
FedSKETCH: Communication-Efficient Federated Learning via Sketching

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Abstract

1 Communication complexity and data privacy are the two key challenges in Federated Learning (FL) where the goal is to perform a distributed learning through a
2 large volume of devices. In this work, we introduce two new algorithms, namely
3 FedSKETCH and FedSKETCHGATE, to address jointly both challenges and which
4 are, respectively, intended to be used for homogeneous and heterogeneous data distribution settings. Our algorithms are based on a key and novel sketching technique,
5 called HEAPRIX that is unbiased, compresses the accumulation of local gradients
6 using count sketch, and exhibits communication-efficiency properties leveraging
7 low-dimensional sketches. We provide sharp convergence guarantees of our
8 algorithms and validate our theoretical findings with various sets of experiments.
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1 Introduction

12 Federated Learning (FL) is a recently emerging framework for distributed large scale machine
13 learning problems. In FL, data is distributed across devices [23, 33] and due to privacy concerns,
14 users are only allowed to communicate with the parameter server. Formally, the optimization problem
15 across p distributed devices is defined as follows:

$$\min_{\mathbf{x} \in \mathbb{R}^d, \sum_{j=1}^p q_j = 1} f(\mathbf{x}) \triangleq \sum_{j=1}^p q_j F_j(\mathbf{x}), \quad (1)$$

16 where $F_j(\mathbf{x}) = \mathbb{E}_{\xi \in \mathcal{D}_j} [L_j(\mathbf{x}, \xi)]$ is the local cost function at device j , $q_j \triangleq \frac{n_j}{n}$, n_j is the number
17 of data shards at device j and $n = \sum_{j=1}^p n_j$ is the total number of data samples, ξ is a random
18 variable distributed according to probability distribution \mathcal{D}_j , and L_j is a loss function that measures
19 the performance of model \mathbf{x} at device j . We note that, while for the homogeneous setting we
20 assume $\{\mathcal{D}_j\}_{j=1}^p$ have the same distribution across devices and $L_i = L_j$, $1 \leq (i, j) \leq p$, in the
21 heterogeneous setting, these distributions and loss functions L_j can vary from a device to another.

22 There are several challenges that need to be addressed in FL in order to efficiently learn a global
23 model that performs well in average for all devices:

24 – *Communication-efficiency*: There are often many devices communicating with the server, thus
25 incurring immense communication overhead. One approach to reduce communication round is using
26 *local SGD with periodic averaging* [50, 39, 48, 44] which periodically averages models after a few
27 local updates, contrary to baseline SGD [6] where gradient averaging is performed at each iteration.
28 Local SGD has been proposed in [33, 23] under the FL setting and its convergence analysis is studied
29 in [39, 44, 50, 48], later on improved in the followup references [3, 12, 21, 40] for homogeneous
30 setting. It is further extended to heterogeneous setting [12, 20, 47, 30, 38, 31]. The second approach to
31 deal with communication cost aims at reducing the size of communicated message per communication
32 round, such as local gradient quantization [1, 4, 42, 45, 46] or sparsification [2, 32, 41, 40].

33 – *Data heterogeneity*: Since locally generated data in each device may come from different distribution,
34 local computations involved in FL setting can lead to poor convergence error in practice [27, 31].

To mitigate the negative impact of data heterogeneity, [13, 16, 31, 20] suggest applying variance reduction or gradient tracking techniques along local computations.

–*Privacy* [11, 14]: Privacy has been widely addressed by injecting an additional layer of randomness to respect differential-privacy property [34] or using cryptography-based approaches under secure multi-party computation [5]. Further study of challenges can be found in recent surveys [27] and [18].

To tackle the aforementioned challenges in FL jointly, sketching based algorithms [7, 9, 22, 25] are promising approaches. For instance, to reduce communication cost, [17] develops a distributed SGD algorithm using sketching along providing its convergence analysis in the homogeneous setting, and establish a communication complexity of order $\mathcal{O}(\log(d))$ per round, where d is the dimension of the vector of parameters compared to $\mathcal{O}(d)$ complexity per round of baseline mini-batch SGD. Yet, the proposed sketching scheme in [17], built from a communication-efficiency perspective, is based on a deterministic procedure which requires access to the exact information of the gradients, thus not meeting the privacy-preserving criteria. This systemic issue is partially addressed in [37].

Focusing on privacy, [26] derives a single framework in order to tackle these issues jointly and introduces `DiffSketch` algorithm, based on the Count Sketch operator, yet does not provide its convergence analysis. Additionally, the estimation error of `DiffSketch` is higher than the sketching scheme in [17] which may end up in poor convergence.

Our main contributions are summarized as follows:

- We provide a new algorithm – `HEAPRIX` – and theoretically show that it reduces the cost of communication between devices and server, based on unbiased sketching without requiring the broadcast of exact values of gradients to the server. Based on `HEAPRIX`, we develop general algorithms for communication-efficient and sketch-based FL, namely `FedSKETCH` and `FedSKETCHGATE` for homogeneous and heterogeneous data distribution settings respectively.
- We establish non-asymptotic convergence bounds for convex, Polyak-Łojasiewicz (PL) and non-convex functions in Theorems 1 and 2 in both homogeneous and heterogeneous cases, and highlight an improvement in the number of iteration to reach a stationary point. We also provide a convergence analysis for the `PRIVIX/DiffSketch`¹ algorithm proposed in [26].
- We illustrate the benefits of `FedSKETCH` and `FedSKETCHGATE` over baseline methods through a set of experiments. The latter shows the advantages of the `HEAPRIX` compression method achieving comparable test accuracy as Federated SGD (`FedSGD`) while compressing the information exchanged between devices and server.

Notation: We denote the number of communication rounds and bits per round and per device by R and B respectively. The count sketch of vector \mathbf{x} is designated by $\mathbf{S}(\mathbf{x})$. $[p]$ denotes the set $\{1, \dots, p\}$.

2 Compression using Count Sketch

In this paper, we exploit the commonly used `Count Sketch` [7] which uses two sets of functions that encode any input vector \mathbf{x} into a hash table $\mathbf{S}_{m \times t}(\mathbf{x})$. Pairwise independent hash functions $\{h_{j,1 \leq j \leq t} : [d] \rightarrow m\}$ are used along with another set of pairwise independent sign hash functions $\{\text{sign}_{j,1 \leq j \leq t} : [d] \rightarrow \{+1, -1\}\}$ to map entries of \mathbf{x} (x_i , $1 \leq i \leq d$) into t different columns of $\mathbf{S}_{m \times t}$, wherein to lower the dimension of the input vector we usually have $d \gg mt$. The final update reads $\mathbf{S}[j][h_j(i)] = \mathbf{S}[j][h_j(i)] + \text{sign}_j(i)x_i$ for any $1 \leq j \leq t$. There are various types of sketching algorithms which are developed based on count sketching that we develop in the following subsections. See the Appendix for the detailed Count Sketch algorithm.

2.1 Sketching based Unbiased Compressor

We define an unbiased compressor as follows:

Definition 1 (Unbiased compressor). *We call randomized function, $C : \mathbb{R}^d \rightarrow \mathbb{R}^d$ an unbiased compression operator with $\Delta \geq 1$, if*

$$\mathbb{E}[C(\mathbf{x})] = \mathbf{x} \quad \text{and} \quad \mathbb{E}[\|C(\mathbf{x})\|_2^2] \leq \Delta \|\mathbf{x}\|_2^2.$$

We denote this class of compressors by $\mathbb{U}(\Delta)$.

¹We use `PRIVIX` and `DiffSketch` [26] interchangeably throughout the paper.

82 This definition leads to the following property

$$\mathbb{E} \left[\|\mathbf{C}(\mathbf{x}) - \mathbf{x}\|_2^2 \right] \leq (\Delta - 1) \|\mathbf{x}\|_2^2 .$$

83 Note that if we let $\Delta = 1$ then our algorithm reduces to the case of no compression. This property
84 allows us to control the noise of the compression.

85 An instance of such unbiased compressor is PRIVIX which obtains an estimate of input \mathbf{x} from a
86 count sketch noted $\mathbf{S}(\mathbf{x})$. In this algorithm, to query the quantity x_i , the i -th element of the vector
87 \mathbf{x} , we compute the median of t approximated values specified by the indices of $h_j(i)$ for $1 \leq j \leq t$,
88 see [26], or Algorithm 6 in the Appendix (for more details). The following property of count sketch
89 would be useful for our theoretical analysis.

90 **Property 1** ([26]). *For any $\mathbf{x} \in \mathbb{R}^d$, we have:*

91 *Unbiased estimation: As in [26], we have $\mathbb{E}_{\mathbf{S}} [\text{PRIVIX}[\mathbf{S}(\mathbf{x})]] = \mathbf{x}$.*

92 *Bounded variance: For the given $m < d$, $t = \mathcal{O}(\ln(\frac{d}{\delta}))$ with probability $1 - \delta$ we have:*

$$\mathbb{E}_{\mathbf{S}} \left[\|\text{PRIVIX}[\mathbf{S}(\mathbf{x})] - \mathbf{x}\|_2^2 \right] \leq \frac{c \times d}{m} \|\mathbf{x}\|_2^2 ,$$

93 where c ($e \leq c < m$) is a positive constant independent of the dimension of the input, d .

94 We note that bounded variance assumption does not necessary implies any compression as d could be
95 relatively large. Thus, with probability $1 - \delta$ we obtain $\text{PRIVIX} \in \mathbb{U}(1 + c\frac{d}{m})$. $\Delta = 1 + c\frac{d}{m}$ implies
96 that if $m \rightarrow d$, then $\Delta \rightarrow 1 + c$, indicating a noisy reconstruction. The reference [26] shows that if the
97 data is normally distributed, PRIVIX is differentially private [10], up to additional assumptions and
98 algorithmic design.

99 2.2 Sketching based Biased Compressor

100 A biased compressor is defined as follows:

101 **Definition 2** (Biased compressor). *A (randomized) function, $C : \mathbb{R}^d \rightarrow \mathbb{R}^d$ belongs to $\mathbb{C}(\Delta, \alpha)$, a*
102 *class of compression operators with $\alpha > 0$ and $\Delta \geq 1$, if*

$$\mathbb{E} \left[\|\alpha \mathbf{x} - C(\mathbf{x})\|_2^2 \right] \leq \left(1 - \frac{1}{\Delta} \right) \|\mathbf{x}\|_2^2 ,$$

103 The reference [15] proves that $\mathbb{U}(\Delta) \subset \mathbb{C}(\Delta, \alpha)$. An example of bi-
104 ased compression via sketching and using top_m operation is given below:
105

106 Following [17], HEAVYMIX with sketch size
107 $\Theta(m \log(\frac{d}{\delta}))$ is a biased compressor with
108 $\alpha = 1$ and $\Delta = d/m$ with probability $\geq 1 - \delta$,
109 meaning that it reconstruct the $\tilde{\mathbf{g}}$ from input
110 vector \mathbf{g} . In other words, with probability
111 $1 - \delta$, $\text{HEAVYMIX} \in \mathbb{C}(\frac{d}{m}, 1)$. We note
112 that Algorithm 1 is a variation of the sketch-
113 ing algorithm developed in [17] with distinc-
114 tion that HEAVYMIX does not require a second
115 round of communication to obtain the exact
116 values of top_m . This is mainly because in
117 SKETCGED-SGD [17] the server has to obtain
118 the exact values of *the average of sketches*; however HEAVYMIX obtains exact value locally, thus
119 does not require a second round of communication. Additionally, while a sketching algorithm
120 implementing HEAVYMIX has smaller estimation error compared to PRIVIX, it requires having access
121 to the exact values of top_m , therefore not benefiting from privacy properties contrary to PRIVIX. In
122 the following we introduce HEAPRIX which is built upon HEAVYMIX and PRIVIX methods.

Algorithm 1 HEAVYMIX

- 1: **Inputs:** $\mathbf{S}(\mathbf{g})$; parameter m
 - 2: Query the vector $\tilde{\mathbf{g}} \in \mathbb{R}^d$ from $\mathbf{S}(\mathbf{g})$:
 - 3: Query $\hat{\ell}_2^2 = (1 \pm 0.5) \|\mathbf{g}\|^2$ from sketch $\mathbf{S}(\mathbf{g})$
 - 4: $\forall j$ query $\hat{\mathbf{g}}_j^2 = \tilde{\mathbf{g}}_j^2 \pm \frac{1}{2m} \|\mathbf{g}\|^2$ from sketch $\mathbf{S}(\mathbf{g})$
 - 5: $H = \{j | \hat{\mathbf{g}}_j \geq \frac{\hat{\ell}_2^2}{m}\}$ and $NH = \{j | \hat{\mathbf{g}}_j < \frac{\hat{\ell}_2^2}{m}\}$
 - 6: $\text{Top}_m = H \cup \text{rand}_{\ell}(NH)$, where $\ell = m - |H|$
 - 7: Get exact values of Top_m
 - 8: **Output:** $\tilde{\mathbf{g}} : \forall j \in \text{Top}_m : \tilde{\mathbf{g}}_j = \mathbf{g}_j$ else $\mathbf{g}_j = 0$
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123 2.3 Sketching based Induced Compressor

124 Due to Theorem 3 in [15], which illustrates that we can convert the biased compressor into an
125 unbiased one such that, for $C_1 \in \mathbb{C}(\Delta_1)$ with $\alpha = 1$, if you choose $C_2 \in \mathbb{U}(\Delta_2)$, then in-
126 duced compressor $C : \mathbf{x} \mapsto C_1(\mathbf{x}) + C_2(\mathbf{x} - C_1(\mathbf{x}))$ belongs to $\mathbb{U}(\Delta)$ with $\Delta = \Delta_2 + \frac{1 - \Delta_2}{\Delta_1}$.

Based on this notion, Algorithm 2 proposes an induced sketching algorithm by utilizing HEAVYMIX and PRIVIX for C_1 and C_2 respectively where the reconstruction of input \mathbf{x} is performed using hash table \mathbf{S} and \mathbf{x} , similar to PRIVIX and HEAVYMIX. Note that if $m \rightarrow d$, then $C(\mathbf{x}) \rightarrow \mathbf{x}$, implying that the convergence rate can be improved by decreasing the size of compression m .

Corollary 1. *Based on Theorem 3 of [15], HEAPRIX in Algorithm 2 satisfies $C(\mathbf{x}) \in \mathcal{U}(c \frac{d}{m})$.*

Benefits of HEAPRIX: Corollary 1 states that, unlike PRIVIX, HEAPRIX compression noise can be made as small as possible using larger hash size. In the distributed setting, contrary to SKETCHED-SGD [17] where decompressing is happening at the server, HEAPRIX does not require having access to exact top _{m} values of the input as it is based on HEAVYMIX, which helps preserving privacy. In other words, HEAPRIX leverages the best of both: the *unbiasedness* of PRIVIX while using *heavy hitters* as in HEAVYMIX.

3 FedSKETCH and FedSKETCHGATE

We introduce two new algorithms for both homogeneous and heterogeneous settings.

3.1 Homogeneous Setting

In FedSKETCH, the number of local updates, between two consecutive communication rounds, at device j is denoted by τ . Unlike [13], server node does not store any global model, rather, device j has two models: $\mathbf{x}^{(r)}$ and $\mathbf{x}_j^{(\ell, r)}$, which are respectively the local and global models. We develop FedSKETCH in Algorithm 3. A variant of this algorithm implementing HEAPRIX is also described in Algorithm 3. We remark that for this variant, we need to have an additional communication round between server and worker j to aggregate $\delta_j^{(r)} \triangleq \mathbf{S}_j [\text{HEAVYMIX}(\mathbf{S}^{(r)})]$ (Lines 3 and 3) to compute $\mathbf{S}^{(r)} = \frac{1}{k} \sum_{j \in \mathcal{K}} \mathbf{S}_j^{(r)}$. The main difference between FedSKETCH and DiffSketch in [26] is that we use distinct local and global learning rates. Furthermore, unlike [26], we do not add local Gaussian noise.

Algorithmic comparison with [13] An important feature of our algorithm is that due to a lower dimension of the count sketch, the resulting averages ($\mathbf{S}^{(r)}$ and $\tilde{\mathbf{S}}^{(r)}$) received by the server are also of lower dimension. Therefore, these algorithms exploit a bidirectional compression

during the communication from server to device back and forth. As a result, for the case of large quantization error $\omega = \theta(\frac{d}{m})$ as shown in [13], our algorithms can outperform FedCOM and FedCOMGATE developed in [13] if sufficiently large hash tables are used and the uplink communication cost is high. Furthermore, while, in [13], server stores a global model and aggregates the partial gradients from devices which can enable the server to extract some information regarding the device's data, in

Algorithm 2 HEAPRIX

1: **Inputs:** $\mathbf{x} \in \mathbb{R}^d, t, m, \mathbf{S}_{m \times t}, h_j (1 \leq i \leq t)$, $\text{sign}_j (1 \leq i \leq t)$, parameter m
2: Approximate $\mathbf{S}(\mathbf{x})$ using HEAVYMIX
3: Approximate $\mathbf{S}(\mathbf{x} - \text{HEAVYMIX}[\mathbf{S}(\mathbf{x})])$ with PRIVIX
4: **Output:**
 $\text{HEAVYMIX}[\mathbf{S}(\mathbf{x})] + \text{PRIVIX}[\mathbf{S}(\mathbf{x} - \text{HEAVYMIX}[\mathbf{S}(\mathbf{x})])]$.

Algorithm 3 FedSKETCH(R, τ, η, γ)

1: **Inputs:** $\mathbf{x}^{(0)}$: initial model shared by local devices, global and local learning rates γ and η , respectively
2: **for** $r = 0, \dots, R - 1$ **do**
3: **parallel for device** $j \in \mathcal{K}^{(r)}$ **do:**
4: **if PRIVIX variant:**
 $\Phi^{(r)} \triangleq \text{PRIVIX}[\mathbf{S}^{(r-1)}]$
5: **if HEAPRIX variant:**
 $\Phi^{(r)} \triangleq \text{HEAVYMIX}[\mathbf{S}^{(r-1)}] + \text{PRIVIX}[\mathbf{S}^{(r-1)} - \tilde{\mathbf{S}}^{(r-1)}]$
6: Set $\mathbf{x}^{(r)} = \mathbf{x}^{(r-1)} - \gamma \Phi^{(r)}$ and $\mathbf{x}_j^{(0, r)} = \mathbf{x}^{(r)}$
7: **for** $\ell = 0, \dots, \tau - 1$ **do**
8: Sample a mini-batch $\xi_j^{(\ell, r)}$ and compute $\tilde{\mathbf{g}}_j^{(\ell, r)}$
9: Update $\mathbf{x}_j^{(\ell+1, r)} = \mathbf{x}_j^{(\ell, r)} - \eta \tilde{\mathbf{g}}_j^{(\ell, r)}$
10: **end for**
11: Device j broadcasts $\mathbf{S}_j^{(r)} \triangleq \mathbf{S}_j(\mathbf{x}_j^{(0, r)} - \mathbf{x}_j^{(\tau, r)})$.
12: Server **computes** $\mathbf{S}^{(r)} = \frac{1}{k} \sum_{j \in \mathcal{K}} \mathbf{S}_j^{(r)}$.
13: Server **broadcasts** $\mathbf{S}^{(r)}$ to devices in randomly drawn devices $\mathcal{K}^{(r)}$.
14: **if HEAPRIX variant:**
15: Second round of communication: $\delta_j^{(r)} := \mathbf{S}_j[\text{HEAVYMIX}(\mathbf{S}^{(r)})]$ and broadcasts $\tilde{\mathbf{S}}^{(r)} \triangleq \frac{1}{k} \sum_{j \in \mathcal{K}} \delta_j^{(r)}$ to devices in set $\mathcal{K}^{(r)}$
16: **end parallel for**
17: **end**
18: **Output:** $\mathbf{x}^{(R-1)}$

contrast, in our algorithms server does not store the global model and only broadcasts the average sketches. Thus, sketching-based server-devices communication algorithms such as ours do not reveal the exact values of the inputs, to preserve privacy as a by-product.

Remark 1. As pointed out in [15], while induced compressors transform a biased compressor into unbiased one, as a drawback it doubles communication cost since the devices need to send $C_1(\mathbf{x})$ and $C_2(\mathbf{x} - C_1(\mathbf{x}))$ separately. We note that in the special case of HEAPRIX, due to the use of sketching, the extra communication round cost is compensated with lower number of bits per round thanks to the lower dimension of sketching.

3.2 Heterogeneous Setting

In this section, we focus on the optimization problem of (1) in the special case of $q_1 = \dots = q_p = \frac{1}{p}$ with full device participation ($k = p$). These results can be extended to the scenario where devices are sampled. For non i.i.d. data, the FedSKETCH algorithm, designed for homogeneous setting, may fail to perform well in practice. The main reason is that in FL, devices are using local stochastic descent direction which could be different than global descent direction when the data distribution are non-identical. Therefore, to mitigate the effect of data heterogeneity, we introduce a new algorithm called FedSKETCHGATE described in Algorithm 4. This algorithm leverages the idea of gradient tracking applied in [13] (with compression) and a special case of $\gamma = 1$ without compression [31]. The main idea is that using an approximation of global gradient, $\mathbf{c}_j^{(r)}$ allows to correct the local gradient direction. For the FedSKETCHGATE with PRIVIX variant, the correction vector $\mathbf{c}_j^{(r)}$ at device j and communication round r is computed in Line 4. While using HEAPRIX compression, FedSKETCHGATE also updates $\tilde{\mathbf{S}}^{(r)}$ via Line 4.

Remark 2. Most of the existing communication-efficient algorithms with compression only consider communication-efficiency from devices to server. However, Algorithms 3 and 4 also improve the communication efficiency from server to devices since it exploits low-dimensional sketches (and averages), communicated from the server to devices.

For both FedSKETCH and FedSKETCHGATE algorithms, unlike PRIVIX, HEAPRIX variant requires a second round of communication. Therefore, in Cross-Device FL setting, where there could be millions of devices, HEAPRIX variant may not be practical, and we note that it could be more suitable for Cross-Silo FL setting.

4 Convergence Analysis

We first state commonly used assumptions required in the following convergence analysis (reminder of our notations can be found Table 1 of the Appendix).

Assumption 1 (Smoothness and Lower Boundedness). *The local objective function $f_j(\cdot)$ of device j is differentiable for $j \in [p]$ and L -smooth, i.e., $\|\nabla f_j(\mathbf{x}) - \nabla f_j(\mathbf{y})\| \leq L\|\mathbf{x} - \mathbf{y}\|$, $\forall \mathbf{x}, \mathbf{y} \in \mathbb{R}^d$. Moreover, the optimal objective function $f(\cdot)$ is bounded below by $f^* := \min_{\mathbf{x}} f(\mathbf{x}) > -\infty$.*

Algorithm 4 FedSKETCHGATE(R, τ, η, γ)

- 1: **Inputs:** $\mathbf{x}^{(0)} = \mathbf{x}_j^{(0)}$ shared by all local devices, global and local learning rates γ and η .
- 2: **for** $r = 0, \dots, R - 1$ **do**
- 3: **parallel for device** $j = 1, \dots, p$ **do:**
- 4: **if PRIVIX variant:**

$$\mathbf{c}_j^{(r)} = \mathbf{c}_j^{(r-1)} - \frac{1}{\tau} \left[\text{PRIVIX}(\mathbf{S}^{(r-1)}) - \text{PRIVIX}(\mathbf{S}_j^{(r-1)}) \right]$$

where $\Phi^{(r)} \triangleq \text{PRIVIX}(\mathbf{S}^{(r-1)})$
- 5: **if HEAPRIX variant:**

$$\mathbf{c}_j^{(r)} = \mathbf{c}_j^{(r-1)} - \frac{1}{\tau} (\Phi^{(r)} - \Phi_j^{(r)})$$
- 6: Set $\mathbf{x}^{(r)} = \mathbf{x}^{(r-1)} - \gamma \Phi^{(r)}$ and $\mathbf{x}_j^{(0,r)} = \mathbf{x}^{(r)}$
- 7: **for** $\ell = 0, \dots, \tau - 1$ **do**
- 8: Sample mini-batch $\xi_j^{(\ell,r)}$ and compute $\tilde{\mathbf{g}}_j^{(\ell,r)}$
- 9: $\mathbf{x}_j^{(\ell+1,r)} = \mathbf{x}_j^{(\ell,r)} - \eta (\tilde{\mathbf{g}}_j^{(\ell,r)} - \mathbf{c}_j^{(r)})$
- 10: **end for**
- 11: Device j broadcasts $\mathbf{S}_j^{(r)} \triangleq \mathbf{S}(\mathbf{x}_j^{(0,r)} - \mathbf{x}_j^{(\tau,r)})$.
- 12: Server **computes** $\mathbf{S}^{(r)} = \frac{1}{p} \sum_{j=1}^p \mathbf{S}_j^{(r)}$ and **broadcasts** $\mathbf{S}^{(r)}$ to all devices.
- 13: **if HEAPRIX variant:**
- 14: Device j computes $\Phi_j^{(r)} \triangleq \text{HEAPRIX}[\mathbf{S}_j^{(r)}]$
- 15: Second round of communication to obtain $\delta_j^{(r)} := \mathbf{S}_j(\text{HEAVYMIX}[\mathbf{S}^{(r)}])$
- 16: Broadcasts $\tilde{\mathbf{S}}^{(r)} \triangleq \frac{1}{p} \sum_{j=1}^p \delta_j^{(r)}$ to devices
- 17: **end parallel for**
- 18: **end**
- 19: **Output:** $\mathbf{x}^{(R-1)}$

Assumption 1 is common in stochastic optimization. We present our results for PL, convex and general non-convex objectives. [19] show that PL condition implies strong convexity property with same module (PL objectives can also be non-convex, hence strong convexity does not imply PL condition necessarily).

4.1 Convergence of FEDSKETCH

We now focus on the homogeneous case where data is i.i.d. among local devices, and therefore, the stochastic local gradient of each worker is an unbiased estimator of the global gradient. We have:

Assumption 2 (Bounded Variance). *For all $j \in [m]$, we can sample an independent mini-batch ℓ_j of size $|\Xi_j^{(\ell,r)}| = b$ and compute an unbiased stochastic gradient $\tilde{\mathbf{g}}_j = \nabla f_j(\mathbf{x}; \Xi_j)$, $\mathbb{E}_{\Xi_j}[\tilde{\mathbf{g}}_j] = \nabla f(\mathbf{x}) = \mathbf{g}$ with the variance bounded is bounded by a constant σ^2 , i.e., $\mathbb{E}_{\Xi_j}[\|\tilde{\mathbf{g}}_j - \mathbf{g}\|^2] \leq \sigma^2$.*

Theorem 1. *Suppose Assumptions 1-2 hold. Given $0 < m \leq d$ and considering Algorithm 3 with sketch size $B = O\left(m \log\left(\frac{dR}{\delta}\right)\right)$ and $\gamma \geq k$, with probability $1 - \delta$ we have:*

*In the **non-convex** case, $\{\mathbf{x}^{(r)}\}_{r \geq 0}$ satisfies $\frac{1}{R} \sum_{r=0}^{R-1} \mathbb{E}[\|\nabla f(\mathbf{x}^{(r)})\|_2^2] \leq \epsilon$ if:*

- **FS-PRIVIX**, for $\eta = \frac{1}{L\gamma} \sqrt{\frac{k}{R\tau(\frac{cd}{mk} + 1)}}$: $R = O(1/\epsilon)$ and $\tau = O((d+m)/(mk\epsilon))$.

- **FS-HEAPRIX**, for $\eta = \frac{1}{L\gamma} \sqrt{\frac{k}{R\tau(\frac{cd-m}{mk} + 1)}}$: $R = O(1/\epsilon)$ and $\tau = O(d/(mk\epsilon))$.

*In the **PL or strongly convex** case, $\{\mathbf{x}^{(r)}\}_{r \geq 0}$ satisfies $\mathbb{E}[f(\mathbf{x}^{(R-1)}) - f(\mathbf{x}^*)] \leq \epsilon$ if we set:*

- **FS-PRIVIX**, for $\eta = \frac{1}{2L(cd/mk+1)\tau\gamma}$: $R = O((d/mk+1)\kappa \log(1/\epsilon))$ and $\tau = O((d/m+1)/(d/m+k)\epsilon)$.

- **FS-HEAPRIX**, for $\eta = \frac{1}{2L((cd-m)/mk+1)\tau\gamma}$: $R = O(((d-m)/mk+1)\kappa \log(1/\epsilon))$ and $\tau = O(d/m/(((d-m-1)+k)\epsilon))$.

*In the **Convex** case, $\{\mathbf{x}^{(r)}\}_{r \geq 0}$ satisfies $\mathbb{E}[f(\mathbf{x}^{(R-1)}) - f(\mathbf{x}^*)] \leq \epsilon$ if we set:*

- **FS-PRIVIX**, for $\eta = \frac{1}{2L(cd/mk+1)\tau\gamma}$: $R = O(L(1+d/mk)/\epsilon \log(1/\epsilon))$ and $\tau = O((d/m+1)^2/(k(d/mk+1)^2\epsilon^2))$.

- **FS-HEAPRIX**, for $\eta = \frac{1}{2L((cd-m)/mk+1)\tau\gamma}$: $R = O(L(1+(d-m)/mk)/\epsilon \log(1/\epsilon))$ and $\tau = O((d/m)^2/(k([d-m]/mk+1)^2\epsilon^2))$.

The bounds in Theorem 1 suggest that in homogeneous setting if we set $d = m$ (no compression), the number of communication rounds to achieve the ϵ error matches with the number of iterations required to achieve the same error under a centralized setting. Additionally, computational complexity scales down with number of sampled devices. To stress on the further impact of using sketching, we also compare our results with prior works in terms of total number of communicated bits per device.

Comparison with [17] From privacy aspect, we note [17] requires for server to have access to exact values of top_m gradients, hence do not preserve privacy, whereas our schemes do not need those exact values. From communication cost point of view, for strongly convex objective and compared to [17], we improve the total communication per worker from $RB = O\left(\frac{d}{\epsilon} \log\left(\frac{d}{\delta\sqrt{\epsilon}} \max\left(\frac{d}{m}, \frac{1}{\sqrt{\epsilon}}\right)\right)\right)$ to

$$RB = O\left(\kappa\left(\frac{d-m}{k} + m\right) \log \frac{1}{\epsilon} \log\left(\frac{\kappa d}{\delta}\left(\frac{d-m}{mk} + 1\right) \log \frac{1}{\epsilon}\right)\right).$$

We note that while reducing communication cost, our scheme requires $\tau = O(d/m(k(\frac{d}{mk}+1)\epsilon)) > 1$, which scales down with the number of sampled devices, k . Moreover, unlike [17], we do not use bounded gradient assumption. Therefore, we obtain stronger result with weaker assumptions. Regarding general non-convex objectives, our result improves the total communication cost per worker in [17] from $RB = O\left(\max(\frac{1}{\epsilon^2}, \frac{d^2}{k^2\epsilon}) \log(\frac{d}{\delta} \max(\frac{1}{\epsilon^2}, \frac{d^2}{k^2\epsilon}))\right)$ for *only one device* to $RB =$

272 $O(\frac{m}{\epsilon} \log(\frac{d}{\epsilon\delta}))$. We also highlight that we can obtain similar rates for Algorithm 3 in heterogeneous
 273 environment if we make the additional assumption of uniformly bounded gradient.

274 **Note:** Such improved communication cost over prior related works is due to joint exploitation of
 275 *sketching*, to reduce the dimension of communicated messages, and the use of *local updates*, to
 276 reduce the total number of communication rounds leading to a specific convergence error.

277 4.2 Convergence of FedSKETCHGATE

278 We start with bounded local variance assumption:

279 **Assumption 3** (Bounded Local Variance). *For all $j \in [p]$, we can sample an independent mini-*
 280 *batch Ξ_j of size $|\xi_j| = b$ and compute an unbiased stochastic gradient $\tilde{\mathbf{g}}_j = \nabla f_j(\mathbf{x}; \Xi_j)$ with*
 281 *$\mathbb{E}_{\Xi}[\tilde{\mathbf{g}}_j] = \nabla f_j(\mathbf{x}) = \mathbf{g}_j$. Moreover, the variance of local stochastic gradients is bounded such that*
 282 *$\mathbb{E}_{\Xi}[\|\tilde{\mathbf{g}}_j - \mathbf{g}_j\|^2] \leq \sigma^2$.*

283 **Theorem 2.** *Suppose Assumptions 1 and 3 hold. Given $0 < m \leq d$, and considering*
 284 *FedSKETCHGATE in Algorithm 4 with sketch size $B = O(m \log(\frac{dR}{\delta}))$ and $\gamma \geq p$ with proba-*
 285 *bility $1 - \delta$ we have*

286 *In the **non-convex** case, $\eta = \frac{1}{L\gamma} \sqrt{\frac{mp}{R\tau(cd)}}$, $\{\mathbf{x}^{(r)}\}_{r=0}^{\infty}$ satisfies $\frac{1}{R} \sum_{r=0}^{R-1} \mathbb{E}[\|\nabla f(\mathbf{x}^{(r)})\|_2^2] \leq \epsilon$ if:*

287 • **FS-PRIVIX:**

$$R = O((d + m)/m\epsilon) \quad \text{and} \quad \tau = O(1/(p\epsilon)).$$

288 • **FS-HEAPRIX:** $R = O(d/m\epsilon)$ and $\tau = O(1/(p\epsilon))$.

289 *In the **PL or Strongly convex** case, $\{\mathbf{x}^{(r)}\}_{r=0}^{\infty}$ satisfies $\mathbb{E}[f(\mathbf{x}^{(R-1)}) - f(\mathbf{x}^{(*)})] \leq \epsilon$ if:*

290 • **FS-PRIVIX**, for $\eta = 1/(2L(\frac{cd}{m} + 1)\tau\gamma)$: $R = O((\frac{d}{m} + 1)\kappa \log(1/\epsilon))$ and $\tau = O(1/(p\epsilon))$

291 • **FS-HEAPRIX**, for $\eta = m/(2cLd\tau\gamma)$: $R = O((\frac{d}{m})\kappa \log(1/\epsilon))$ and $\tau = O(1/(p\epsilon))$.

292 *In the **convex** case, $\{\mathbf{x}^{(r)}\}_{r=0}^{\infty}$ satisfies $\mathbb{E}[f(\mathbf{x}^{(R-1)}) - f(\mathbf{x}^{(*)})] \leq \epsilon$ if:*

293 • **FS-PRIVIX**, for $\eta = 1/(2L(cd/m + 1)\tau\gamma)$: $R = O(L(d/m + 1)\epsilon \log(1/\epsilon))$ and $\tau =$
 294 $O(1/(p\epsilon^2))$.

295 • **FS-HEAPRIX**, for $\eta = m/(2Lcd\tau\gamma)$: $R = O(L(d/m)\epsilon \log(1/\epsilon))$ and $\tau = O(1/(p\epsilon^2))$.

296 Theorem 2 implies that the number of communication rounds and local updates are similar to the
 297 corresponding quantities in homogeneous setting except for the non-convex case where the number
 298 of rounds also depends on the compression rate (summarized Table 2-3 of the Appendix).

299 4.3 Comparison with Prior Methods

300 Before comparing with prior works, we highlight that privacy is another purpose of using unbiased
 301 sketching in addition to communication efficiency. Therefore, our main competing schemes are
 302 distributed algorithms based on sketching. Nonetheless, for the sake of showing the effectiveness of
 303 our algorithms, we also compare with prior non-sketching based distributed algorithms ([20, 3, 36,
 304 13]) in Section B of Appendix.

305 **Comparison with [26].** Note that our convergence analysis does not rely on the bounded gradient
 306 assumption. We also improve both the number of communication rounds R and the size of transmitted
 307 bits B per communication round. Additionally, we highlight that, while [26] provides a convergence
 308 analysis for convex objectives, our analysis holds for PL (thus strongly convex case), general convex
 309 and general non-convex objectives.

310 **Comparison with [37].** Due to gradient tracking, our algorithm tackles data heterogeneity issue,
 311 while algorithms in [37] does not particularly. As a consequence, in FedSKETCHGATE each device
 312 has to store an additional state vector compared to [37]. Yet, as our method is built upon an
 313 unbiased compressor, server does not need to store any additional error correction vector. The
 314 convergence results for both of two variants of FetchSGD in [37] rely on the uniform bounded gradient
 315 assumption which may not be applicable with L -smoothness assumption when data distribution
 316 is highly heterogeneous, as in FL, see [21], while our bounds do not assume such boundedness.

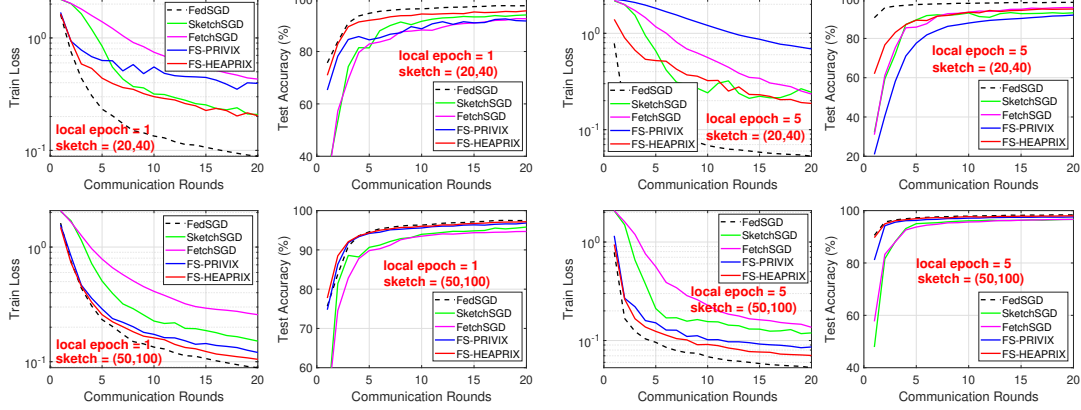


Figure 1: Homogeneous case: Comparison of compressed optimization methods on LeNet CNN.

Besides, Theorem 1 [37] assumes that *Contraction Holds* for the sequence of gradients which may not hold in practice, yet based on this strong assumption, their total communication cost (RB) in order to achieve ϵ error is $RB = O\left(m \max\left(\frac{1}{\epsilon^2}, \frac{d^2 - dm}{m^2 \epsilon}\right) \log\left(\frac{d}{\delta} \max\left(\frac{1}{\epsilon^2}, \frac{d^2 - dm}{m^2 \epsilon}\right)\right)\right)$. For the sake of comparison we let the compression ratio in [37] to be $\frac{m}{d}$. In contrast, without any extra assumptions, our results in Theorem 2 for PRIVIX and HEAPRIX are respectively $RB = O\left(\frac{(d+m)}{\epsilon} \log\left(\frac{d^2}{\epsilon \delta} + d\right)\right)$ and $RB = O\left(\frac{d}{\epsilon} \log\left(\frac{d^2}{\epsilon m \delta}\right)\right)$ which improves the total communication cost of Theorem 1 in [37] under regimes such that $\frac{1}{\epsilon} \geq d$ or $d \gg m$. Theorem 2 in [37] is based the *Sliding Window Heavy Hitters* assumption, which is similar to the gradient diversity assumption in [29, 12]. Under that assumption the total communication cost is shown to be $RB = O\left(\frac{m \max(I^{2/3}, 2 - \alpha)}{\epsilon^3 \alpha} \log\left(\frac{d \max(I^{2/3}, 2 - \alpha)}{\epsilon^3 \delta}\right)\right)$ where I is a constant related to the window of gradients. We improve this bound under weaker assumptions in a regime where $\frac{I^{2/3}}{\epsilon^2} \geq d$. We also provide bounds for PL, convex and non-convex objectives contrary to [37]. Finally, we note that algorithms in [37] are using momentum at server. While we do not use it explicitly, we can modify our algorithms to include momentum easily.

5 Numerical Study

In this section, we provide empirical results on MNIST benchmark dataset to demonstrate the effectiveness of our proposed algorithms. We train LeNet-5 Convolutional Neural Network (CNN) architecture introduced in [24], with 60 000 parameters. We compare Federated SGD (FedSGD) as the full-precision baseline, along with four sketching methods SketchSGD [17], FetchSGD [37], and two FedSketch variants FS-PRIVIX and FS-HEAPRIX. Note that in Algorithm 3, FS-PRIVIX with global learning rate $\gamma = 1$ is equivalent to the DiffSketch algorithm proposed in [29]. Also, SketchSGD is slightly modified to compress the change in local weights (instead of local gradient in every iteration), and FetchSGD is implemented with second round of communication for fairness. (The original proposal does not include second round of communication, which performs worse with small sketch size.) As suggested in [37], the momentum factor of FetchSGD is set to 0.9, and we also follow some recommended implementation tricks to improve its performance, which are detailed in the Appendix. The number of workers is set to 50 and we report the results for 1 and 5 local epochs. A local epoch is finished when all workers go through their local data samples once. The local batch size is 30. In each round, we randomly choose half of the devices to be active. We tune the learning rates (η and γ , if applicable) over log-scale and report the best results, for both *homogeneous* and *heterogeneous* setting. In the former case, each device receives uniformly drawn data samples, and in the latter, it only receives samples from one or two classes among ten.

Homogeneous case. In Figure 1, we provide the training loss and test accuracy with different number of local epochs and sketch size, $(t, k) = (20, 40)$ and $(50, 100)$. Note that, these two choices of sketch size correspond to a $75\times$ and $12\times$ compression ratio, respectively. We conclude

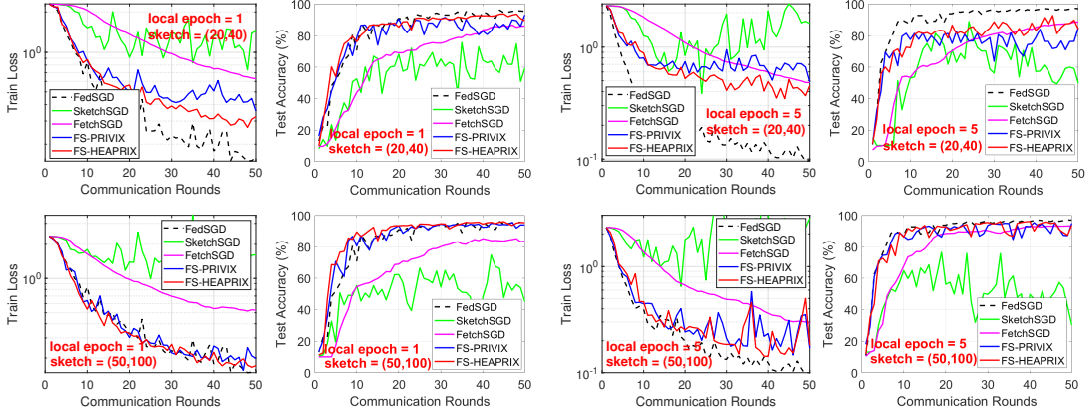


Figure 2: Heterogeneous case: Comparison of compressed optimization algorithms on LeNet CNN.

- In general, increasing compression ratio would sacrifice learning performance. In all cases, FS-HEAPRIX performs the best in terms of both training objective and test accuracy, among all compressed methods.
- FS-HEAPRIX is better than FS-PRIVIX, especially with small sketches (high compression ratio). FS-HEAPRIX yields acceptable extra test error compared to full-precision FedSGD, particularly when considering the high compression ratio (e.g., $75\times$).
- From the training loss, we see that the performance of FS-HEAPRIX improves when the number of local updates increases. *That is, the proposed method is able to further reduce the communication cost by reducing the number of rounds required for communication.* This is also consistent with our theoretical findings.

In general, our proposed FS-HEAPRIX outperforms all competing methods, and a sketch size of $(50, 100)$ is sufficient to approach the accuracy of full-precision FedSGD.

Heterogeneous case. We plot similar set of results in Figure 2 for non-i.i.d. data distribution, which leads to more twists and turns in the training curves. We see that SketchSGD performs very poorly in the heterogeneous case, which is improved by error tracking and momentum in FetchSGD, as expected. However, both of these methods are worse than our proposed FedSketchGATE methods, which can achieve similar generalization accuracy as full-precision FedSGD, even with small sketch size (i.e., $75\times$ compression with 1 local epoch). Note that, slower convergence and worse generalization of FedSGD in non-i.i.d. data distribution case is also reported in e.g. [33, 8].

We also notice in Figure 2 the edge of FS-HEAPRIX over FS-PRIVIX in terms of training loss and test accuracy. However, we see that in the heterogeneous setting, more local updates tend to undermine the learning performance, especially with small sketch size. Nevertheless, when the sketch size is not too small, i.e., $(50, 100)$, FS-HEAPRIX can still provide comparable test accuracy as FedSGD in both cases. Our empirical study demonstrates that FedSketch (and FedSketchGATE) frameworks are able to perform well in homogeneous (resp. heterogeneous) settings, with high compression rate. In particular, FedSketch methods are beneficial over SketchSGD [17] and FetchSGD [37] in all cases. FS-HEAPRIX performs the best among all the tested compressed algorithms, which in many cases achieves similar generalization accuracy as full-precision FedSGD with small sketch size.

6 Conclusion

In this paper, we introduced FedSKETCH and FedSKETCHGATE algorithms for homogeneous and heterogeneous data distribution setting respectively for Federated Learning wherein communication between server and devices is only performed using count sketch. Our algorithms, thus, provide communication-efficiency and privacy, through random hashes based sketches. We analyze the convergence error for *non-convex*, *PL* and *general convex* objective functions in the scope of Federated Optimization. We provide insightful numerical experiments showcasing the advantages of our FedSKETCH and FedSKETCHGATE methods over current federated optimization algorithm. The proposed algorithms outperform competing compression method and can achieve comparable test accuracy as Federated SGD, with high compression ratio.

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524 Checklist

- 525 1. For all authors...
- 526 (a) Do the main claims made in the abstract and introduction accurately reflect the paper's
527 contributions and scope? [Yes]
- 528 (b) Did you describe the limitations of your work? [Yes]
- 529 (c) Did you discuss any potential negative societal impacts of your work? [N/A]
- 530 (d) Have you read the ethics review guidelines and ensured that your paper conforms to
531 them? [Yes]
- 532 2. If you are including theoretical results...
- 533 (a) Did you state the full set of assumptions of all theoretical results? [Yes]
- 534 (b) Did you include complete proofs of all theoretical results? [Yes]
- 535 3. If you ran experiments...
- 536 (a) Did you include the code, data, and instructions needed to reproduce the main exper-
537 imental results (either in the supplemental material or as a URL)? [Yes] The code is
538 available upon demand.
- 539 (b) Did you specify all the training details (e.g., data splits, hyperparameters, how they
540 were chosen)? [Yes]
- 541 (c) Did you report error bars (e.g., with respect to the random seed after running experi-
542 ments multiple times)? [No] Results are averaged over several runs.
- 543 (d) Did you include the total amount of compute and the type of resources used (e.g., type
544 of GPUs, internal cluster, or cloud provider)? [N/A]
- 545 4. If you are using existing assets (e.g., code, data, models) or curating/releasing new assets...
- 546 (a) If your work uses existing assets, did you cite the creators? [Yes]
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- 549
- 550 (d) Did you discuss whether and how consent was obtained from people whose data you're
551 using/curating? [N/A]
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553 information or offensive content? [N/A]
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- 555 (a) Did you include the full text of instructions given to participants and screenshots, if
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- 557 (b) Did you describe any potential participant risks, with links to Institutional Review
558 Board (IRB) approvals, if applicable? [N/A]
- 559 (c) Did you include the estimated hourly wage paid to participants and the total amount
560 spent on participant compensation? [N/A]

561 Appendix for FedSKETCH: Communication-Efficient Federated 562 Learning via Sketching

563 A Notations and Definitions

564 **Notation.** Here we denote the count sketch of the vector \mathbf{x} by $\mathbf{S}(\mathbf{x})$ and with an abuse of notation,
565 we indicate the expectation over the randomness of count sketch with $\mathbb{E}_{\mathbf{S}}[\cdot]$. We illustrate the random
566 subset of the devices selected by the central server with \mathcal{K} with size $|\mathcal{K}| = k \leq p$, and we represent
567 the expectation over the device sampling with $\mathbb{E}_{\mathcal{K}}[\cdot]$.

Table 1: Table of Notations

p	\triangleq	Number of devices
k	\triangleq	Number of sampled devices for homogeneous setting
$\mathcal{K}^{(r)}$	\triangleq	Set of sampled devices in communication round r
d	\triangleq	Dimension of the model
τ	\triangleq	Number of local updates
R	\triangleq	Number of communication rounds
B	\triangleq	Size of transmitted bits
$R \times B$	\triangleq	Total communication cost per device
κ	\triangleq	Condition number
ϵ	\triangleq	Target accuracy
μ	\triangleq	PL constant
m	\triangleq	Number of bins of hash tables
$\mathbf{S}(\mathbf{x})$	\triangleq	Count sketch of the vector \mathbf{x}
$\mathbb{U}(\Delta)$	\triangleq	Class of unbiased compressor, see Definition 1

568 **Definition 3** (Polyak-Łojasiewicz). *A function $f(\mathbf{x})$ satisfies the Polyak-Łojasiewicz(PL) condition*
569 *with constant μ if $\frac{1}{2}\|\nabla f(\mathbf{x})\|_2^2 \geq \mu(f(\mathbf{x}) - f(\mathbf{x}^*))$, $\forall \mathbf{x} \in \mathbb{R}^d$ with \mathbf{x}^* is an optimal solution.*

570 A.1 Count sketch

571 In this paper, we exploit the commonly used Count Sketch [7] which is described in Algorithm 5.

Algorithm 5 Count Sketch (CS) [7]

```

1: Inputs:  $\mathbf{x} \in \mathbb{R}^d, t, k, \mathbf{S}_{m \times t}, h_j(1 \leq i \leq t), \text{sign}_j(1 \leq i \leq t)$ 
2: Compress vector  $\mathbf{x} \in \mathbb{R}^d$  into  $\mathbf{S}(\mathbf{x})$ :
3: for  $x_i \in \mathbf{x}$  do
4:   for  $j = 1, \dots, t$  do
5:      $\mathbf{S}[j][h_j(i)] = \mathbf{S}[j-1][h_{j-1}(i)] + \text{sign}_j(i) \cdot x_i$ 
6:   end for
7: end for
8: return  $\mathbf{S}_{m \times t}(\mathbf{x})$ 

```

572 A.2 PRIVIX and compression error of HEAPRIX

573 For the sake of completeness we review PRIVIX algorithm that is also mentioned in [26] as follows:

Algorithm 6 PRIVIX/DiffSketch [26]: Unbiased compressor based on sketching.

```

1: Inputs:  $x \in \mathbb{R}^d, t, m, \mathbf{S}_{m \times t}, h_j(1 \leq i \leq t), \text{sign}_j(1 \leq i \leq t)$ 
2: Query  $\tilde{x} \in \mathbb{R}^d$  from  $\mathbf{S}(x)$ :
3: for  $i = 1, \dots, d$  do
4:    $\tilde{x}[i] = \text{Median}\{\text{sign}_j(i) \cdot \mathbf{S}[j][h_j(i)] : 1 \leq j \leq t\}$ 
5: end for
6: Output:  $\tilde{x}$ 

```

Regarding the compression error of sketching we restate the following Corollary from the main body of this paper:

Corollary 2. *Based on Theorem 3 of [15] and using Algorithm 2, we have $C(x) \in \mathbb{U}(c \frac{d}{m})$. This shows that unlike PRIVIX (Algorithm 6) the compression noise can be made as small as possible using large size of hash table.*

Proof. The proof simply follows from Theorem 3 in [15] and Algorithm 2 by setting $\Delta_1 = c \frac{d}{m}$ and $\Delta_2 = 1 + c \frac{d}{m}$ we obtain $\Delta = \Delta_2 + \frac{1-\Delta_2}{\Delta_1} = c \frac{d}{m} = O(\frac{d}{m})$ for the compression error of HEAPRIX. \square

B Summary of comparison of our results with prior works

For the purpose of further clarification, we summarize the comparison of our results with related works. We recall that p is the number of devices, d is the dimension of the model, κ is the condition number, ϵ is the target accuracy, R is the number of communication rounds, and τ is the number of local updates. We start with the homogeneous setting comparison. Comparison of our results and existing ones for homogeneous and heterogeneous setting are given respectively Table 2 and Table 3.

Table 2: Comparison of results with compression and periodic averaging in the homogeneous setting. UG and PP stand for Unbounded Gradient and Privacy Property respectively.

Reference	PL/Strongly Convex	UG	PP
Ivkin et al. [17]	$R = O\left(\max\left(\frac{d}{m\sqrt{\epsilon}}, \frac{1}{\epsilon}\right)\right), \tau = 1, B = O\left(m \log\left(\frac{dR}{\delta}\right)\right)$ $pRB = O\left(\frac{pd}{m\epsilon} \log\left(\frac{d}{\delta\sqrt{\epsilon}} \max\left(\frac{d}{m}, \frac{1}{\sqrt{\epsilon}}\right)\right)\right)$	\times	\times
Theorem 1	$R = O\left(\kappa\left(\frac{d-m}{mk} + 1\right) \log\left(\frac{1}{\epsilon}\right)\right), \tau = O\left(\frac{d}{\kappa\left(\frac{d}{k} + m\right)\epsilon}\right), B = O\left(m \log\left(\frac{dR}{\delta}\right)\right)$ $kRB = O\left(m\kappa(d-m+mk) \log\frac{1}{\epsilon} \log\left(\frac{\kappa(d\frac{d-m}{mk} + d) \log\frac{1}{\epsilon}}{\delta}\right)\right)$	\checkmark	\checkmark

Comparison with [13] and [36] Convergence analysis of algorithms in [13] relies on unbiased compression, while in this paper our FL algorithm based on HEAPRIX enjoys from unbiased compression with equivalent biased compression variance. Moreover, we highlight that the convergence analysis of FedCOMGATE is based on the extra assumption of boundedness of the difference between the average of compressed vectors and compressed averages of vectors. However, we do not need this extra assumption as it is satisfied naturally due to linearity of sketching. Finally, as pointed out in Remark 2, our algorithms enjoy from a bidirectional compression property, unlike FedCOMGATE in general. Furthermore, since results in [13] improve the communication complexity of FedPAQ algorithm, developed in [36], hence FedSKETCH and FedSKETCHGATE improves the communication complexity obtained in [36].

Comparison with [3]. We note that the algorithm in [3] uses a composed compression and quantization while our algorithm is solely based on compression. So, in order to compare with algorithms in [3] we only consider Qsparse-local-SGD with compression and we let compression factor $\gamma = \frac{m}{d}$ (to compare with the same compression ratio induced with sketch size of mt). For strongly convex objective in Qsparse-local-SGD to achieve convergence error of ϵ they require $R = O\left(\kappa \frac{d}{m\sqrt{\epsilon}}\right)$ and $\tau = O\left(\frac{m}{pd\sqrt{\epsilon}}\right)$, which is improved to $R = O\left(\frac{\kappa d}{m} \log(1/\epsilon)\right)$ and $\tau = O\left(\frac{1}{p\epsilon}\right)$ for PL objectives.

Table 3: Comparison of results with compression and periodic averaging in the heterogeneous setting. UG and PP stand for Unbounded Gradient and Privacy Property respectively.

Reference	non-convex	General Convex	UG	PP
Basu et al. [3] (with $\gamma = m/d$)	$R = O\left(\frac{d}{m\epsilon^{1.5}}\right)$ $\tau = O\left(\frac{m}{pd\sqrt{\epsilon}}\right)$ $B = O(d)$ $RB = O\left(\frac{d^2}{m\epsilon^{1.5}}\right)$	—	✗	✗
Li et al. [26]	—	$R = O\left(\frac{d}{m\epsilon^2}\right)$ $\tau = 1$ $B = O\left(m \log\left(\frac{d^2}{m\epsilon^2\delta}\right)\right)$	✗	✓
Rothchild et al. [37]	$R = O\left(\max\left(\frac{1}{\epsilon^2}, \frac{d^2 - md}{m^2\epsilon}\right)\right)$ $\tau = 1$ $B = O\left(m \log\left(\frac{d}{\delta} \max\left(\frac{1}{\epsilon^2}, \frac{d^2 - md}{m^2\epsilon}\right)\right)\right)$ $RB = O\left(m \max\left(\frac{1}{\epsilon^2}, \frac{d^2 - md}{m^2\epsilon}\right) \log\left(\frac{d}{\delta} \max\left(\frac{1}{\epsilon^2}, \frac{d^2 - md}{m^2\epsilon}\right)\right)\right)$	—	✗	✗
Rothchild et al. [37]	$R = O\left(\frac{\max(I^{2/3}, 2 - \alpha)}{\epsilon^3}\right)$ $\tau = 1$ $B = O\left(\frac{m}{\alpha} \log\left(\frac{d \max(I^{2/3}, 2 - \alpha)}{\epsilon^3\delta}\right)\right)$ $RB = O\left(\frac{m \max(I^{2/3}, 2 - \alpha)}{\epsilon^3\alpha} \log\left(\frac{d \max(I^{2/3}, 2 - \alpha)}{\epsilon^3\delta}\right)\right)$	—	✗	✗
Theorem 2	$R = O\left(\frac{d}{m\epsilon}\right)$ $\tau = O\left(\frac{1}{p\epsilon}\right)$ $B = O\left(m \log\left(\frac{d^2}{m\epsilon\delta}\right)\right)$ $RB = O\left(\frac{d}{\epsilon} \log\left(\frac{d^2}{m\epsilon\delta} \log\left(\frac{1}{\epsilon}\right)\right)\right)$	$R = O\left(\frac{d}{m\epsilon} \log\left(\frac{1}{\epsilon}\right)\right)$ $\tau = O\left(\frac{1}{p\epsilon^2}\right)$ $B = O\left(m \log\left(\frac{d^2}{m\epsilon\delta}\right)\right)$	✓	✓

Similarly, for non-convex objective [3] requires $R = O\left(\frac{d}{m\epsilon^{1.5}}\right)$ and $\tau = O\left(\frac{m}{pd\sqrt{\epsilon}}\right)$, which is improved to $R = O\left(\frac{d}{m\epsilon}\right)$ and $\tau = O\left(\frac{1}{p\epsilon}\right)$. We note that we reduce communication rounds at the cost of increasing number of local updates (which scales down with number of devices, p). Additionally, we highlight that our FedSKETCHGATE exploits the gradient tracking idea to deal with data heterogeneity, while algorithms in [3] does not develop such mechanism and may suffer from poor convergence in heterogeneous setting. We also note that setting $\tau = 1$ and using top_m compressor, the QSPARSE-local-SGD algorithm becomes similar to distributed SGD with sketching as they both use the error feedback framework to improve the compression variance. Finally, since the average of sparse vectors may not be sparse in general the number of transmitted bits from server to devices in QSPARSE-Local-SGD in [3] may not be sparse in general ($B = O(d)$), however our algorithms enjoy from bidirectional compression properly due to lower dimension and linearity properties of sketching ($B = O(m \log(\frac{Rd}{\delta}))$). Therefore, the total number of bits per device for strongly convex and non-convex objective is improved respectively from $RB = O\left(\kappa \frac{d^2}{m\sqrt{\epsilon}}\right)$ and $RB = O\left(\frac{d^2}{m\epsilon^{1.5}}\right)$ in [3] to $RB = O\left(\kappa d \log\left(\frac{\kappa d^2}{m\delta} \log\left(\frac{1}{\epsilon}\right)\right) \log(1/\epsilon)\right) = O\left(\kappa d \max\left(\log\left(\frac{\kappa d^2}{m\delta}\right), \log^2(1/\epsilon)\right)\right)$ and $RB = O\left(\log\left(\frac{d^2}{m\epsilon\delta}\right) \frac{d}{\epsilon}\right)$.

Additionally, as we noted using sketching for transmission implies two way communication from master to devices and vice versa. Therefore, in order to show efficacy of our algorithm we compare our convergence analysis with the obtained rates in the following related work:

Comparison with [35]. The reference [35] considers two-way compression from parameter server to devices and vice versa. They provide the convergence rate of $R = O\left(\frac{\omega^{\text{Up}} \omega^{\text{Down}}}{\epsilon^2}\right)$ for strongly-objective functions where ω^{Up} and ω^{Down} are uplink and downlink's compression noise (specializing to our case for the sake of comparison $\omega^{\text{Up}} = \omega^{\text{Down}} = \theta(d)$) for general heterogeneous data distribution. In contrast, while our algorithms are using bidirectional compression due to use of sketching for communication, our convergence rate for strongly-convex objective is $R = O(\kappa \mu^2 d \log(\frac{1}{\epsilon}))$ with probability $1 - \delta$.

629 We would like to also mention that there prior studies such as [43] and [49] that analyze the two-way
 630 compression, but since [35] is the state-of-the-art on this topic we only compared our results with
 631 these papers.

632 C Theoretical Proofs

633 We will use the following fact (which is also used in [30, 12]) in proving results.

634 **Fact 3** ([30, 12]). *Let $\{x_i\}_{i=1}^p$ denote any fixed deterministic sequence. We sample a multiset \mathcal{P} (with*
 635 *size K) uniformly at random where x_j is sampled with probability q_j for $1 \leq j \leq p$ with replacement.*
 636 *Let $\mathcal{P} = \{i_1, \dots, i_K\} \subset [p]$ (some i_j s may have the same value). Then*

$$\mathbb{E}_{\mathcal{P}} \left[\sum_{i \in \mathcal{P}} x_i \right] = \mathbb{E}_{\mathcal{P}} \left[\sum_{k=1}^K x_{i_k} \right] = K \mathbb{E}_{\mathcal{P}} [x_{i_k}] = K \left[\sum_{j=1}^p q_j x_j \right] \quad (2)$$

637 For the sake of the simplicity, we review an assumption for the quantization/compression, that
 638 naturally holds for PRIVIX and HEAPRIX.

639 **Assumption 4** ([13]). *The output of the compression operator $Q(\mathbf{x})$ is an unbiased estimator of*
 640 *its input \mathbf{x} , and its variance grows with the squared of the squared of ℓ_2 -norm of its argument, i.e.,*
 641 $\mathbb{E}[Q(\mathbf{x})] = \mathbf{x}$ *and* $\mathbb{E}[\|Q(\mathbf{x}) - \mathbf{x}\|^2] \leq \omega \|\mathbf{x}\|^2$.

642 We note that the sketching PRIVIX and HEAPRIX, satisfy Assumption 4 with $\omega = c \frac{d}{m}$ and $\omega =$
 643 $c \frac{d}{m} - 1$ respectively with probability $1 - \frac{\delta}{R}$ per communication round. Therefore, all the results in
 644 Theorem 1, by taking union over the all probabilities of each communication rounds, are concluded
 645 with probability $1 - \delta$ by plugging $\omega = c \frac{d}{m}$ and $\omega = c \frac{d}{m} - 1$ respectively into the corresponding
 646 convergence bounds.

647 C.1 Proof of Theorem 1

648 In this section, we study the convergence properties of our FedSKETCH method presented in Algo-
 649 rithm 3. Before developing the proofs for FedSKETCH in the homogeneous setting, we first mention
 650 the following intermediate lemmas.

651 **Lemma 1.** *Using unbiased compression and under Assumption 2, we have the following bound:*

$$\mathbb{E}_{\mathcal{K}} \left[\mathbb{E}_{\mathbf{S}, \xi^{(r)}} \left[\|\tilde{\mathbf{g}}_{\mathbf{S}}^{(r)}\|^2 \right] \right] = \mathbb{E}_{\xi^{(r)}} \mathbb{E}_{\mathbf{S}} \left[\|\tilde{\mathbf{g}}_{\mathbf{S}}^{(r)}\|^2 \right] \leq \tau \left(\frac{\omega}{k} + 1 \right) \sum_{j=1}^m q_j \left[\sum_{c=0}^{\tau-1} \|\mathbf{g}_j^{(c,r)}\|^2 + \sigma^2 \right] \quad (3)$$

Proof.

$$\begin{aligned} & \mathbb{E}_{\xi^{(r)} | \mathbf{w}^{(r)}} \mathbb{E}_{\mathcal{K}} \left[\mathbb{E}_{\mathbf{S}} \left[\left\| \frac{1}{k} \sum_{j \in \mathcal{K}} \mathbf{S} \left(\sum_{c=0}^{\tau-1} \tilde{\mathbf{g}}_j^{(c,r)} \right) \right\|^2 \right] \right] \\ &= \mathbb{E}_{\xi^{(r)}} \left[\mathbb{E}_{\mathcal{K}} \left[\mathbb{E}_{\mathbf{S}} \left[\left\| \frac{1}{k} \sum_{j \in \mathcal{K}} \mathbf{S} \left(\underbrace{\sum_{c=0}^{\tau-1} \tilde{\mathbf{g}}_j^{(c,r)}}_{\tilde{\mathbf{g}}_{\mathbf{S}_j}^{(r)}} \right) \right\|^2 \right] \right] \right] \\ &\stackrel{\textcircled{1}}{=} \mathbb{E}_{\xi^{(r)}} \left[\mathbb{E}_{\mathcal{K}} \left[\left\| \frac{1}{k} \sum_{j \in \mathcal{K}} \tilde{\mathbf{g}}_{\mathbf{S}_j}^{(r)} - \frac{1}{k} \sum_{j \in \mathcal{K}} \mathbb{E}_{\mathbf{S}} \left[\tilde{\mathbf{g}}_{\mathbf{S}_j}^{(r)} \right] \right\|^2 + \left\| \mathbb{E}_{\mathbf{S}} \left[\frac{1}{k} \sum_{j \in \mathcal{K}} \tilde{\mathbf{g}}_{\mathbf{S}_j}^{(r)} \right] \right\|^2 \right] \right] \end{aligned}$$

$$\begin{aligned}
& \stackrel{\textcircled{2}}{=} \mathbb{E}_{\xi^{(r)}} \left[\mathbb{E}_{\mathcal{K}} \left[\mathbb{E}_{\mathbf{S}} \left[\left\| \frac{1}{k} \left[\sum_{j \in \mathcal{K}} \tilde{\mathbf{g}}_{\mathbf{S}j}^{(r)} - \sum_{j \in \mathcal{K}} \tilde{\mathbf{g}}_j^{(r)} \right] \right\|^2 \right] + \left\| \frac{1}{k} \sum_{j \in \mathcal{K}} \tilde{\mathbf{g}}_j^{(r)} \right\|^2 \right] \right] \\
& = \mathbb{E}_{\xi^{(r)}} \left[\mathbb{E}_{\mathcal{K}} \left[\left[\text{Var}_{\mathbf{S}} \left[\frac{1}{k} \sum_{j \in \mathcal{K}} \tilde{\mathbf{g}}_{\mathbf{S}j}^{(r)} \right] + \left\| \frac{1}{k} \sum_{j \in \mathcal{K}} \tilde{\mathbf{g}}_j^{(r)} \right\|^2 \right] \right] \right] \\
& = \mathbb{E}_{\xi^{(r)}} \left[\mathbb{E}_{\mathcal{K}} \left[\left[\frac{1}{k^2} \sum_{j \in \mathcal{K}} \text{Var}_{\mathbf{S}_j} [\tilde{\mathbf{g}}_{\mathbf{S}j}^{(r)}] + \left\| \frac{1}{k} \sum_{j \in \mathcal{K}} \tilde{\mathbf{g}}_j^{(r)} \right\|^2 \right] \right] \right] \\
& \leq \mathbb{E}_{\xi^{(r)}} \left[\mathbb{E}_{\mathcal{K}} \left[\left[\frac{1}{k^2} \sum_{j \in \mathcal{K}} \omega \left\| \tilde{\mathbf{g}}_j^{(r)} \right\|^2 + \left\| \frac{1}{k} \sum_{j \in \mathcal{K}} \tilde{\mathbf{g}}_j^{(r)} \right\|^2 \right] \right] \right] \\
& = \left[\mathbb{E}_{\xi} \left[\frac{1}{k} \sum_{j \in \mathcal{K}} \omega \left\| \tilde{\mathbf{g}}_j^{(r)} \right\|^2 + \mathbb{E}_{\mathcal{K}} \mathbb{E}_{\xi^{(r)}} \left\| \frac{1}{k} \sum_{j \in \mathcal{K}} \tilde{\mathbf{g}}_j^{(r)} \right\|^2 \right] \right] \\
& = \left[\mathbb{E}_{\xi} \left[\left[\frac{\omega}{k} \sum_{j=1}^p q_j \left\| \tilde{\mathbf{g}}_j^{(r)} \right\|^2 + \mathbb{E}_{\mathcal{K}} \left[\text{Var} \left(\frac{1}{k} \sum_{j \in \mathcal{K}} \tilde{\mathbf{g}}_j^{(r)} \right) + \left\| \frac{1}{k} \sum_{j \in \mathcal{K}} \mathbf{g}_j^{(r)} \right\|^2 \right] \right] \right] \right] \\
& = \frac{\omega}{k} \sum_{j=1}^p q_j \mathbb{E}_{\xi} \left\| \tilde{\mathbf{g}}_j^{(r)} \right\|^2 + \mathbb{E}_{\mathcal{K}} \left[\frac{1}{k^2} \sum_{j \in \mathcal{K}} \text{Var} \left(\tilde{\mathbf{g}}_j^{(r)} \right) + \left\| \frac{1}{k} \sum_{j \in \mathcal{K}} \mathbf{g}_j^{(r)} \right\|^2 \right] \\
& \leq \frac{\omega}{k} \sum_{j=1}^p q_j \mathbb{E}_{\xi} \left\| \tilde{\mathbf{g}}_j^{(r)} \right\|^2 + \mathbb{E}_{\mathcal{K}} \left[\frac{1}{k^2} \sum_{j \in \mathcal{K}} \tau \sigma^2 + \frac{1}{k} \sum_{j \in \mathcal{K}} \left\| \mathbf{g}_j^{(r)} \right\|^2 \right] \\
& = \frac{\omega}{k} \sum_{j=1}^p q_j \left[\text{Var} \left(\tilde{\mathbf{g}}_j^{(r)} \right) + \left\| \mathbf{g}_j^{(r)} \right\|^2 \right] + \left[\frac{\tau \sigma^2}{k} + \sum_{j=1}^p q_j \left\| \mathbf{g}_j^{(r)} \right\|^2 \right] \\
& \leq \frac{\omega}{k} \sum_{j=1}^p q_j \left[\tau \sigma^2 + \left\| \mathbf{g}_j^{(r)} \right\|^2 \right] + \left[\frac{\tau \sigma^2}{k} + \sum_{j=1}^p q_j \left\| \mathbf{g}_j^{(r)} \right\|^2 \right] \\
& = (\omega + 1) \frac{\tau \sigma^2}{k} + \left(\frac{\omega}{k} + 1 \right) \left[\sum_{j=1}^p q_j \left\| \mathbf{g}_j^{(r)} \right\|^2 \right] \tag{4}
\end{aligned}$$

652 where ① holds due to $\mathbb{E} [\|x\|^2] = \text{Var}[x] + \|\mathbb{E}[x]\|^2$, ② is due to $\mathbb{E}_{\mathbf{S}} \left[\frac{1}{p} \sum_{j=1}^p \tilde{\mathbf{g}}_{\mathbf{S}j}^{(r)} \right] = \frac{1}{p} \sum_{j=1}^m \tilde{\mathbf{g}}_j^{(r)}$.

653 Next we show that from Assumptions 3, we have

$$\mathbb{E}_{\xi^{(r)}} \left[\left\| \tilde{\mathbf{g}}_j^{(r)} - \mathbf{g}_j^{(r)} \right\|^2 \right] \leq \tau \sigma^2 \tag{5}$$

654 To do so, note that

$$\begin{aligned}
\text{Var} \left(\tilde{\mathbf{g}}_j^{(r)} \right) & = \mathbb{E}_{\xi^{(r)}} \left[\left\| \tilde{\mathbf{g}}_j^{(r)} - \mathbf{g}_j^{(r)} \right\|^2 \right] \stackrel{\textcircled{1}}{=} \mathbb{E}_{\xi^{(r)}} \left[\left\| \sum_{c=0}^{\tau-1} \left[\tilde{\mathbf{g}}_j^{(c,r)} - \mathbf{g}_j^{(c,r)} \right] \right\|^2 \right] = \text{Var} \left(\sum_{c=0}^{\tau-1} \tilde{\mathbf{g}}_j^{(c,r)} \right) \\
& \stackrel{\textcircled{2}}{=} \sum_{c=0}^{\tau-1} \text{Var} \left(\tilde{\mathbf{g}}_j^{(c,r)} \right) \\
& = \sum_{c=0}^{\tau-1} \mathbb{E} \left[\left\| \tilde{\mathbf{g}}_j^{(c,r)} - \mathbf{g}_j^{(c,r)} \right\|^2 \right] \\
& \stackrel{\textcircled{3}}{\leq} \tau \sigma^2 \tag{6}
\end{aligned}$$

where in ① we use the definition of $\tilde{\mathbf{g}}_j^{(r)}$ and $\mathbf{g}_j^{(r)}$, in ② we use the fact that mini-batches are chosen in i.i.d. manner at each local machine, and ③ immediately follows from Assumptions 2.

Replacing $\mathbb{E}_{\xi^{(r)}} [\|\tilde{\mathbf{g}}_j^{(r)} - \mathbf{g}_j^{(r)}\|^2]$ in (4) by its upper bound in (5) implies that

$$\mathbb{E}_{\xi^{(r)}|\mathbf{w}^{(r)}} \mathbb{E}_{\mathbf{S}, \mathcal{K}} \left[\left\| \frac{1}{k} \sum_{j \in \mathcal{K}} \mathbf{S} \left(\sum_{c=0}^{\tau-1} \tilde{\mathbf{g}}_j^{(c,r)} \right) \right\|^2 \right] \leq (\omega + 1) \frac{\tau \sigma^2}{k} + \left(\frac{\omega}{k} + 1 \right) \sum_{j=1}^p q_j \|\mathbf{g}_j^{(r)}\|^2 \quad (7)$$

Further note that we have

$$\left\| \mathbf{g}_j^{(r)} \right\|^2 = \left\| \sum_{c=0}^{\tau-1} \mathbf{g}_j^{(c,r)} \right\|^2 \leq \tau \sum_{c=0}^{\tau-1} \|\mathbf{g}_j^{(c,r)}\|^2 \quad (8)$$

where the last inequality is due to $\left\| \sum_{i=1}^n \mathbf{a}_i \right\|^2 \leq n \sum_{i=1}^n \|\mathbf{a}_i\|^2$, which together with (7) leads to the following bound:

$$\mathbb{E}_{\xi^{(r)}|\mathbf{w}^{(r)}} \mathbb{E}_{\mathbf{S}} \left[\left\| \frac{1}{k} \sum_{j \in \mathcal{K}} \mathbf{S} \left(\sum_{c=0}^{\tau-1} \tilde{\mathbf{g}}_j^{(c,r)} \right) \right\|^2 \right] \leq (\omega + 1) \frac{\tau \sigma^2}{k} + \tau \left(\frac{\omega}{k} + 1 \right) \sum_{j=1}^p q_j \|\mathbf{g}_j^{(c,r)}\|^2, \quad (9)$$

and the proof is complete. \square

Lemma 2. Under Assumption 1, and according to the FedCOM algorithm the expected inner product between stochastic gradient and full batch gradient can be bounded with:

$$-\mathbb{E}_{\xi, \mathbf{S}, \mathcal{K}} \left[\left\langle \nabla f(\mathbf{w}^{(r)}), \tilde{\mathbf{g}}^{(r)} \right\rangle \right] \leq \frac{1}{2} \eta \frac{1}{m} \sum_{j=1}^m \sum_{c=0}^{\tau-1} \left[-\|\nabla f(\mathbf{w}^{(r)})\|_2^2 - \|\nabla f(\mathbf{w}_j^{(c,r)})\|_2^2 + L^2 \|\mathbf{w}^{(r)} - \mathbf{w}_j^{(c,r)}\|_2^2 \right] \quad (10)$$

Proof. We have:

$$\begin{aligned} & -\mathbb{E}_{\{\xi_1^{(t)}, \dots, \xi_m^{(t)} | \mathbf{w}_1^{(t)}, \dots, \mathbf{w}_m^{(t)}\}} \mathbb{E}_{\mathbf{S}, \mathcal{K}} \left[\left\langle \nabla f(\mathbf{w}^{(r)}), \tilde{\mathbf{g}}_{\mathbf{S}, \mathcal{K}}^{(r)} \right\rangle \right] \\ &= -\mathbb{E}_{\{\xi_1^{(t)}, \dots, \xi_m^{(t)} | \mathbf{w}_1^{(t)}, \dots, \mathbf{w}_m^{(t)}\}} \left[\left\langle \nabla f(\mathbf{w}^{(r)}), \eta \sum_{j \in \mathcal{K}} q_j \sum_{c=0}^{\tau-1} \tilde{\mathbf{g}}_j^{(c,r)} \right\rangle \right] \\ &= -\left\langle \nabla f(\mathbf{w}^{(r)}), \eta \sum_{j=1}^m q_j \sum_{c=0}^{\tau-1} \mathbb{E}_{\xi, \mathbf{S}} [\tilde{\mathbf{g}}_{j, \mathbf{S}}^{(c,r)}] \right\rangle \\ &= -\eta \sum_{c=0}^{\tau-1} \sum_{j=1}^m q_j \left\langle \nabla f(\mathbf{w}^{(r)}), \mathbf{g}_j^{(c,r)} \right\rangle \\ &\stackrel{\textcircled{1}}{=} \frac{1}{2} \eta \sum_{c=0}^{\tau-1} \sum_{j=1}^m q_j \left[-\|\nabla f(\mathbf{w}^{(r)})\|_2^2 - \|\nabla f(\mathbf{w}_j^{(c,r)})\|_2^2 + \|\nabla f(\mathbf{w}^{(r)}) - \nabla f(\mathbf{w}_j^{(c,r)})\|_2^2 \right] \\ &\stackrel{\textcircled{2}}{\leq} \frac{1}{2} \eta \sum_{c=0}^{\tau-1} \sum_{j=1}^m q_j \left[-\|\nabla f(\mathbf{w}^{(r)})\|_2^2 - \|\nabla f(\mathbf{w}_j^{(c,r)})\|_2^2 + L^2 \|\mathbf{w}^{(r)} - \mathbf{w}_j^{(c,r)}\|_2^2 \right] \end{aligned} \quad (11)$$

where ① is due to $2\langle \mathbf{a}, \mathbf{b} \rangle = \|\mathbf{a}\|^2 + \|\mathbf{b}\|^2 - \|\mathbf{a} - \mathbf{b}\|^2$, and ② follows from Assumption 1. \square

The following lemma bounds the distance of local solutions from global solution at r th communication round.

Lemma 3. Under Assumptions 2 we have:

$$\mathbb{E} \left[\|\mathbf{w}^{(r)} - \mathbf{w}_j^{(c,r)}\|_2^2 \right] \leq \eta^2 \tau \sum_{c=0}^{\tau-1} \left\| \mathbf{g}_j^{(c,r)} \right\|_2^2 + \eta^2 \tau \sigma^2$$

669 *Proof.* Note that

$$\begin{aligned}
\mathbb{E} \left[\left\| \mathbf{w}^{(r)} - \mathbf{w}_j^{(c,r)} \right\|_2^2 \right] &= \mathbb{E} \left[\left\| \mathbf{w}^{(r)} - \left(\mathbf{w}^{(r)} - \eta \sum_{k=0}^c \tilde{\mathbf{g}}_j^{(k,r)} \right) \right\|_2^2 \right] \\
&= \mathbb{E} \left[\left\| \eta \sum_{k=0}^c \tilde{\mathbf{g}}_j^{(k,r)} \right\|_2^2 \right] \\
&\stackrel{\textcircled{1}}{=} \mathbb{E} \left[\left\| \eta \sum_{k=0}^c \left(\tilde{\mathbf{g}}_j^{(k,r)} - \mathbf{g}_j^{(k,r)} \right) \right\|_2^2 \right] + \mathbb{E} \left[\left\| \eta \sum_{k=0}^c \mathbf{g}_j^{(k,r)} \right\|_2^2 \right] \\
&\stackrel{\textcircled{2}}{=} \eta^2 \sum_{k=0}^c \mathbb{E} \left[\left\| \left(\tilde{\mathbf{g}}_j^{(k,r)} - \mathbf{g}_j^{(k,r)} \right) \right\|_2^2 \right] + (c+1) \eta^2 \sum_{k=0}^c \mathbb{E} \left[\left\| \mathbf{g}_j^{(k,r)} \right\|_2^2 \right] \\
&\leq \eta^2 \sum_{k=0}^{\tau-1} \mathbb{E} \left[\left\| \left(\tilde{\mathbf{g}}_j^{(k,r)} - \mathbf{g}_j^{(k,r)} \right) \right\|_2^2 \right] + \tau \eta^2 \sum_{k=0}^{\tau-1} \mathbb{E} \left[\left\| \mathbf{g}_j^{(k,r)} \right\|_2^2 \right] \\
&\stackrel{\textcircled{3}}{\leq} \eta^2 \sum_{k=0}^{\tau-1} \sigma^2 + \tau \eta^2 \sum_{k=0}^{\tau-1} \mathbb{E} \left[\left\| \mathbf{g}_j^{(k,r)} \right\|_2^2 \right] \\
&= \eta^2 \tau \sigma^2 + \eta^2 \sum_{k=0}^{\tau-1} \tau \left\| \mathbf{g}_j^{(k,r)} \right\|_2^2 \tag{12}
\end{aligned}$$

670 where $\textcircled{1}$ comes from $\mathbb{E}[\mathbf{x}^2] = \text{Var}[\mathbf{x}] + [\mathbb{E}[\mathbf{x}]]^2$ and $\textcircled{2}$ holds because $\text{Var}\left(\sum_{j=1}^n \mathbf{x}_j\right) =$
671 $\sum_{j=1}^n \text{Var}(\mathbf{x}_j)$ for i.i.d. vectors \mathbf{x}_i (and i.i.d. assumption comes from i.i.d. sampling), and fi-
672 nally $\textcircled{3}$ follows from Assumption 2. \square

673 C.1.1 Main result for the non-convex setting

674 Now we are ready to present our result for the homogeneous setting. We first state and prove the
675 result for the general non-convex objectives.

676 **Theorem 4** (non-convex). *For FedSKETCH(τ, η, γ), for all $0 \leq t \leq R\tau - 1$, under Assumptions 1*
677 *to 2, if the learning rate satisfies*

$$1 \geq \tau^2 L^2 \eta^2 + \left(\frac{\omega}{k} + 1 \right) \eta \gamma L \tau \tag{13}$$

678 *and all local model parameters are initialized at the same point $\mathbf{w}^{(0)}$, then the average-squared*
679 *gradient after τ iterations is bounded as follows:*

$$\frac{1}{R} \sum_{r=0}^{R-1} \left\| \nabla f(\mathbf{w}^{(r)}) \right\|_2^2 \leq \frac{2(f(\mathbf{w}^{(0)}) - f(\mathbf{w}^{(*)}))}{\eta \gamma \tau R} + \frac{L \eta \gamma (\omega + 1)}{k} \sigma^2 + L^2 \eta^2 \tau \sigma^2, \tag{14}$$

680 *where $\mathbf{w}^{(*)}$ is the global optimal solution with function value $f(\mathbf{w}^{(*)})$.*

681 *Proof.* Before proceeding with the proof of Theorem 4, we would like to highlight that

$$\mathbf{w}^{(r)} - \mathbf{w}_j^{(\tau,r)} = \eta \sum_{c=0}^{\tau-1} \tilde{\mathbf{g}}_j^{(c,r)}. \tag{15}$$

682 From the updating rule of Algorithm 3 we have

$$\mathbf{w}^{(r+1)} = \mathbf{w}^{(r)} - \gamma \eta \left(\frac{1}{k} \sum_{j \in \mathcal{K}} \mathbf{s} \left(\sum_{c=0, r}^{\tau-1} \tilde{\mathbf{g}}_j^{(c,r)} \right) \right) = \mathbf{w}^{(r)} - \gamma \left[\frac{\eta}{k} \sum_{j \in \mathcal{K}} \mathbf{s} \left(\sum_{c=0}^{\tau-1} \tilde{\mathbf{g}}_j^{(c,r)} \right) \right].$$

In what follows, we use the following notation to denote the stochastic gradient used to update the global model at r th communication round

$$\tilde{\mathbf{g}}_{\mathbf{S},\mathcal{K}}^{(r)} \triangleq \frac{\eta}{p} \sum_{j=1}^p \mathbf{S} \left(\frac{\mathbf{w}^{(r)} - \mathbf{w}_j^{(\tau,r)}}{\eta} \right) = \frac{1}{k} \sum_{j \in \mathcal{K}} \mathbf{S} \left(\sum_{c=0}^{\tau-1} \tilde{\mathbf{g}}_j^{(c,r)} \right).$$

683 and notice that $\mathbf{w}^{(r)} = \mathbf{w}^{(r-1)} - \gamma \tilde{\mathbf{g}}^{(r)}$.

684 Then using the unbiased estimation property of sketching we have:

$$\mathbb{E}_{\mathbf{S}} [\tilde{\mathbf{g}}_{\mathbf{S}}^{(r)}] = \frac{1}{k} \sum_{j \in \mathcal{K}} \left[-\eta \mathbb{E}_{\mathbf{S}} \left[\mathbf{S} \left(\sum_{c=0}^{\tau-1} \tilde{\mathbf{g}}_j^{(c,r)} \right) \right] \right] = \frac{1}{k} \sum_{j \in \mathcal{K}} \left[-\eta \left(\sum_{c=0}^{\tau-1} \tilde{\mathbf{g}}_j^{(c,r)} \right) \right] \triangleq \tilde{\mathbf{g}}_{\mathbf{S},\mathcal{K}}^{(r)}.$$

685 From the L -smoothness gradient assumption on global objective, by using $\tilde{\mathbf{g}}^{(r)}$ in inequality (15) we
686 have:

$$f(\mathbf{w}^{(r+1)}) - f(\mathbf{w}^{(r)}) \leq -\gamma \langle \nabla f(\mathbf{w}^{(r)}), \tilde{\mathbf{g}}^{(r)} \rangle + \frac{\gamma^2 L}{2} \|\tilde{\mathbf{g}}^{(r)}\|^2 \quad (16)$$

687 By taking expectation on both sides of above inequality over sampling, we get:

$$\begin{aligned} \mathbb{E} \left[\mathbb{E}_{\mathbf{S}} \left[f(\mathbf{w}^{(r+1)}) - f(\mathbf{w}^{(r)}) \right] \right] &\leq -\gamma \mathbb{E} \left[\mathbb{E}_{\mathbf{S}} \left[\langle \nabla f(\mathbf{w}^{(r)}), \tilde{\mathbf{g}}_{\mathbf{S}}^{(r)} \rangle \right] \right] + \frac{\gamma^2 L}{2} \mathbb{E} \left[\mathbb{E}_{\mathbf{S}} \|\tilde{\mathbf{g}}_{\mathbf{S}}^{(r)}\|^2 \right] \\ &\stackrel{(a)}{=} \underbrace{-\gamma \mathbb{E} \left[\langle \nabla f(\mathbf{w}^{(r)}), \tilde{\mathbf{g}}^{(r)} \rangle \right]}_{\text{(I)}} + \frac{\gamma^2 L}{2} \underbrace{\mathbb{E} \left[\mathbb{E}_{\mathbf{S}} \left[\|\tilde{\mathbf{g}}_{\mathbf{S}}^{(r)}\|^2 \right] \right]}_{\text{(II)}}. \end{aligned} \quad (17)$$

688 We proceed to use Lemma 1, Lemma 2, and Lemma 3, to bound terms (I) and (II) in right hand side
689 of (17), which gives

$$\begin{aligned} &\mathbb{E} \left[\mathbb{E}_{\mathbf{S}} \left[f(\mathbf{w}^{(r+1)}) - f(\mathbf{w}^{(r)}) \right] \right] \\ &\leq \gamma \frac{1}{2} \eta \sum_{j=1}^p q_j \sum_{c=0}^{\tau-1} \left[-\left\| \nabla f(\mathbf{w}^{(r)}) \right\|_2^2 - \left\| \mathbf{g}_j^{(c,r)} \right\|_2^2 + L^2 \eta^2 \sum_{c=0}^{\tau-1} \left[\tau \left\| \mathbf{g}_j^{(c,r)} \right\|_2^2 + \sigma^2 \right] \right] \\ &\quad + \frac{\gamma^2 L (\frac{\omega}{k} + 1)}{2} \left[\eta^2 \tau \sum_{j=1}^p q_j \sum_{c=0}^{\tau-1} \left\| \mathbf{g}_j^{(c,r)} \right\|_2^2 \right] + \frac{\gamma^2 \eta^2 L (\omega + 1)}{2} \frac{\tau \sigma^2}{k} \\ &\stackrel{\textcircled{1}}{\leq} \frac{\gamma \eta}{2} \sum_{j=1}^p q_j \sum_{c=0}^{\tau-1} \left[-\left\| \nabla f(\mathbf{w}^{(r)}) \right\|_2^2 - \left\| \mathbf{g}_j^{(c,r)} \right\|_2^2 + \tau L^2 \eta^2 \left[\tau \left\| \mathbf{g}_j^{(c,r)} \right\|_2^2 + \sigma^2 \right] \right] \\ &\quad + \frac{\gamma^2 L (\frac{\omega}{k} + 1)}{2} \left[\eta^2 \tau \sum_{j=1}^p q_j \sum_{c=0}^{\tau-1} \left\| \mathbf{g}_j^{(c,r)} \right\|_2^2 \right] + \frac{\gamma^2 \eta^2 L (\omega + 1)}{2} \frac{\tau \sigma^2}{k} \\ &= -\eta \gamma \frac{\tau}{2} \left\| \nabla f(\mathbf{w}^{(r)}) \right\|_2^2 \\ &\quad - \left(1 - \tau L^2 \eta^2 \tau - \left(\frac{\omega}{k} + 1 \right) \eta \gamma L \tau \right) \frac{\eta \gamma}{2} \sum_{j=1}^p q_j \sum_{c=0}^{\tau-1} \left\| \mathbf{g}_j^{(c,r)} \right\|_2^2 + \frac{L \tau \gamma \eta^2}{2k} (k L \tau \eta + \gamma (\omega + 1)) \sigma^2 \\ &\stackrel{\textcircled{2}}{\leq} -\eta \gamma \frac{\tau}{2} \left\| \nabla f(\mathbf{w}^{(r)}) \right\|_2^2 + \frac{L \tau \gamma \eta^2}{2k} (k L \tau \eta + \gamma (\omega + 1)) \sigma^2, \end{aligned} \quad (18)$$

690 where in $\textcircled{1}$ we incorporate outer summation $\sum_{c=0}^{\tau-1}$, and $\textcircled{2}$ follows from condition

$$1 \geq \tau L^2 \eta^2 \tau + \left(\frac{\omega}{k} + 1 \right) \eta \gamma L \tau.$$

691 Summing up for all R communication rounds and rearranging the terms gives:

$$\frac{1}{R} \sum_{r=0}^{R-1} \left\| \nabla f(\mathbf{w}^{(r)}) \right\|_2^2 \leq \frac{2(f(\mathbf{w}^{(0)}) - f(\mathbf{w}^{(*)}))}{\eta \gamma \tau R} + \frac{L \eta \gamma (\omega + 1)}{k} \sigma^2 + L^2 \eta^2 \tau \sigma^2.$$

692 From the above inequality, is it easy to see that in order to achieve a linear speed up, we need to have
 693 $\eta\gamma = O\left(\frac{\sqrt{k}}{\sqrt{R\tau}}\right)$. \square

694 **Corollary 3** (Linear speed up). *In (14) for the choice of $\eta\gamma = O\left(\frac{1}{L}\sqrt{\frac{k}{R\tau(\omega+1)}}\right)$, and $\gamma \geq k$ the*
 695 *convergence rate reduces to:*

$$\frac{1}{R} \sum_{r=0}^{R-1} \left\| \nabla f(\mathbf{w}^{(r)}) \right\|_2^2 \leq O \left(\frac{L\sqrt{(\omega+1)} (f(\mathbf{w}^{(0)}) - f(\mathbf{w}^*))}{\sqrt{kR\tau}} + \frac{(\sqrt{(\omega+1)})^2 \sigma^2}{\sqrt{kR\tau}} + \frac{k\sigma^2}{R\gamma^2} \right). \quad (19)$$

696 *Note that according to (19), if we pick a fixed constant value for γ , in order to achieve an ϵ -accurate*
 697 *solution, $R = O\left(\frac{1}{\epsilon}\right)$ communication rounds and $\tau = O\left(\frac{\omega+1}{k\epsilon}\right)$ local updates are necessary. We*
 698 *also highlight that (19) also allows us to choose $R = O\left(\frac{\omega+1}{\epsilon}\right)$ and $\tau = O\left(\frac{1}{k\epsilon}\right)$ to get the same*
 699 *convergence rate.*

700 **Remark 3.** *Condition in (13) can be rewritten as*

$$\begin{aligned} \eta &\leq \frac{-\gamma L\tau \left(\frac{\omega}{k} + 1\right) + \sqrt{\gamma^2 \left(L\tau \left(\frac{\omega}{k} + 1\right)\right)^2 + 4L^2\tau^2}}{2L^2\tau^2} \\ &= \frac{-\gamma L\tau \left(\frac{\omega}{k} + 1\right) + L\tau \sqrt{\left(\frac{\omega}{k} + 1\right)^2 \gamma^2 + 4}}{2L^2\tau^2} \\ &= \frac{\sqrt{\left(\frac{\omega}{k} + 1\right)^2 \gamma^2 + 4} - \left(\frac{\omega}{k} + 1\right) \gamma}{2L\tau}. \end{aligned} \quad (20)$$

701 *So based on (20), if we set $\eta = O\left(\frac{1}{L\gamma} \sqrt{\frac{k}{R\tau(\omega+1)}}\right)$, it implies that:*

$$R \geq \frac{\tau k}{(\omega+1)\gamma^2 \left(\sqrt{\left(\frac{\omega}{k} + 1\right)^2 \gamma^2 + 4} - \left(\frac{\omega}{k} + 1\right) \gamma \right)^2}. \quad (21)$$

702 *We note that $\gamma^2 \left(\sqrt{\left(\frac{\omega}{k} + 1\right)^2 \gamma^2 + 4} - \left(\frac{\omega}{k} + 1\right) \gamma \right)^2 = \Theta(1) \leq 5$ therefore even for $\gamma \geq m$ we*
 703 *need to have*

$$R \geq \frac{\tau k}{5(\omega+1)} = O\left(\frac{\tau k}{(\omega+1)}\right). \quad (22)$$

704 *Therefore, for the choice of $\tau = O\left(\frac{\omega+1}{k\epsilon}\right)$, due to condition in (22), we need to have $R = O\left(\frac{1}{\epsilon}\right)$.*
 705 *Similarly, we can have $R = O\left(\frac{\omega+1}{\epsilon}\right)$ and $\tau = O\left(\frac{1}{k\epsilon}\right)$.*

706 **Corollary 4** (Special case, $\gamma = 1$). *By letting $\gamma = 1$, $\omega = 0$ and $k = p$ the convergence rate in (14)*
 707 *reduces to*

$$\frac{1}{R} \sum_{r=0}^{R-1} \left\| \nabla f(\mathbf{w}^{(r)}) \right\|_2^2 \leq \frac{2(f(\mathbf{w}^{(0)}) - f(\mathbf{w}^{(*)}))}{\eta R\tau} + \frac{L\eta}{p} \sigma^2 + L^2 \eta^2 \tau \sigma^2,$$

708 *which matches the rate obtained in [44]. In this case the communication complexity and the number*
 709 *of local updates become*

$$R = O\left(\frac{p}{\epsilon}\right), \quad \tau = O\left(\frac{1}{\epsilon}\right),$$

710 *which simply implies that in this special case the convergence rate of our algorithm reduces to the*
 711 *rate obtained in [44], which indicates the tightness of our analysis.*

712 **C.1.2 Main result for the PL/Strongly convex setting**

713 We now turn to stating the convergence rate for the homogeneous setting under PL condition which
 714 naturally leads to the same rate for strongly convex functions.

715 **Theorem 5** (PL or strongly convex). *For FedSKETCH(τ, η, γ), for all $0 \leq t \leq R\tau - 1$, under
 716 Assumptions 1 to 2 and 3, if the learning rate satisfies*

$$1 \geq \tau^2 L^2 \eta^2 + \left(\frac{\omega}{k} + 1\right) \eta \gamma L \tau$$

717 and if the all the models are initialized with $\mathbf{w}^{(0)}$ we obtain:

$$\mathbb{E}\left[f(\mathbf{w}^{(R)}) - f(\mathbf{w}^{(*)})\right] \leq (1 - \eta \gamma \mu \tau)^R \left(f(\mathbf{w}^{(0)}) - f(\mathbf{w}^{(*)})\right) + \frac{1}{\mu} \left[\frac{1}{2} L^2 \tau \eta^2 \sigma^2 + (1 + \omega) \frac{\gamma \eta L \sigma^2}{2k}\right]$$

718 *Proof.* From (18) under condition:

$$1 \geq \tau L^2 \eta^2 \tau + \left(\frac{\omega}{k} + 1\right) \eta \gamma L \tau$$

719 we obtain:

$$\begin{aligned} \mathbb{E}\left[f(\mathbf{w}^{(r+1)}) - f(\mathbf{w}^{(r)})\right] &\leq -\eta \gamma \frac{\tau}{2} \left\|\nabla f(\mathbf{w}^{(r)})\right\|_2^2 + \frac{L \tau \gamma \eta^2}{2k} (k L \tau \eta + \gamma(\omega + 1)) \sigma^2 \\ &\leq -\eta \mu \gamma \tau \left(f(\mathbf{w}^{(r)}) - f(\mathbf{w}^{(*)})\right) + \frac{L \tau \gamma \eta^2}{2k} (k L \tau \eta + \gamma(\omega + 1)) \sigma^2 \end{aligned} \quad (23)$$

720 which leads to the following bound:

$$\mathbb{E}\left[f(\mathbf{w}^{(r+1)}) - f(\mathbf{w}^{(*)})\right] \leq (1 - \eta \mu \gamma \tau) \left[f(\mathbf{w}^{(r)}) - f(\mathbf{w}^{(*)})\right] + \frac{L \tau \gamma \eta^2}{2k} (k L \tau \eta + (\omega + 1) \gamma) \sigma^2$$

721 By setting $\Delta = 1 - \eta \mu \gamma \tau$ we obtain the following bound:

$$\begin{aligned} &\mathbb{E}\left[f(\mathbf{w}^{(R)}) - f(\mathbf{w}^{(*)})\right] \\ &\leq \Delta^R \left[f(\mathbf{w}^{(0)}) - f(\mathbf{w}^{(*)})\right] + \frac{1 - \Delta^R}{1 - \Delta} \frac{L \tau \gamma \eta^2}{2k} (k L \tau \eta + (\omega + 1) \gamma) \sigma^2 \\ &\leq \Delta^R \left[f(\mathbf{w}^{(0)}) - f(\mathbf{w}^{(*)})\right] + \frac{1}{1 - \Delta} \frac{L \tau \gamma \eta^2}{2k} (k L \tau \eta + (\omega + 1) \gamma) \sigma^2 \\ &= (1 - \eta \mu \gamma \tau)^R \left[f(\mathbf{w}^{(0)}) - f(\mathbf{w}^{(*)})\right] + \frac{1}{\eta \mu \gamma \tau} \frac{L \tau \gamma \eta^2}{2k} (k L \tau \eta + (\omega + 1) \gamma) \sigma^2 \end{aligned} \quad (24)$$

722 □

723 **Corollary 5.** *If we let $\eta \gamma \mu \tau \leq \frac{1}{2}$, $\eta = \frac{1}{2L(\frac{\omega}{k} + 1)\tau \gamma}$ and $\kappa = \frac{L}{\mu}$ the convergence error in Theorem 5,*

724 *with $\gamma \geq k$ results in:*

$$\begin{aligned} &\mathbb{E}\left[f(\mathbf{w}^{(R)}) - f(\mathbf{w}^{(*)})\right] \\ &\leq e^{-\eta \gamma \mu \tau R} \left(f(\mathbf{w}^{(0)}) - f(\mathbf{w}^{(*)})\right) + \frac{1}{\mu} \left[\frac{1}{2} \tau L^2 \eta^2 \sigma^2 + (1 + \omega) \frac{\gamma \eta L \sigma^2}{2k}\right] \\ &\leq e^{-\frac{R}{2(\frac{\omega}{k} + 1)\kappa}} \left(f(\mathbf{w}^{(0)}) - f(\mathbf{w}^{(*)})\right) + \frac{1}{\mu} \left[\frac{1}{2} L^2 \frac{\tau \sigma^2}{L^2 (\frac{\omega}{k} + 1)^2 \gamma^2 \tau^2} + \frac{(1 + \omega) L \sigma^2}{2 (\frac{\omega}{k} + 1) L \tau k}\right] \\ &= O\left(e^{-\frac{R}{2(\frac{\omega}{k} + 1)\kappa}} \left(f(\mathbf{w}^{(0)}) - f(\mathbf{w}^{(*)})\right) + \frac{\sigma^2}{(\frac{\omega}{k} + 1)^2 \gamma^2 \mu \tau} + \frac{(\omega + 1) \sigma^2}{\mu (\frac{\omega}{k} + 1) \tau k}\right) \\ &= O\left(e^{-\frac{R}{2(\frac{\omega}{k} + 1)\kappa}} \left(f(\mathbf{w}^{(0)}) - f(\mathbf{w}^{(*)})\right) + \frac{\sigma^2}{\gamma^2 \mu \tau} + \frac{(\omega + 1) \sigma^2}{\mu (\frac{\omega}{k} + 1) \tau k}\right) \end{aligned} \quad (25)$$

725 which indicates that to achieve an error of ϵ , we need to have $R = O\left(\left(\frac{\omega}{k} + 1\right) \kappa \log\left(\frac{1}{\epsilon}\right)\right)$ and $\tau =$
726 $\frac{(\omega+1)}{k\left(\frac{\omega}{k}+1\right)\epsilon}$. Additionally, we note that if $\gamma \rightarrow \infty$, yet $R = O\left(\left(\frac{\omega}{k} + 1\right) \kappa \log\left(\frac{1}{\epsilon}\right)\right)$ and $\tau = \frac{(\omega+1)}{k\left(\frac{\omega}{k}+1\right)\epsilon}$
727 will be necessary.

728 C.1.3 Main result for the general convex setting

729 **Theorem 6** (Convex). For a general convex function $f(\mathbf{w})$ with optimal solution $\mathbf{w}^{(*)}$, using
730 *FedSKETCH*(τ, η, γ) to optimize $\tilde{f}(\mathbf{w}, \phi) = f(\mathbf{w}) + \frac{\phi}{2} \|\mathbf{w}\|^2$, for all $0 \leq t \leq R\tau - 1$, under
731 Assumptions 1 to 2, if the learning rate satisfies

$$1 \geq \tau^2 L^2 \eta^2 + \left(\frac{\omega}{k} + 1\right) \eta \gamma L \tau$$

732 and if the all the models initiate with $\mathbf{w}^{(0)}$, with $\phi = \frac{1}{\sqrt{k\tau}}$ and $\eta = \frac{1}{2L\gamma\tau(1+\frac{\omega}{k})}$ we obtain:

$$\begin{aligned} \mathbb{E}\left[f(\mathbf{w}^{(R)}) - f(\mathbf{w}^{(*)})\right] &\leq e^{-\frac{R}{2L(1+\frac{\omega}{k})\sqrt{m\tau}}} \left(f(\mathbf{w}^{(0)}) - f(\mathbf{w}^{(*)})\right) \\ &\quad + \left[\frac{\sqrt{k}\sigma^2}{8\sqrt{\tau}\gamma^2(1+\frac{\omega}{k})^2} + \frac{(\omega+1)\sigma^2}{4\left(\frac{\omega}{k}+1\right)\sqrt{k\tau}}\right] + \frac{1}{2\sqrt{k\tau}} \|\mathbf{w}^{(*)}\|^2 \end{aligned} \quad (26)$$

733 We note that above theorem implies that to achieve a convergence error of ϵ we need to have
734 $R = O\left(L\left(1 + \frac{\omega}{k}\right) \frac{1}{\epsilon} \log\left(\frac{1}{\epsilon}\right)\right)$ and $\tau = O\left(\frac{(\omega+1)^2}{k\left(\frac{\omega}{k}+1\right)^2\epsilon}\right)$.

735 *Proof.* Since $\tilde{f}(\mathbf{w}^{(r)}, \phi) = f(\mathbf{w}^{(r)}) + \frac{\phi}{2} \|\mathbf{w}^{(r)}\|^2$ is ϕ -PL, according to Theorem 5, we have:

$$\begin{aligned} &\tilde{f}(\mathbf{w}^{(R)}, \phi) - \tilde{f}(\mathbf{w}^{(*)}, \phi) \\ &= f(\mathbf{w}^{(r)}) + \frac{\phi}{2} \|\mathbf{w}^{(r)}\|^2 - \left(f(\mathbf{w}^{(*)}) + \frac{\phi}{2} \|\mathbf{w}^{(*)}\|^2\right) \\ &\leq (1 - \eta\gamma\phi\tau)^R \left(f(\mathbf{w}^{(0)}) - f(\mathbf{w}^{(*)})\right) + \frac{1}{\phi} \left[\frac{1}{2} L^2 \tau \eta^2 \sigma^2 + (1 + \omega) \frac{\gamma\eta L \sigma^2}{2k}\right] \end{aligned} \quad (27)$$

736 Next rearranging (27) and replacing μ with ϕ leads to the following error bound:

$$\begin{aligned} &f(\mathbf{w}^{(R)}) - f^* \\ &\leq (1 - \eta\gamma\phi\tau)^R \left(f(\mathbf{w}^{(0)}) - f(\mathbf{w}^{(*)})\right) + \frac{1}{\phi} \left[\frac{1}{2} L^2 \tau \eta^2 \sigma^2 + (1 + \omega) \frac{\gamma\eta L \sigma^2}{2k}\right] \\ &\quad + \frac{\phi}{2} \left(\|\mathbf{w}^*\|^2 - \|\mathbf{w}^{(r)}\|^2\right) \\ &\leq e^{-(\eta\gamma\phi\tau)R} \left(f(\mathbf{w}^{(0)}) - f(\mathbf{w}^{(*)})\right) + \frac{1}{\phi} \left[\frac{1}{2} L^2 \tau \eta^2 \sigma^2 + (1 + \omega) \frac{\gamma\eta L \sigma^2}{2k}\right] + \frac{\phi}{2} \|\mathbf{w}^{(*)}\|^2 \end{aligned}$$

737 Next, if we set $\phi = \frac{1}{\sqrt{k\tau}}$ and $\eta = \frac{1}{2(1+\frac{\omega}{k})L\gamma\tau}$, we obtain that

$$\begin{aligned} &f(\mathbf{w}^{(R)}) - f^* \\ &\leq e^{-\frac{R}{2(1+\frac{\omega}{k})L\sqrt{m\tau}}} \left(f(\mathbf{w}^{(0)}) - f(\mathbf{w}^{(*)})\right) + \sqrt{k\tau} \left[\frac{\sigma^2}{8\tau\gamma^2(1+\frac{\omega}{k})^2} + \frac{(\omega+1)\sigma^2}{4\left(\frac{\omega}{k}+1\right)\tau k}\right] + \frac{1}{2\sqrt{k\tau}} \|\mathbf{w}^{(*)}\|^2, \end{aligned}$$

738 thus the proof is complete. \square

C.2 Proof of Theorem 2

The proof of Theorem 2 follows directly from the results in [13]. We first mention the general Theorem 7 from [13] for general compression noise ω . Next, since the sketching PRIVIX and HEAPRIX, satisfy Assumption 4 with $\omega = c \frac{d}{m}$ and $\omega = c \frac{d}{m} - 1$ respectively with probability $1 - \frac{\delta}{R}$ per communication round, all the results in Theorem 2, conclude from Theorem 7 with probability $1 - \delta$ (by taking union over the all probabilities of each communication rounds with probability $1 - \delta/R$) and plugging $\omega = c \frac{d}{m}$ and $\omega = c \frac{d}{m} - 1$ respectively into the corresponding convergence bounds. For the heterogeneous setting, the results in [13] requires the following extra assumption that naturally holds for the sketching:

Assumption 5 ([13]). *The compression scheme Q for the heterogeneous data distribution setting satisfies the following condition $\mathbb{E}_Q[\|\frac{1}{m} \sum_{j=1}^m Q(\mathbf{x}_j)\|^2 - \|Q(\frac{1}{m} \sum_{j=1}^m \mathbf{x}_j)\|^2] \leq G_q$.*

We note that since sketching is a linear compressor, in the case of our algorithms for heterogeneous setting we have $G_q = 0$.

Next, we restate the Theorem in [13] here as follows:

Theorem 7. *Consider FedCOMGATE in [13]. If Assumptions 1, 3, 4 and 5 hold, then even for the case the local data distribution of users are different (heterogeneous setting) we have*

- **non-convex:** By choosing stepsizes as $\eta = \frac{1}{L\gamma} \sqrt{\frac{p}{R\tau(\omega+1)}}$ and $\gamma \geq p$, we obtain that the iterates satisfy $\frac{1}{R} \sum_{r=0}^{R-1} \|\nabla f(\mathbf{w}^{(r)})\|_2^2 \leq \epsilon$ if we set $R = O\left(\frac{\omega+1}{\epsilon}\right)$ and $\tau = O\left(\frac{1}{p\epsilon}\right)$.
- **Strongly convex or PL:** By choosing stepsizes as $\eta = \frac{1}{2L(\frac{\omega}{p}+1)\tau\gamma}$ and $\gamma \geq \sqrt{p\tau}$, we obtain that the iterates satisfy $\mathbb{E}\left[f(\mathbf{w}^{(R)}) - f(\mathbf{w}^{(*)})\right] \leq \epsilon$ if we set $R = O\left((\omega+1)\kappa \log\left(\frac{1}{\epsilon}\right)\right)$ and $\tau = O\left(\frac{1}{p\epsilon}\right)$.
- **Convex:** By choosing stepsizes as $\eta = \frac{1}{2L(\omega+1)\tau\gamma}$ and $\gamma \geq \sqrt{p\tau}$, we obtain that the iterates satisfy $\mathbb{E}\left[f(\mathbf{w}^{(R)}) - f(\mathbf{w}^{(*)})\right] \leq \epsilon$ if we set $R = O\left(\frac{L(1+\omega)}{\epsilon} \log\left(\frac{1}{\epsilon}\right)\right)$ and $\tau = O\left(\frac{1}{p\epsilon^2}\right)$.

Proof. Since the sketching methods PRIVIX and HEAPRIX, satisfy the Assumption 4 with $\omega = c \frac{d}{m}$ and $\omega = c \frac{d}{m} - 1$ respectively with probability $1 - \frac{\delta}{R}$ per communication round, we conclude the proofs of Theorem 2 using Theorem 7 with probability $1 - \delta$ (by taking union over all communication rounds) and plugging $\omega = c \frac{d}{m}$ and $\omega = c \frac{d}{m} - 1$ respectively into the convergence bounds. \square

D Numerical Experiments and Additional Results

D.1 Implementation of FetchSGD

Our implementation of FetchSGD basically follows the original paper (Algorithm 1 in [37]). The only difference is that, in the original algorithm, the local workers compress the gradient (in every local step) and transmit it to the central server. In our setting, we extend to the case with multiple local updates, where the difference in local weights are transmitted (same as the standard FL framework). Also, TopK compression is used to decode the sketches at the central server. We apply the same implementation trick that when accumulating the errors, we only count the non-zero coordinates and leave other coordinates zero for the accumulator. This greatly improves the empirical performance.

775 D.2 Additional Plots for the MNIST Experiments

776 D.2.1 Homogeneous setting

777 In the homogeneous case, each node has same data distribution. To achieve this setting, we randomly
 778 choose samples uniformly from 10 classes of hand-written digits. The train loss and test accuracy
 779 are provided in Figure 3, where we report local epochs $\tau = 2$ in addition to the main context (single
 780 local update). The number of users is set to 50, and in each round of training we randomly pick half
 781 of the nodes to be active (i.e., receiving data and performing local updates). We can draw similar
 782 conclusion: FS-HEAPRIX consistently performs better than other competing methods. The test
 783 accuracy increases with larger τ in homogeneous setting.

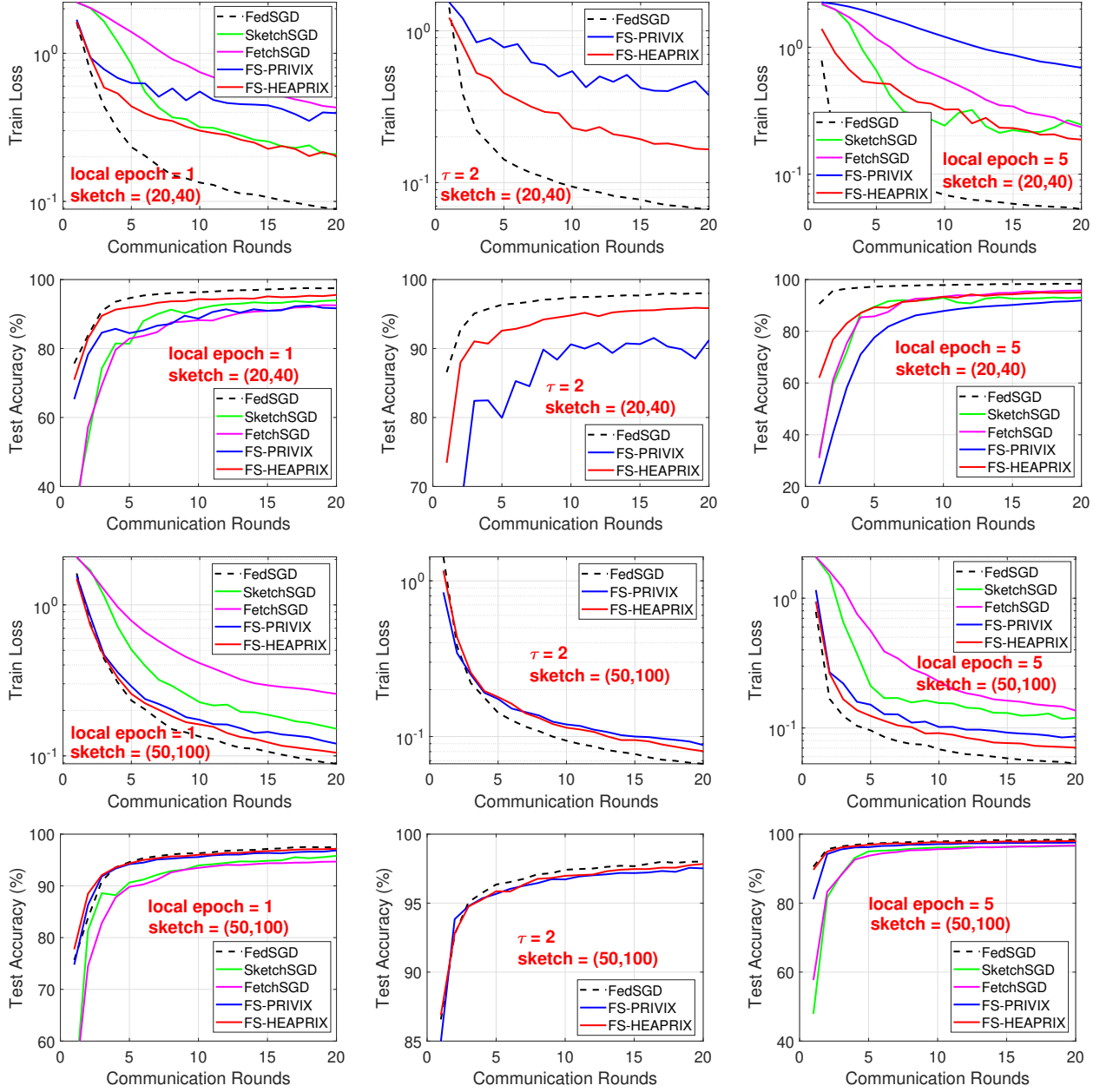


Figure 3: MNIST Homogeneous case: Comparison of compressed optimization methods on LeNet CNN architecture.

784 D.2.2 Heterogeneous setting

785 Analogously, we present experiments on MNIST dataset under heterogeneous data distribution,
 786 including $\tau = 2$. We simulate the setting by only sending samples from one digit to each local
 787 worker (very few nodes get two classes). We see from Figure 4 that FS-HEAPRIX shows consistent
 788 advantage over competing methods. SketchedSGD performs poorly in this case.

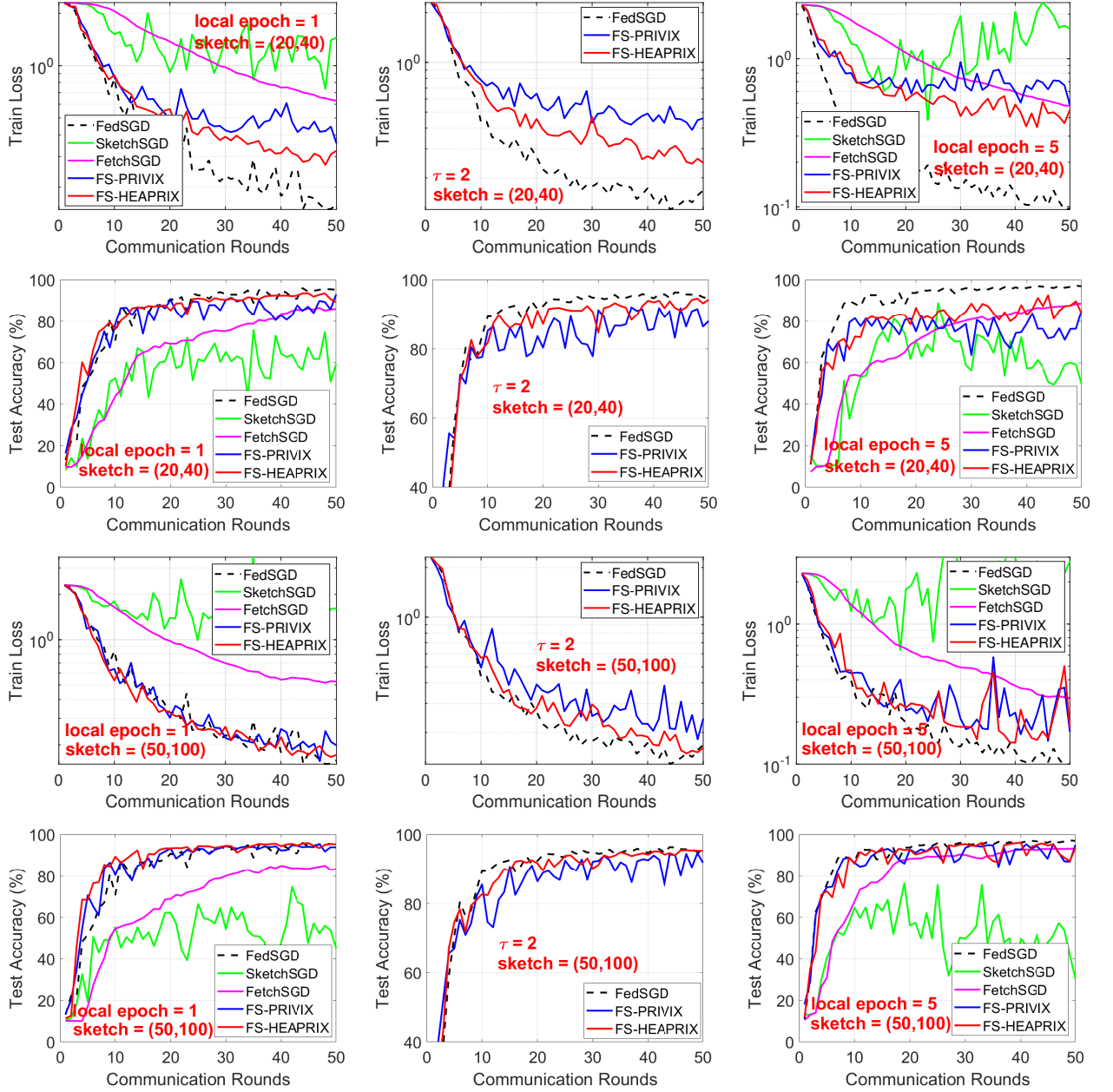


Figure 4: MNIST Heterogeneous case: Comparison of compressed optimization algorithms on LeNet CNN architecture.

789 D.3 Additional Experiments: CIFAR-10

790 We conduct similar sets of experiments on CIFAR10 dataset. We also use the simple LeNet CNN
 791 structure, as in practice small models are more favorable in federated learning, due to the limitation of
 792 mobile devices. The test accuracy is presented in Figure 5 and Figure 6, for respectively homogeneous
 793 and heterogeneous data distribution. In general, we retrieve similar information as from MNIST
 794 experiments: our proposed FS-HEAPRIX improves FS-PRIVIX and SketchedSGD in all cases. We
 795 note that although the test accuracy provided by LeNet cannot reach the state-of-the-art accuracy
 796 given by some huge models, it is also informative in terms of comparing the relative performance of
 797 different sketching methods.

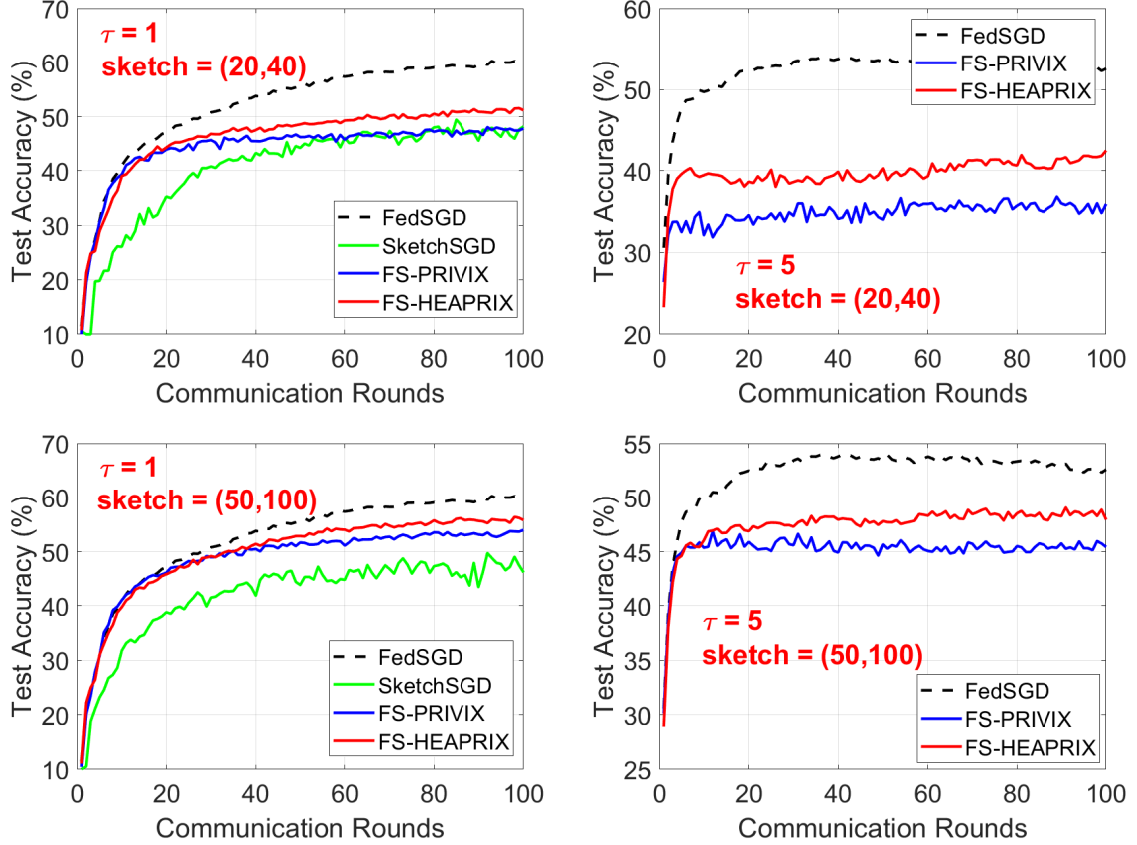


Figure 5: Homogeneous case: CIFAR10: Comparison of compressed optimization methods on LeNet CNN.

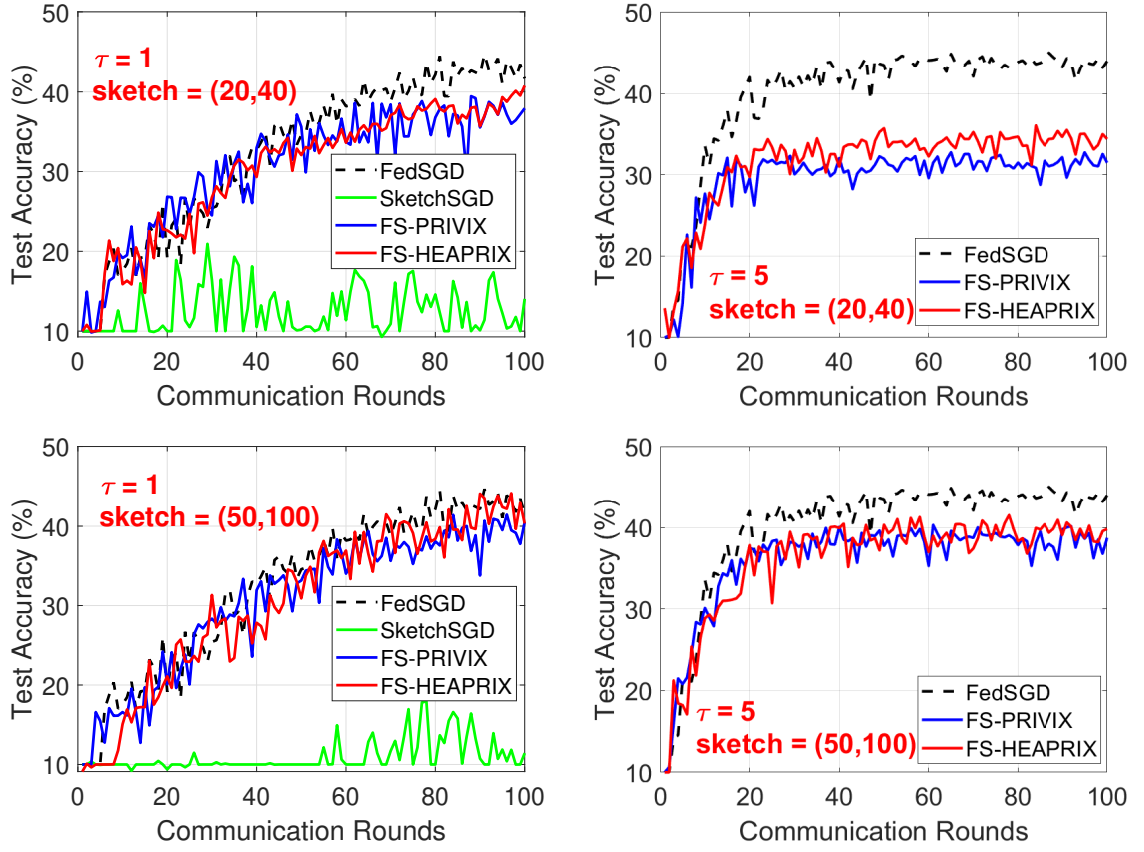


Figure 6: Heterogeneous case: CIFAR10: Comparison of compressed optimization methods on LeNet CNN.