + \frac{1}{2}L_3Q_{ij}\left(\partial_iQ_{kl}\right)\left(\partial_jQ_{kl}\right) + \frac{4\pi}{P_0}L_q\epsilon_{ijk}Q_{ij}\left(\partial_jQ_{ik}\right) \quad (4)$$

where $Q_{ij,k} = \partial Q_{ij}/\partial x_k$, ϵ_{ijk} is the Levi-Civita tensor, $\{L_1, L_2, L_3, L_q, L_s\}$ are the elastic constants, $\{a, B, C\}$ are thermodynamic constants related to the nematic isotropic transition and T and T^* are the system temperature and the virtual nematic-isotropic phase transition temperature, respectively. Here we used Einstein summation convention in repeated indexes.

The elastic constants of the Frank energy $\{K_{11}, K_{22}, K_{33}, K_{24}\}$ and the Landau-De Gennes ones $\{L_1, L_2, L_3, L_q, L_s\}$ are related by the expressions:

$$\begin{aligned} L_1 &= \frac{2.0(k33 - k11 + 3.0k22)}{(27.0{S_{eq}}^2)} \\ L_2 &= \frac{4.0(k11 - k22 - k24)}{(9.0{S_{eq}}^2)} \\ L_3 &= \frac{4.0(k33 - k11)}{(27.0{S_{eq}}^3)} \\ L_q &= \frac{2.0(k22)}{(9.0{S_{eq}}^2)} \\ L_s &= \frac{4(k24)}{(9.0{S_{eq}}^2)} \end{aligned} \quad (5)$$

The dielectric energy density is given by:

$$f_e(\mathbf{Q}) = -\frac{1}{3}\epsilon_0\Delta\epsilon^m E_iE_jQ_{ij} + \frac{\epsilon_0}{2}\mathbf{E}\cdot\mathbf{E}, \quad (6)$$

where e_0 is the vacuum dieletric constant, $\Delta\epsilon = \epsilon_{\|} - \epsilon_{\perp}$ is the dielectric anisotropy which measures the diference between the dieletric constant parallel($\epsilon_{\|}$) and perpendicular ($\epsilon_{\perp}$) to the liquid crystal director. The volume energy density (f_v) will be given by the sum of all energy terms being considered:

$$f_v(\mathbf{Q}) = f_{ldg}(\mathbf{Q}) + f_e(\mathbf{Q}) \quad (7)$$

The liquid crystal also interacts with the confining surfaces, which can induce an order parameter at the surface and a proffered direction for the direction, which is called surface easy axis n_0. One of the simplest form of the surface energy density between a liquid crystal and a surface is given by the Rapini-Papoular (also called Nobili-Durant) which is given by:

$$f_{rp}((Q)) = \frac{1}{2}W_1(Q_{ij} - Q^0_{ij})(Q_{ij} - Q^0_{ij}) \quad (8)$$

where Q^0_{ij} is the surface induced order parameter, which can be given in its tensorial form, or constructed using the induced scalar order parameters $\{S^0, P^0\}$ and the induced easy axis $\{\vec{n}, \vec{l}\}$ using expression 3.

When the liquid crystals boundary is a liquid or gas, instead of inducing a preferred direction the surface may induces a a preferred plane of orientation perpendicular to the surface. Any variation of the director inside this plane gives the same energy. This type of surface energy is described by the Fournier-Galatola anchoring energy given by [7]:

$$f_{FG}(\mathbf{Q}) = W\left(\tilde{Q_{ij}}(\mathbf{Q}) - \tilde{Q}^{\perp}_{ij}(\mathbf{Q})\right)\left(\tilde{Q_{ij}}(\mathbf{Q}) - \tilde{Q}^{\perp}_{ij}(Q)\right) \quad (9)$$

where W is the anchoring strength constant, $\tilde{Q_{ij}}(\mathbf{Q}) = Q_{ij} + \frac{S_0}{3}\delta_{ij}$ and $\tilde{Q_{ij}}^{\perp}(\mathbf{Q}) = (\delta_{ik} - \nu_i\nu_k)Q_{kl}(\delta_{lj} - \nu_l\nu_j)$. Here $\vec{\nu} = \{\nu_1, \nu_2, \nu_2\}$ is the normal surface vector.

The time evolution of the order parameter is given by the Beris-Edwards set of equations. If we neglect the liquid crystal flow, the time evolution of Q_{ij} in the bulk will be given by:

$$\frac{\partial Q_{ij}}{\partial t} = -\frac{1}{\mu}\left(\frac{\partial f_V(\mathbf{Q})}{\partial Q_{ij}} - \frac{\partial}{\partial x_k}\frac{\partial f_V(\mathbf{Q})}{\partial Q_{ij,k}}\right) = F_{ij}(\mathbf{Q}), \quad (10)$$

where μ is the bulk viscosity and $\partial Q_{ij}\partial Qkl = (\delta_{ik}\delta_{jl} + \delta_{jk}\delta_{kl} - 2\delta_{ij}\delta_{kl}/3)$. Meanwhile the dynamics in the bulk will be given by

$$\frac{\partial Q_{ij}}{\partial t} = -\frac{1}{\mu_s}\left(\nu_k\frac{\partial f_{LDG}(\mathbf{Q})}{\partial Q_{ij,k}} - \frac{\partial f_{pen}(\mathbf{Q})}{\partial Q_{ij}}\right) = F^s_{ij}(\mathbf{Q}), \quad (11)$$

here μ_s is the surface viscosity.

We solve solve the system of equations using the method of lines [8]. In this method the spatial and time discretization of the governing equations are performed separated and independently, being the spatial dimensions of the equations discretized first. We discretize the R.H.S of (11) and (11) by finite diferences. In the bulk we have taken centered diferences for both first order and second order derivatives. In the surface we have taken centered diferences for derivatives perpendicular to the normal and first order to derivatives parallel to the surface normal.

In the methods of lines the temporal discretization depends on the kind of integrator intendend to propagate the solution. In the current version of marlics, we have implemented only explicit integrators, thus we have aproxiameted the time derivative by forward diferences. As an example we will take the Euler method; assuming Q^t_{ij} is the value Q_{ij} at time t, the value of $Q^{t+\Delta t}_{ij}$ can be calculated by:

$$Q^{t+\Delta t}_{ij} = Q^t_{ij} + \Delta t F_{ij}(\mathbf{Q}) \quad (12)$$

Although the Euler method is convergent and can be used in some cases, it has its drawbacks. The value of Δt is fixed during the simulation, and the allowed timestep size is to small for some applications. As an alternative we have provided another 2 explicit integrator with the program: xplicit second order Runge-Kutta, which also has a fixed timestep but is more stable and the Dormand-Prince 5(4), which has a self adaptive timestep. We implemented the time adaption as proposed in reference [9].

3. Software Usage:

Here we present the basic information necessary to install and use the software. The complete information about software usage can be found in the supplementary material "Userguide".

3.1. Installation:

The installation of Marlics in Unix systems is very straightforward. The source code comes with a *makefile* to help user compile it in its computer. Actually the makefile provides an automatic installation for two compilers (being one of them free). The user will need just to assure he/she has the *make* software and the necessary external libraries and their respective developer files installed. These libraries are: GSL (Gnu Scientific library), OpenMp(optional, but highly recomended) and a CBLAS implementation (you can use the GSL implementation for example). If everything is present, the user just need to open a terminal in the program folder and type:

```
make
```

to compile the program. Once the compilation is done, the user will find a executable named *marlics* in the instalation folder. For ease of use, this executable can be added to one of the system search paths for binaries files, or the user can add the installation folder to the list of search-able paths. Its is also possible to run the simulations in the same path that the software is installed, but we strong recommend against, since it can be become very confuse.

3.2. Simulation set up and Execution:

To execute marlics you must call the program passing an input file which sets the simulation parameter as follows:

```
marlics intput_file
```

being *input_file* the file containing the simulation parameters.

We already provide input files for some situations that the user can use to test the program, or as a base to their own simulations. All the parameters necessary to set up the simulation must be passed to the program via the input file. An entry in the input file is set by passing the parameter name followed by the parameter required values:

```
parameter value
```

Comments can be placed in the input file starting a line with "#", everything in this line will be ignored by marlics. Also, everything following the required parameters will also be ignored by marlics. We found it quite useful to let the parameters units after its values. Some paramters must be set, while others can have standard values assciated with them. Whenever marlics use an standard value, it inform the user in the standard output whch value was used. The paramerters standard values and its units can be found in the table Appendix B.

The simulation constants must be filled with a real number. The user has two options to pass the elastic constants: the user can pass the L_i values, which are the constants actually used in the calculations, or pass the k_{ii} values and let marlics calculate

L_i. Also the chirality power can be passed in two different ways, as p_0, i.e the helix passe, or as q_0, i.e the helix vector.

We provide the most common initial conditions: random, homogeneous oriented along a direction $\vec{v}$. We have also provide an initial condition set as read_from_file and pass an file containing the initial condition. The refereed file must be formated as the standard output file presented in section 3.3. The remaining initial co nditions can be checked in the user manual.

We provided 3 defferent geometries in marlics: bulk, sphere and slab. Each geometry has its number of boundary conditions, in the case, 0, 1 and 2 respectively. To define a boundary the user must start a line with the *boundary* keyword followed by the boundary name and its number. There is 4 forms of boundary conditions implemented in marlics: Rapinni-Papoular (in Nobili-Durant) form, Fournier-Galatola, strong boundaries and homeotropic.

For more details see the supplementary file "Userguide". For reference, we also include an example of input file in the appendix Appendix C.

3.3. Output files:

The software produces two outputs: an log of the program execution, and a series of files containing the spatial distribution of the order parameters. The log has the function of informing the user about the parameters read by the program, so it can be used as reference in the future, and informing the current state of the simulation. The log is printed in the standard output, which can aid the preparation of the input file, but we strong recommend redirecting it to a separate file for reference in future.

The main output of the software are the files containing the spatial distribution of the order parameter: the main LC director director $\vec{n}$, the co-director $\vec{l}$, the uniaxial order parameter S and biaxial order parameter P. We decided to output the order parameters in this form instead of the elements of Q_{ij}, since the former are more ready to use and interpret than the later. We also preferred to refer to the position in space using the lattice numbers instead of the space position in the Cartesian frame. The actual position can be easily reprieved multiplying the column by its referred grid spacing (dx, dy or dz). Although every output file has associate with it an time t, this number is not output in the file. Instead the output file number and is referred output time is informed in the log output.

An example of a truncated output file can be viewed in appendix Appendix D. More information can be found the in the supplementary material"Userguide".

4. Test problems:

To validate our code we performed a few standard simulations. Even though we are presenting a source code which can be used as a framework for user implemented situations, here we wanted to show out of the box capabilities. In this way we chose some of well documented scenarios performed in the marlics framework.

4.1. Bulk Nematic:

4.2. Cholesteric Slab:

It is know that a cholesteric liquid crystal with pitch p_0 placed inside slab with planar anchoring in both substrates will organize itself with the profile

4.3. Nematic sphere with strong anchoring

4.4. Cholesteric sphere with weak anchoring

5. Conclusions

In conclusion, we have presented marlics. Marlics is open source and offers some of the most common cases for device simulation. Moreover it provides a class organized framework where the user can program its on cases if necessary.

Appendix A. Software implementation and extension:

Marlics is subdivided in a series of C++ classes. There are 4 main super classes which confers most of marlics usability : energy, boundary, geometry and integrator. Each specific functionality must be implemented deriving one of these classes, for instance, the Landau-de Gennes energy is implemented through a class derived from the energy super class.

Boundary is a derivation of energy and contains the same functions plus some members containing information about which boundary it is associated.

Integrator contains the routine to evolve the system of equations.

Geometry contain the information about the geometry of the system and their boundary conditions. Here the user must provide a way to calculate the boundaries normals, and the parameter field derivatives. With all inofrmation present in the geometry class, we put the calculation of RHS in it, therefore, there is a routine to calculate the evolution RHS. To do this the geoemtry has as member an energy instance and a linked list containing the boundaries pointers.

There is an final class called driver which parses the input file and assembly the simulation pieces, except for the boundaries, it setups all the other classes. In the appendix **??** we presents a short description showing how the user can implement their new situations (boundaries, integrators and geometries).

Appendix B. List of available parameters and its units:

Parameter name	variable type	units	mandatory/standard value
Geometry	string		Yes
Nx	integer		Yes
Ny	integer		Yes
Nz	integer		Yes
dx	real	nm	Yes
dy	real	nm	Yes
dz	real	nm	Yes
integrator	string		Yes
facmin	real		0.4
facmax	real		3
prefac	real		0.8
Atol	real		0.001
Rtol	real		0.001
a	real	$MJ/(m^2K)$	Yes
b	real	MJ/m^2	Yes
c	real	MJ/m^2	Yes
K1	real	pN	See 3
K2	real	pN	See 3
K3	real	pN	See 3
L1	real	pN	See 3
L2	real	pN	No
L3	real	pN	No
Ls	real	pN	No
Lq	real	pN/m	No
p0 or q0	real	nm or 1/nm	No
T	real	K	No
Mu or gamma	real	Pa/s	Yes
Mu_s or gamma_s	real	nm Pa m/s	See 3
ti	real	μs	0.0
tf	real	μs	Yes
dt	real	μs	Tf/1e6
timeprint	real	μs	Tf/20
timeprint_type	string		Linear
timeprint_increase_factor	real		Tf/20
output_folder	string		.
output_fname	string		director_field_$$.csv
initial_output_file_number	int		0
initial_conditions	string		yes
initial_file_name	string		See 3
theta_i	real	degrees	See 3
phi_i	real	degrees	See 3
anchoring_type	int + string		See 3
Wo1	int + real		See 3
theta_0	int + real		See 3
phi_0	int + real		See 3

Appendix C. Input File example:

Here you can find the complete input file:

```
281
282     #Geometry Parameters:
283     geometry   slab
284     Nx   200                    /*       grid size       */
285     Ny   200                    /*       grid size       */
286     Nz   100                    /*       grid size       */
287     dx   10.0                   /*       10^-9 m          */
288     dy   10.0                   /*       10^-9 m          */
289     dz   10.0                   /*       10^-9 m          */
290
291
292    #Integrator parameters:
293    integrator  DP5
294    atol 0.005
295    rtol 0.005
296    facmax 3.0
297    facmin 0.4
298    prefac 0.8
299
300
301    #Liquid crystal parameters:
302    a    0.182
303    b   -2.12
304    c   1.73
305    T -1              Kelvin
306    k11   16.7          /*    pN    */
307    k22   7.8           /*    pN    */
308    k33   18.1          /*    pN    */
309    k24   0             /*    pN    */
310    p0   500
311    mu_1      0.3           /*      Pa s              */
312    mu_1_s    30.0           /*     Pa nm s      */
313
314
315    #Time parameters:
316    dt   0.001              /*      10^-6 s          */
317    ti   0.0                /*      10^-6 s          */
318    tf   5000.0             /*      10^-6 s          */
319
320    #Output Parameters:
321     time_print_type              logarithmic
322     timeprint                    50.         /*   10^-6 s    */
323     timeprint_increase_factor    1.16
324     output_folder                .
325     output_fname                 output_$$.csv
326     initial_output_file_number   0
327
328     #Initial conditions:
329     initial_conditions   random
330
331
332     #Boundaries conditions
333
334     #Bottom boundaries:
335     anchoring_type 0   Rapini-Papoular
336     Wo1            0   1000.0
337     theta_0        0   45.0
```

```
phi_0              0   45.0

#Top boundaries:
anchoring_type 1   Fournier-Galatola
Wo1                1   1000.0
```

Appendix D. Output file:

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