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# Sparsified Distributed Adaptive Learning with Error Feedback

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## Abstract

1 To be completed...

## 2 1 Introduction

3 Some related work:

4 [17] develops variant of signSGD (as a biased compression schemes) for distributed optimization.  
5 Contributions are mainly on this error feedback variant. In [24], the authors provide theoretical  
6 results on the convergence of sparse Gradient SGD for distributed optimization (we want that for  
7 AMS here). [25] develops a variant of distributed SGD with sparse gradients too. Contributions  
8 include a memory term used while compressing the gradient (using top k for instance). Speeding up  
9 the convergence in  $\frac{1}{T^3}$ .

## 10 2 Preliminaries

### 11 Sparse Optimization Methods.

12 **Distributed Learning.** When a large number of compute engines is available, being able to  
13 train global machine learning models while mutualizing the available and *decentralized* source of  
14 computation has been a growing focus for the community.

15 Decentralized optimization methods include methods such as ADMM [6], Distributed Subgradient  
16 Descent [22], Dual Averaging [10], Prox-PDA [13], GNSD [20], and Choco-SGD [19].

17 A recent work [7], which focuses on adaptive gradient methods, namely the Adam [18] and the  
18 AMSGrad [23] optimization methods, develops a decentralized variant of gradient based and adap-  
19 tive methods in the context of gossip protocols. To date, very few contributions provided attempt  
20 to efficiently run adaptive gradient method in such a distributed setting. Apart from [7], [?] pro-  
21 poses a decentralized version of AMSGrad [23] which provably satisfies some non-standard regret.  
22 Though, no sparsified variants of them have been proposed for practical purposes nor been studied  
23 in the literature.

24 **Sparse Distributed Optimization.** While the capabilities of the compute powers is exploding,  
25 the communication complexity between either the central server and the decentralized workers or  
26 among workers is becoming ineffectively large [? 21]. Gradient sparsification constitutes one pop-  
27 ular method to induce sparsity through the optimization procedure and reduce the number of bits  
28 transmitted at each iteration. Extensive works have studied this technique to improve the communi-  
29 cation efficiency of SGD-based methods such as distributed SGD. This large class of sparsification  
30 techniques include gradient quantization leveraging quantized vector of gradients in the communi-  
31 cation phase [2, 27, 15, 26, 12, 8, 14], gradient sparsification generally selection top k components

of the vector to be communicated, see [25, 1], or variants of the particular SGD algorithm such as low-precision SGD [4, 17] proposing a trade-off between communication cost and precision, and signSGD [9, 28] where only the signs of the gradient vectors are communicated. Most of these works apply to the SGD method [5] as a prototype where a novel method and some convergence results are presented with a rate of  $\mathcal{O}(\frac{1}{\sqrt{T}})$  where  $T$  denotes the total number of iterations, see [3], thus achieving the same rate as plain SGD, see [11, 16].

Yet these communication reduction techniques, still presents a negative dependence on the number of workers, typically a linear dependence. Hence the need for even more efficient techniques which constitutes the object of our paper.

### 3 Method

Consider standard synchronous distributed optimization setting. AMSGrad is used as the prototype, and the local workers is only in charge of gradient computation.

#### 3.1 TopK AMSGrad with Error Feedback

The key difference (and interesting part) of our TopK AMSGrad compared with the following arxiv paper “Quantized Adam” <https://arxiv.org/pdf/2004.14180.pdf> is that, in our model only gradients are transmitted. In “QAdam”, each local worker keeps a local copy of moment estimator  $m$  and  $v$ , and compresses and transmits  $m/v$  as a whole. Thus, that method is very much like the sparsified distributed SGD, except that  $g$  is changed into  $m/v$ . In our model, the moment estimates  $m$  and  $v$  are computed only at the central server, with the compressed gradients instead of the full gradient. This would be the key (and difficulty) in convergence analysis.

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#### Algorithm 1 SPARS-AMS for Federated Learning

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1: Input: parameter  $\beta_1, \beta_2$ , learning rate  $\eta_t$ .
2: Initialize: central server parameter  $\theta_0 \in \Theta \subseteq \mathbb{R}^d$ ;  $e_{t,i} = 0$  the error accumulator for each
   worker; sparsity parameter  $k$ ;  $N$  local workers;  $m_0 = 0, v_0 = 0, \hat{v}_0 = 0$ 
3: for  $t = 1$  to  $T$  do
4:   parallel for worker  $i \in [n]$  do:
5:     Receive model parameter  $\theta_{t-1}$  from central server
6:     Compute stochastic gradient  $g_{t,i}$  at  $\theta_t$ 
7:     Compute  $\tilde{g}_{t,i} = \text{TopK}(g_{t,i} + e_{t,i}, k)$ 
8:     Update the error  $e_{t+1,i} = e_{t,i} + g_{t,i} - \tilde{g}_{t,i}$ 
9:     Send  $\tilde{g}_{t,i}$  back to central server
10:  end parallel
11:  Central server do:
12:     $\bar{g}_t = \frac{1}{N} \sum_{i=1}^N \tilde{g}_{t,i}$ 
13:     $m_t = \beta_1 m_{t-1} + (1 - \beta_1) \bar{g}_t$ 
14:     $v_t = \beta_2 v_{t-1} + (1 - \beta_2) \bar{g}_t^2$ 
15:     $\hat{v}_t = \max(v_t, \hat{v}_{t-1})$ 
16:    Update global model  $\theta_t = \theta_{t-1} - \eta_t \frac{m_t}{\sqrt{\hat{v}_t}}$ 
17: end for

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#### 3.2 Convergence Analysis

Several mild assumptions to make: Nonconvex and smooth loss function, unbiased stochastic gradient, bounded variance of the gradient, bounded norm of the gradient, control of the distance between the true gradient and its sparse variant.

Check [7] for proofs starting with single machine and extending to distributed settings (several machines).

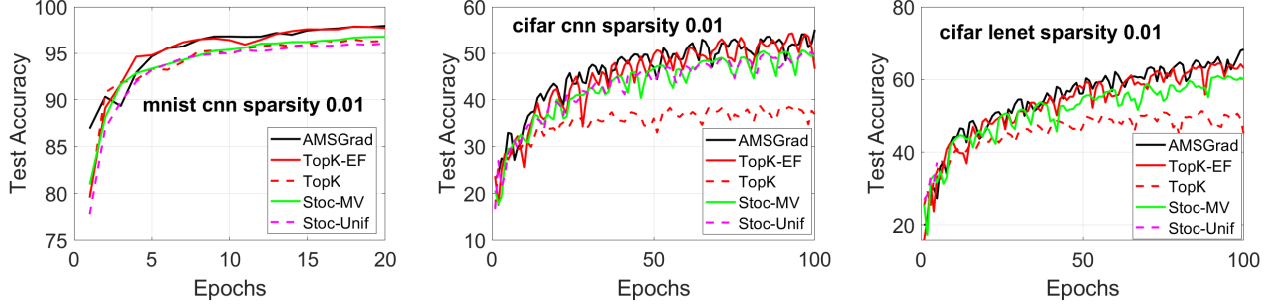


Figure 1: Test accuracy.

### 3.2.1 Single machine

Under the centralized setting, the goal is to derive an upper bound to the second order moment of the gradient of the objective function at some iteration  $T_f \in [1, T]$ .

We first define multiple auxiliary sequences. For the first moment, define

$$\begin{aligned}\bar{m}_t &= m_t + \mathcal{E}_t, \\ \mathcal{E}_t &= \beta_1 \mathcal{E}_{t-1} + (1 - \beta_1)(e_{t+1} - e_t),\end{aligned}$$

such that

$$\begin{aligned}\bar{m}_t &= \bar{m}_t + \mathcal{E}_t \\ &= \beta_1(m_t + \mathcal{E}_t) + (1 - \beta_1)(\bar{g}_t + e_{t+1} - e_1) \\ &= \beta_1 \bar{m}_{t-1} + (1 - \beta_1)g_t.\end{aligned}$$

### 3.2.2 Multiple machine

## 4 Experiments

Our proposed TopK-EF with AMSGrad matches that of full AMSGrad, in distributed learning. Number of local workers is 20. Error feedback fixes the convergence issue of using solely the TopK gradient.

## 5 Conclusion

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