# Sparsified Distributed Adaptive Learning with Error Feedback

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#### **Abstract**

To be completed...

#### 1 Introduction

- 3 Some related work:
- 4 [12] develops variant of signSGD (as a biased compression schemes) for distributed optimization.
- 5 Contributions are mainly on this error feedback variant. In [14], the authors provide theoretical
- 6 results on the convergence of sparse Gradient SGD for distributed optimization (we want that for
- 7 AMS here). [15] develops a variant of distributed SGD with sparse gradients too. Contributions
- 8 include a memory term used while compressing the gradient (using top k for instance). Speeding up
- 9 the convergence in  $\frac{1}{T^3}$ .

#### 10 2 Preliminaries

- Distributed Learning. Extensive literature in distributed (synch or asynch) SGD. Some distributed Adaptive method (DAMS and FODS (for FL) papers) but not much. Even less sparsified variants of them.
- Sparse Optimization. Gradient sparsification constitutes one popular method to induce sparsity
- through the optimization procedure and reduce the number of bits transmitted at each iteration.
- 16 Extensive works have studied this technique to improve the communication efficiency of SGD-
- based methods such as distributed SGD. This large class of sparsification techniques include gradient
- quantization leveraging quantized vector of gradients in the communication phase [2, 17, 10, 16, 9],
- gradient sparsification generally selection top k components of the vector to be communicated, see
- 20 [15, 1], or variants of the particular SGD algorithm such as low-precision SGD [4, 12] proposing
- 21 a trade-off between communication cost and precision, and signSGD [7, 18] where only the signs
- 22 of the gradient vectors are communicated. Most of these works apply to the SGD method [5] as a
- prototype where a novel method and some convergence results are presented with a rate of  $\mathcal{O}(\frac{1}{\sqrt{T}})$
- where T denotes the total number of iterations, see [3], thus achieving the same rate as plain SGD,
- 25 see [8, 11].
- 26 A recent work [6] focuses on adaptive gradient method, in particular the Adam [13] optimization
- 27 method.

#### 8 Compression based Distributed Optimization.

#### 29 3 Method

32

30 Consider standard synchronous distributed optimization setting. AMSGrad is used as the prototype, 31 and the local workers is only in charge of gradient computation.

#### 3.1 TopK AMSGrad with Error Feedback

The key difference (and interesting part) of our TopK AMSGrad compared with the following arxiv paper "Quantized Adam" https://arxiv.org/pdf/2004.14180.pdf is that, in our model only gradients are transmitted. In "QAdam", each local worker keeps a local copy of moment estimator m and v, and compresses and transmits m/v as a whole. Thus, that method is very much like the sparsified distributed SGD, except that g is changed into m/v. In our model, the moment estimates m and v are computed only at the central server, with the compressed gradients instead of the full gradient. This would be the key (and difficulty) in convergence analysis.

#### Algorithm 1 SPARS-AMS for Federated Learning

```
1: Input: parameter \beta_1, \beta_2, learning rate \eta_t.
 2: Initialize: central server parameter \theta_0 \in \Theta \subseteq \mathbb{R}^d; e_{t,i} = 0 the error accumulator for each
     worker; sparsity parameter k; N local workers; m_0 = 0, v_0 = 0, \hat{v}_0 = 0
 3: for t = 1 to T do
 4:
         parallel for worker i \in [n] do:
 5:
            Receive model parameter \theta_{t-1} from central server
 6:
            Compute stochastic gradient g_{t,i} at \theta_t
 7:
             Compute \tilde{g}_{t,i} = TopK(g_{t,i} + e_{t,i}, k)
             Update the error e_{t+1,i} = e_{t,i} + g_{t,i} - \tilde{g}_{t,i}
 8:
 9:
            Send \tilde{g}_{t,i} back to central server
10:
         end parallel
         Central server do:
11:
         \bar{g}_t = \frac{1}{N} \sum_{i=1}^{N} \tilde{g}_{t,i}
12:
13:
         m_t = \beta_1 \overline{m_{t-1}} + (1 - \beta_1) \overline{g}_t
14:
         v_t = \beta_2 v_{t-1} + (1 - \beta_2) \bar{g}_t^2
15:
         \hat{v}_t = \max(v_t, \hat{v}_{t-1})
         Update global model \theta_t = \theta_{t-1} - \eta_t \frac{m_t}{\sqrt{\hat{\eta}_t}}
16:
17: end for
```

#### 40 3.2 Convergence Analysis

- Several mild assumptions to make: Nonconvex and smooth loss function, unbiased stochastic gradient, bounded variance of the gradient, bounded norm of the gradient, control of the distance between
- 43 the true gradient and its sparse variant.
- 44 Check [6] for proofs starting with single machine and extending to distributed settings (several machines).

#### 46 3.2.1 Single machine

- Under the centralized setting, the goal is to derive an upper bound to the second order moment of the gradient of the objective function at some iteration  $T_f \in [1, T]$ .
- We first define multiple auxiliary sequences. For the first moment, define

$$\bar{m}_t = m_t + \mathcal{E}_t,$$
  
$$\mathcal{E}_t = \beta_1 \mathcal{E}_{t-1} + (1 - \beta_1)(e_{t+1} - e_t),$$

50 such that

$$\bar{m}_t = \bar{m}_t + \mathcal{E}_t 
= \beta_1 (m_t + \mathcal{E}_t) + (1 - \beta_1) (\bar{g}_t + e_{t+1} - e_1) 
= \beta_1 \bar{m}_{t-1} + (1 - \beta_1) q_t.$$

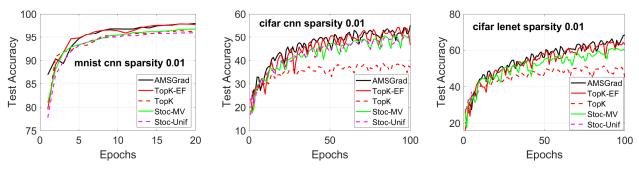


Figure 1: Test accuracy.

## 51 3.2.2 Multiple machine

# 2 4 Experiments

- Our proposed TopK-EF with AMSGrad matches that of full AMSGrad, in distributed learning.
- Number of local workers is 20. Error feedback fixes the convergence issue of using solely the
- 55 TopK gradient.

## 56 5 Conclusion

#### 7 References

- [1] Alham Fikri Aji and Kenneth Heafield. Sparse communication for distributed gradient descent.
   arXiv preprint arXiv:1704.05021, 2017.
- [2] Dan Alistarh, Demjan Grubic, Jerry Li, Ryota Tomioka, and Milan Vojnovic. Qsgd: Communication-efficient sgd via gradient quantization and encoding. In *Advances in Neu-* ral Information Processing Systems, pages 1709–1720, 2017.
- [3] Dan Alistarh, Torsten Hoefler, Mikael Johansson, Sarit Khirirat, Nikola Konstantinov, and Cédric Renggli. The convergence of sparsified gradient methods. *arXiv preprint* arXiv:1809.10505, 2018.
- [4] Jeremy Bernstein, Yu-Xiang Wang, Kamyar Azizzadenesheli, and Animashree Anandkumar.
   signsgd: Compressed optimisation for non-convex problems. In *International Conference on Machine Learning*, pages 560–569. PMLR, 2018.
- [5] Léon Bottou and Olivier Bousquet. The tradeoffs of large scale learning. In J. C. Platt,
   D. Koller, Y. Singer, and S. T. Roweis, editors, Advances in Neural Information Processing
   Systems 20, pages 161–168. Curran Associates, Inc., 2008.
- Congliang Chen, Li Shen, Haozhi Huang, Qi Wu, and Wei Liu. Quantized adam with error feedback. *arXiv preprint arXiv:2004.14180*, 2020.
- [7] Christopher De Sa, Matthew Feldman, Christopher Ré, and Kunle Olukotun. Understanding
   and optimizing asynchronous low-precision stochastic gradient descent. In *Proceedings of the* 44th Annual International Symposium on Computer Architecture, pages 561–574, 2017.
- 77 [8] Saeed Ghadimi and Guanghui Lan. Stochastic first-and zeroth-order methods for nonconvex stochastic programming. *SIAM Journal on Optimization*, 23(4):2341–2368, 2013.
- Farzin Haddadpour, Mohammad Mahdi Kamani, Mehrdad Mahdavi, and Viveck Cadambe.
  Trading redundancy for communication: Speeding up distributed sgd for non-convex optimization. In *International Conference on Machine Learning*, pages 2545–2554. PMLR, 2019.
- Peng Jiang and Gagan Agrawal. A linear speedup analysis of distributed deep learning with
   sparse and quantized communication. In *Proceedings of the 32nd International Conference on Neural Information Processing Systems*, pages 2530–2541, 2018.
- Belhal Karimi, Blazej Miasojedow, Eric Moulines, and Hoi-To Wai. Non-asymptotic analysis
   of biased stochastic approximation scheme. In *Conference on Learning Theory*, pages 1944–1974. PMLR, 2019.
- Sai Praneeth Karimireddy, Quentin Rebjock, Sebastian U Stich, and Martin Jaggi. Error feed back fixes signsgd and other gradient compression schemes. arXiv preprint arXiv:1901.09847,
   2019.
- 91 [13] Diederik P Kingma and Jimmy Ba. Adam: A method for stochastic optimization. *arXiv* preprint arXiv:1412.6980, 2014.
- 93 [14] Shaohuai Shi, Kaiyong Zhao, Qiang Wang, Zhenheng Tang, and Xiaowen Chu. A convergence 94 analysis of distributed sgd with communication-efficient gradient sparsification. In *IJCAI*, 95 pages 3411–3417, 2019.
- [15] Sebastian U Stich, Jean-Baptiste Cordonnier, and Martin Jaggi. Sparsified sgd with memory.
   In Advances in Neural Information Processing Systems, pages 4447–4458, 2018.
- 98 [16] Jianqiao Wangni, Jialei Wang, Ji Liu, and Tong Zhang. Gradient sparsification for communication-efficient distributed optimization. *arXiv preprint arXiv:1710.09854*, 2017.
- 100 [17] Wei Wen, Cong Xu, Feng Yan, Chunpeng Wu, Yandan Wang, Yiran Chen, and Hai Li. Tern-101 grad: Ternary gradients to reduce communication in distributed deep learning. *arXiv preprint* 102 *arXiv:1705.07878*, 2017.

[18] Guandao Yang, Tianyi Zhang, Polina Kirichenko, Junwen Bai, Andrew Gordon Wilson, and
 Chris De Sa. Swalp: Stochastic weight averaging in low precision training. In *International Conference on Machine Learning*, pages 7015–7024. PMLR, 2019.

# 106 A Appendix