

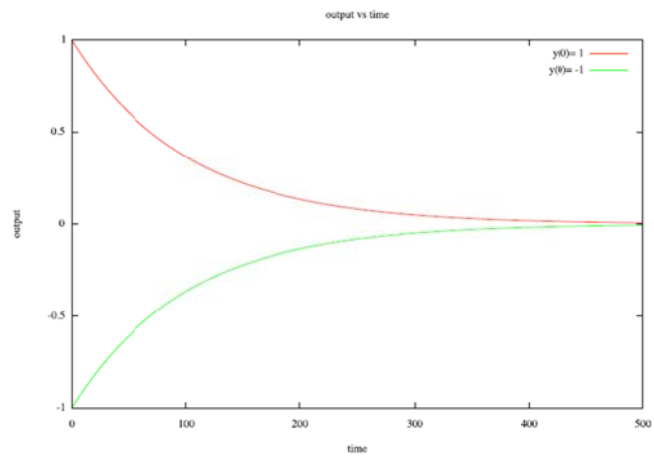
## Nodo

```
%% Use initial value of x=1 and x=-1
%% h is the time step for integration
%% t is the length of simulation
```

```
function SimpleNode (h, t)
    NumTimeSteps = t/h;

    x = 1;
    SimulateNode(x, 1, h, t);
    hold on;

    x = -1;
    SimulateNode(x, 1, h, t);
    hold off;
```



end

```
%% Use euler to integrate
%% x is the initial value
%% tau is the time constant
%% h is the step size for integration
%% t is the simulation length
```

```
function SimulateNode (x, tau, h, t)

    NumTimeSteps = t/h;

    x = zeros(1, NumTimeSteps);
    oldx = x;
    x(1) = oldx;
    tau_value = 1/tau;

    for TStep = 1:NumTimeSteps
        newx = oldx + (h * (tau_value * -oldx));
        x(TStep+1) = newx;
        oldx = newx;
    end

    % Now display
    t = 0:NumTimeSteps;
    max(x);

    % str = sprintf('x= %g;',x);
    xlabel('time'), ylabel('output'), title('output vs time');
    plot(t, x);
```

end

$$\frac{dy}{dt} = \frac{1}{\tau_i}(-y_i)$$

en discreto es:

$$(newX - oldX)/h = -\tau\_value * oldX$$

Condiciones iniciales oldX=x(1);

Desde ahí calculamos recurrentemente newX que va llenando de valores

$$x = \text{zeros}(1, \text{NumTimeSteps});$$

## Efecto de tau

---

```
%% Minimal CTRNN - investigates the effect of tau on the system
%% x is the initial value of x
%% h is the time step for integration
%% t is the length of simulation
```

```
function MinimalCTRNN (x, h, t)

    NumTimeSteps = t/h;

    for tau = 0.2:0.2:2.0
        x = zeros(1, NumTimeSteps);
        oldx = x;
        x(1) = oldx;
        tau_value = 1/tau;

        for TStep = 1:NumTimeSteps
            newx = oldx + (h * (tau_value * -oldx));
            x(TStep+1) = newx;
            oldx = newx;
        end

        % Now display
        t = 0:NumTimeSteps;
        max(x);
        % str = sprintf('T_i= %g;', tau);
        xlabel('time'), ylabel('output'), title('output vs time');
        plot(t, x);
        hold on;
    end
    hold off;

end
```

$$y(t+1) = y(t) + (h * \frac{1}{\tau_i} * (-y(t)))$$

La misma ecuación del caso anterior con

tau\_value = 1/tau; donde tau=0.2, 0.4, 0.6, 0.8, 1, 1.2, 1.4, 1.6, 1.8, 2

## Efecto de la unidad temporal

---

```
%% Measures the effect of t -> h
%% h is the time step for integration
%% t is the length of simulation
```

```
function MinimalCTRNN2 (x, h, t)

    NumTimeSteps = t/h;

    tau = h/2;
    x = zeros(1, NumTimeSteps);
    oldx = x;
    X(1) = oldx;
    tau_value = 1/tau;

    for TStep = 1:NumTimeSteps
        newx = oldx + (h * (tau_value * -oldx));
        X(TStep+1) = newx;
        oldx = newx;
    end

    % Now display
    t = 0:NumTimeSteps;
```

When tau = h/2 the system now oscillates between -1 and 1 during integration steps.

```

max(x);

% str = sprintf('T_i= %g;', tau);

xlabel('time'), ylabel('output'), title('output vs time');
plot(t, X, str);
hold on;

tau = h;
x = zeros(1, NumTimeSteps);
oldx = x;
x(1)= oldx;
tau_value= 1/tau;

for TStep = 1:NumTimeSteps
    newx = oldx + (h * (tau_value * -oldx));
    x(TStep+1) = newx;
    oldx = newx;
end

% Now display
t = 0:NumTimeSteps;
max(X);

% str = sprintf('T_i= %g;', tau);

xlabel('time'), ylabel('output'), title('output vs time');
plot(t, x);
hold on;
hold off;

end

```

When  $\tau = h$  the output of the system follows the input, which in this case is zero. So the node 'decays' instantly

## Añadir estímulos

---

```

%% Minimal CTRNN3
%% Shows the effect of varying Tau and input for a CTRNN Node
%% h is the time step for integration
%% t is the length of simulation

function MinimalCTRNN3 (x, h, t)

    NumTimeSteps = t/h; % Num of integration steps
    HalfTimeStep = 2/h % halfway point
    tau = [0.5, 1, 2];
    I = [-4, 4];
    t = 0:NumTimeSteps;
    for i=1:3
        for j=1:2
            x = zeros(1, NumTimeSteps);
            oldx = x;
            x(1) = oldx;
            tau_value = 1 / tau(i);

            for TStep = 1:NumTimeSteps
                if (TStep < HalfTimeStep)
                    delta_x = tau_value * (-oldx + I(j));
                else

```

```

        delta_x = tau_value * (-oldx);
    %% l == 0
    end

    newx = oldx + (h * delta_x);
    x(TStep+1) = newx;
    oldx = newx;

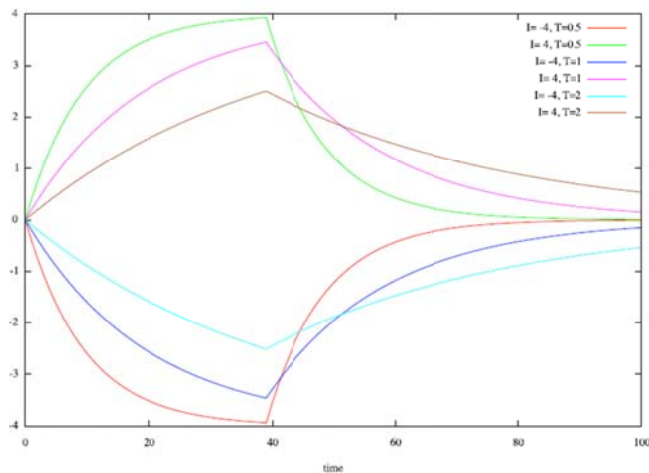
end

% Now display
% str = sprintf('l= %g, T=%g;', l(j), tau(i))
xlabel('time'), ylabel('output'), title('output vs time');
plot(t, x);
hold on;
end

end
hold off;

end

```



Se muestran los casos de  $\tau = [0.5, 1, 2]$  combinados con  $l = [-4, 4]$  para los casos en que  $l > 0$  (y cuando  $l=0$ )

## Función de activación (sigmoide)

Used Sigmoid Plot with bias = 0, and gain = 1

% Standard logistic activation function

```

function s = Sigmoid (x)
    s = 1;
    s = s / (1 + exp(-x));
end

```

```

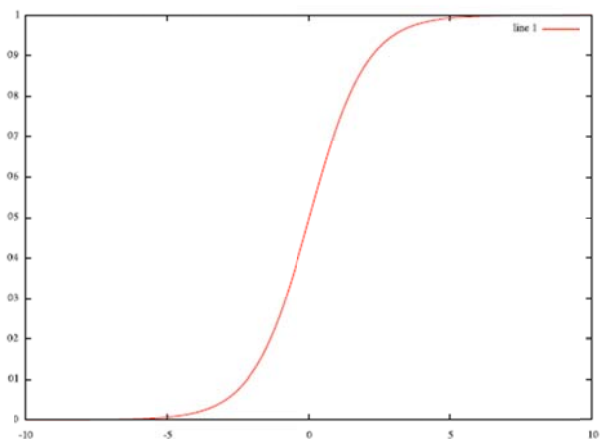
%% Plots the response of the Sigmoid function
%% b is the bias
%% g is the gain

```

```

function SigmoidPlot(b, g)
    X = -10:0.1:10;
    dim = size(X);
    length = dim(1,2);
    S = zeros(1, length);

```



```

% Calculate Sigmoid
    for i=1:length
        S(i) = Sigmoid(g * (X(i) + b));
    end
% str = sprintf('g=%g b=%g;', g, b);
    plot(X, S);
end

```

---

```

% Standard logistic activation function
% Sigmoid

```

```

function s = SigmoidDerivative (x)
    s = exp(-x);
    d = (1 + exp(-x)) ^ 2;
    s = s / d;
end

```

```

%% Plots the response of the derivative of the
%% Sigmoid function
%% b is the bias
%% g is the gain
function SigmoidDerivativePlot (b, g)

```

```

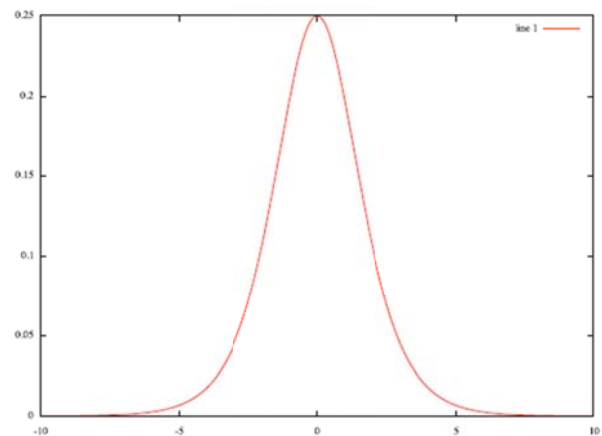
    X = -10:0.1:10;
    dim = size(X);
    length = dim(1,2);

    % Calculate change in sigmoid
    dS = zeros(1, length);

    %% Plot derivative
    %hold on;

    for i=1:length
        dS(i) = SigmoidDerivative(g *
(X(i) + b));
    end
    plot (X, dS);
end

```




---

## Efecto del umbral

Used SigmoidPlot to show effect of bias on Sigmoid Function.

```

%% Single recurrent CTRNN node with Bias
%% x is the initial value of the node
%% h is the time step for integration
%% t is the length of simulation

```

```

function BiasNode (x, h, t)

```

```

    NumTimeSteps = t/h;           % Num of integration steps
    tau = 1;

```

```

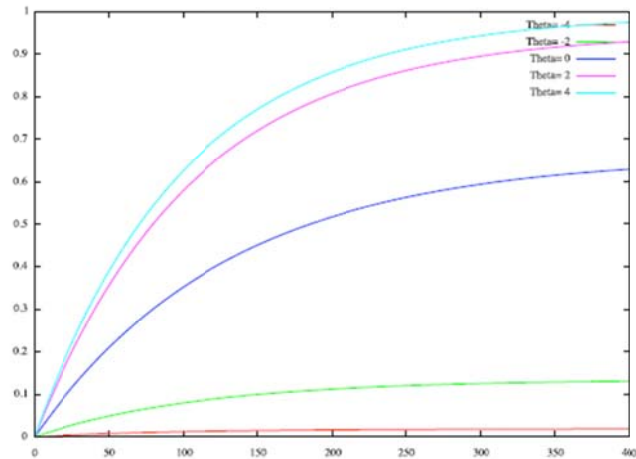
for theta = -4:2:4
    X = zeros(1, NumTimeSteps);
    oldx = x;
    X(1) = oldx;
    I = 0; %% no input
    w = 1; %% connection is on but no multiplier

    for TStep = 1:NumTimeSteps
        delta_x = -oldx + (w * Sigmoid(oldx + theta)) + I;
        newx = oldx + (h * delta_x);
        X(TStep+1) = newx;
        oldx = newx;
    end

    % Now display
    t = 0:NumTimeSteps;
    str = sprintf(';Theta= %g;', theta);
    %xlabel('time'), ylabel('output'), title('output vs time');
    plot(t, X, str);
    hold on;
end
hold off;

end

```



## Efecto del factor de escala de ganancia

Used SigmoidPlot to show effect of gain on Sigmoid Function.

```

%% Single recurrent CTRNN node with Gain
%% x is the initial value of the node
%% h is the time step for integration
%% t is the length of simulation

```

```

function GainNode (x, h, t)

    NumTimeSteps = t/h; % Num of integration steps
    tau = 1;

    for g = -5:5
        X = zeros(1, NumTimeSteps);
        oldx = x;
    end

```

```

X(1)      = oldx;
I         = 0;          %% no input
w         = 1;          %% connection is on but no multiplier
theta     = 1;

```

```

for TStep = 1:NumTimeSteps
    delta_x = -oldx + (w * Sigmoid(g*(oldx + theta))) + I;
    newx = oldx + (h * delta_x);
    X(TStep+1) = newx;
    oldx = newx;

```

```

end

```

```

% Now display
t = 0:NumTimeSteps;
str = sprintf('g= %g;', g);
xlabel('time'), ylabel('output'), title('output vs time');
plot(t, X, str);
hold on;

```

```

end

```

```

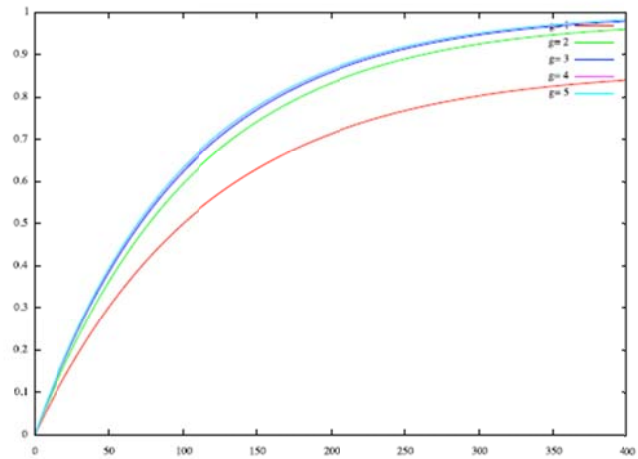
hold off;

```

```

end

```



## Un modelo completo

```
%% Minimal CTRNN node with recurrent connection
%% x is the initial value
%% h is the time step for integration
%% t is the length of simulation
```

```
function RecurrentNode (x, h, t)
    NumTimeSteps = t/h;    % Num of integration steps
    tau = 1;

    for w = -4:2:4
        X = zeros(1, NumTimeSteps);
        oldx = x;
        X(1) = oldx;
        I = 0;    % No input
        theta = 0;    % No bias

        for TStep = 1:NumTimeSteps

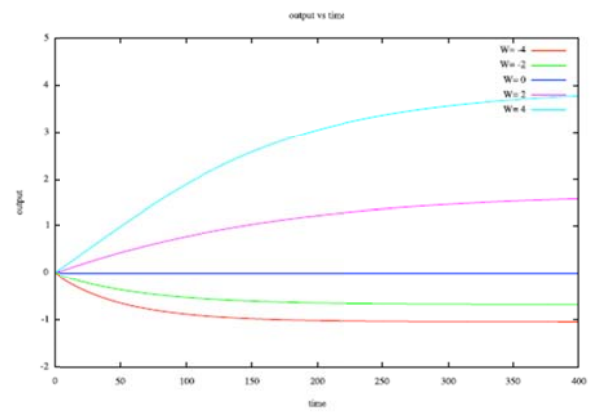
            delta_x = -oldx + (w * Sigmoid(oldx + theta)) + I;

            newx = oldx + (h * delta_x);
            X(TStep+1) = newx;
            oldx = newx;

            % Now display
            t = 0:NumTimeSteps;
            %str = sprintf('W= %g;', w);

            %xlabel('time'), ylabel('output'), title('output vs time');
            plot(t, X);
            hold on;

        end
        hold off;
    end
```



```
%% Shows phase portrait for a given weight and bias
```

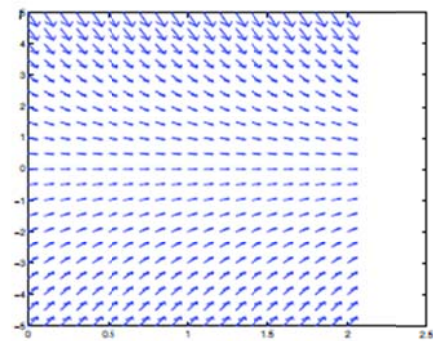
```
function FullNodePhase (w, bias)

    [T, X] = meshgrid([0:0.1:2], [-5:0.5:5]);
    dT = ones(size(T));
    %dX = -X + (w * ((1-exp(-X)).^(-1)));

    dX = -X + (w * ((1 + exp(-(X + bias))).^(-1)) );

    quiver(T,X,dT,dX)

end
```



[w = -5, bias = -5]

quiver(X,Y,U,V) es una función de Matlab que dibuja los vectores U, V con flechas en los puntos X, Y. Las matrices X, Y, U, V deben tener el mismo tamaño.



## Análisis de una CTRNN como un Sistema dinámico

```
% Plots the equilibria for %  $I = y - w * \sigma(y + bias)$ 
% w is the weight of the connection
% b is the bias

function PlotEquilibria (w, b)

    %  $I = y - w * \sigma(y + bias)$ 
    Y=-20:0.1:20;
    % plot for weight and bias
    str = sprintf('b=%g;', b);
    xlabel("I");
    ylabel("y");
    plot (Y - (w * ((1 + exp(-(Y + b))))).^(-1)) , Y, str);
end



---


% Plots the equilibrium as a function of I
% Will plot for a bias = -5, 0 and 5
% W is the weight of the connection

function PlotEquilibriaRange (w)

    %  $I = y - w * \sigma(y + bias)$ 
    Y=-20:0.1:20;

    % plot for bias = -5 b
    = -5;
    PlotEquilibria(w, -5);
    hold on
    % plot for bias = -0
    PlotEquilibria(w, 0);

    % plot for bias = 5
    PlotEquilibria(w, 5);
    hold off
end

% Plots the equilibrium as a function of I
% where b is the bias
% and w is the weight of the connection

function PlotEquilibria2 (w, bias)

    %  $I = y - w * \sigma(y + bias)$ 
    %  $y = f(x)$ 
    X=-20:0.1:20;

    % plot for bias
    str = sprintf('w=%g, b=%g;', w, bias);
    plot (X, X - (w * ((1 + exp(-(X + bias))))).^(-1)) , str)

    hold on
    %  $I = y$ 
```

```

plot(X, X);

end

hold off

function WeightMesh

    % I = y - w * sigma (y + bias)
    [y, w] = meshgrid(-20:20, -20:20);
    %i = y - (w * ((exp(-y) * (((1 + exp(-(y)))^(-1))^2) )));
    i = y - (w * ((1 + exp(-(y + 0)))^(-1)));
    xlabel('i');
    ylabel('w');
    mesh(i, w, y);

end

```