Modern Digital Communications: A Hands-On Approach

GPS: The Big Picture

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Positioning: The Basic Idea

The basic idea is quite simple and relies on the following fact: The intersection of two spheres is (at most) a circle and the intersection of this circle with another sphere gives (at most) two points. Hence:

- If you know the position of 3 satellites
- and you know the distance that separates your receiver from each of the 3 satellites
- then you are at the intersection of three spheres of which you know the centers and the radii. Hence you are in one of two possible locations. Only one of them is reasonably near the surface of the earth.

(You may want to draw a picture of the 2-dimensional case with two satellites.)

Mathematically

If

- $p^{(k)} \in \mathbb{R}^3$ is the position of satellite k in some coordinate system, $k = 1, 2, 3$
- $r^{(k)} \in \mathbb{R}_+$ is the *range*, i.e., the distance between $p^{(k)}$ and your unknown position $p \in \mathbb{R}^3$
- then your position p is one of the two solutions to the following system (three equations and three unknowns)

$$r^{(k)} = \|p^{(k)} - p\|, \qquad k = 1, 2, 3.$$

One Step Further

Each satellite broadcasts enough information to determine its position, i.e., the ephemeris. We will see later where this information is to be found in the bitstream that you obtain from decoding the satellite.

In principle, we measure the distance to a satellite by multiplying by the speed of light the time it takes for the signal to travel from the satellite to the receiver. There is a slight complication though.

We know precisely when a satellite sends a specific signal with respect to the satellite clock but the satellite clock $t^s = t + \delta t^s$ has an offset δt^s with respect to a reference GPS time t. This is not a problem because the satellite broadcasts the information needed to determine δt^s. So once we have decoded the satellite we know the correction.[1]

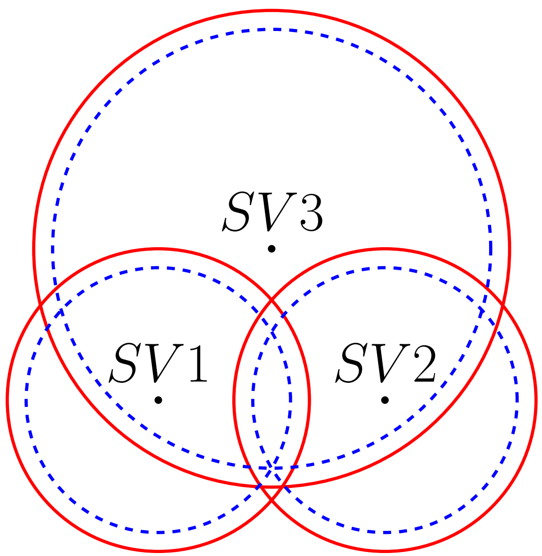
[1] δt^s is described by a Taylor expansion for the clock offset plus a relativistic term that depends on the orbit and on the satellite's position within the orbit.

A slightly more serious complication comes from the fact that the receiver clock has also an offset δt_r with respect to the GPS time t. If we neglect to account for this offset, the distance obtained will have an offset and this offset is the same for all satellites. The distance obtained this way is called *corrected pseudorange*.[2]

By using 4 (instead of 3) satellites we will be able to determine the additional unknown offset (and thereby obtain enough information to synchronize the receiver clock with the GPS time if we wish).

[2]This is not standard terminology. The literature calls *pseudorange* what we obtain if we neglect both offsets. Hence *corrected* refers to the fact that we are accounting for the satellite offset.

The following figure helps us visualize that corrected pseudoranges are enough, provided we have an additional satellite. For illustration we assume that we are in two dimensions with three satellites (instead of three dimensions with four satellites). We denote by b the difference between the range and the pseudorange. When $b = 0$ (blue dotted lines) we can determine the receiver position as the only point at the intersection of the three dotted circles. When $b \neq 0$ (red solid lines) the intersection of the three circles is empty. If one is given the solid circles, one can find out b as the amount by which one has to reduce the radii so that the circles intersect in one point.



From Pseudoranges to Ranges

Mathematically

If we manage two write four equations of the form

$$\rho_c^{(k)} + b = \|p^{(k)} - p\|, \qquad k = 1, 2, 3, 4, \tag{1}$$

where the superscript (k) refers to the kth satellite and b is any constant, then we can solve for the receiver position p (which has three coordinates) as well as for b. Hence we have four equations and four unknowns.

Things Are A Bit More Complicated

- We tacitly assumed that the satellites and the receiver were stationary. The fact that the satellites move and the earth rotates makes things more interesting. The satellite essentially sends the parameters of its ellipse (shape of the ellipse and orientation in space) as well as the satellite position on the ellipse. It is up to us to find the satellite position with respect to a reference system that rotates with the earth.

- It is not immediately obvious how to measure the pseudorange at any desired receiver-time.