

RESEARCH SEMINAR

Forecasting with Deep Temporal Hierarchies

Personal Details

Filotas Theodosiou: MSc Data Science

Research Associate at VIVES University of Applied Sciences

Email: filotas.thedosiou@vives.be

LinkedIn: https://www.linkedin.com/in/filotas-theodosiou-4916881a1/

Main Resources:

1. Forecasting with Deep Temporal Hierarchies: A novel way for forecasting with temporal hierarchies based on deep learning models. - Master Thesis -

- Theodosiou, Filotas & Kourentzes, Nikolas. (2021) Forecasting with Deep Temporal Hierarchies.
 SSRN Scholarly Paper ID 3918315. Social Science Research
- 3. Theodosiou, Filotas & Kourentzes, Nikolaos. (2021). Deep Learning Temporal Hierarchies for Interval Forecasts. International Conference of AI in Finance. Time Series Workshop



Content

- Why Temporal Hierarchies?
- Temporal Hierarchies in a Nutshell
- Non-Linear Data-Driven Reconciliation
 - Positional Information
 - Variance Scaling
 - Weight Optimization
- Experiment
- End-to-End Framework
- Prediction Intervals
- Conclusions

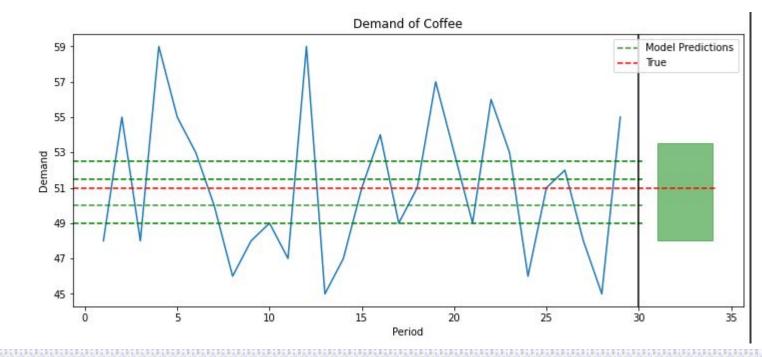
Why Temporal Hierarchies? Model Selection Uncertainty

We forecast the demand of coffee.

- We assume demand is independent of weather/price/location
- Constant demand with mean = 51 with some noise

We generate predictions using different models But we get high forecast uncertainty

- Model selection uncertainty
- Parametric uncertainty
- Forecast uncertainty originated from noise



Why Temporal Hierarchies? Forecast Combinations

In every forecasting problem we look for a "good-enough" model.

- A non trivial and usually difficult task
- Let's avoid it!

Forecast Combinations

- Instead of looking for the "best-model", combine diverse predictions
- Simple combinations show accuracy benefits

How to get diverse predictions?

Very bad forecasts will harm the combined predictions





Temporal Hierarchies in a Nutshell Introduction to Temporal Hierarchies

How to get diverse predictions?

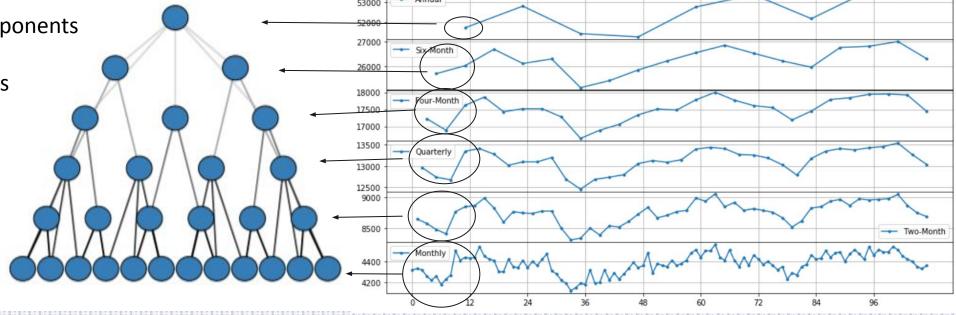
Change your viewpoint through Temporal Aggregations

Temporal Aggregations:

 Filter high frequency components (seasonality)

 Boost high frequency ones (trend)

Reveal hidden structural information





Temporal Hierarchies in a Nutshell Introduction to Temporal Hierarchies

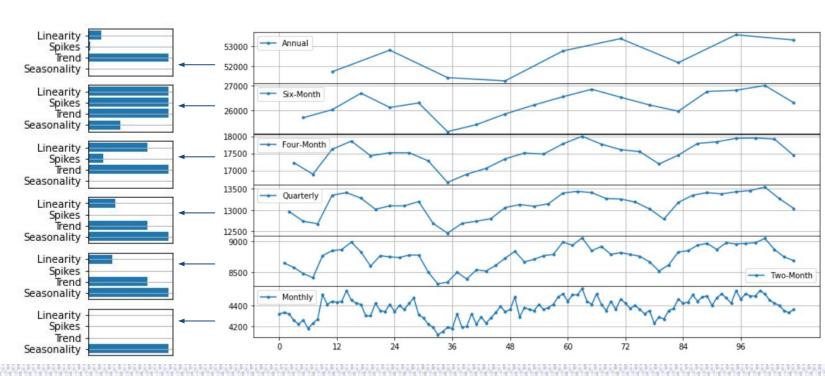
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Change your viewpoint through Temporal Aggregations

The Diversity Among Levels

Features are strengthen/weaken at each level

What if we combine information from each level into a single forecast?



Features/components estimated using tsfeatures in R

- Spikes measures spikness. Measured as the variance of the leave-one-out variances of the remainder component after STL
- Linearity is measured based on the coefficients of an orthogonal quadratic regression



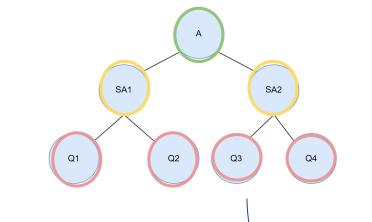
Temporal Hierarchies in a Nutshell How to Forecast with Temporal Hierarchies (THieF)

Hierarchical Forecasting.

- Bottom up
- Top down -> Biased

Temporal Hierarchy Forecasts (THieF)

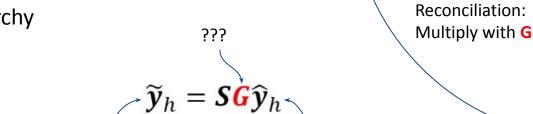
- 1. Generate Base Forecasts \hat{y}_h
- 2. Reconcile (Just a fancy word for combine) into the bottom level
- 3. Reconstruct the entire hierarchy



$$b = (Q1, Q2, Q3, Q4)$$

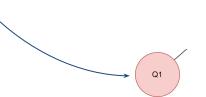
$$y = Sb$$

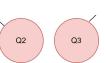




Reconciled coherent / predictions

Base forecasts









Panagiotelis, A., Athanasopoulos, G., Gamakumara, P., & Hyndman, R.J. (2021). Forecast reconciliation: A geometric view with new insights on bias correction. International Journal of Forecasting, 37, 343–359.

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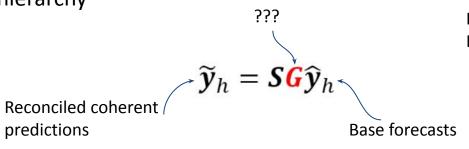
Q1 Q2 Q3 Q4

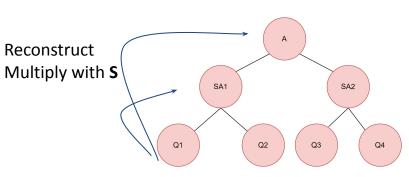
b = (Q1, Q2, Q3, Q4) **y** = (A, SA1, SA2, **b**) **y** = **Sb**

$$S = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$
 Top level
Middle level(s

Temporal Hierarchy Forecasts (THieF)

- 1. Generate Base Forecasts $\,\widehat{oldsymbol{y}}_h$
- 2. Reconcile (Just a fancy word for combine) into the bottom level
- 3. Reconstruct the entire hierarchy









Temporal Hierarchies in a Nutshell Estimate G

Optimal Reconciliation Matrix G: $G = (S'W_h^{-1}S)^{-1}S'W_h^{-1}$ (in terms of least squares)

 \boldsymbol{W}_h is the positive definite covariance matrix of base forecast errors.

Problem: Estimating W_h in practise is very challenging.

Solution: Again, avoid the problem and rely on approximations

Two diagonal estimators which will be useful later:

- 1. Structural Scaling: A diagonal matrix containing the number of errors contributing to each level.
- 2. Variance Scaling: A diagonal matrix with the in-sample one-step-ahead error variance of each level.

Possible Limitations:

- 1. Strict Linearity → limited capacity
- 2. Partial Covariance Estimation partial consideration of relationships between levels
- 3. Assumptions each estimators relies on a number of assumptions

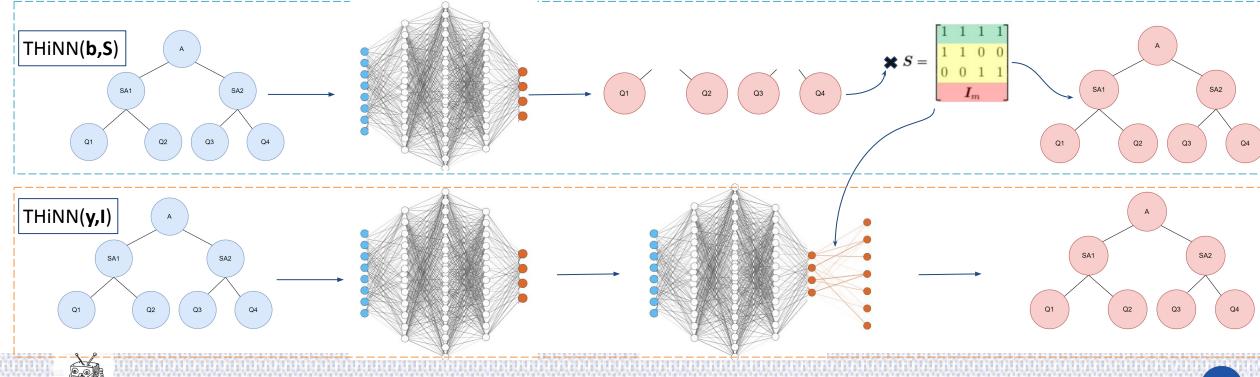




Temporal Hierarchy Neural Network (THINN) A Non-Linear Data-Driven Reconciler

Approximate G with Temporal Hierarchy Neural Network (THINN)

- NNs: Non-Linear function approximators with success in numerous tasks
- Relax classical restrictions
- Preference over classical ML (Gradient Boosted Decision Trees) due to architectural freedom.

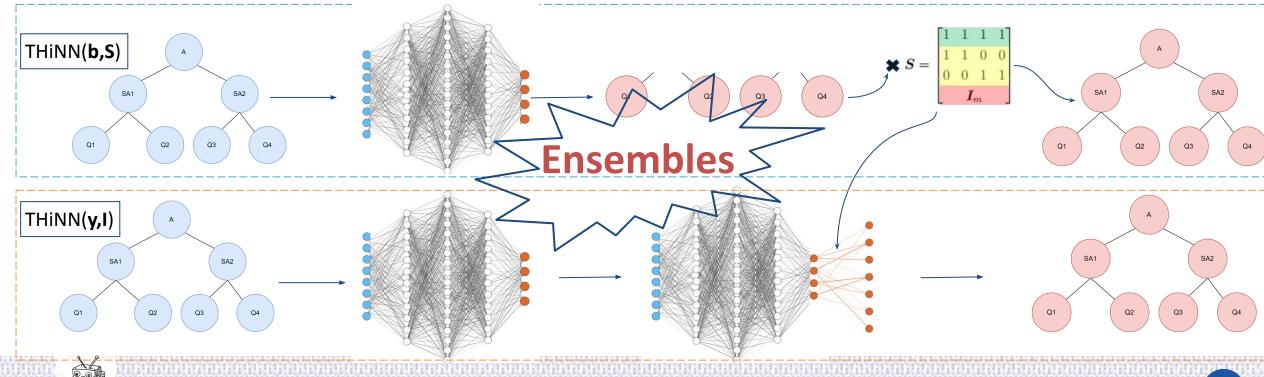




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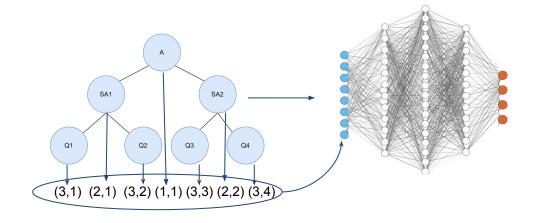
Temporal Hierarchy Neural Network (THiNN) Positional Information

Limitation of Feed Forward NNs:

- Input is random flatten values
- No positional information
- No level-specific information
- Hard time understanding values

Positional Encoding:

- Inspired by Transformers
- Described the location (level) and the position (time-step) of the forecast.
- Pair of type (level, time-step) -> fails on large hierarchies
- Pairs of (cos,sin)
- One-hot-encoded values





Temporal Hierarchy Neural Network (THiNN) Connection between Base and Reconciled Forecasts

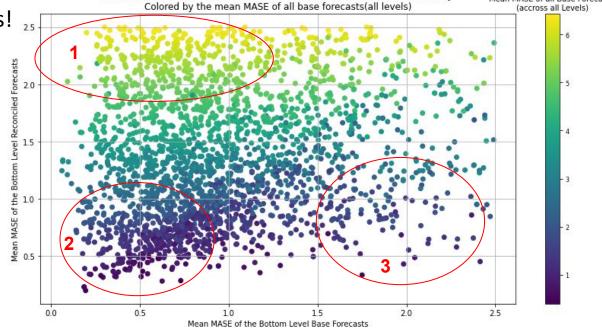
Inaccurate initial forecasts can lead to bad forecast combinations! So, Bad base forecasts can negatively affect reconciliation?

 Measure MASE of bottom level (MASE_b) base forecasts (ETS)

- Measure MASE of all (MASE tot) base forecasts (ETS)
- Measure MASE of bottom level (MASE rec) reconciled
 - Use Structural Scaling (no parametric uncertainty)
 - Accurate reconciled bottom level =>
 Accurate reconciled hierarchy (coherence)

Results:

- Low MASE_B & High MASE_tot => High MASE_rec
- 2. Low MASE_B & Low MASE_tot = > Low MASE_rec
- 3. High MASE_B & Low MASE_tot => Low MASE_rec



Connection Between Bottom Level Base and Reconciled Predictions Accuracy





Temporal Hierarchy Neural Network (THINN) Base Forecasts Accuracy "Hints"

Insights:

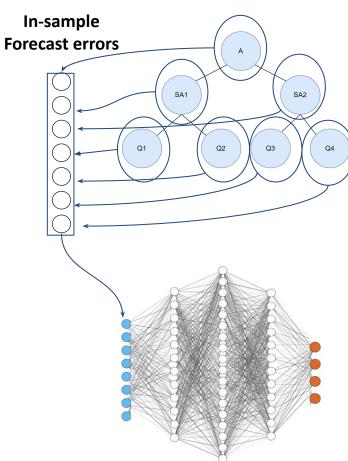
- Having one/two accurate base forecasts does not guarantee good reconciliation
- Bad forecasts drag reconciled predictions outside the accurate subspace
- Problems with model misspecifications on base forecasts (eg ETS for all base forecasts on "difficult" frequencies?)

Ideally THiNN should focus on more accurate base forecasts. Not possible.

Assumption: in-sample performance gives an adequate picture for out-of-sample

Include mean in-sample accuracy of each base forecast

- THiNN corrects base forecasts based on their in-sample accuracy
- THINN is not overly affected by inaccurate base forecasts
- Inaccurate base forecasts will be dragged "more" into the accurate subspace





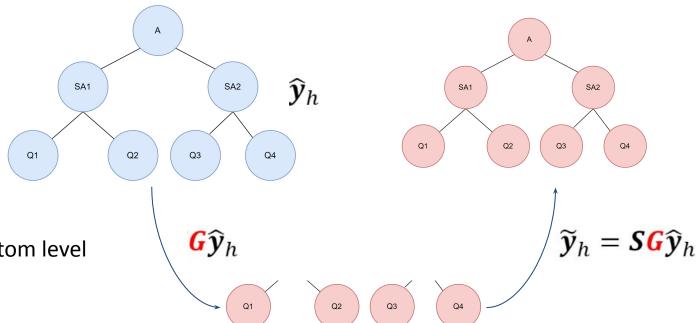
Temporal Hierarchy Neural Network (THiNN) Training the models

Let's review THieF:

- 1. Get base forecasts for each level
- 2. Reconcile at the bottom level
- 3. Reconstruct the hierarchy with bottom-up

Bottom-up:

- Unbiased
- Ensures coherence
- Good hierarchical results given accurate bottom level

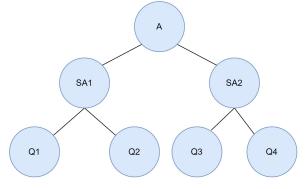


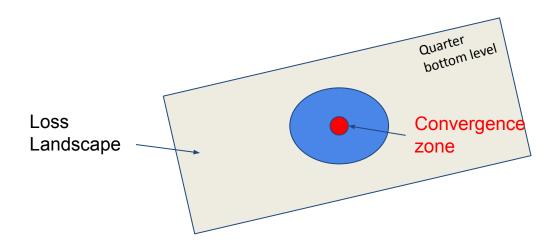
Temporal Hierarchy Neural Network (THINN)

Training the models

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Temporal Hierarchy Neural Network (THINN)

Training the models

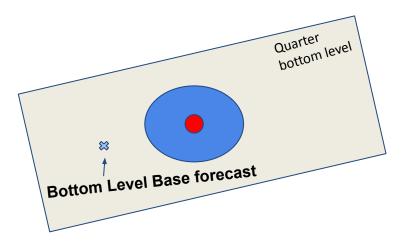
Bottom-up:

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Q1 Q2 Q3 Q4

Basic THINN:

Gets a set of base forecasts



Temporal Hierarchy Neural Network (THiNN)

Training the models

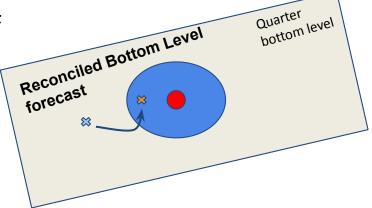
Bottom-up:

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Basic THINN:

- Gets a set of base forecasts
- Tunes weights to reconcile on bottom level (THieF style)



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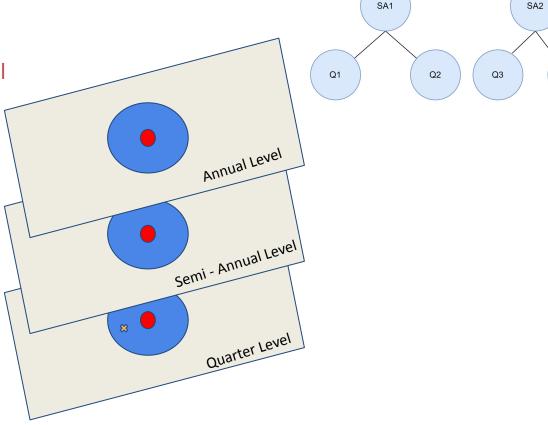
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Basic THINN:

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- Through coherence constraints builds the complete hierarchy





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Training the models

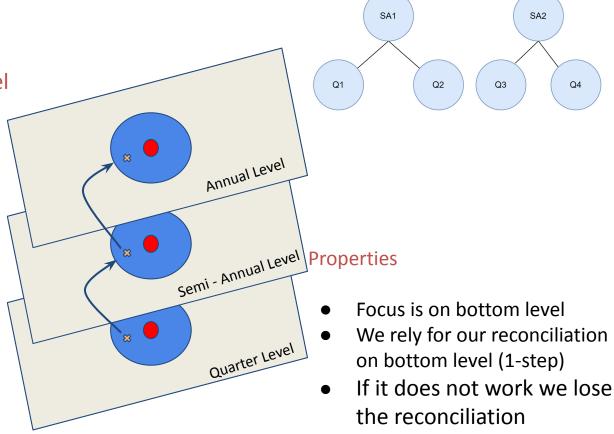
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If bottom level is accurate = whole hierarchy is accurate





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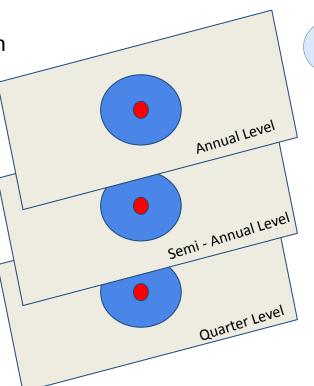
Training the models

Multi - Level Reconciliation

Instead of tuning weights to only reconcile on bottom level

Simultaneously optimize on every level and then align

Objectives of reconciling at each level are identical.



SA2

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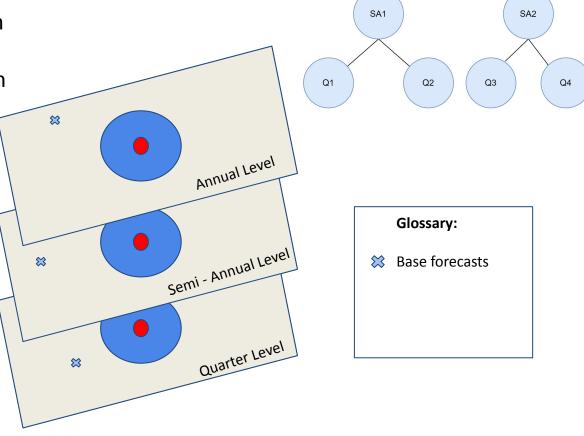
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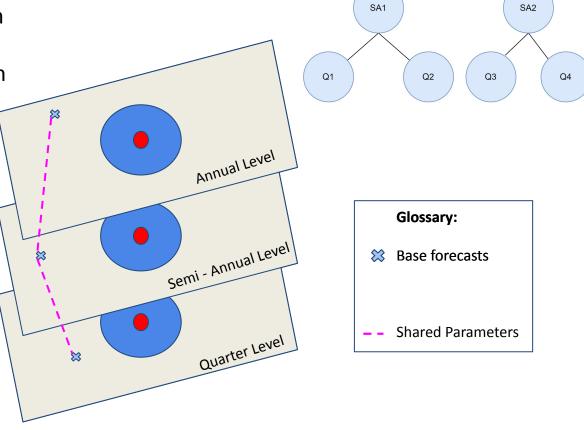
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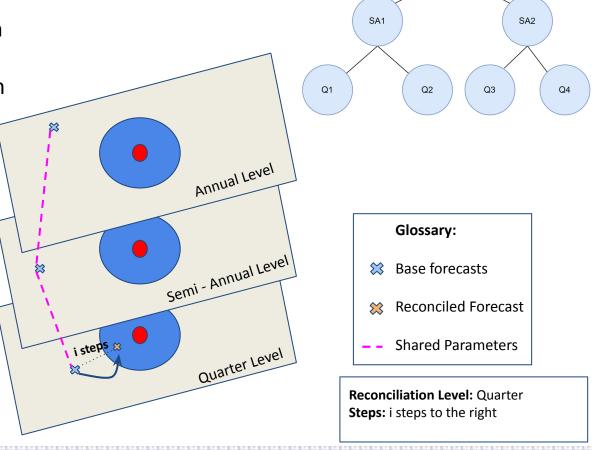
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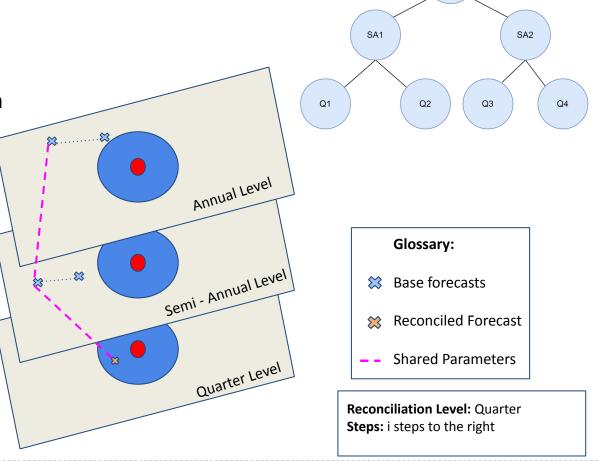
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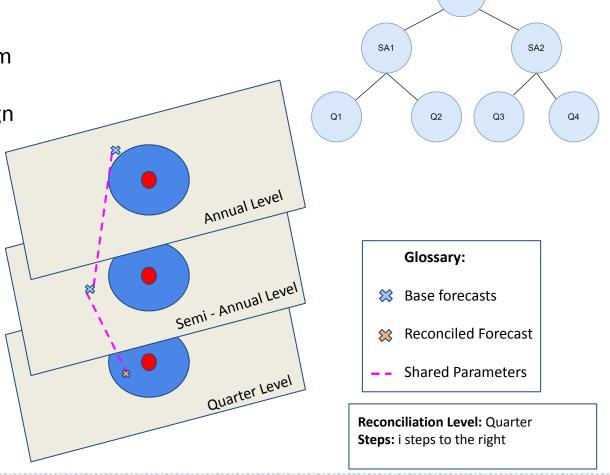
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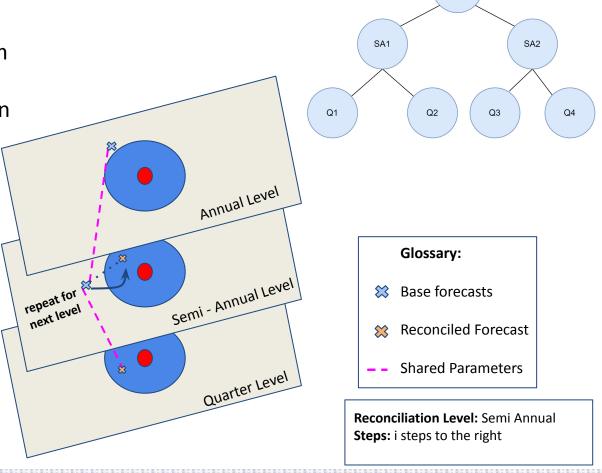
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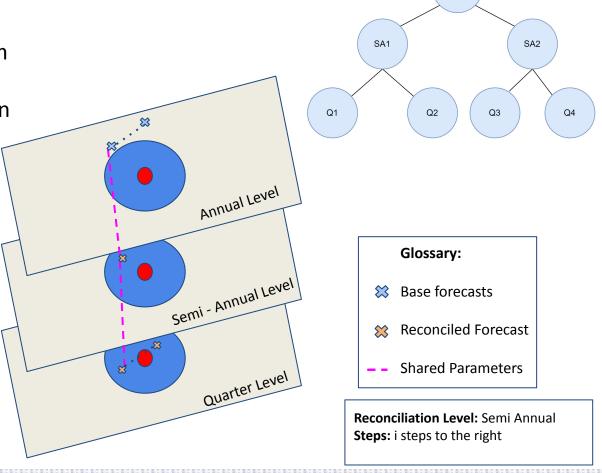
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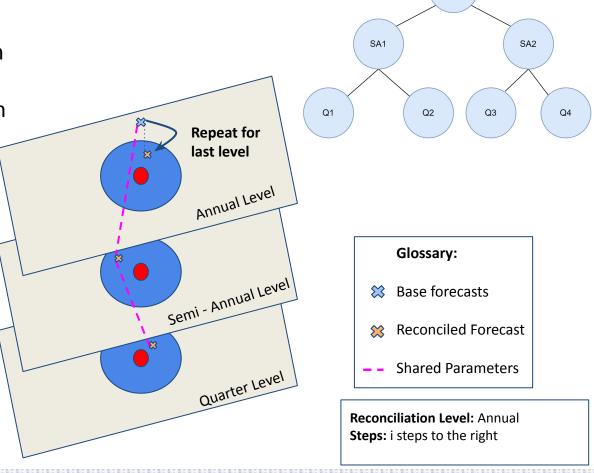
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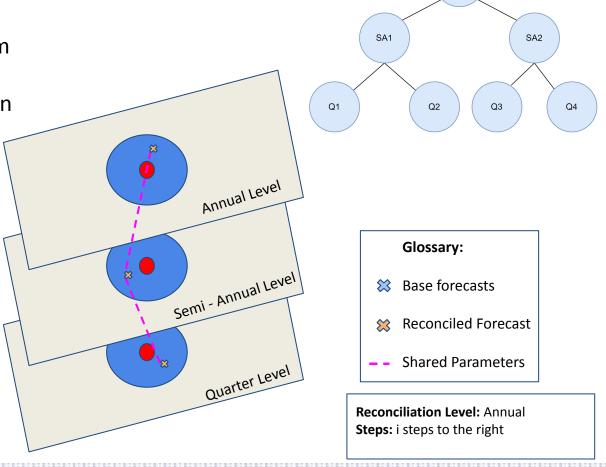
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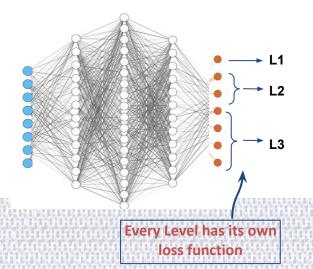
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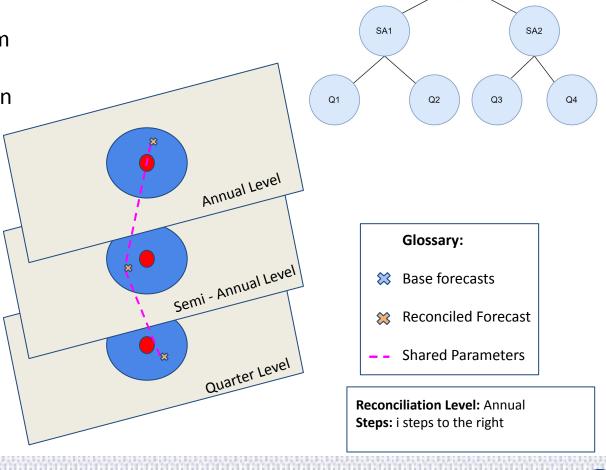
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Multi - Task Learning







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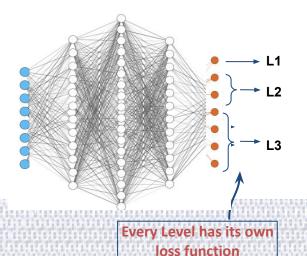
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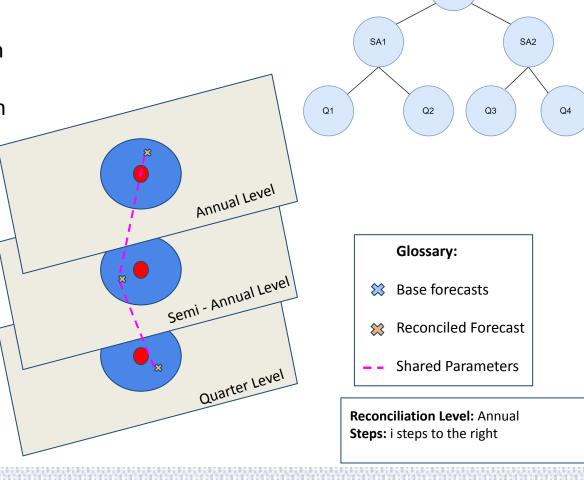
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Multi - Task Learning



Limitation:

Predictions are not coherent.



Temporal Hierarchy Neural Network (THiNN)

Training the models

Multi - Level Reconciliation

Every Level has its own loss function

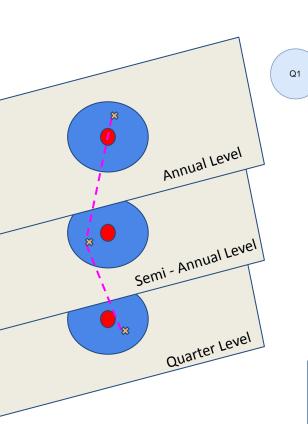
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Multi - Task Learning G on Structural Scaling: Fixed Equal to errors on each level Benefits accurate base forecasts Limitation: Predictions are not coherent.



Glossary:

Base forecasts

Reconciled Forecast

- Shared Parameters

Reconciliation Level: Annual
Steps: i steps to the right

SA2



Temporal Hierarchy Neural Network (THiNN)

Training the models

Multi - Level Reconciliation

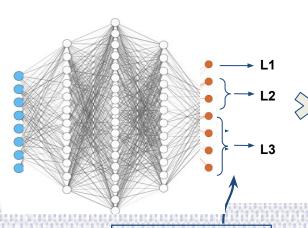
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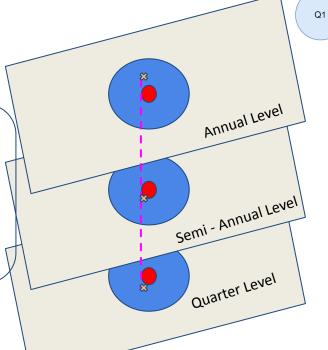
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G on Structural Scaling:

- Fixed
- Equal to errors on each level
- Benefits accurate base forecasts

Limitation:

Predictions are not coherent.



Glossary:

- Base forecasts
- Reconciled Forecast

Shared Parameters

Reconciliation Level: Structural Scaling

SA2

Steps: <u>ALIGNED</u>



Temporal Hierarchy Neural Network (THiNN) A Quick Experiment

Accuracy benefits from each addition

- Data: 10.000 monthly time series from M4 (random selection)
 Last 12 months kept as test set
- Accuracy Metric: Mean Absolute Scaled Error (MASE)
- Base Forecasts: ETS
- Benchmarks: Structural Scaling

Results:

- NN reconciles outperform benchmarks in general
- THiN(y,I) is not performing well
- Positional information not helpful → misused??
- In-Sample errors show benefits
- THINN MT and THINN(y,S) perform the best

| Model | Bottom Level MASE | Complete Hierarchy MASE | | |
|--|-------------------|-------------------------|--|--|
| Base Forecasts | 1.1861 | 0.9314 | | |
| Structural Scalling | 1,1772 | 0.91487 | | |
| THIN(b,S) | 1.1744 | 0.86714 | | |
| THiN(y,I) | 1.3524 | 0.9974 | | |
| THIN - MT | 1.1752 | 0.8599 | | |
| THiN(b,S) + Pos | 1.7952 | 1.2145 | | |
| THiN(y,I) + Pos | 1.6222 | 1.594 | | |
| THiN - MT + Pos | 1.6612 | 1.2138 | | |
| THiN(b,S) + Errors | 1.1651 | 0.8554 | | |
| $\mathrm{THiN}(y,\!I) + \mathrm{Errors}$ | 1.5021 | 1.1102 | | |
| THiN - MT + Errors | 1.1616 | 0.8334 | | |

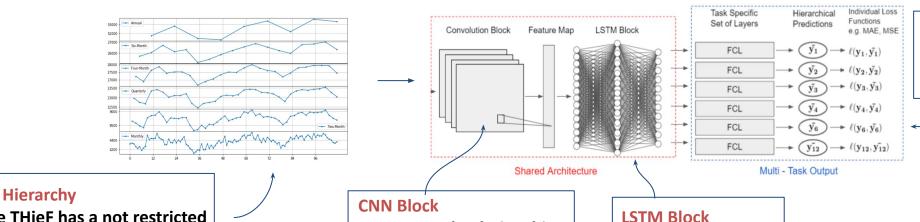


End-to-End Architectures DeepTHieF

Deep Temporal Hierarchies Forecasting (DeepTHieF)

- **End-to-end learning**
 - Single-Model _____Base Forecasts + Reconciles is computationally heavy
 - Shared parameters ____ End-to-end architectures perform better in such occasions
- All about representation manipulation
- Novel, informative way to represent a single time series

Higher capacity models with Better understanding of time series features



Multi - Head Output

Multi-Task module optimized each for each level and then reconciles



Unlike THieF has a not restricted number of temporal levels

Extract Local Relationships among levels

Models sequential part of time series



Theodosiou, Filotas & Kourentzes, Nikolaos. (2021). Deep Learning Temporal Hierarchies



End-to-End Architectures Another Quick Experiment

Experimental Setup:

- 12.000 time series from M4
- 12-fold Cross-Validation
- Evaluate Bottom Level & Complete Hierarchy
- Metric MASE

Findings:

- 1. DeepTHieF outperforms all methods
- 2. THiNN reconcilers perform better than benchmarks
- 3. THiNN(y,I) performs the best among reconcilers

| Forecast | Original | frequency | Complete Hierarchy | | |
|--|----------|-----------|--------------------|--------|--|
| Torcoast | Mean | Median | Mean | Median | |
| Base | | | | | |
| ETS | 2.6267 | 1.2742 | 0.3485 | 0.2095 | |
| Reconciliation & | ETS | | | | |
| Structural | 2.6590 | 1.2973 | 0.3223 | 0.1898 | |
| $\text{THiNN}(\hat{\boldsymbol{b}}, \boldsymbol{S})$ | 2.6374 | 1.2823 | 0.3066 | 0.1791 | |
| $\text{THiNN}(\hat{y}, I)$ | 2.6246 | 1.2663 | 0.3040 | 0.1766 | |
| $\text{THiNN}(\hat{\boldsymbol{b}}, \hat{\boldsymbol{w}})$ | 2.6294 | 1.2649 | 0.3091 | 0.1845 | |
| $\mathrm{THiNN}(\hat{\boldsymbol{y}}, \hat{\boldsymbol{w}})$ | 2.7352 | 1.3361 | 0.3173 | 0.1848 | |
| DeepTHieF | 1.5479 | 1.0616 | 0.2077 | 0.1659 | |

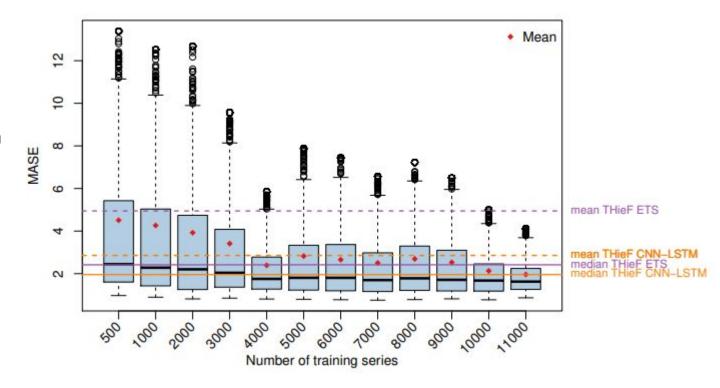




End-to-End Architectures DeepTHieF Data Requirements

Requirements for Global Training:

- Global DL Forecasting models performance increases linearly to data size
- Still DeepTHieF shows a robust performance on a reduced number of time series
- THieF has an advantage on a small number of time series.







End-to-End Architectures Prediction Intervals (PIs)

THieF by construction is optimized for Point Forecast Reconciliation.

Approaches in Literature for Reconciling PIs

- Naive approach: Reconcile lower and upper prediction interval
- Amazon: Sample from a distribution and map to the coherent subspace
- Empirical Methods

The Flexibility of NNs

- NNs can be optimized on producing PIs
- Appropriate Loss Function
- THiNN can produce upper and lower quantiles

| Model | Method | Daily | Weekly | Monthly | Quarterly | Mean |
|-----------|-----------------------|-------|--------|---------|-----------|-------|
| DeepTHieF | Empirical - Direct | 3.997 | 3.993 | 2.774 | 3.144 | 3.477 |
| | Empirical - KDE | 5.430 | 5.306 | 3.589 | 3.358 | 4.421 |
| | Interval Optimization | 1.125 | 2.138 | 1.774 | 2.500 | 1.884 |
| ETS | Analytical | 4.023 | 4.081 | 4.239 | 7.034 | 4.844 |
| | Simulation | 4.023 | 4.082 | 4.240 | 7.039 | 4.846 |
| Naïve | Analytical | 5.714 | 5.747 | 5.866 | 9.601 | 6.731 |

Table 1: Interval Scores comparison for the proposed methods

Optimized on Interval Score



Theodosiou, Filotas & Kourentzes, Nikolaos. (2021). Deep Learning Temporal Hierarchies for Interval Forecasts. International Conference of AI in Finance. Time Series Workshop



Conclusions & Future of NNs and Temporal Hierarchies

- THieF's success is based on combining diverse information
- Different viewpoints produce different features
- THieF struggles when base forecasts are very inaccurate.

We proposed THiNN -> A Data-Driven Non-Linear Reconciler based on THieF

- Positional Information
- Base forecasts accuracy
- Multi-task Learning

DeepTHieF & End-to-End Architectures

- Novel Input representation based on Temporal Hierarchies
- Single NN does not work at each task (not in forecasting)
- Change how global NNs view time series



MEER INFO

Research Associate

Filotas, Theodosiou filotas.theodosiou@vives.be +306975721445 Campus Brugge Xaverianenstraat Xaverianenstraat 12 050 30 51 00 **Campus Kortrijk**

Doorniksesteenweg 145 056 26 41 60

facebook.com/viveshogeschool



