FedSKETCH: Communication-Efficient Federated Learning via Sketching

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Abstract

Communication complexity and data privacy are the two key challenges in Federated Learning (FL) where the goal is to perform a distributed learning through a large volume of devices. In this work, we introduce two new algorithms, namely FedSKETCH and FedSKETCHGATE, to address jointly both challenges and which are, respectively, intended to be used for homogeneous and heterogeneous data distribution settings. Our algorithms are based on a key and novel sketching technique, called HEAPRIX that is unbiased, compresses the accumulation of local gradients using count sketch, and exhibits communication-efficiency properties leveraging low-dimensional sketches. We provide sharp convergence guarantees of our algorithms and validate our theoretical findings with various sets of experiments.

1. Introduction

Federated Learning (FL) is a recently emerging framework for distributed large scale machine learning problems. In FL, data is distributed across devices (McMahan et al., 2017; Konečnỳ et al., 2016) and due to privacy concerns, users are only allowed to communicate with the parameter server. Formally, the optimization problem across p distributed devices is defined as follows:

$$\min_{\boldsymbol{x}\in\mathbb{R}^d,\, \sum_{j=1}^{p} q_j=1} f(\boldsymbol{x}) \triangleq \sum_{j=1}^{p} q_j F_j(\boldsymbol{x}) \,, \qquad (1)$$

where $F_j(\boldsymbol{x}) = \mathbb{E}_{\xi\in\mathcal{D}_j}\left[L_j\left(\boldsymbol{x},\xi\right)\right]$ is the local cost function at device j, $q_j \triangleq \frac{n_j}{n}$, n_j is the number of data shards at device j and $n = \sum_{j=1}^{p} n_j$ is the total number of data samples, ξ is a random variable distributed according to probability distribution $\mathcal{D}_j$, and L_j is a loss function that measures the performance of model $\boldsymbol{x}$ at device j. We note that, while for the homogeneous setting we assume $\{\mathcal{D}_j\}_{j=1}^{p}$ have the same distribution across devices and $L_i = L_j$, $1 \leq (i,j) \leq p$, in the heterogeneous setting, these distributions and loss functions L_j can vary from a device to another.

There are several challenges that need to be addressed in FL in order to efficiently learn a global model that performs well in average for all devices:

– *Communication-efficiency*: There are often many devices communicating with the server, thus incurring immense communication overhead. One approach to reduce communication round is using *local SGD with periodic averaging* (Zhou & Cong, 2018; Stich, 2019; Yu et al., 2019b; Wang & Joshi, 2018) which periodically averages models after few local updates, contrary to baseline SGD (Bottou & Bousquet, 2008) where model averaging is performed at each iteration. Local SGD has been proposed in McMahan et al. (2017); Konečnỳ et al. (2016) under the FL setting and its convergence analysis is studied in Stich (2019); Wang & Joshi (2018); Zhou & Cong (2018); Yu et al. (2019b), later on improved in the follow up references (Basu et al., 2019; Haddadpour & Mahdavi, 2019; Khaled et al., 2020; Stich & Karimireddy, 2019) for homogeneous setting. It is further extended to heterogeneous setting (Yu et al., 2019a; Li et al., 2020d; Sahu et al., 2018; Liang et al., 2019; Haddadpour & Mahdavi, 2019; Karimireddy et al., 2019). Second approach to deal with communication cost aims at reducing the size of communicated message per communication round, such as local gradient quantization (Alistarh et al., 2017; Bernstein et al., 2018; Tang et al., 2018; Wen et al., 2017; Wu et al., 2018) or sparsification (Alistarh et al., 2018; Lin et al., 2018; Stich et al., 2018; Stich & Karimireddy, 2019).

–*Data heterogeneity*: Since locally generated data in each device may come from different distribution, local computations involved in FL setting can lead to poor convergence error in practice (Li et al., 2020a; Liang et al., 2019). To mitigate the negative impact of data heterogeneity, (Haddadpour et al., 2020; Horváth et al., 2019; Liang et al., 2019; Karimireddy et al., 2019) suggest applying variance reduction or gradient tracking techniques along local computations.

–*Privacy* (Geyer et al., 2017; Hardy et al., 2017): Privacy has been widely addressed by injecting an additional layer of randomness to respect differential-privacy property (McMahan et al., 2018) or using cryptography-based approaches under secure multi-party computation (Bonawitz et al., 2017).

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Further study of challenges can be found in recent surveys (Li et al., 2020b) and (Kairouz et al., 2019).

To tackle all major aforementioned challenges in FL jointly, sketching based algorithms (Charikar et al., 2004; Cormode & Muthukrishnan, 2005; Kleinberg, 2003; Li et al., 2008) are promising approaches. For instance, to reduce communication cost, (Ivkin et al., 2019) develop a distributed SGD algorithm using sketching along providing its convergence analysis in the homogeneous setting, and establish a communication complexity of order $\mathcal{O}(\log(d))$ per round, where d is the dimension of the vector of parameters compared to $\mathcal{O}(d)$ complexity per round of baseline mini-batch SGD. Yet, the proposed sketching scheme in Ivkin et al. (2019), built from a communication-efficiency perspective, is based on a deterministic procedure which requires access to the exact information of the gradients, thus not meeting the crucial privacy-preserving criteria. This systemic flaw is partially addressed in Rothchild et al. (2020).

Focusing on privacy, (Li et al., 2019) derive a single framework in order to tackle these issues jointly and introduces `DiffSketch` algorithm, based on the Count Sketch operator, yet does not provide its convergence analysis. Additionally, the estimation error of `DiffSketch` is higher than the sketching scheme in Ivkin et al. (2019) which may end up in poor convergence.

In this paper, we propose new sketching algorithms to address the aforementioned challenges simultaneously. Our main contributions are summarized as:

- We provide a new algorithm – `HEAPRIX` – and theoretically show that it reduces the cost of communication between devices and server, which is based on unbiased sketching without requiring the broadcast of exact values of gradients to the server. Based on `HEAPRIX`, we develop general algorithms for communication-efficient and sketch-based FL, namely `FedSKETCH` and `FedSKETCHGATE` for both homogeneous and heterogeneous data distribution settings respectively.

- We establish non-asymptotic convergence bounds for convex, Polyak-Łojasiewicz (PL) and non-convex functions in Theorems 1 and 2 in both homogeneous and heterogeneous cases, and highlight an improvement in the number of iteration to reach a stationary point. We also provide a convergence analysis for the `PRIVIX` algorithm proposed in Li et al. (2019).

- We illustrate the benefits of `FedSKETCH` and `FedSKETCHGATE` over baseline methods through a set of experiments. The latter shows the advantages of the `HEAPRIX` compression method achieving comparable test accuracy as Federated SGD (`FedSGD`) while compressing the information exchanged between devices and server.

Notation: We denote the number of communication rounds and bits per round and per device by R and B respectively. The count sketch of any vector $\boldsymbol{x}$ is designated by $\mathbf{S}(\boldsymbol{x})$. $[p]$ denotes the set $\{1, \ldots, p\}$.

2. Compression using Count Sketch

In this paper, we exploit the commonly used `Count Sketch` (Charikar et al., 2004) which uses two sets of functions that encode any input vector $\boldsymbol{x}$ **into a hash table** $\boldsymbol{S}_{m\times t}(\boldsymbol{x})$. Pairwise independent hash functions $\{h_{j,1\leq j\leq t} : [d] \to m\}$ are used along with another set of pairwise independent sign hash functions $\{\text{sign}_{j,1\leq j\leq t} : [d] \to \{+1,-1\}\}$ to map entries of $\boldsymbol{x}$ (x_i, $1 \leq i \leq d$) into t different columns of $\mathbf{S}_{m\times t}$, wherein to lower the dimension of the input vector we usually have $d \gg mt$. The final update reads $\mathbf{S}[j][h_j(i)] = \mathbf{S}[j-1][h_{j-1}(i)] + \text{sign}_j(i).x_i$ for any $1 \leq j \leq t$. There are various types of sketching algorithms which are developed based on count sketching that we develop in the following subsections. See the Appendix for the detailed Count Sketch algorithm.

2.1. Sketching based Unbiased Compressor

We define an unbiased compressor as follows:

Definition 1 (Unbiased compressor)**.** *A randomized function, $C : \mathbb{R}^d \to \mathbb{R}^d$ is called an unbiased compression operator with $\Delta \geq 1$, if we have*

$$\mathbb{E}\left[C(\boldsymbol{x})\right] = \boldsymbol{x} \quad \textit{and} \quad \mathbb{E}\left[\|C(\boldsymbol{x})\|_2^2\right] \leq \Delta \|\boldsymbol{x}\|_2^2 \ .$$

We denote this class of compressors by $\mathbb{U}(\Delta)$.

This definition leads to the following property

$$\mathbb{E}\left[\|\mathrm{C}(\boldsymbol{x}) - \boldsymbol{x}\|_2^2\right] \leq (\Delta - 1) \|\boldsymbol{x}\|_2^2 \ .$$

Note that if we let $\Delta = 1$ then our algorithm reduces to the case of no compression. This property allows us to control the noise of the compression.

An instance of such unbiased compressor is `PRIVIX` which obtains an estimate of input $\boldsymbol{x}$ from a count sketch noted $\boldsymbol{S}(\boldsymbol{x})$. In this algorithm, to query the quantity x_i, the $i-th$ element of the vector $\boldsymbol{x}$, we compute the median of t approximated values specified by the indices of $h_j(i)$ for $1 \leq j \leq t$, see (Li et al., 2019) or Algorithm 6 in the Appendix (for more details). For the purpose of our proof, we state the following crucial properties of the count sketch:

Property 1 ((Li et al., 2019))**.** *For any $\boldsymbol{x} \in \mathbb{R}^d$, we have:*

Unbiased estimation: As in Li et al. (2019), we have:

$$\mathbb{E}_{\mathbf{S}}\left[\mathtt{PRIVIX}[\mathbf{S}(\boldsymbol{x})]\right] = \boldsymbol{x} \, .$$

Bounded variance: For the given $m < d$, $t = \mathcal{O}\left(\ln\left(\frac{d}{\delta}\right)\right)$ with probability $1-\delta$ we have:

$$\mathbb{E}_{\mathbf{S}}\left[\|\mathtt{PRIVIX}[\mathbf{S}(\boldsymbol{x})] - \boldsymbol{x}\|_2^2\right] \leq c\frac{d}{m} \|\boldsymbol{x}\|_2^2 \ ,$$

where c ($e \leq c < m$) is a positive constant independent of the dimension of the input, d.

Thus, with probability $1-\delta$ we obtain that $\texttt{PRIVIX} \in \mathbb{U}(1+c\frac{d}{m})$. Note $\Delta = 1 + c\frac{d}{m}$ implies that if $m \to d$, then $\Delta \to 1 + c$, indicating a noisy reconstruction. Exploiting this noisy reconstruction, Li et al. (2019) show that if the data is normally distributed, PRIVIX is differentially private (Dwork, 2006), up to additional assumptions and algorithmic design.

2.2. Sketching based Biased Compressor

A biased compressor is defined as follows:

Definition 2 (Biased compressor)**.** *A (randomized) function, $C : \mathbb{R}^d \to \mathbb{R}^d$ belongs to $\mathbb{C}(\Delta, \alpha)$, a class of compression operators with $\alpha > 0$ and $\Delta \geq 1$, if*

$$\mathbb{E}\left[\|\alpha \boldsymbol{x} - C(\boldsymbol{x})\|_2^2\right] \leq \left(1 - \frac{1}{\Delta}\right)\|\boldsymbol{x}\|_2^2 ,$$

The reference (Horváth & Richtárik, 2020) proves that $\mathbb{U}(\Delta) \subset \mathbb{C}(\Delta, \alpha)$. An example of biased compression via sketching and using top_m operation is given below:

Algorithm 1 HEAVYMIX

1: **Inputs:** $\mathbf{S}(\mathbf{g})$; parameter m
2: Query the vector $\tilde{\mathbf{g}} \in \mathbb{R}^d$ from $\mathbf{S}(\mathbf{g})$:
3: Query $\hat{\ell}_2^2 = (1 \pm 0.5)\|\mathbf{g}\|^2$ from sketch $\mathbf{S}(\mathbf{g})$
4: $\forall j$ query $\hat{\mathbf{g}}_j^2 = \hat{\mathbf{g}}_j^2 \pm \frac{1}{2m}\|\mathbf{g}\|^2$ from sketch $\mathbf{S}(\mathbf{g})$
5: $H = \{j|\hat{\mathbf{g}}_j \geq \frac{\hat{\ell}_2^2}{m}\}$ and $NH = \{j|\hat{\mathbf{g}}_j < \frac{\hat{\ell}_2^2}{m}\}$
6: $\text{Top}_m = H \cup \text{rand}_\ell(NH)$, where $\ell = m - |H|$
7: Get exact values of Top_m
8: **Output:** $\tilde{\mathbf{g}} : \forall j \in \text{Top}_m : \tilde{\mathbf{g}}_i = \mathbf{g}_i$ else $\mathbf{g}_i = 0$

Following Ivkin et al. (2019), HEAVYMIX with sketch size $\Theta\left(m \log\left(\frac{d}{\delta}\right)\right)$ is a biased compressor with $\alpha = 1$ and $\Delta = d/m$ with probability $\geq 1-\delta$. In other words, with probability $1-\delta$, $\texttt{HEAVYMIX} \in C(\frac{d}{m}, 1)$. We note that Algorithm 1 is a variation of the sketching algorithm developed in Ivkin et al. (2019) with distinction that HEAVYMIX does not require a second round of communication to obtain the exact values of top_m. Additionally, while a sketching algorithm implementing HEAVYMIX has smaller estimation error compared to PRIVIX, it requires having access to the exact values of top_m, therefore not benefiting from privacy properties contrary to PRIVIX. In the following we introduce our sketching scheme – HEAPRIX – as a combination of those two methods.

2.3. Sketching based Induced Compressor

Due to Theorem 3 in Horváth & Richtárik (2020), which illustrates that we can convert the biased compressor into an unbiased one such that, for $C_1 \in \mathbb{C}(\Delta_1)$ with $\alpha = 1$, if you choose $C_2 \in \mathbb{U}(\Delta_2)$, then induced compressor $C : x \mapsto C_1(\mathbf{x}) + C_2(\mathbf{x} - C_1(\mathbf{x}))$ belongs to $\mathbb{U}(\Delta)$ with $\Delta = \Delta_2 + \frac{1-\Delta_2}{\Delta_1}$. Based on this notion, Algorithm 2 proposes an induced sketching algorithm by utilizing HEAVYMIX and PRIVIX for C_1 and C_2 respectively where the reconstruction of input $\mathbf{x}$ is performed using hash table $\mathbf{S}$ and $\mathbf{x}$, similar to PRIVIX and HEAVYMIX.

Algorithm 2 HEAPRIX

1: **Inputs:** $\boldsymbol{x} \in \mathbb{R}^d, t, m, \mathbf{S}_{m\times t}, h_j (1 \leq i \leq t), \text{sign}_j (1 \leq i \leq t)$, parameter m
2: Approximate $\mathbf{S}(\boldsymbol{x})$ using HEAVYMIX
3: Approximate $\mathbf{S}(\boldsymbol{x} - \texttt{HEAVYMIX}[\mathbf{S}(\boldsymbol{x})])$ using PRIVIX
4: **Output:**

$$\texttt{HEAVYMIX}[\mathbf{S}(\boldsymbol{x})] + \texttt{PRIVIX}[\mathbf{S}(\boldsymbol{x} - \texttt{HEAVYMIX}[\mathbf{S}(\boldsymbol{x})])] .$$

Note that if $m \to d$, then $C(\boldsymbol{x}) \to \boldsymbol{x}$, which implies that the convergence rate of the algorithm can be improved by decreasing the size of compression m.

Corollary 1. *Based on Theorem 3 of (Horváth & Richtárik, 2020), HEAPRIX in Algorithm 2 satisfies $C(\boldsymbol{x}) \in \mathbb{U}(c\frac{d}{m})$.*

Benefits of HEAPRIX: Corollary 1 states that, unlike PRIVIX, HEAPRIX compression noise can be made as small as possible using larger hash size. Contrary to HEAVYMIX, HEAPRIX does not require having access to exact top_m values of the input, thus helps preserving privacy. In other words, HEAPRIX leverages the best of both worlds: the *unbiasedness* of PRIVIX while using *heavy hitters* as in HEAVYMIX.

3. FedSKETCH and FedSKETCHGATE

We define two general frameworks for different sketching algorithms for homogeneous and heterogeneous settings.

3.1. Homogeneous Setting

In FedSKETCH, the number of local updates, between two consecutive communication rounds, at device j is denoted by τ. Unlike Haddadpour et al. (2020), server node does not store any global model, rather, device j has two models: $\boldsymbol{x}^{(r)}$ and $\boldsymbol{x}_j^{(\ell,r)}$, which are respectively the local and global models. We develop FedSKETCH in Algorithm 3. A variant of this algorithm implementing HEAPRIX is also described in Algorithm 3. We note that for this variant, we need to have an additional communication round between server and worker j to aggregate $\delta_j^{(r)} \triangleq \mathbf{S}_j\left[\texttt{HEAVYMIX}(\mathbf{S}^{(r)})\right]$, see Lines 3 and 3. The main difference between our FedSKETCH and the DiffSketch algorithm in Li et al. (2019) is that we use distinct local and global learning rates.

Algorithm 3 FedSKETCH(R, τ, η, γ)

1: **Inputs:** $\boldsymbol{x}^{(0)}$: initial model shared by all local devices, global and local learning rates γ and η, respectively
2: **for** $r = 0, \ldots, R-1$ **do**
3: **parallel for device** $j \in \mathcal{K}^{(r)}$ **do**:
4: **if PRIVIX variant:**

$$\mathbf{\Phi}^{(r)} \triangleq \texttt{PRIVIX}\left[\mathbf{S}^{(r-1)}\right]$$

5: **if HEAPRIX variant:**

$$\mathbf{\Phi}^{(r)} \triangleq \texttt{HEAVYMIX}\left[\mathbf{S}^{(r-1)}\right] + \texttt{PRIVIX}\left[\mathbf{S}^{(r-1)} - \tilde{\mathbf{S}}^{(r-1)}\right]$$

6: Set $\boldsymbol{x}^{(r)} = \boldsymbol{x}^{(r-1)} - \gamma \mathbf{\Phi}^{(r)}$ and $\boldsymbol{x}_j^{(0,r)} = \boldsymbol{x}^{(r)}$
7: **for** $\ell = 0, \ldots, \tau - 1$ **do**
8: Sample a mini-batch $\xi_j^{(\ell,r)}$ and compute $\tilde{\mathbf{g}}_j^{(\ell,r)}$
9: Update $\boldsymbol{x}_j^{(\ell+1,r)} = \boldsymbol{x}_j^{(\ell,r)} - \eta\, \tilde{\mathbf{g}}_j^{(\ell,r)}$
10: **end for**
11: Device j broadcasts $\mathbf{S}_j^{(r)} \triangleq \mathbf{S}_j\left(\boldsymbol{x}_j^{(0,r)} - \boldsymbol{x}_j^{(\tau,r)}\right)$.
12: Server **computes** $\mathbf{S}^{(r)} = \frac{1}{k}\sum_{j \in \mathcal{K}} \mathbf{S}_j^{(r)}$.
13: Server **broadcasts** $\mathbf{S}^{(r)}$ to devices in randomly drawn devices $\mathcal{K}^{(r)}$.
14: **if HEAPRIX variant:**
15: Second round of communication: $\delta_j^{(r)} := \mathbf{S}_j\left[\texttt{HEAVYMIX}(\mathbf{S}^{(r)})\right]$ and broadcasts $\tilde{\mathbf{S}}^{(r)} \triangleq \frac{1}{k}\sum_{j \in \mathcal{K}} \delta_j^{(r)}$ to devices in set $\mathcal{K}^{(r)}$
16: **end parallel for**
17: **end**
18: **Output:** $\boldsymbol{x}^{(R-1)}$

Furthermore, unlike Li et al. (2019), we do not add local Gaussian noise.

Algorithmic comparison with Haddadpour et al. (2020) An important feature of our algorithm is that due to a lower dimension of the count sketch, the resulting averages ($\mathbf{S}^{(r)}$ and $\tilde{\mathbf{S}}^{(r)}$) received by the server, are also of lower dimension. Therefore, these algorithms exploit a bidirectional compression during the communication from server to device back and forth. As a result, due to this bidirectional property of communicating sketching for the case of large quantization error $\omega = \theta(\frac{d}{m})$ as shown in Haddadpour et al. (2020), our algorithms can outperform FedCOM and FedCOMGATE developed in Haddadpour et al. (2020) if sufficiently large hash tables are used and the uplink communication cost is high. Furthermore, while, in Haddadpour et al. (2020), server stores a global model and aggregates the partial gradients from devices which can enable the server to extract some information regarding the device's data, in contrast, in our algorithms server does not store the global model and only broadcasts the average sketches. Thus, sketching-based server-devices communication algorithms such as ours do not reveal the exact values of the inputs, to preserve privacy as a by-product.

Remark 1. *As pointed out in (Horváth & Richtárik, 2020), while induced compressors transform a biased compressor into unbiased one, as a drawback it doubles communication cost since the devices need to send $C_1(\boldsymbol{x})$ and $C_2(\boldsymbol{x} - C_1(\boldsymbol{x}))$ separately. We note that in the special case of HEAPRIX, due to the use of sketching, the extra communication round cost is compensated with lower number of bits per round thanks to the lower dimension of sketching.*

3.2. Heterogeneous Setting

In this section, we focus on the optimization problem of (1) in the special case of $q_1 = \ldots = q_p = \frac{1}{p}$ with full device participation ($k = p$). These results can be extended to the scenario where devices are sampled. For non i.i.d. data, the FedSKETCH algorithm, designed for homogeneous setting, may fail to perform well in practice. The main reason is that in FL, devices are using local stochastic descent direction which could be different than global descent direction when the data distribution are non-identical. Therefore, to mitigate the effect of data heterogeneity, we introduce a new algorithm called FedSKETCHGATE described in Algorithm 4. This algorithm leverages the idea of gradient tracking applied in Haddadpour et al. (2020) (with compression) and a special case of $\gamma = 1$ without compression (Liang et al., 2019). The main idea is that using an approximation of global gradient, $\mathbf{c}_j^{(r)}$ allows to correct the local gradient direction. For the FedSKETCHGATE with PRIVIX variant, the correction vector $\mathbf{c}_j^{(r)}$ at device j and communication round r is computed in Line 4. While using HEAPRIX compression, FedSKETCHGATE also updates $\tilde{\mathbf{S}}^{(r)}$ via Line 4.

Remark 2. *Most of the existing communication-efficient algorithms with compression only consider communication-efficiency from devices to server. However, Algorithms 3 and 4 also improve the communication efficiency from server to devices since it exploits low-dimensional sketches (and averages), communicated from the server to devices.*

For both FedSKETCH and FedSKETCHGATE algorithms, unlike PRIVIX, HEAPRIX variant requires a second round of communication. Therefore, in Cross-Device FL setting, where there could be millions of devices, HEAPRIX variant may not be practical, and we note that it could be more suitable for Cross-Silo FL setting.

4. Convergence Analysis

We first state commonly used assumptions required in the following convergence analysis (reminder of our notations can be found Table 1 of the Appendix).

Assumption 1 (Smoothness and Lower Boundedness)**.** *The*

Algorithm 4 $\texttt{FedSKETCHGATE}(R, \tau, \eta, \gamma)$

Inputs: $\boldsymbol{x}^{(0)} = \boldsymbol{x}_j^{(0)}$ shared by all local devices, global and local learning rates γ and η.
for $r = 0, \ldots, R-1$ **do**
parallel for device $j = 1, \ldots, p$ **do**:
  if PRIVIX variant:

$$\mathbf{c}_j^{(r)} = \mathbf{c}_j^{(r-1)} - \frac{1}{\tau}\left[\texttt{PRIVIX}\left(\mathbf{S}^{(r-1)}\right) - \texttt{PRIVIX}\left(\mathbf{S}_j^{(r-1)}\right)\right]$$

where $\boldsymbol{\Phi}^{(r)} \triangleq \texttt{PRIVIX}(\mathbf{S}^{(r-1)})$
  if HEAPRIX variant:

$$\mathbf{c}_j^{(r)} = \mathbf{c}_j^{(r-1)} - \frac{1}{\tau}\left(\boldsymbol{\Phi}^{(r)} - \boldsymbol{\Phi}_j^{(r)}\right)$$

Set $\boldsymbol{x}^{(r)} = \boldsymbol{x}^{(r-1)} - \gamma \boldsymbol{\Phi}^{(r)}$ and $\boldsymbol{x}_j^{(0,r)} = \boldsymbol{x}^{(r)}$
  for $\ell = 0, \ldots, \tau - 1$ **do**
    Sample mini-batch $\xi_j^{(\ell,r)}$ and compute $\tilde{\mathbf{g}}_j^{(\ell,r)}$
    $\boldsymbol{x}_j^{(\ell+1,r)} = \boldsymbol{x}_j^{(\ell,r)} - \eta\left(\tilde{\mathbf{g}}_j^{(\ell,r)} - \mathbf{c}_j^{(r)}\right)$
  end for
Device j broadcasts $\mathbf{S}_j^{(r)} \triangleq \mathbf{S}\left(\boldsymbol{x}_j^{(0,r)} - \boldsymbol{x}_j^{(\tau,r)}\right)$.
Server **computes** $\mathbf{S}^{(r)} = \frac{1}{p}\sum_{j=1} \mathbf{S}_j^{(r)}$ and **broadcasts** $\mathbf{S}^{(r)}$ to all devices.
 if HEAPRIX variant:
Device j computes $\boldsymbol{\Phi}_j^{(r)} \triangleq \texttt{HEAPRIX}[\mathbf{S}_j^{(r)}]$
Second round of communication to obtain $\delta_j^{(r)} := \mathbf{S}_j\left(\texttt{HEAVYMIX}[\mathbf{S}^{(r)}]\right)$
Broadcasts $\tilde{\mathbf{S}}^{(r)} \triangleq \frac{1}{p}\sum_{j=1}^{p} \delta_j^{(r)}$ to devices
end parallel for
end
Output: $\boldsymbol{x}^{(R-1)}$

local objective function $f_j(\cdot)$ of device j is differentiable for $j \in [p]$ and L-smooth, i.e., $\|\nabla f_j(\boldsymbol{x}) - \nabla f_j(\mathbf{y})\| \leq L\|\boldsymbol{x} - \mathbf{y}\|$, $\forall\, \boldsymbol{x}, \mathbf{y} \in \mathbb{R}^d$. Moreover, the optimal objective function $f(\cdot)$ is bounded below by $f^* := \min_{\boldsymbol{x}} f(\boldsymbol{x}) > -\infty$.

Assumption 1 is common in stochastic optimization. We present our results for PL, convex and general non-convex objectives. The reference Karimi et al. (2016) show that PL condition implies strong convexity property with same module (PL objectives can also be non-convex, hence strong convexity does not imply PL condition necessarily).

4.1. Convergence of FEDSKETCH

We now focus on the homogeneous case where data is i.i.d. among local devices, and therefore , the stochastic local gradient of each worker is an unbiased estimator of the global gradient. We have:

Assumption 2 (Bounded Variance). *For all $j \in [m]$, we can sample an independent mini-batch ℓ_j of size $|\Xi_j^{(\ell,r)}| = b$ and compute an unbiased stochastic gradient $\tilde{\mathbf{g}}_j = \nabla f_j(\boldsymbol{x}; \Xi_j)$, $\mathbb{E}_{\xi_j}[\tilde{\mathbf{g}}_j] = \nabla f(\boldsymbol{x}) = \mathbf{g}$ with the variance bounded is bounded by a constant σ^2, i.e., $\mathbb{E}_{\Xi_j}\left[\|\tilde{\mathbf{g}}_j - \mathbf{g}\|^2\right] \leq \sigma^2$.*

Theorem 1. *Suppose Assumptions 1-2 hold. Given $0 < m \leq d$ and considering Algorithm 3 with sketch size $B = O\left(m \log\left(\frac{dR}{\delta}\right)\right)$ and $\gamma \geq k$, with probability $1-\delta$ we have: In the **non-convex** case, $\{\boldsymbol{x}^{(r)}\}_{r=>0}$ satisfies $\frac{1}{R}\sum_{r=0}^{R-1}\left\|\nabla f(\boldsymbol{x}^{(r)})\right\|_2^2 \leq \epsilon$ if:*

- *FS-PRIVIX, for $\eta = \frac{1}{L\gamma}\sqrt{\frac{k}{R\tau\left(\frac{cd}{mk}+1\right)}}$:*

$$R = O\left(1/\epsilon\right) \quad \text{and} \quad \tau = O\left((d+m)/(mk\epsilon)\right) .$$

- *FS-HEAPRIX, for $\eta = \frac{1}{L\gamma}\sqrt{\frac{k}{R\tau\left(\frac{cd-m}{mk}+1\right)}}$:*

$$R = O\left(1/\epsilon\right) \quad \text{and} \quad \tau = O\left(d/(mk\epsilon)\right) .$$

*In the **PL or strongly convex** case, $\{\boldsymbol{x}^{(r)}\}_{r=>0}$ satisfies $\mathbb{E}[f(\boldsymbol{x}^{(R-1)}) - f(\boldsymbol{x}^{(*)})] \leq \epsilon$ if we set:*

- *FS-PRIVIX, for $\eta = \frac{1}{2L(cd/mk+1)\tau\gamma}$:*

$$R = O\left(\left(d/mk + 1\right)\kappa \log\left(1/\epsilon\right)\right) ,$$
$$\tau = O\left(\left(d/m+1\right)\Big/\left(d/m+k\right)\epsilon\right) .$$

- *FS-HEAPRIX, for $\eta = \frac{1}{2L((cd-m)/mk+1)\tau\gamma}$:*

$$R = O\left(\left((d-m)/mk + 1\right)\kappa \log\left(1/\epsilon\right)\right) ,$$
$$\tau = O\left(d/m\Big/\left(\left((d/m-1)+k\right)\epsilon\right)\right) .$$

*In the **Convex** case, $\{\boldsymbol{x}^{(r)}\}_{r=>0}$ satisfies $\mathbb{E}\Big[f(\boldsymbol{x}^{(R-1)}) - f(\boldsymbol{x}^{(*)})\Big] \leq \epsilon$ if we set:*

- *FS-PRIVIX, for $\eta = \frac{1}{2L(cd/mk+1)\tau\gamma}$:*

$$R = O\left(L\left(1 + d/mk\right)/\epsilon \log\left(1/\epsilon\right)\right) ,$$
$$\tau = O\left(\left(d/m+1\right)^2 / \left(k\left(d/mk+1\right)^2 \epsilon^2\right)\right) .$$

- *FS-HEAPRIX, for $\eta = \frac{1}{2L((cd-m)/mk+1)\tau\gamma}$:*

$$R = O\left(L\left(1 + (d-m)/mk\right)/\epsilon \log\left(1/\epsilon\right)\right) ,$$
$$\tau = O\left(\left(d/m\right)^2 \Big/ \left(k\left([d-m]/mk+1\right)^2 \epsilon^2\right)\right) .$$

The bounds in Theorem 1 suggest that in homogeneous setting if we set $d = m$ (no compression), the number of communication rounds to achieve the ϵ error matches with the number of iterations required to achieve the same error under a centralized setting. Additionally, computational complexity scales down with number of sampled devices. To stress on the further impact of using sketching, we also compare our results with prior works in terms of total number of communicated bits per device as follows:

Comparison with Ivkin et al. (2019) From privacy aspect, we note Ivkin et al. (2019) requires for server to have access to exact values of top$_m$ gradients, hence do not preserve privacy, whereas our schemes do not need those exact values. From communication cost point of view, for strongly convex objective and compared to Ivkin et al. (2019), we improve the total communication per worker from $RB = O\left(\frac{d}{\epsilon}\log\left(\frac{d}{\delta\sqrt{\epsilon}}\max\left(\frac{d}{m},\frac{1}{\sqrt{\epsilon}}\right)\right)\right)$ to

$$RB = O\left(\kappa(\frac{d-m}{k}+m)\log\frac{1}{\epsilon}\log\left(\frac{\kappa d}{\delta}(\frac{d-m}{mk}+1)\log\frac{1}{\epsilon}\right)\right).$$

We note that while reducing communication cost, our scheme requires $\tau = O(d/m(k(\frac{d}{mk}+1)\epsilon)) > 1$, which scales down with the number of sampled devices, k. Moreover, unlike Ivkin et al. (2019), we do not use bounded gradient assumption. Therefore, we obtain stronger result with weaker assumptions. Regarding general non-convex objectives, our result improves the total communication cost per worker in Ivkin et al. (2019) from $RB = O\left(\max(\frac{1}{\epsilon^2},\frac{d^2}{k^2\epsilon}\right)\log(\frac{d}{\delta}\max(\frac{1}{\epsilon^2},\frac{d^2}{k^2\epsilon})))$ for *only one device* to $RB = O(\frac{m}{\epsilon}\log(\frac{d}{\epsilon\delta}))$. We also highlight that we can obtain similar rates for Algorithm 3 in heterogeneous environment if we make the additional assumption of uniformly bounded gradient.

Note: Such improved communication cost over prior related works is due to joint exploitation of *sketching*, to reduce the dimension of communicated messages, and the use of *local updates*, to reduce the total number of communication rounds leading to a specific convergence error.

4.2. Convergence of FedSKETCHGATE

We start with bounded local variance assumption:

Assumption 3 (Bounded Local Variance). *For all $j \in [p]$, we can sample an independent mini-batch Ξ_j of size $|\xi_j| = b$ and compute an unbiased stochastic gradient $\tilde{\mathbf{g}}_j = \nabla f_j(\boldsymbol{x};\Xi_j)$ with $\mathbb{E}_\xi[\tilde{\mathbf{g}}_j] = \nabla f_j(\boldsymbol{x}) = \mathbf{g}_j$. Moreover, the variance of local stochastic gradients is bounded such that $\mathbb{E}_\Xi\left[\|\tilde{\mathbf{g}}_j - \mathbf{g}_j\|^2\right] \le \sigma^2$.*

Theorem 2. *Suppose Assumptions 1 and 3 hold. Given $0 < m \le d$, and considering FedSKETCHGATE in Algorithm 4 with sketch size $B = O\left(m\log\left(\frac{dR}{\delta}\right)\right)$ and $\gamma \ge p$ with probability $1-\delta$ we have*

*In the **non-convex** case, $\eta = \frac{1}{L\gamma}\sqrt{\frac{mp}{R\tau(cd)}}$, $\{\boldsymbol{x}^{(r)}\}_{r=>0}$ satisfies $\frac{1}{R}\sum_{r=0}^{R-1}\left\|\nabla f(\boldsymbol{x}^{(r)})\right\|_2^2 \le \epsilon$ if:*

- *FS-PRIVIX:*

$$R = O((d+m)/m\epsilon) \quad \text{and} \quad \tau = O(1/(p\epsilon)).$$

- *FS-HEAPRIX:*

$$R = O(d/m\epsilon) \quad \text{and} \quad \tau = O(1/(p\epsilon)).$$

*In the **PL or Strongly convex** case, $\{\boldsymbol{x}^{(r)}\}_{r=>0}$ satisfies $\mathbb{E}\Big[f(\boldsymbol{x}^{(R-1)}) - f(\boldsymbol{x}^{(*)})\Big] \le \epsilon$ if:*

- *FS-PRIVIX, for $\eta = 1/(2L(\frac{cd}{m}+1)\tau\gamma)$:*

$$R = O\left((\frac{d}{m}+1)\kappa\log(1/\epsilon)\right) \quad \text{and} \quad \tau = O\left(1/(p\epsilon)\right).$$

- *FS-HEAPRIX, for $\eta = m/(2cLd\tau\gamma)$:*

$$R = O\left((\frac{d}{m})\kappa\log(1/\epsilon)\right) \quad \text{and} \quad \tau = O\left(1/(p\epsilon)\right).$$

*In the **convex** case, $\{\boldsymbol{x}^{(r)}\}_{r=>0}$ satisfies $\mathbb{E}[f(\boldsymbol{x}^{(R-1)}) - f(\boldsymbol{x}^{(*)})] \le \epsilon$ if:*

- *FS-PRIVIX, for $\eta = 1/(2L(cd/m+1)\tau\gamma)$:*

$$R = O\left(L(d/m+1)\epsilon\log(1/\epsilon)\right) \quad \text{and} \quad \tau = O\left(1/(p\epsilon^2)\right).$$

- *FS-HEAPRIX, for $\eta = m/(2Lcd\tau\gamma)$:*

$$R = O\left(L(d/m)\epsilon\log(1/\epsilon)\right) \quad \text{and} \quad \tau = O\left(1/(p\epsilon^2)\right).$$

Theorem 2 implies that the number of communication rounds and local updates are similar to the corresponding quantities in homogeneous setting except for the non-convex case where the number of communication rounds also depends on the compression rate.

These results are summarized in Table 2-3 of the Appendix.

4.3. Comparison with Prior Methods

Before comparing with prior works, we highlight that privacy is another purpose of using unbiased sketching in addition to communication efficiency. Therefore, our main competing schemes are distributed algorithms based on sketching. Nonetheless, for the sake of showing the effectiveness of our algorithms, we also compare with prior non-sketching based distributed algorithms ((Karimireddy et al., 2019; Basu et al., 2019; Reisizadeh et al., 2020; Haddadpour et al., 2020)) in Section B of Appendix.

Comparison with Li et al. (2019). Note that our convergence analysis does not rely on the bounded gradient assumption. We also improve both the number of communication rounds R and the size of transmitted bits B per communication round. Additionally, we highlight that, while (Li et al., 2019) provides a convergence analysis for convex objectives, our analysis holds for PL (thus strongly convex case), general convex and general non-convex objectives.

Comparison with Rothchild et al. (2020). Due to gradient tracking, our algorithm tackles data heterogeneity issue, while algorithms in Rothchild et al. (2020) does not particularly. As a consequence, in FedSKETCHGATE each device has to store an additional state vector compared to Rothchild et al. (2020). Yet, as our method is built upon an unbiased compressor, server does not need to store any additional error correction vector. The convergence results for both of two variants of `FetchSGD` in Rothchild et al. (2020) rely on the uniform bounded gradient assumption which may not be applicable with L-smoothness assumption when data distribution is highly heterogeneous, as in FL, see (Khaled et al., 2020), while our bounds do not assume such boundedness. Besides, Theorem 1 (Rothchild et al., 2020) assumes that *Contraction Holds* for the sequence of gradients which may not hold in practice, yet based on this strong assumption, their total communication cost (RB) in order to achieve ϵ error is $RB = O\left(m\max(\frac{1}{\epsilon^2}, \frac{d^2-dm}{m^2\epsilon})\log\left(\frac{d}{\delta}\max(\frac{1}{\epsilon^2}, \frac{d^2-dm}{m^2\epsilon})\right)\right)$. For the sake of comparison we let the compression ratio in Rothchild et al. (2020) to be $\frac{m}{d}$. In contrast, without any extra assumptions, our results in Theorem 2 for `PRIVIX` and `HEAPRIX` are respectively $RB = O(\frac{(d+m)}{\epsilon}\log(\frac{(\frac{d^2}{m})+d}{\epsilon\delta}))$ and $RB = O(\frac{d}{\epsilon}\log(\frac{d^2}{\epsilon m\delta}))$ which improves the total communication cost of Theorem 1 in Rothchild et al. (2020) under regimes such that $\frac{1}{\epsilon} \geq d$ or $d \gg m$. Theorem 2 in Rothchild et al. (2020) is based the *Sliding Window Heavy Hitters* assumption, which is similar to the gradient diversity assumption in Li et al. (2020c); Haddadpour & Mahdavi (2019). Under that assumption the total communication cost is shown to be $RB = O\left(\frac{m\max(I^{2/3}, 2-\alpha)}{\epsilon^3\alpha}\log\left(\frac{d\max(I^{2/3}, 2-\alpha)}{\epsilon^3\delta}\right)\right)$ where I is a constant related to the window of gradients. We improve this bound under weaker assumptions in a regime where $\frac{I^{2/3}}{\epsilon^2} \geq d$. We also provide bounds for PL, convex and non-convex objectives contrary to Rothchild et al. (2020). Finally, we note that algorithms in Rothchild et al. (2020) are using momentum at server. While we do not use it explicitly, we can modify our algorithms to include momentum easily.

5. Numerical Study

In this section, we provide empirical results on MNIST benchmark dataset to demonstrate the effectiveness of our proposed algorithms. We train LeNet-5 Convolutional Neural Network (CNN) architecture introduced in LeCun et al. (1998), with 60 000 parameters. We compare Federated SGD (`FedSGD`) as the full-precision baseline, along with four sketching methods `SketchSGD` (Ivkin et al., 2019), `FetchSGD` (Rothchild et al., 2020), and two FedSketch variants `FS-PRIVIX` and `FS-HEAPRIX`. Note that in Algorithm 3, `FS-PRIVIX` with global learning rate $\gamma = 1$ is equivalent to the `DiffSketch` algorithm proposed in Li et al. (2020c). Also, `SketchSGD` is slightly modified to compress the change in local weights (instead of local gradient in every iteration), and `FetchSGD` is implemented

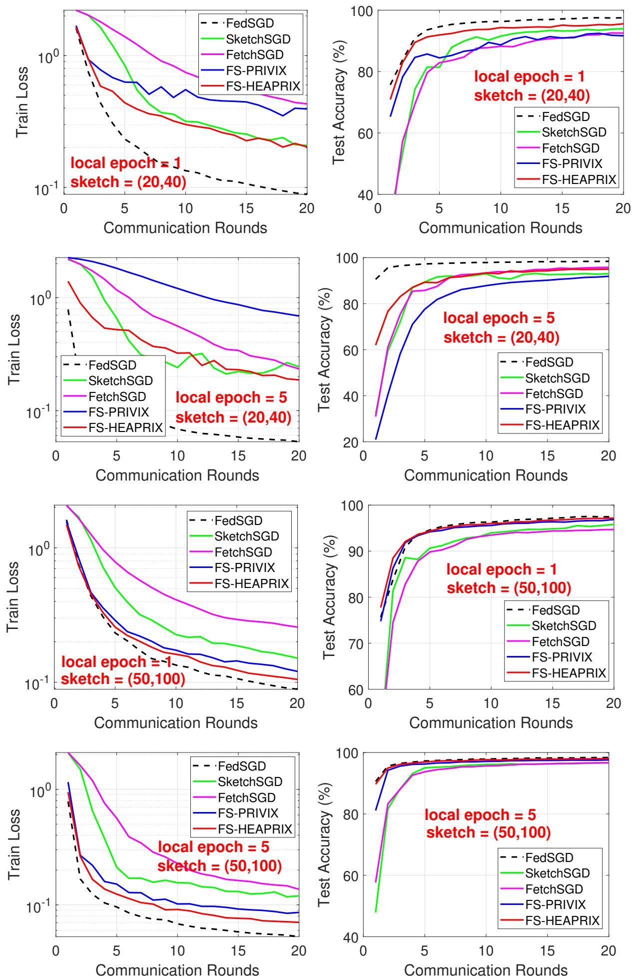

Figure 1. Homogeneous case: Comparison of compressed optimization methods on LeNet CNN.

with second round of communication for fairness. (The original proposal does not include second round of communication, which performs worse with small sketch size.) As suggested in (Rothchild et al., 2020), the momentum factor of `FetchSGD` is set to 0.9, and we also follow some recommended implementation tricks to improve its performance, which are detailed in the Appendix. The number of workers is set to 50 and we report the results for 1 and 5 local epochs. A local epoch is finished when all workers go through their local data samples once. The local batch size is 30. In each round, we randomly choose half of the devices to be active. We tune the learning rates (η and γ, if applicable) over log-scale and report the best results, for both *homogeneous* and *heterogeneous* setting. In the former case, each device receives uniformly drawn data samples, and in the latter, it only receives samples from one or two classes among ten.

Homogeneous case. In Figure 1, we provide the training loss and test accuracy with different number of local epochs and sketch size, $(t, k) = (20, 40)$ and $(50, 100)$. Note that, these two choices of sketch size correspond to a $75\times$ and $12\times$ compression ratio, respectively. We conclude

- In general, increasing compression ratio would sacrifice learning performance. In all cases, FS−HEAPRIX performs the best in terms of both training objective and test accuracy, among all compressed methods.
- FS−HEAPRIX is better than FS−PRIVIX, especially with small sketches (high compression ratio). FS−HEAPRIX yields acceptable extra test error compared to full-precision FedSGD, particularly when considering the high compression ratio (e.g. $75\times$).
- From the training loss, we see that the performance of FS−HEAPRIX improves when the number of local updates increases. *That is, the proposed method is able to further reduce the communication cost by reducing the number of rounds required for communication.* This is also consistent with our theoretical findings.

In general, our proposed FS−HEAPRIX outperforms all competing methods, and a sketch size of $(50, 100)$ is sufficient to approach the accuracy of full-precision FedSGD.

Heterogeneous case. We plot similar set of results in Figure 2 for non-i.i.d. data distribution, which leads to more twists and turns in the training curves. We see that SketchSGD performs very poorly in the heterogeneous case, which is improved by error tracking and momentum in FetchSGD, as expected. However, both of these methods are worse than our proposed FedSketchGATE methods, which can achieve similar generalization accuracy as full-precision FedSGD, even with small sketch size (i.e. $75\times$ compression with 1 local epoch). Note that, slower convergence and worse generalization of FedSGD in non-i.i.d. data distribution case is also reported in e.g. McMahan et al. (2017); Chen et al. (2020).

We also notice in Figure 2 the advantage of FS−HEAPRIX over FS−PRIVIX in terms of training loss and test accuracy. However, empirically we see that in the heterogeneous setting, more local updates tend to undermine the learning performance, especially with small sketch size. Nevertheless, when the sketch size is not too small, i.e. $(50, 100)$, FS−HEAPRIX can still provide comparable test accuracy as FedSGD in both cases. Our empirical study demonstrates that our proposed FedSketch (and FedSketchGATE) frameworks are able to perform well in homogeneous (resp. heterogeneous) setting, with high compression rate. In particular, FedSketch methods are advantageous over recent SketchedSGD (Ivkin et al., 2019) and FetchSGD (Rothchild et al., 2020) in all cases. FS−HEAPRIX performs the best among all the tested compressed optimization algorithms, which in many cases

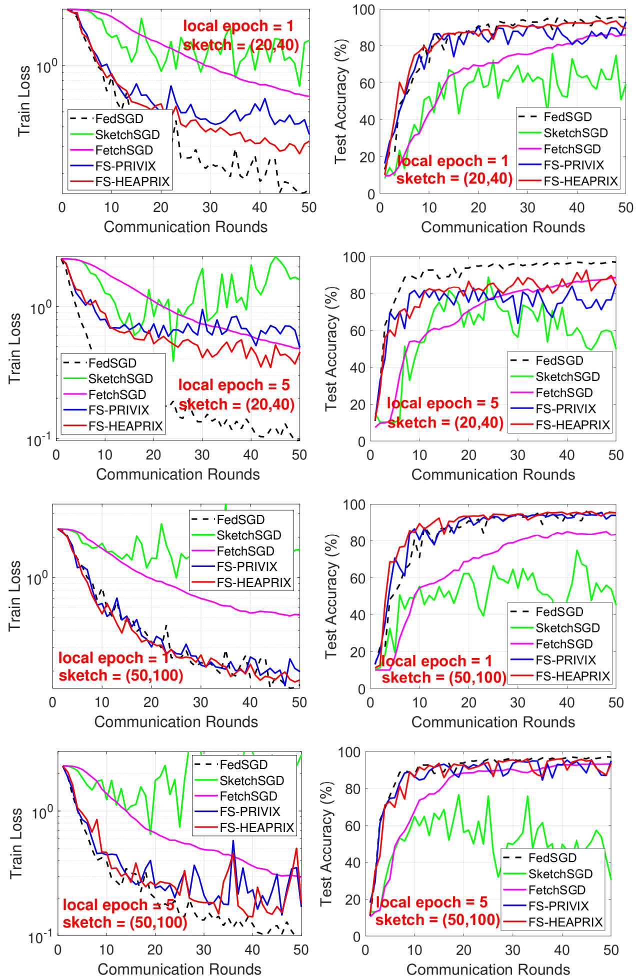

Figure 2. Heterogeneous case: Comparison of compressed optimization algorithms on LeNet CNN.

achieves similar generalization accuracy as full-precision FedSGD with small sketch size.

6. Conclusion

In this paper, we introduced FedSKETCH and FedSKETCHGATE algorithms for homogeneous and heterogeneous data distribution setting respectively for Federated Learning wherein communication between server and devices is only performed using count sketch. Our algorithms, thus, provide communication-efficiency and privacy, through random hashes based sketches. We analyze the convergence error for *non-convex*, *PL* and *general convex* objective functions in the scope of Federated Optimization. We provide insightful numerical experiments showcasing the advantages of our FedSKETCH and FedSKETCHGATE methods over current federated optimization algorithm. The proposed algorithms outperform competing compression method and can achieve comparable test accuracy as Federated SGD, with high compression ratio.

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