Exercise 4:

① $r_{\varepsilon,\varepsilon}(t) = r(t) \times \sqrt{2} \exp(-j2\pi f_c(1+\varepsilon)t) = r(t)\times\sqrt{2}\exp(-j2\pi f_c t)\cdot\exp(-j2\pi f_c \varepsilon t)$

$r_{\varepsilon,\varepsilon}(t) = r_\varepsilon(t)\exp(-j2\pi f_c \varepsilon t)$.

②

a) $q(t)$ is Nyquist pulse $\iff \langle q(t-kT), q(t-k'T)\rangle = 0 \quad \forall k \neq k' \in \mathbb{Z}$, T symbol period.

$$\langle q_\varepsilon(t-kT), q_\varepsilon(t-k'T)\rangle = \int e^{j2\pi f_c \varepsilon t} q(t-kT)\cdot e^{-j2\pi f_c \varepsilon t} q^*(t-k'T)\,dt.$$

$$= \int q(t-kT)q^*(t-k'T)\,dt = 0 \quad \forall k, k' \in \mathbb{Z}.$$

b) $y(t) = r_{\varepsilon,\varepsilon}(t) * q^*(-t)$

$$e^{-j2\pi\varepsilon f_c t} r_\varepsilon(t) * q_\varepsilon^*(-t) = e^{-j2\pi\varepsilon f_c t}\int r_\varepsilon(\tau)\, q_\varepsilon^*(\tau-t)\,d\tau = e^{-j2\pi\varepsilon f_c t}\int r_\varepsilon(\tau) e^{-j2\pi\varepsilon f_c(\tau-t)} q^*(\tau-t)\,d\tau$$

$$= \int r_\varepsilon(\tau) e^{-j2\pi\varepsilon f_c \tau} q^*(\tau-t)\,d\tau = r_{\varepsilon,\varepsilon}(t) * q^*(-t) = y(t)$$

c) $$y[n] = y(nT) = e^{-j2\pi\varepsilon f_c nT}\left(r_\varepsilon(t) * q_\varepsilon^*(-t)\right)\Big|_{t=nT} = e^{-j2\pi f_o n}\left\{\sum_k h_\varepsilon[k]a[n-k] + z[n]\right\}.$$

so that $z[n] = N(t)\sqrt{2}\,e^{-j2\pi f_c t} * q_\varepsilon^*(-t)\Big|_{t=nT}$. But $N(t)\sqrt{2}\,e^{-j2\pi f_c t}$ is the baseband equivalent AWGN. Thus using hint, $z[n]$ is white.

3) $\tilde{y}[n] = e^{j2\pi f_o n} y[n] = \sum_k h_\varepsilon[k]a[n-k] + z[n]$. using c).