ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE

School of Computer and Communication Sciences

Handout 23 Modern Digital Communications

Final Exam Solutions December 20, 2017

PROBLEM 1. (Paper and Pencil / MATLAB)

1.
$$C = \begin{pmatrix} x_0 & 0 & \dots & 0 \\ x_1 & x_0 & \dots & 0 \\ \vdots & & & \\ x_{N-1} & x_{N-2} & \dots & 0 \\ 0 & x_{N-1} & \dots & 0 \\ \vdots & & & \\ 0 & 0 & \dots & x_{N-1} \end{pmatrix}.$$

2. The MMSE estimator is $\hat{\boldsymbol{h}}(\boldsymbol{y}) = K_{\boldsymbol{HY}} K_{\boldsymbol{Y}}^{-1} \boldsymbol{y}$, where $K_{\boldsymbol{HY}} = E\left[\boldsymbol{H}\boldsymbol{Y}^\dagger\right] = K_{\boldsymbol{H}} C^\dagger$ and $K_{\boldsymbol{Y}} = E\left[\boldsymbol{Y}\boldsymbol{Y}^\dagger\right] = C K_{\boldsymbol{H}} C^\dagger + K_{\boldsymbol{Z}}$.
3. See the attached MATLAB routine.

PROBLEM 2. (MATLAB)

See the attached MATLAB routine.

PROBLEM 3. (Paper and Pencil / MATLAB)

1. $\mathcal{F}\{p(t-\tau)\} = p_{\mathcal{F}}(f) e^{-j2\pi f\tau}$. So $r_{\mathcal{F}}(f) = \mathcal{F}\{p(t-\tau)e^{j2\pi dt}\} = p_{\mathcal{F}}(f-d)e^{-j2\pi(f-d)\tau}$.
2. $|r_{\mathcal{F}}(f)| = |p_{\mathcal{F}}(f-d)|$. The correlation between $|r_{\mathcal{F}}(f)|$ and $|p_{\mathcal{F}}(f)|$ will have its maximum at d.
3. With this method, we can estimate d with a resolution of $f_s/N = 1/(NT_s)$. For a resolution of 1 Hz or better, we need to take $N = \lceil 1/T_s \rceil$.
4. See the attached MATLAB routine.

PROBLEM 4. (Paper and Pencil / MATLAB)

1. See the attached MATLAB routine.
2.
$$C = \begin{pmatrix} 1 & 1 \\ 2 & 1 \\ \vdots & \\ N & 1 \end{pmatrix}.$$

3. $\boldsymbol{u}_1 = (1, \dots, N)^T$ and $\boldsymbol{u}_2 = (1, \dots, 1)^T$.
4. The projection theorem requires $\boldsymbol{y} - H\boldsymbol{v}$ to be orthogonal to $\boldsymbol{u}_1$ and $\boldsymbol{u}_2$: $(\boldsymbol{y} - H\boldsymbol{v})^T \boldsymbol{u}_1 = 0$ and $(\boldsymbol{y} - H\boldsymbol{v})^T \boldsymbol{u}_2 = 0$.
5. In matrix form, we have $(\boldsymbol{y} - H\boldsymbol{v})^T H = 0$. Hence, $\boldsymbol{y}^T H = \boldsymbol{v}^T H^T H$. Transposing the previous relation we get $H^T \boldsymbol{y} = H^T H \boldsymbol{v}$. Thus, $\boldsymbol{v}_{LS}(\boldsymbol{y}) = (H^T H)^{-1} H^T \boldsymbol{y}$.
6. See the attached MATLAB routine.