



Network Design Problems with Traffic Capture

Gabriel Gutiérrez-Jarpa 2020



Overview

- Rapid transit network design with modal competition
- Corridor-based metro network design with travel flow capture
- Bikeways and funiculars network design
- More research if we have more time.



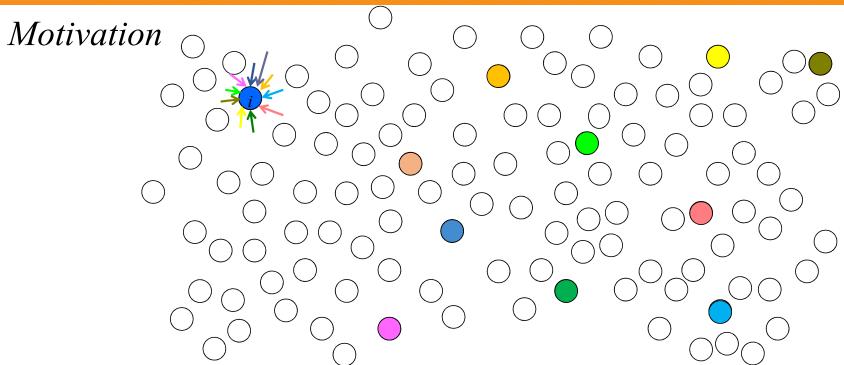


Rapid transit network design with modal competition

Luigi Moccia, Gilbert Laporte, Vladimir Marianov, and Gabriel Gutiérrez-Jarpa



Introduction



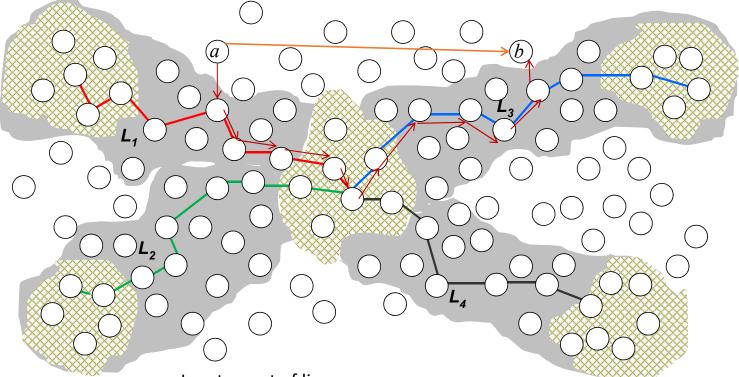


Introduction

Motivation O



Motivation



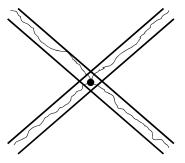
Locate a set of lines:

Maximizing traffic capture, maximizing the time savings of the captured flows, and minimizing total cost (Example: 4 lines.)

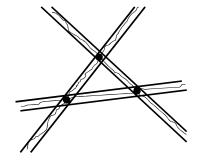


Introduction

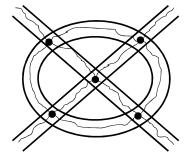
Motivation



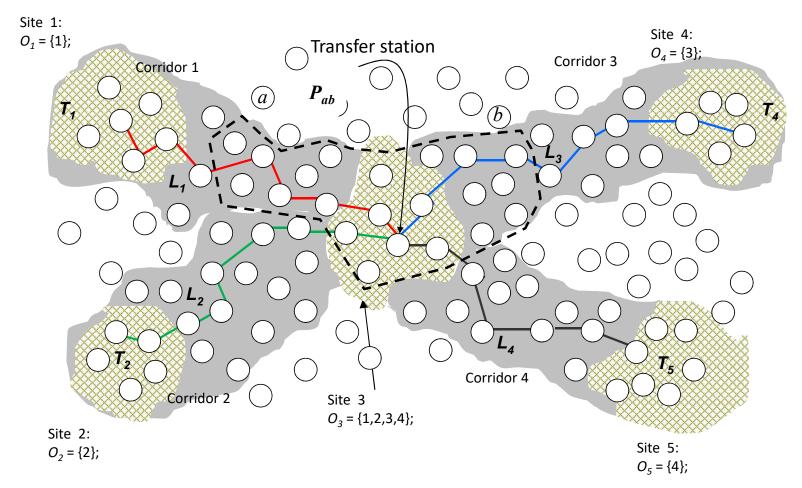
a) Four lines and one transfer point



b) Nine lines and tree transfer points



c) Twelve lines and five transfer points





Formulation of the RTNDC

Let G(N,E), where:

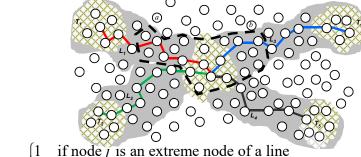
N set of nodes and E set of edges.

Variables of structure

$$x_{ij}^{s} = \begin{cases} 1 & \text{If edge } (i, j) \text{ belongs to the line } s \\ 0 & \text{otherwise} \end{cases}$$

$$w_i^s = \begin{cases} 1 & \text{if node } i \text{ is visited by line } s \\ 0 & \text{otherwise} \end{cases}$$

$$z_{ik} = \begin{cases} 1 & \text{if node } i \text{ is assigned to station } k \\ 0 & \text{otherwise} \end{cases}$$



$$y_{j} = \begin{cases} 1 & \text{if node } j \text{ is an extreme node of a line} \\ 0 & \text{otherwise} \end{cases}$$

$$ws_{i} = \begin{cases} 1 & \text{if node } i \text{ is a station} \\ 0 & \text{otherwise} \end{cases}$$

$$wt_{i} = \begin{cases} 1 & \text{if node } i \text{ is a transfer station} \\ 0 & \text{otherwise} \end{cases}$$

Variables of traffic captured and Travel Time

$$v_{ij} = \begin{cases} 1 & \text{if the demand of traffic between node} \\ i & \text{and } j \text{ is captured} \\ 0 & \text{otherwise} \end{cases}$$

$$r_{ab} = \begin{cases} 1 & \text{if travel time difference is positive,} \\ 0 & \text{otherwise} \end{cases}$$

 $\beta_{ab} \ge 0$, the travel time difference between car and transit for a passenger,



Formulation of the RTNDC

Objectives:

minimize
$$Z_c = \sum_{s \in S} \sum_{(i,j) \in E_s} c_{ij} x_{ij}^s + \sum_{i \in N^S} (cs_i ws_i + ct_i wt_i)$$
 Construction Cost

maximize
$$Z_{ts} = \sum_{a,b \in N: a < b} t_{ab} \beta_{ab}$$
 The time savings of the captured flows

maximize
$$Z_p = \sum_{a,b \in N: a < b} t_{ab} v_{ab}$$
 Captured flows

$$\sum_{i \in T_k} \sum_{j \in N_s \setminus T_k} \left(x_{ij}^s + x_{ji}^s \right) \ge 1$$

$$\sum_{j \in N_s} \left(x_{ij}^s + x_{ji}^s \right) = 2w_i^s - y_i$$

$$\sum_{i \in T_k} y_i = 1$$

$$ws_i + wt_i \le 1$$

$$w_i^s + w_i^m \le ws_i + 2wt_i$$

$$\sum_{i,j \in Q} x_{ij}^s \leq \sum_{t \in Q \setminus \{q\}} w_t^s \qquad \qquad \text{doctorado en ingeniería industrial}$$

Structure of the network



Formulation of the RTNDC

$$\sum_{k \in N(i)} z_{ik} \le 1$$

$$z_{ik} \le \sum_{s \in S} w_k^s$$

$$v_{ij} \le \sum_{k \in N(i)} Z_{ik}$$

$$v_{ij} \le \sum_{k \in N(j)} Z_{jk}$$

Assignment

Traffic capture

Time

$$\beta_{ab} \leq T_{ab} - \left[\sum_{s \in S_{ab}} \sum_{(i,j) \in P_{ab}} \tau_{ij} x_{ij}^s + \sum_{k \in N(a)} \delta_{ak} z_{ak} + \right]$$

$$\sum_{k \in N(\mathbf{b})} \delta_{bk} z_{bk} + \phi_{ab} \left[+ M_{ab}^{-} \left(1 - r_{ab} \right) \right]$$

$$\beta_{ab} \leq M_{ab}^+ v_{ab}$$

$$v_{ab} \le r_{ab}$$

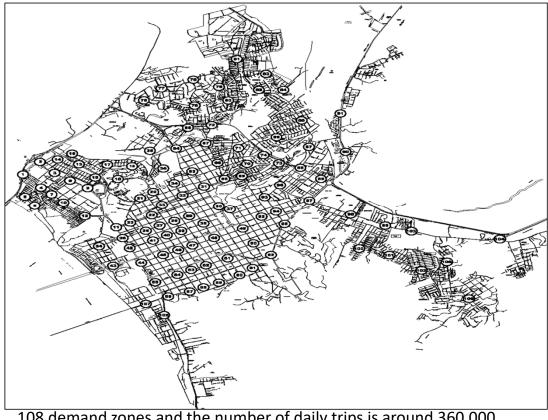
Traffic captured

Value of the variables



Result Concepción, Chile Concepción City, Chile

Parameter	Value	Unit of measure
Walking speed	5	km/h
Bus speed	15	km/h
Car speed downtown	10	km/h
Car speed peripheral area	20	km/h
Metro speed	40	km/h
Time length of a metro stop	60	S
Transfer time metro-metro	180	s
Transfer time walk-metro	60	s
Transfer time bus-metro	60	s

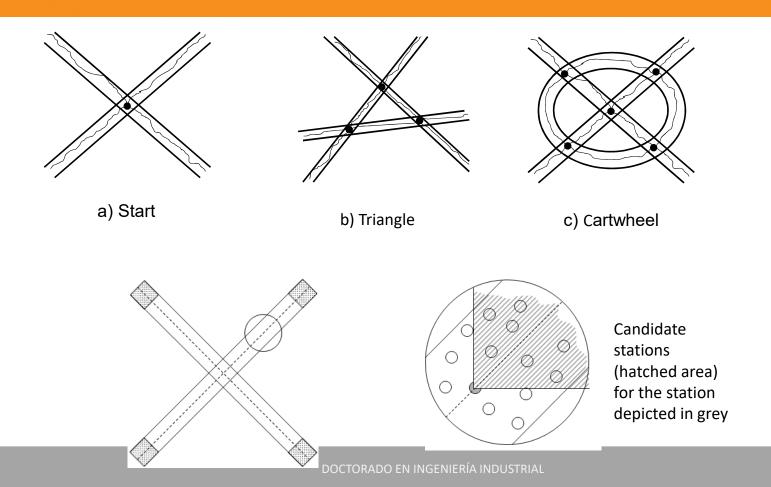


Parameter	Value	Unit of measure
Line cost	60	million US\$/km
Station cost	10	million US\$ per station
Transfer station cost	25	million US\$ per station

108 demand zones and the number of daily trips is around 360,000.



Result Concepción, Chile





Result Concepción, Chile

CPLEX 12.6.1 with AMPL, on a PC Intel Core i7 at 3.4GHz, with 8 GB RAM, and Windows 7 as OS.

maximize
$$Z_{ts} = \sum_{a,b \in N: a < b} t_{ab} \beta_{ab}$$
 (2) Or

maximize
$$Z_p = \sum_{a,b \in N: a < b} t_{ab} v_{ab}$$
 (3)

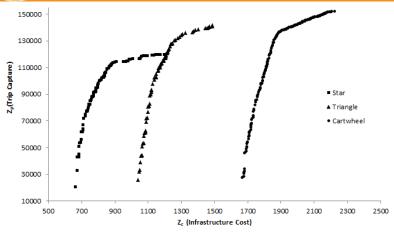
$$Z_{c} = \sum_{s \in S} \sum_{(i,j) \in E_{s}} c_{ij} x_{ij}^{s} + \sum_{i \in N^{S}} \left(cs_{i} ws_{i} + ct_{i} wt_{i} \right) \leq Z_{c}^{\max}$$

(4) - (16), value of variables

Structure	Number	of variables		# constraints	Z_p		Z_{ts}				
	Binary	Continuous	Total		Inst.	Time	Inst.	Time			
Star	10,457	3,788	14,245	25,462	55	2,920	12	29,809			
Triangle	9,689	3,788	13,477	24,734	51	2,593	12	18,563			
Cartwheel	8,966	3,788	12,754	24,029	57	2,249	12	15,756			

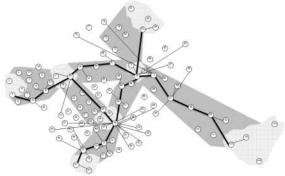


Results $(Z_c v/s Z_p)$

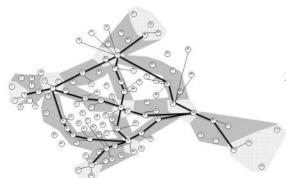


 Z_p =113,770, Z_c =888, and Z_{ts} =89,452

Trade-off curves, Z_p vs $Z_{c\prime}$ for the three configurations



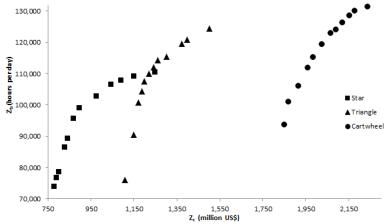




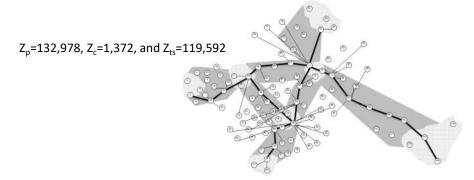
 Z_p =138,208, Z_c =1,905, and Z_{ts} =105,040

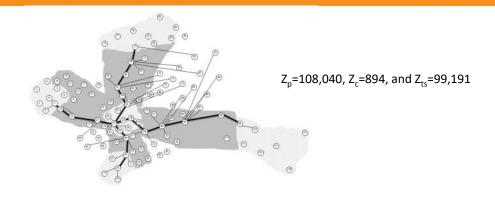


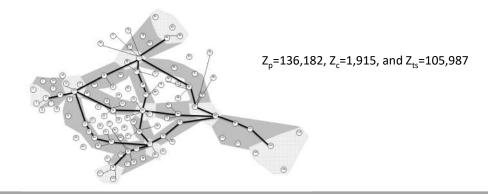
Results (Z_c v/s Z_{ts})



Trade-off curves, Z_{ts} vs $Z_{c\prime}$ for the three configurations









Multi-objective rapid transit network design with modal competition: The case of Concepción, Chile

Computers & Operations Research, , Volume 78, February 2017, Pages 27-43 Gabriel Gutiérrez-Jarpa, Gilbert Laporte, Vladimir Marianov, Luigi Moccia





Corridor-based metro network design with travel flow capture

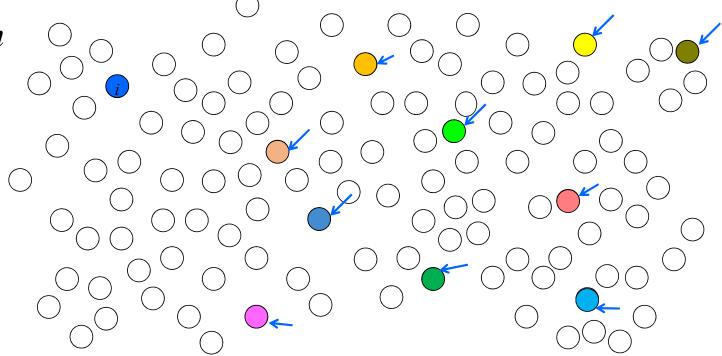
by

Gilbert Laporte, Vladimir Marianov, and Gabriel Gutiérrez-Jarpa



Introduction

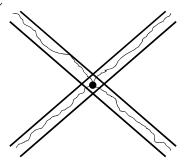
Motivation



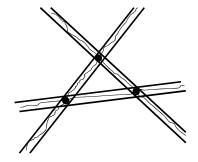


Introduction

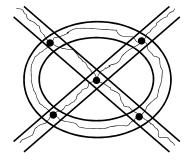
Motivation



a) Four Lines and one transfer point



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Stage I: Greedy generation heuristic

For each $(i,j) \in N_{EX}$, compute the travel time between each pair of nodes $a, b \in N_{O/D}$ lying inside the corridor spanned by (i,j) Compute the traffic capture of each corridor

 $S = \emptyset$

repeat

Add those corridors to S for which a set of rules holds, **until** No more corridors can be added for which the rules hold.

Stage II: Estimation of O/D travel times, captured traffic and captured O/D pairs

for each O/D pair contained in the corridors in S do

Estimate the travel times between all the O/D pairs in the corridor

if Travel time by metro \leq Travel time by other mode then

O/D pair is candidate to be captured

The traffic volume captured depends on the ratio of travel times by alternative modes

end if

end for



Stage III: Selection of corridors and adjustment of real capture

repeat

Select the maximum capture subset of corridors using an Integer Programming Formulation (P)

z = Solution of the IP Formulation

Recalculate all travel times using the selected corridors, update captured O/D-pairs, and compute z_{actual} = actual capture using selected corridors. For each O/D pair, update capture using the actual capture.

until
$$|z-z_{actual}|/z \le \varepsilon$$

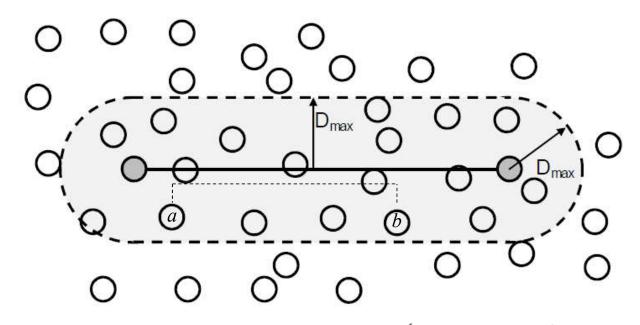
Stage IV: Opening of transfer stations on corridor crossings

Open new transfer stations at corridor crossings.

Recalculate all travel times, captured OD-pairs demand, and zactual



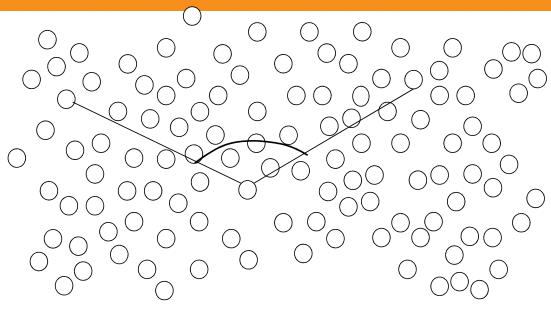
Stage I: Greedy generation heuristic



$$\beta = \frac{\text{Metro_Time(a,b)}}{\text{Alternative_Time(a,b)}} \quad \textbf{(1)} \quad \text{\%Traffic _ Captured } (a,b) = \begin{cases} 0 & 1.00 < \beta \\ 25 & 0.75 \le \beta \le 1.00 \\ 50 & 0.50 < \beta \le 0.75 \\ 75 & 0.25 < \beta \le 0.50 \\ 100 & 0.00 < \beta < 0.25. \end{cases} \quad \textbf{(2)}$$



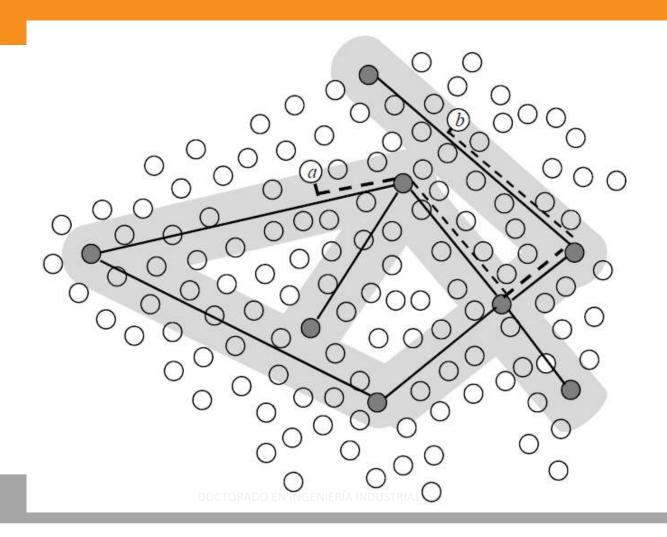
Stage I: Greedy generation heuristic



- 1) At least one of its extremes must coincide with an extreme of a corridor already in S,
- 2) The angle it makes with any of the other corridors having the same extreme nodes, must be at least equal to a prespecified minimum value θ^{min} ,
- 3) Its construction cost plus the total cost of the corridors already in S cannot exceed αC_{max} , where α is an input parameter and C_{max} is a budged, and
- 4) Adding any other corridor that is feasible in the sense of points 1–3 to the set, instead of s, will not result in a higher captured traffic.



Stage II: Estimation of O/D travel times, captured traffic and captured O/D pairs





Stage III: Selection of corridors and adjustment of real capture

(P) maximize
$$z = \sum_{(a,b)\in\psi} t_{ab}v_{ab}$$
 (3)

$$\sum_{s \in S} c_s y_s + \sum_{i \in N_c} g_i w_i \le C_{max}$$
(4)

$$\sum_{i \in N_c} g_i = 1 - max \\
y_s + y_m \le 1 + w_i \quad i \in N_c, s, m \in P_i \\
h_{ms} \le y_s \quad s, m \in S : s \le m$$
(5)
$$\sum_{\substack{(t,i) \in A_c^0 : t \ne i_s, j_s \\ f_{imjm}^s + f_{jmim}^s}} f_{it}^s = \sum_{\substack{(i,t) \in A_c^0 : t \ne i_s, j_s \\ f_{imjm}^s + f_{jmim}^s}} f_{it}^s \quad s \in S, i \in N_c : i \ne i_s, j_s$$
(13)

$$h_{ms} \le y_s$$
 $s, m \in S : s \le m$ (6) $f_{i-1}^s + f_{i-1}^s \le y_m$ $s, m \in S$ (14)

$$h_{ms} \le y_s$$
 $s, m \in S : s \le m$ (6) $f_{i_m j_m}^s + f_{j_m i_m}^s \le y_m$ $s, m \in S$ (14) $h_{ms} \le y_m$ $s, m \in S : s \le m$ (7) $f_{0j}^s \le x_j$ $s \in S, j \in N_c$ (15)

$$r_{ab}^{sm} \le h_{sm}$$
 $(a, b) \in \psi, (s, m) \in R_{ab}$ (8)
$$\sum_{j \in N} x_j \le 1$$
 (16)

$$\sum_{(s,m)\in R_{ab}} r_{ab}^{sm} \le 1 \quad (a,b) \in \psi \qquad (9)$$

$$\sum_{i \in N_c} r_j^{sm} \le 1 \quad (a,b) \in \psi \qquad (10)$$

$$y_m + y_s \le 1 \quad i \in N_c, m, s \in P_i : \theta_{ms} < \theta^{min} \qquad (17)$$

$$v_{ab} \le \sum_{(s,m) \in R_{ab}} p_{ab}^{sm} r_{ab}^{sm} \quad (a,b) \in \psi$$
 (10) $y_s \in \{0,1\} \quad s \in S$

$$\sum_{i \in N_c} f_{0j}^s = y_s \quad s \in S \tag{11}$$

$$w_i \in \{0, 1\} \quad i \in N_c \tag{19}$$

$$v_{ab} \in \{0, 1\}$$
 $v_{ab} \in \{0, 1\}$ $v_{ab} \in \{0, 1\}$

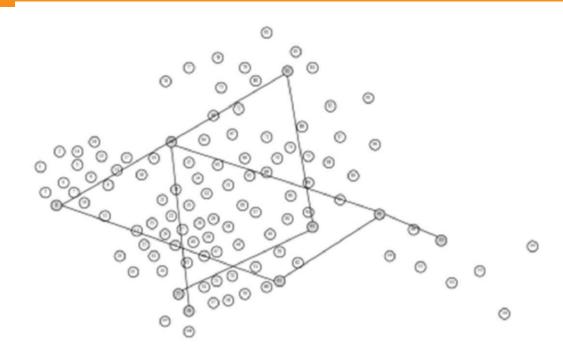
$$\sum_{t,i_s) \in A_c^0} f_{ti_s}^s + \sum_{(t,j_s) \in A_c^0} f_{tj_s}^s = y_s \qquad s \in S$$

$$h_{sm} \in \{0,1\} \qquad s, m \in S : s \le m$$
(21)

$$\sum_{(t,i)\in A_{2}^{0}:t\neq i_{s},j_{s}} f_{ti}^{s} = \sum_{(i,t)\in A_{2}^{0}} f_{it}^{s} \quad s \in S, i \in N_{c}: i \neq i_{s},j_{s}$$
(13)
$$0 \leq f_{ij}^{s} \leq 1 \quad s \in S, (i,j) \in A_{c}^{0}$$
(22)



Stage IV: Opening of transfer stations on corridor crossings





Result Concepción, Chile

Concepción City, Chile

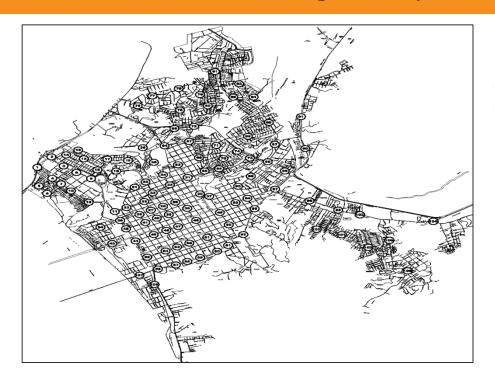


Table 1. Parameters.

Description	Value
$\mathcal{D}_{ ext{max}}$	500 m
L	500 m
Total time to take the metro	120 s
Transfer time	240 s
Walking speed	4 km/h
Metro speed	60 km/h
Stop time at station	60 s
ϵ	5 %
Line cost	60 millions US\$/km
Transfer station cost	25 millions US\$

108 demand zones and the number of daily trips is around 360,000.

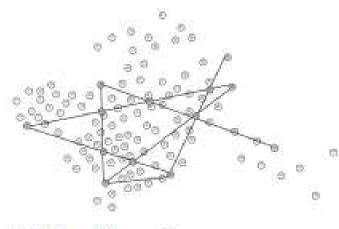


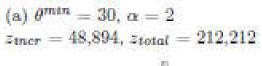
Solutions

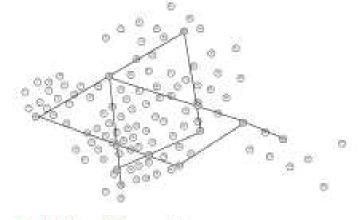
#	$oldsymbol{ heta}^{ ext{min}}$	Stage II	ige III Stage IV								s															#	$oldsymbol{ heta}^{ ext{min}}$	Stage l	II				St	age IV	7			s		
		Cactual	Pr	Zactual	z	Er	Itr 1	vs g	gi	z_{iner}	z_{total}	%C																		Cactual	Pr	Zactual	z	Er I	r N	g_i	z_{incr}	z_{total}	%C	
1	0	4,340	0.9	120,510	120,510	0	1 5	1	125	54,646	175,156	45	24.2															1	0	4,471	1	137,056	138,798	1.3 1	10	25	0 46,700	183,756	34	36.3
2	10	4,537	1	144,894	144,964	0	1 7	1	175	37,588	182,482	26	27.8															2	10	4,537	1	144,894	145,094	0.1 1	7	17	5 37,588	182,482	26	38
3	20	4,616	1	170,302	170,302	0	1 7	1	175	43,420	213,722	26	28.1															3	20	4,616	1	170,302	170,302	0 1	7	17	5 43,420	213,722	26	38.2
4	30	4,438	1	163,318	163,318	0	1 6	5 1	150	48,894	212,212	30	27.1															4	30	4,438	1	163,318	163,318	0 1	6	15	0 48,894	212,212	30	35.1
5	40	4,494	1	117,188	117,188	0	2 4	1	100	29,206	146,394	25	34.1															5	40	4,569	1	116,986	119,346	2 2	7	17	5 39,768	156,754	34	52.9
6	50	3,644	0.8	113,688	117,126	2.9	1 4	1	100	43,080	156,768	38	23.8															6	50	4,221	0.9	151,186	156,560	3.4 1	4	10	0 54,770	205,956	36	42.6
7	60	4,077	0.9	124,956	126,892	1.5	1 4	1	100	39,280	164,236	31	25.6	#	$oldsymbol{ heta}^{ ext{min}}$	Stage I	п					Stag	ge IV				s	7	60	4,606	1	130,392	135,504	3.8 1	6	15	0 47,694	178,086	37	54.3
8	70	2,354	0.5	79,534	79,534	0	1 0) (0	0	79,534	0	23.2			Cactual	Pr	z_{actual}	z	Er	Itr	NS	gi	z_{iner}	z_{total}	%C		8	70	4,569	1	117,096	121,826	3.9 2	3	75	39,426	156,522	34	52.1
9	80	2,950	0.6	89,464	90,570	1.2	1 2	5	50	11,724	101,188	13	22.6	1	0	4,570	1	149,942	149,942	0	1	6	150	65,502	215,444	44	76.3	9	80	4,542	1	138,916	142,724	2.7 2	4	10	0 38,502	177,418	28	64.8
10	90	3,574	0.8	83,558	87,764	4.8	1 1	. 2	25	10,884	94,442	13	22.5	2	10	4,587	1	151,594	153,820	1.4	1	8	200	43,272	194,866	29	81.3	10	90	4,437	1	111,590	113,668	1.8 3	7	17	5 21,214	132,804	19	117.6
11	100	3,960	0.9	79,480	79,480	0	2 2		50	6,996	86,476	9	35.3	3	20	4,616	1	170,302	170,302	0	1	7	175	43,420	213,722	26	59	11	100	3,960	0.9	79,480	79,480	0 2	2	50	6,996	86,476	9	35.4
	Av.	3,908	0.8	116,990	117,968	1	1 4	9	95	29,611	146,601	23	26.8	4	30	4,556	1	177,420	178,238	0.5	1	7	175	40,084	217,504	23	96.3		Av.	4,452	1	132,838	135,147	1.7 2	6	14	38,634	171,472	28	51.6
		Table 2	2. Cap	ture for	$\alpha = 2, \beta \ge 0$).75.								5	40	4,354	0.9	118,196	122,224	3.3	1	5	125	47,214	165,410	40	65.6	Table 3. Capture for $\alpha = 4$, $\beta \ge 0.75$.												
														6	50	4,619	1	156,434	161,444	3.1	1	5	125	60,242	216,676	39	158.7													
														7	60	4,606	1	130,392	135,048	3.4	2	6	150	47,694	178,086	37	361.7													
														8	70	4,569	1	117,096	122,466	4.4	4	3	75	39,426	156,522	34	517.9													
														9	80	4,564	1	140,260	144,906	3.2	3	6	150	42,310	182,570	30	694.9													
														10	90	4,477	1	123,756	126,906	2.5	4	5	125	40,750	164,506	33	1,908.8													
														11	100	3,960	0.9	79,480	79,480	0	2	2	50	6,996	86,476	9	36.4													
															Av.	4,498	1	137,716	140,434	2	2	5	136	43,355	181,071	31	368.8													
																		7	Гable 4. Са	pture	for α	τ = 6,	β≥0.	75.																



Network structures



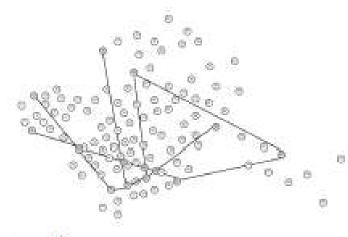




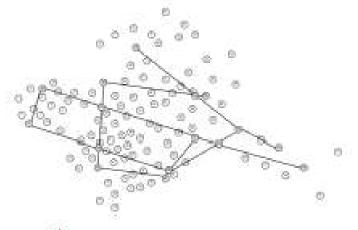
(b)
$$\theta^{min} = 60$$
, $\alpha = 2$
 $z_{incr} = 39,280$, $z_{total} = 164,236$



Network structures



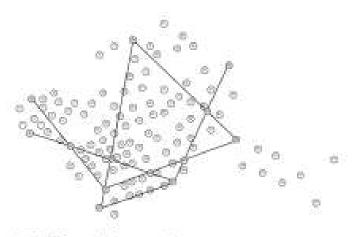
(c)
$$\theta^{min} = 50$$
, $\alpha = 4$
 $z_{incr} = 54,770$, $z_{total} = 205,956$

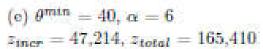


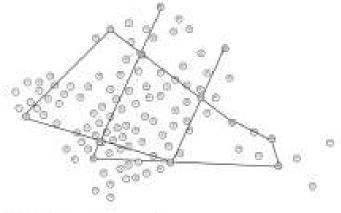
(d)
$$\theta^{min} = 80$$
, $\alpha = 4$
 $z_{incr} = 38,502$, $z_{total} = 177,418$



Network structures



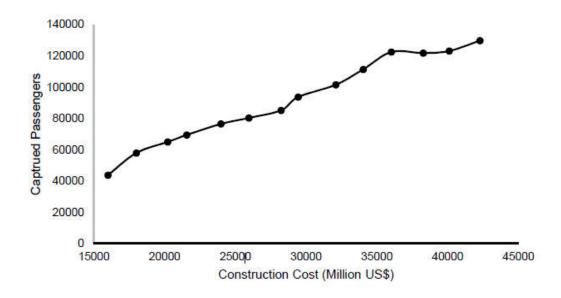




(f)
$$\theta^{min} = 70$$
, $\alpha = 6$
 $z_{iner} = 39,426$, $z_{total} = 156,522$



Construction Cost v/s Captured Passengers





Corridor-based metro network design with travel flow capture

Computers & Operations Research, Volume 89, January 2018, Pages 58-67 Gabriel Gutiérrez-Jarpa, Gilbert Laporte, Vladimir Marianov





Bikeways and funiculars network design

by

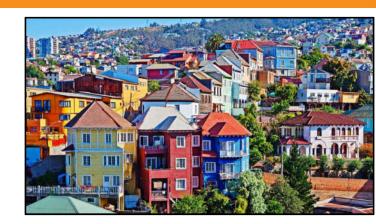
Pablo Torrealba, German Paredes and Gabriel Gutiérrez-Jarpa





Radio Cooperativa

Parkin (2014) and Gonzalo-Orden et al(2014)





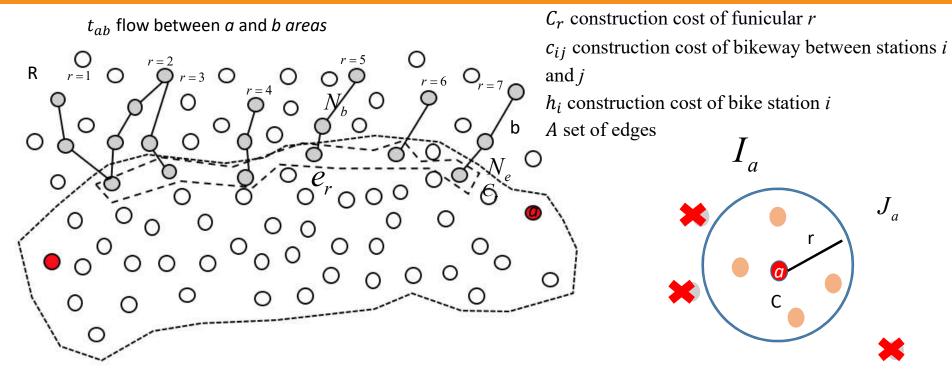




Google Earth Pro



Formulation





Formulation

$$Min Z_1 = \sum_{(i,j)\in A} c_{ij} x_{ij} + \sum_{r\in R} C_r y_r + \sum_{i\in N_b} h_i g_i \sum_{(i,j)\in A} x_{ij} \ge \sum_{i\in N_b} (g_i + w_i) - 1$$

$$Mix Z_2 = \sum_{(a,b)\in\Psi} t_{ab} v_{ab}$$

$$\sum_{(i,j)\in A} x_{ij} \ge \sum_{i\in N_b} (g_i + w_i) - 1 \tag{3}$$

$$\sum_{i/(i,j)\in A} x_{ij} + \sum_{i/(i,j)\in A} x_{ij} \ge g_i + w_i \qquad \forall i \in N_b$$
 (4)

$$\sum_{j/(i,j)\in A} x_{ij} + \sum_{j/(j,i)\in A} x_{ij} \le |M_i| g_i + w_i \qquad \forall i \in N_b$$
 (5)

$$g_i + w_i \le 1 \qquad \forall i \in N_b \tag{6}$$

$$y_r \le g_{e_n} \qquad \forall r \in R \tag{7}$$

$$v_{ab} \le \sum_{i \in I_{-}} g_i + \sum_{r \in J_{-}} y_r \qquad \forall (a,b) \in \Psi$$
 (8)

$$v_{ab} \le \sum_{i \in I_b} g_i + \sum_{i \in J_b} y_r \qquad \forall (a, b) \in \Psi$$
 (9)

$$\sum_{i \in S, j \in S: (i,j) \in A} x_{ij} \le |S| - 1 \qquad \forall S \subseteq N_b / |S| > 2 \qquad (10)$$

Binary value of decision variables

Formulation

$$\sum_{\substack{i \in S, j \in S \\ /(i,j) \in \mathcal{A}}} x_{ij} \le |S| - 1$$

$$; \forall S \subset N_b/3 \le |S| < |N_b| \tag{9}$$

- Branch and Cut Method
- Separation algorithm to identify subtours constraints





http://www.sectra.gob.cl/encuestas_movilidad/encuestas_movilidad.htm

220 demand zones and the number of daily trips is around 835,000



Network





Bikeways

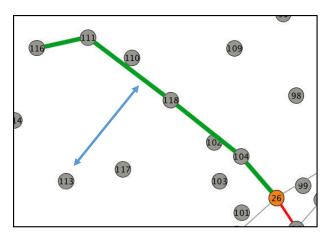




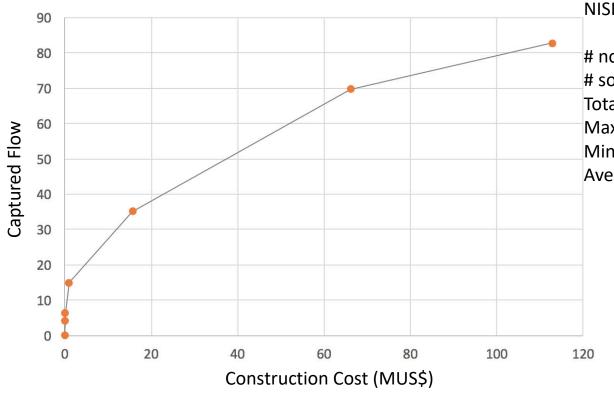
Funiculars

- Each funicular includes stop stations
- The distance between a node and a funicular was calculated using google map.









NISE Method to identify non-inferior solutions.

non-inferior solutions : 9

solved Instances: 12 Total Time (s): 81,048.

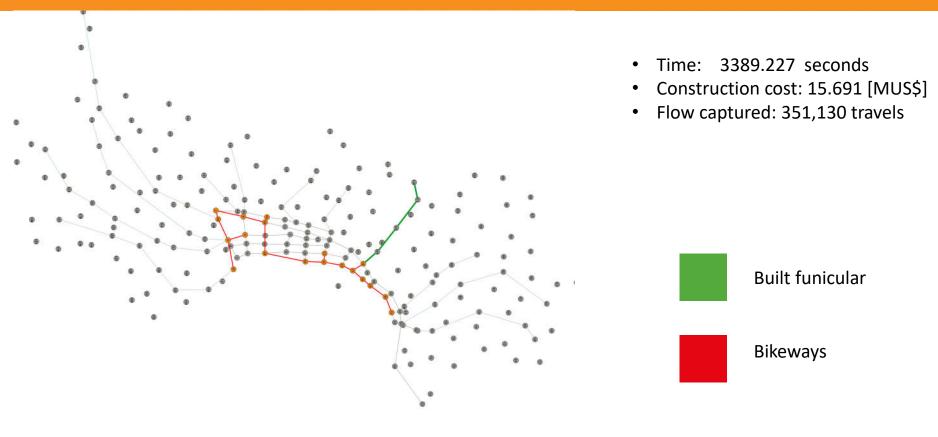
Maximum : 25,200

Minimum: 3,001

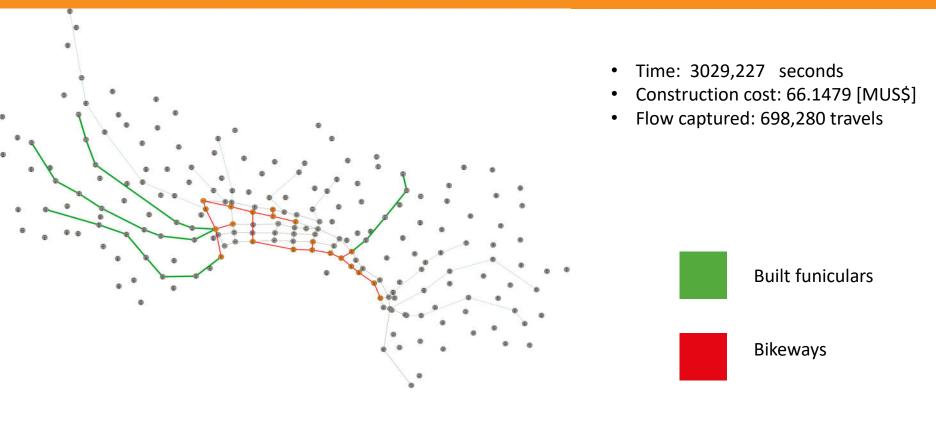
Average time: 6,754

Eclipse IDE for Java Developers, Version:
Oxygen.1a Release (4.7.1a)
CPLEX 12.8.0 with AMPL, on a PC Intel
Core i7-4770CPU, with 8 GB RAM, and
Windows 7 as OS.

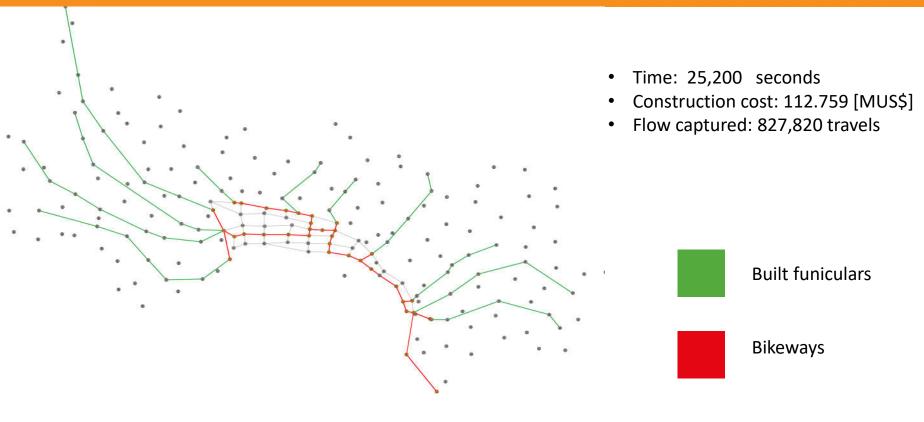














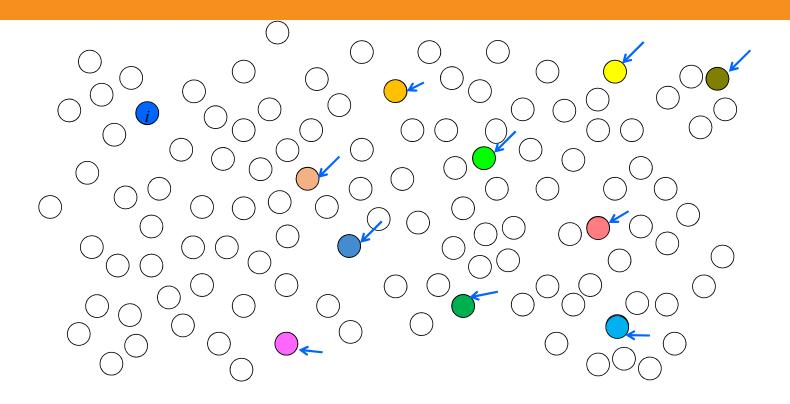
Conclusion and future research

- A first formulation was presented for bikeways and funiculars network design.
- The average time to solve the instances is too large.

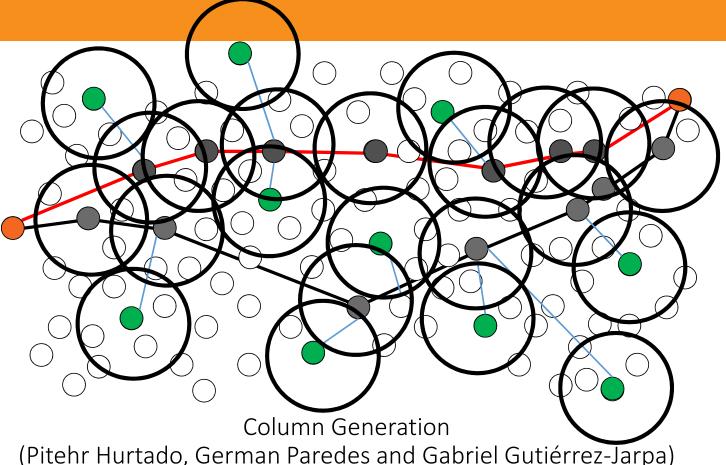
FUTURE RESEARCH

- e-constraints method to identify more non-inferior solutions.
- Include competition to design the network (bus).
- Design each funicular identifying stop stations.
- Distribution network design.









(Pitehr Hurtado, German Paredes and Gabriel Gutiérrez-Jarpa)



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