Ex 2:

$$H^{-1} = 100 \times \begin{bmatrix} 0.0050 + 5i & -5i \\ -5i & 0.0050 + 5i \end{bmatrix}.$$

$$K_V = E[vv^+] = E[H^{-1} z z^+ (H^{-1})^+] = H^{-1} E[zz^+](H^{-1})^+ = 6^2 H^{-1}.(H^{-1})^+.$$

$$K_V = 6^2 . \begin{bmatrix} 500\,000 & -500\,000 \\ -500\,000 & 500\,000 \end{bmatrix} \Rightarrow \quad \mathrm{var}(v_1) = \mathrm{var}(v_2) = 500\,000\, 6^2.$$

1) Orthogonality principle: $\hat{x}$ LMMSE of x $(\hat{x} = Hx + c) \Rightarrow \begin{cases} E[(\hat{x} - x) y^+] = 0 \\ E[(\hat{x} - x)] = 0 \end{cases}$

Here $\hat{x} = By$.

$$E[(By - x)y^+] = 0 \iff B\,E[yy^+] = E[xy^+] \iff B K_y = K_{xy}.$$

$$K_y = E[(Hx+z)(Hx+z)^+] = E[Hxx^+H^+] + E[zz^+] = HH^+ + K_z = HH^+ + 6^2 I_R$$

x and z are statistically indep. & $E[z] = 0$.

$$K_{xy} = E[xx^+H^+] = H^+$$

$$\Rightarrow B[HH^+ + 6^2 I_R] = H^+ \iff \boxed{B = H^+ [HH^+ + 6^2 I_R]^{-1}}$$