

# AniLA: Anisotropic Langevin Dynamics for training Energy-Based Models

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**Abstract**

## 1 Introduction

Given a stream of input data noted  $x$ , the energy-based model (EBM) is a Gibbs distribution defined as:

$$p_{\theta}(x) = \frac{1}{Z(\theta)} \exp(f_{\theta}(x)) \quad (1)$$

## 2 MCMC based EBM

**Energy Based Models:** Energy based models [LeCun et al.(2006)LeCun, Chopra, Hadsell, Ranzato, and Huang, Ngiam et al.(2011)Ngiam, Chen, Koh, and Ng] are a class of generative models that leverages the power of Gibbs potential and high dimensional sampling techniques to produce high quality synthetic image samples. Training of such models occurs via Maximum Likelihood (ML).

**MCMC procedures:**

## 3 AniLA sampler based EBM

### 3.1 Preliminaries and Bottlenecks of Langevin MCMC based EBM

### 3.2 Curvature informed MCMC

We introduce a new sampler based on the Langevin updates presented above.

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**Algorithm 1** STANLEY FOR ENERGY-BASED MODEL

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- 1: **Input:** Total number of iterations  $T$ , number of MCMC transitions  $K$  and of samples  $M$  learning rate  $\eta$ , initial values  $\theta_0$ ,  $\{z_0^m\}_{m=1}^M$  and  $n$  observations  $\{x_i\}_{i=1}^n$ .
- 2: **for**  $t = 1$  to  $T$  **do**
- 3:   Compute the anisotropic stepsize as follows:

$$\gamma_t = \frac{b}{\max(b, |\nabla f_{\theta_t}(x)|)} \quad (2)$$

- 4:   Draw  $m$  samples  $\{z_t^m\}_{m=1}^M$  from the objective potential (1) via Langevin diffusion:

$$z_t^m = z_t^m + \gamma_t/2 \nabla f_{\theta_t}(x) + \sqrt{\gamma} \mathbf{B}_t \quad (3)$$

where  $\mathbf{B}_t$  is the brownian motion, drawn from a Normal distribution.

- 5:   Samples  $m$  positive observations  $\{x_i\}_{i=1}^m$  from the empirical data distribution
- 6:   Compute the gradient of the empirical log-EBM (1) as follows:

$$\nabla \sum_{i=1}^m \log p_{\theta_t}(x_i) = \mathbb{E}_{p_{\text{data}}} [\nabla_{\theta} f_{\theta_t}(x)] - \mathbb{E}_{p_{\theta}} [\nabla_{\theta} f_{\theta}(z_t^m)] \approx \frac{1}{m} \sum_{i=1}^m \nabla_{\theta} f_{\theta_t}(x_i) - \frac{1}{m} \sum_{i=1}^m \nabla_{\theta} f_{\theta_t}(z_t^m) \quad (4)$$

- 7:   Update the vector of global parameters of the EBM:

$$\theta_{t+1} = \theta_{t+1} + \eta \nabla \sum_{i=1}^m \log p_{\theta_t}(x_i) \quad (5)$$

8: **end for**

- 9: **Output:** Generated samples  $\{z_T^m\}_{m=1}^M$
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## 4 Geometric ergodicity of AniLA sampler

We will present in this section, our theoretical analysis for the Markov Chain constructed using Line 3-4.

## 5 Numerical Experiments

### 5.1 Application on Toy Example: Gaussian Mixture Model

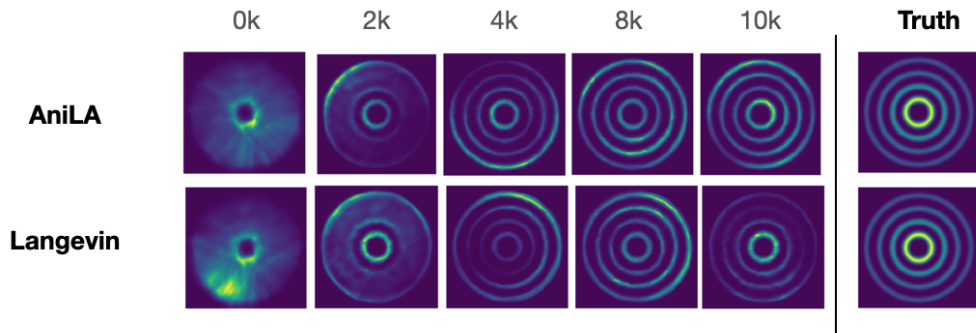


Figure 1: (Rings Toy Dataset)

## 5.2 Flowers Dataset



Figure 2: (Flowers Dataset). Left: Langevin Method. Right: AniLA method. After 100k iterations.

## 5.3 CIFAR Dataset



Figure 3: (CIFAR Dataset). Left: Langevin Method. Right: AniLA method. After 100k iterations.

## 6 Conclusion

## References

- [LeCun et al.(2006)LeCun, Chopra, Hadsell, Ranzato, and Huang] Y. LeCun, S. Chopra, R. Hadsell, M. Ranzato, and F. Huang. A tutorial on energy-based learning. *Predicting structured data*, 1(0), 2006.
- [Ngiam et al.(2011)Ngiam, Chen, Koh, and Ng] J. Ngiam, Z. Chen, P. W. Koh, and A. Y. Ng. Learning deep energy models. In *Proceedings of the 28th international conference on machine learning (ICML-11)*, pages 1105–1112, 2011.