Ex 4:

① $r(t) = s(t) * h(t) + N(t)$.

$$s(t) * h(t) = \sum_k a[k]\, p(t-kT) * h(t)$$

$$s(t) * h(t) = \sum_k a[k] \int h(\tau)\, p(t-kT-\tau)\, d\tau = \sum_k a[k] \cdot (p*h)(t-kT)$$

$$\Rightarrow r(t) = \sum_k a[k] (p*h)(t-kT) + N(t)$$

② $$y(t) = \sum_n a[n] (p*h)(t-nT) * q^*(-t) + N(t) * q^*(-t).$$

$$y(t) = \sum_n a[n] (p*h*q^*)(t-nT) + N(t) * q^*(-t).$$

$$y[k] = \sum_n a[n] (p*h*q^*)((k-n)T) + (N*q^*)(kT).$$

Let $h[n] = (p*h*q^*)(nT)$ and $(N*q^*)(kT) = z[k]$

$$y[k] = \sum_n a[n] h[k-n] + z[k] = a[k] * h[k] + z[k]$$

$$y[k] = \sum_n a[n] h[k-n] + z[k]$$

assume $N(t) \sim \mathcal{N}(0, \sigma^2)$.

- $z_k = z[k] = \int N(\tau)\, q^*(\tau-kT)\, d\tau. \Rightarrow E[z_k] = \int E[N(\tau)]\, q^*(\tau-kT)\, d\tau = 0$

$$E[z_k z_{k'}^*] = E\left[\int N(\tau) q^*(\tau-kT) d\tau \cdot \left(\int N(\tau') q^*(\tau'-k'T) d\tau'\right)^* \right].$$

$$= \iint E[N(\tau) \cdot N(\tau')]\, q^*(\tau-kT) \cdot q(\tau'-k'T) \cdot d\tau\, d\tau'.$$

$$= \iint \sigma^2 \cdot \delta(\tau-\tau')\, q^*(\tau-kT) \cdot q(\tau'-k'T)\, d\tau\, d\tau'.$$

$$= \sigma^2 \int q^*(\tau'-kT)\, q(\tau'-k'T)\, d\tau'.$$

$$\Rightarrow E[z_k z_{k'}^*] = \sigma^2 \cdot \langle q(t-k'T), q(t-kT) \rangle$$

③ z_k is AWGN if it is gaussian, with the following 2 properties:

$$E[z_k] = 0$$

$$E[z_k z_{k'}^*] = \sigma^2 \cdot \delta[k-k']$$

Thus if $q(t)$ is Nyquist pulse, we have $\langle q(t-k'T), q(t-kT) \rangle = \delta[k-k']$ //

④ As seen previously, if $q(t) = p(t)$, $y[k]$ will be sufficient statistics for deciding on $a[k]$.

⑤ We have: $h(t) = \sum_{l=0}^{M-1} \alpha_l \delta(t-\tau_l)$

$$p(t) * h(t) * q^*(-t) = \sum_{l=0}^{M} \alpha_l \delta(t-\tau_l) * \underbrace{p(t) * q^*(-t)}_{f(t)} = \sum_{l=0}^{M} \alpha_l f(t-\tau_l).$$

$$\Rightarrow h[n] = p(t) * h(t) * q^*(-t) \Big|_{t=nT} = \sum_{l=0}^{M} \alpha_l f(nT-\tau_l) \quad //$$