`FedSKETCH`: Communication-Efficient Federated Learning via Sketching

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Abstract

Communication complexity and user privacy are the two key challenges in Federated Learning (FL) where the goal is to perform a distributed learning through a large volume of devices. In this paper, we introduce two new algorithms, namely `FedSKETCH` and `FedSKETCHGATE`, to address jointly both challenges and which are, respectively, intended to be used for homogeneous and heterogeneous data distribution settings. Our algorithms are based on a novel sketching technique, called `HEAPRIX` that is unbiased, compresses the accumulation of local gradients using count sketch, and exhibits communication-efficiency properties leveraging low-dimensional sketches. We provide sharper convergence guarantees compared to state-of-the-art algorithms based on sketching for FL setting and validate our theoretical findings with various sets of experiments.

1 Introduction

Federated Learning (FL) is an emerging framework for distributed large scale machine learning problems. In FL, data is distributed across devices [Konečnỳ et al., 2016, McMahan et al., 2017] and users are only allowed to communicate with the parameter server. Formally, the optimization problem across p distributed devices is defined as follows:

$$\min_{\boldsymbol{x}\in\mathbb{R}^d,\, \sum_{j=1}^{p} q_j=1} f(\boldsymbol{x}) \triangleq \sum_{j=1}^{p} q_j f_j(\boldsymbol{x})\,, \tag{1}$$

where for device $j \in \{1,\ldots,p\}$, $f_j(\boldsymbol{x}) = \mathbb{E}_{\xi\in\mathcal{D}_j}\left[L_j\left(\boldsymbol{x},\xi\right)\right]$, L_j is a loss function that measures the performance of model $\boldsymbol{x}$, ξ is a random variable distributed according to probability distribution $\mathcal{D}_j$, $q_j \triangleq \frac{n_j}{n}$ indicates the portion of data samples, n_j is the number of data shards and $n = \sum_{j=1}^{p} n_j$ is the total number of data samples. Note that contrary to the homogeneous setting where we assume $\{\mathcal{D}_j\}_{j=1}^{p}$ have the same distribution across devices and $L_i = L_j$, $1 \leq (i,j) \leq p$, in the heterogeneous setting these distributions and loss functions L_j can vary from a device to another.

There are several challenges that need to be addressed in FL in order to efficiently learn a global model that performs well in average for all devices:

– *Communication-efficiency*: There are often many devices communicating with the server, thus incurring immense communication overhead. One approach to reduce the number of communication rounds is using *local SGD with periodic averaging* [Zhou and Cong, 2018, Stich, 2019, Yu et al., 2019b, Wang and Joshi, 2018] which periodically averages local models after a few local updates, contrary to baseline SGD [Bottou and Bousquet, 2008] where gradient averaging is performed at each iteration. Local SGD has been proposed in [McMahan et al., 2017, Konečnỳ et al., 2016] under the FL setting and its convergence analysis is studied in [Stich, 2019, Wang and Joshi, 2018, Zhou and Cong, 2018, Yu et al., 2019b], later on improved in the followup references [Basu et al., 2019, Haddadpour and Mahdavi, 2019, Khaled et al., 2020, Stich and Karimireddy, 2019] for homogeneous setting. It is further extended to heterogeneous setting [Sahu et al., 2018, Haddadpour and Mahdavi, 2019, Karimireddy et al., 2019, Yu et al., 2019a, Li et al., 2020c, Liang et al., 2019]. The second approach dealing with communication cost aims at reducing the size of communicated message per communication round, such as gradient quantization [Alistarh et al., 2017, Bernstein et al., 2018, Tang et al., 2018, Wen et al., 2017, Wu et al., 2018] or sparsification [Stich et al., 2018, Alistarh et al., 2018, Lin et al., 2018, Stich and Karimireddy, 2019].

–*Data heterogeneity*: Since locally generated data in each device may come from different distribution, local computations involved in FL setting can lead to poor

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convergence error in practice [Li et al., 2020a, Liang et al., 2019]. To mitigate the negative impact of data heterogeneity, [Horváth et al., 2019, Liang et al., 2019, Karimireddy et al., 2019, Haddadpour et al., 2020] suggest applying variance reduction or gradient tracking techniques along local computations.

–*Privacy* [Geyer et al., 2017, Hardy et al., 2017]: Privacy has been widely addressed by injecting an additional layer of randomness to respect differential-privacy property [McMahan et al., 2018] or using cryptography-based approaches under secure multi-party computation [Bonawitz et al., 2017]. Further study related to FL setting can be found in recent surveys [Li et al., 2020a] and [Kairouz et al., 2019].

To jointly tackle the aforementioned challenges in FL, sketching based algorithms [Charikar et al., 2004, Cormode and Muthukrishnan, 2005, Kleinberg, 2003, Li et al., 2008] are promising methods. For instance, to reduce the communication cost, [Ivkin et al., 2019] develops a distributed SGD algorithm using sketching, provides its convergence analysis in the homogeneous setting, and establishes a communication complexity of order $\mathcal{O}(\log(d))$ per round, where d is the dimension of the gradient vector compared to $\mathcal{O}(d)$ complexity per round of baseline mini-batch SGD. Nonetheless, the proposed sketching scheme in [Ivkin et al., 2019], built from a communication-efficiency perspective, is based on a deterministic procedure which requires access to the exact information of the gradients, thus not meeting the privacy-preserving criteria. This systemic issue is partially addressed in [Rothchild et al., 2020].

Focusing on privacy, [Li et al., 2019] derives a single framework in order to address these issues and introduces `DiffSketch` algorithm, based on the Count Sketch operator, yet does not provide its convergence analysis. Besides, the estimation error of `DiffSketch` is higher than the sketching scheme in [Ivkin et al., 2019] which could lead to poor convergence.

Our main contributions are summarized as follows:

- We provide a new algorithm – `HEAPRIX` – and theoretically show that it reduces the cost of communication, based on unbiased sketching without requiring the broadcast of exact values of gradients to the server. Based on `HEAPRIX`, we develop general algorithms for communication-efficient and sketch-based FL, namely `FedSKETCH` and `FedSKETCHGATE` for homogeneous and heterogeneous data distribution settings respectively.

- We establish non-asymptotic convergence bounds for Polyak-Lojasiewicz (PL), convex and non-convex functions in Theorems 1 and 2 in both homogeneous and heterogeneous cases, and highlight an improvement in the number of iterations to reach a stationary point. We also provide *sharper* convergence analysis for the `PRIVIX`/`DiffSketch`[1] algorithm proposed in [Li et al., 2019].

- We illustrate the benefits of `FedSKETCH` and `FedSKETCHGATE` over baseline methods through a number of experiments. The latter shows the advantages of the `HEAPRIX` compression method achieving comparable test accuracy as Federated SGD (`FedSGD`) while compressing the information exchanged between devices and server.

Notation: We denote the number of communication rounds and bits per round and per device by R and B respectively. The count sketch of vector $\boldsymbol{x}$ is designated by $\mathbf{S}(\boldsymbol{x})$. $[p]$ denotes the set $\{1, \ldots, p\}$.

2 Compression using Count Sketch

Throughout the paper, we employ the commonly used `Count Sketch` [Charikar et al., 2004] for developing our algorithms. Please refer to the Appendix for the detailed Count Sketch algorithm.

There are various types of sketching algorithms which are developed based on count sketching that we develop in the following subsections.

2.1 Sketching based Unbiased Compressor

We define an unbiased compressor as follows:

Definition 1 (Unbiased compressor)**.** *We call randomized function, $\mathcal{C} : \mathbb{R}^d \to \mathbb{R}^d$ an unbiased compression operator with $\Delta \geq 1$, if*

$$\mathbb{E}\left[\mathcal{C}(\boldsymbol{x})\right] = \boldsymbol{x} \quad \textit{and} \quad \mathbb{E}\left[\left\|\mathcal{C}(\boldsymbol{x})\right\|_2^2\right] \leq \Delta \left\|\boldsymbol{x}\right\|_2^2 .$$

We denote this class of compressors by $\mathbb{U}(\Delta)$.

This definition leads to the following property

$$\mathbb{E}\left[\left\|\mathcal{C}(\boldsymbol{x}) - \boldsymbol{x}\right\|_2^2\right] \leq (\Delta - 1) \left\|\boldsymbol{x}\right\|_2^2 .$$

Note that if we let $\Delta = 1$ then our algorithm reduces to the case of no compression. This property allows us to control the noise of the compression.

An instance of such unbiased compressor is `PRIVIX` which obtains an estimate of input $\boldsymbol{x}$ from a count sketch noted $\boldsymbol{S}(\boldsymbol{x})$. In this algorithm, to query the quantity x_i, the i-th element of the vector $\boldsymbol{x}$, we compute the median of t approximated values specified by the indices of $h_j(i)$ for $1 \leq j \leq t$, see [Li et al., 2019], or Algorithm 6 in the Appendix (for more details). The following property of count sketch would be useful for our theoretical analysis.

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[1] We use `PRIVIX` and `DiffSketch` [Li et al., 2019] interchangeably throughout the paper.

Property 1 ([Li et al., 2019])**.** *For any* $\boldsymbol{x} \in \mathbb{R}^d$*:*

Unbiased estimation: As in [Li et al., 2019], we have $\mathbb{E}_{\mathbf{S}}\left[\mathit{PRIVIX}[\mathbf{S}\left(\boldsymbol{x}\right)]\right] = \boldsymbol{x}$ *.*

Bounded variance: For the given $m < d$*,* $t = \mathcal{O}\left(\ln\left(\frac{d}{\delta}\right)\right)$ *with probability* $1 - \delta$ *we have:*

$$\mathbb{E}_{\mathbf{S}}\left[\left\|\mathit{PRIVIX}[\mathbf{S}\left(\boldsymbol{x}\right)] - \boldsymbol{x}\right\|_2^2\right] \leq c\frac{d}{m}\left\|\boldsymbol{x}\right\|_2^2 \ ,$$

where c *(*$e \leq c < m$*) is a positive constant independent of the dimension of the input,* d*.*

We note that this bounded variance assumption does not necessary imply any compression as d may be relatively large. Thus, with probability $1 - \delta$, we obtain PRIVIX $\in \mathbb{U}(1+c\frac{d}{m})$. $\Delta = 1+c\frac{d}{m}$ implies that if $m \to d$, then $\Delta \to 1 + c$, indicating a noisy reconstruction. [Li et al., 2019] shows that if the data is normally distributed, PRIVIX is differentially private [Dwork, 2006], up to additional assumptions and algorithmic design choices that this aspect of the sketching is beyond the scope of this paper.

2.2 Sketching based Biased Compressor

A biased compressor is defined as follows:

Definition 2 (Biased compressor)**.** *A (randomized) function,* $\mathcal{C} : \mathbb{R}^d \to \mathbb{R}^d$ *belongs to* $\mathbb{C}(\Delta, \alpha)$*, a class of compression operators with* $\alpha > 0$ *and* $\Delta \geq 1$*, if*

$$\mathbb{E}\left[\left\|\alpha\boldsymbol{x} - \mathcal{C}(\boldsymbol{x})\right\|_2^2\right] \leq \left(1 - \frac{1}{\Delta}\right)\left\|\boldsymbol{x}\right\|_2^2 \ ,$$

It is proven in [Horváth and Richtárik, 2020] that $\mathbb{U}(\Delta) \subset \mathbb{C}(\Delta, \alpha)$. An example of biased compression via sketching and using top_m operation is given below:

Algorithm 1 HEAVYMIX (Modified [Ivkin et al., 2019])

```
Inputs: $\mathbf{S}(\mathbf{g})$; parameter $m$
Query the vector $\tilde{\mathbf{g}} \in \mathbb{R}^d$ from $\mathbf{S}(\mathbf{g})$:
Query $\hat{\ell}_2^2 = (1 \pm 0.5)\left\|\mathbf{g}\right\|^2$ from sketch $\mathbf{S}(\mathbf{g})$
$\forall j$ query $\hat{\mathbf{g}}_j^2 = \hat{\mathbf{g}}_j^2 \pm \frac{1}{2m}\left\|\mathbf{g}\right\|^2$ from sketch $\mathbf{S}(\mathbf{g})$
$H = \{j | \hat{\mathbf{g}}_j \geq \frac{\hat{\ell}_2^2}{m}\}$ and $NH = \{j | \hat{\mathbf{g}}_j < \frac{\hat{\ell}_2^2}{m}\}$
$\text{Top}_m = H \cup \text{rand}_\ell(NH)$, where $\ell = m - |H|$
Get exact values of $\text{Top}_m$
Output: $\tilde{\mathbf{g}} : \forall j \in \text{Top}_m : \tilde{\mathbf{g}}_i = \mathbf{g}_i$ else $\mathbf{g}_i = 0$
```

Following [Ivkin et al., 2019], HEAVYMIX with sketch size $\Theta\left(m \log\left(\frac{d}{\delta}\right)\right)$ is a biased compressor with $\alpha = 1$ and $\Delta = d/m$ with probability $\geq 1 - \delta$. In other words, with probability $1 - \delta$, HEAVYMIX $\in \mathcal{C}(\frac{d}{m}, 1)$. We note that Algorithm 1 is a variation of the sketching algorithm developed in [Ivkin et al., 2019] with distinction that HEAVYMIX does not require a second round of communication to obtain the exact values of top_m. This is mainly because in SKETCGED-SGD [Ivkin et al., 2019] the server receives the exact values of *the average of the sketches*; **however HEAVYMIX obtains exact value locally, thus *does not require a second round of communication at the same time does not violate user privacy*.** Additionally, while sketching algorithm implementing HEAVYMIX has a smaller estimation error compared to PRIVIX, in PRIVIX server does need to have access to the exact values of local gradient providing user privacy Li et al. [2019].

In the following, we introduce HEAPRIX which is built upon HEAVYMIX and PRIVIX methods.

2.3 Sketching based Induced Compressor

Due to Theorem 3 in [Horváth and Richtárik, 2020], which presents that we can convert the biased compressor into an unbiased one such that, for $\mathcal{C}_1 \in \mathbb{C}(\Delta_1)$ with $\alpha = 1$, if you choose $\mathcal{C}_2 \in \mathbb{U}(\Delta_2)$, then induced compressor $\mathcal{C} : x \mapsto \mathcal{C}_1(\mathbf{x}) + \mathcal{C}_2\left(\mathbf{x} - \mathcal{C}_1\left(\mathbf{x}\right)\right)$ belongs to $\mathbb{U}(\Delta)$ with $\Delta = \Delta_2 + \frac{1-\Delta_2}{\Delta_1}$.

Algorithm 2 HEAPRIX

```
Inputs: $\boldsymbol{x} \in \mathbb{R}^d, t, m, \mathbf{S}_{m \times t}, h_j(1 \leq i \leq t), \text{sign}_j(1 \leq i \leq t)$, parameter $m$
Approximate $\mathbf{S}(\boldsymbol{x})$ using HEAVYMIX
Approximate $\mathbf{S}\left(\boldsymbol{x} - \text{HEAVYMIX}[\mathbf{S}(\boldsymbol{x})]\right)$ with PRIVIX
Output:
    $\text{HEAVYMIX}\left[\mathbf{S}\left(\boldsymbol{x}\right)\right] + \text{PRIVIX}\left[\mathbf{S}\left(\boldsymbol{x} - \text{HEAVYMIX}\left[\mathbf{S}\left(\boldsymbol{x}\right)\right]\right)\right]$ .
```

Based on this notion, Algorithm 2 proposes an induced sketching algorithm by utilizing HEAVYMIX and PRIVIX for $\mathcal{C}_1$ and $\mathcal{C}_2$ respectively where the reconstruction of input $\mathbf{x}$ is performed using hash table $\mathbf{S}$ and $\mathbf{x}$, similar to PRIVIX and HEAVYMIX. Note that if $m \to d$, then $\mathcal{C}(\boldsymbol{x}) \to \boldsymbol{x}$, implying that the convergence rate can be improved by decreasing the size of compression m.

Corollary 1. *Based on Theorem 3 of [Horváth and Richtárik, 2020], HEAPRIX in Algorithm 2 satisfies* $\mathcal{C}(\boldsymbol{x}) \in \mathbb{U}(c\frac{d}{m})$*.*

3 FedSKETCH and FedSKETCHGATE

We introduce two new algorithms for both homogeneous and heterogeneous settings.

3.1 Homogeneous Setting

In FedSKETCH, the number of local updates, between two consecutive communication rounds, at device j is denoted by τ. Unlike [Haddadpour et al., 2020], the server does not store any global model, rather, device j has two models: $\boldsymbol{x}^{(r)}$ and $\boldsymbol{x}_j^{(\ell,r)}$, which are respectively the global and local models. We develop FedSKETCH

Algorithm 3 `FedSKETCH`(R, τ, η, γ)

```
1:  Inputs: $\boldsymbol{x}^{(0)}$: initial model shared by local devices, global and local learning rates $\gamma$ and $\eta$, respectively
2:  for $r = 0, \ldots, R-1$ do
3:  parallel for device $j \in \mathcal{K}^{(r)}$ do:
4:      if PRIVIX variant:
            $\mathbf{\Phi}^{(r)} \triangleq \texttt{PRIVIX}\left[\mathbf{S}^{(r-1)}\right]$
5:      if HEAPRIX variant:
            $\mathbf{\Phi}^{(r)} \triangleq \texttt{HEAVYMIX}\left[\mathbf{S}^{(r-1)}\right] + \texttt{PRIVIX}\left[\mathbf{S}^{(r-1)} - \tilde{\mathbf{S}}^{(r-1)}\right]$
6:      Set $\boldsymbol{x}^{(r)} = \boldsymbol{x}^{(r-1)} - \gamma\mathbf{\Phi}^{(r)}$ and $\boldsymbol{x}_j^{(0,r)} = \boldsymbol{x}^{(r)}$
7:      for $\ell = 0, \ldots, \tau - 1$ do
8:          Sample a mini-batch $\xi_j^{(\ell,r)}$ and compute $\tilde{\mathbf{g}}_j^{(\ell,r)}$
9:          Update $\boldsymbol{x}_j^{(\ell+1,r)} = \boldsymbol{x}_j^{(\ell,r)} - \eta\, \tilde{\mathbf{g}}_j^{(\ell,r)}$
10:     end for
11:     Device $j$ broadcasts $\mathbf{S}_j^{(r)} \triangleq \mathbf{S}_j\left(\boldsymbol{x}_j^{(0,r)} - \boldsymbol{x}_j^{(\tau,r)}\right)$.
12:     Server computes $\mathbf{S}^{(r)} = \frac{1}{k}\sum_{j\in\mathcal{K}} \mathbf{S}_j^{(r)}$ .
13:     Server broadcasts $\mathbf{S}^{(r)}$ to devices in randomly drawn devices $\mathcal{K}^{(r)}$.
14:     if HEAPRIX variant:
15:         Second round of communication for computing $\delta_j^{(r)} := \mathbf{S}_j\left[\texttt{HEAVYMIX}(\mathbf{S}^{(r)})\right]$ and broadcasts $\tilde{\mathbf{S}}^{(r)} \triangleq \frac{1}{k}\sum_{j\in\mathcal{K}} \delta_j^{(r)}$ to devices in set $\mathcal{K}^{(r)}$
16: end parallel for
17: end
18: Output: $\boldsymbol{x}^{(R-1)}$
```

in Algorithm 3 with a variant of this algorithm implementing `HEAPRIX`. For this variant, we need to have an additional communication round between the server and worker j to aggregate $\delta_j^{(r)} \triangleq \mathbf{S}_j\left[\texttt{HEAVYMIX}(\mathbf{S}^{(r)})\right]$ (Lines 5 and 14) to compute $\mathbf{S}^{(r)} = \frac{1}{k}\sum_{j\in\mathcal{K}} \mathbf{S}_j^{(r)}$. The main difference between `FedSKETCH` and `DiffSketch` in [Li et al., 2019] is that we use distinct local and global learning rates. Furthermore, unlike [Li et al., 2019], we do not add local Gaussian noise *as privacy is not the main focus of this paper.*

Algorithmic comparison with [Haddadpour et al., 2020] An important feature of our algorithm is that due to a lower dimension of the count sketch, the resulting averages ($\mathbf{S}^{(r)}$ and $\tilde{\mathbf{S}}^{(r)}$) received by the server are also of lower dimension. Therefore, these algorithms exploit a bidirectional compression during the communication from server to device back and forth. As a result, for the case of large quantization error $\omega = \theta(\frac{d}{m})$ as shown in [Haddadpour et al., 2020], our algorithms can outperform those in [Haddadpour et al., 2020] if sufficiently large hash tables are used and the uplink communication cost is high. Furthermore, while, in [Haddadpour et al., 2020], server stores a global model and aggregates the partial gradients from devices which can enable the server to extract some information regarding the device's data, in contrast, in our algorithms server does not store the global model and only broadcasts the average sketches. Thus, sketching-based server-devices communication algorithms such as ours do not reveal the exact values of the inputs.

Remark 1. *As discussed in [Horváth and Richtárik, 2020], while induced compressors transform a biased compressor into an unbiased one at the cost of doubling communication cost since the devices need to send $\mathcal{C}_1(\boldsymbol{x})$ and $\mathcal{C}_2(\boldsymbol{x} - \mathcal{C}_1(\boldsymbol{x}))$ separately. We emphasize that in `HEAPRIX` variant of `FedSKETCH`, due to the use of sketching, the extra communication round cost is compensated with lower number of bits per round thanks to the lower dimension of sketching.*

Benefits of `HEAPRIX` based algorithms: Corollary 1 states that, unlike `PRIVIX`, `HEAPRIX` compression noise can be made as small as possible using larger hash size. In the distributed setting, contrary to `SKETCHED-SGD` [Ivkin et al., 2019] where decompressing is happening in the server, `HEAPRIX` does not require to have access to exact top_m values of the input. This is because `HEAPRIX` uses `HEAVYMIX` *where decompressing is performed at each device locally, thus not requiring server to have exact values of gradients of each device.* In other words, `HEAPRIX` based FL algorithm leverages the best of both: the *unbiasedness* of `PRIVIX` while using *heavy hitters* as in `HEAVYMIX`.

3.2 Heterogeneous Setting

In this section, we focus on the optimization problem of (1) where $q_1 = \ldots = q_p = \frac{1}{p}$ with full device participation ($k = p$). These results can be extended to the scenario with devices sampling. For non i.i.d. data, the `FedSKETCH` algorithm, designed for homogeneous setting, may fail to perform well in practice. The main reason is that in FL, devices are using local stochastic descent direction which could be different than global descent direction when the data distribution are non-identical. Therefore, to mitigate the negative impact of data heterogeneity, we introduce a new algorithm called `FedSKETCHGATE` described in Algorithm 4. This algorithm leverages the idea of gradient tracking applied in [Liang et al., 2019, Haddadpour et al., 2020]. The main idea is that using an approximation of the global gradient, $\mathbf{c}_j^{(r)}$ allows to correct the local gradient direction. For the `FedSKETCHGATE` with `PRIVIX` variant, the correction vector $\mathbf{c}_j^{(r)}$ at device j and communication round r is computed in Line 4. While using `HEAPRIX` compression, `FedSKETCHGATE` also up-

Algorithm 4 `FedSKETCHGATE`(R, τ, η, γ)

```
1: **Inputs:** $\boldsymbol{x}^{(0)} = \boldsymbol{x}_j^{(0)}$ shared by all local devices, global and local learning rates $\gamma$ and $\eta$.
2: **for** $r = 0, \dots, R-1$ **do**
3: **parallel for device** $j = 1, \dots, p$ **do**:
4:    **if PRIVIX variant:**
         $\mathbf{c}_j^{(r)} = \mathbf{c}_j^{(r-1)} - \frac{1}{\tau}\left[\texttt{PRIVIX}\left(\mathbf{S}^{(r-1)}\right) - \texttt{PRIVIX}\left(\mathbf{S}_j^{(r-1)}\right)\right]$
       where $\mathbf{\Phi}^{(r)} \triangleq \texttt{PRIVIX}(\mathbf{S}^{(r-1)})$
5:    **if HEAPRIX variant:**
         $\mathbf{c}_j^{(r)} = \mathbf{c}_j^{(r-1)} - \frac{1}{\tau}\left(\mathbf{\Phi}^{(r)} - \mathbf{\Phi}_j^{(r)}\right)$
6:    Set $\boldsymbol{x}^{(r)} = \boldsymbol{x}^{(r-1)} - \gamma\mathbf{\Phi}^{(r)}$ and $\boldsymbol{x}_j^{(0,r)} = \boldsymbol{x}^{(r)}$
7:    **for** $\ell = 0, \dots, \tau-1$ **do**
8:       Sample mini-batch $\xi_j^{(\ell,r)}$ and compute $\tilde{\mathbf{g}}_j^{(\ell,r)}$
9:       $\boldsymbol{x}_j^{(\ell+1,r)} = \boldsymbol{x}_j^{(\ell,r)} - \eta\left(\tilde{\mathbf{g}}_j^{(\ell,r)} - \mathbf{c}_j^{(r)}\right)$
10:   **end for**
11:   Device $j$ broadcasts $\mathbf{S}_j^{(r)} \triangleq \mathbf{S}\left(\boldsymbol{x}_j^{(0,r)} - \boldsymbol{x}_j^{(\tau,r)}\right)$.
12:   Server **computes** $\mathbf{S}^{(r)} = \frac{1}{p}\sum_{j=1}\mathbf{S}_j^{(r)}$ and **broadcasts** $\mathbf{S}^{(r)}$ to all devices.
13:   **if HEAPRIX variant:**
14:      Device $j$ computes $\mathbf{\Phi}_j^{(r)} \triangleq \texttt{HEAPRIX}[\mathbf{S}_j^{(r)}]$.
15:      Second round of communication to obtain $\delta_j^{(r)} := \mathbf{S}_j\left(\texttt{HEAVYMIX}[\mathbf{S}^{(r)}]\right)$.
16:      Broadcasts $\tilde{\mathbf{S}}^{(r)} \triangleq \frac{1}{p}\sum_{j=1}^{p}\delta_j^{(r)}$ to devices.
17: **end parallel for**
18: **end**
19: **Output:** $\boldsymbol{x}^{(R-1)}$
```

dates $\tilde{\mathbf{S}}^{(r)}$ via Line 15.

Remark 2. *Most of the existing communication-efficient algorithms with compression only consider gradient-compression from devices to server. However, Algorithms 3 and 4 also improve the communication efficiency from server to devices as it exploits low-dimensional sketches (and averages), communicated from the server to devices.*

For both `FedSKETCH` and `FedSKETCHGATE` algorithms, unlike `PRIVIX`, `HEAPRIX` variant requires a second round of communication. Therefore, in Cross-Device FL setting, where there could be millions of devices, `HEAPRIX` variant may not be practical, and we note that it could be more suitable for Cross-Silo FL setting.

4 Convergence Analysis

We first state commonly used assumptions required in the following convergence analysis (reminder of our notations can be found Table 1 of the Appendix).

Assumption 1 (Smoothness and Lower Boundedness)**.** *The local objective function $f_j(\cdot)$ of device j is differentiable for $j \in [p]$ and L-smooth, i.e., $\|\nabla f_j(\boldsymbol{x}) - \nabla f_j(\mathbf{y})\| \le L\|\boldsymbol{x} - \mathbf{y}\|$, $\forall\ \boldsymbol{x}, \mathbf{y} \in \mathbb{R}^d$. Moreover, the optimal objective function $f(\cdot)$ is bounded below by $f^* := \min_{\boldsymbol{x}} f(\boldsymbol{x}) > -\infty$.*

We present our results for PL, convex and general non-convex objectives. [Karimi et al., 2016] show that PL condition implies strong convexity property with same module (PL objectives can also be non-convex, hence strong convexity does not imply PL condition necessarily).

4.1 Convergence of `FEDSKETCH`

We start with the homogeneous case where data is i.i.d. among local devices, and therefore, the stochastic local gradient of each worker is an unbiased estimator of the global gradient. Hence, for this scenario we can make the following assumption:

Assumption 2 (Bounded Variance)**.** *For all $j \in [m]$, we can sample an independent mini-batch ℓ_j of size $|\xi_j^{(\ell,r)}| = b$ and compute an unbiased stochastic gradient $\tilde{\mathbf{g}}_j = \nabla f_j(\boldsymbol{x}; \xi_j)$, $\mathbb{E}_{\xi_j}[\tilde{\mathbf{g}}_j] = \nabla f(\boldsymbol{x}) = \mathbf{g}$ with the variance bounded by a constant σ^2, i.e., $\mathbb{E}_{\xi_j}\left[\|\tilde{\mathbf{g}}_j - \mathbf{g}\|^2\right] \le \sigma^2$.*

Theorem 1. *Suppose Assumptions 1-2 holds. Given $0 < m \le d$ and considering Algorithm 3 with sketch size $B = O\left(m\log\left(\frac{dR}{\delta}\right)\right)$ and $\gamma \ge k$, with probability $1 - \delta$ we have:*

In the ***non-convex*** *case, $\{\boldsymbol{x}^{(r)}\}_{r=>0}$ satisfies $\frac{1}{R}\sum_{r=0}^{R-1}\mathbb{E}\left[\left\|\nabla f(\boldsymbol{x}^{(r)})\right\|_2^2\right] \le \epsilon$ if:*

- *`FS-PRIVIX`, for $\eta = \frac{1}{L\gamma}\sqrt{\frac{1}{R\tau\left(\frac{cd}{m}+\frac{1}{k}\right)}}$: $R = O\left(1/\epsilon\right)$ and $\tau = O\left(\left(\frac{d}{m} + \frac{1}{k}\right)/(\epsilon)\right)$.*
- *`FS-HEAPRIX`, for $\eta = \frac{1}{L\gamma}\sqrt{\frac{1}{R\tau\left(\frac{cd-m}{m}+\frac{1}{k}\right)}}$: $R = O\left(1/\epsilon\right)$ and $\tau = O\left(\left(\frac{cd-m}{m} + \frac{1}{k}\right)/\epsilon\right)$.*

In the ***PL or strongly convex*** *case, $\{\boldsymbol{x}^{(r)}\}_{r=>0}$ satisfies $\mathbb{E}[f(\boldsymbol{x}^{(R-1)}) - f(\boldsymbol{x}^{(*)})] \le \epsilon$ if we set:*

- *`FS-PRIVIX`, for $\eta = \frac{1}{2L(cd/mk+1)\tau\gamma}$: $R = O\left(\left(d/m + \frac{1}{k}\right)\kappa\log\left(1/\epsilon\right)\right)$ and $\tau = O\left(\left(d/m+1\right)\Big/\left(d/m + 1/k\right)\epsilon\right)$.*
- *`FS-HEAPRIX`, for $\eta = \frac{1}{2L((cd-m)/m+1/k)\tau\gamma}$: $R = O\left(\left((d-m)/m + 1/k\right)\kappa\log\left(1/\epsilon\right)\right)$ and $\tau = O\left(d/m\Big/\left(\left((d/m-1) + 1/k\right)\epsilon\right)\right)$.*

The bounds in Theorem 1 suggest that, under the homogeneous setting, if we set $d = m$ (no compression), the number of communication rounds to achieve the ϵ error matches with the number of iterations required to achieve the same error under a centralized setting. Furthermore, we can see that `FS-HEAPRIX` either improves communication or computational complexity over `FS-PRIVIX`. Additionally, computational complexity scales down (partially) with number of sampled devices. To stress on the further impact of using sketching methods, we also compare our results with prior works in terms of total number of communicated bits per device. For the convergence results of convex objectives please see Section 3 in the Appendix.

Comparison with [Ivkin et al., 2019] From a privacy aspect, we note that [Ivkin et al., 2019] requires for the central server to have access to exact values of top$_m$ gradients, hence does not preserve privacy, whereas our schemes do not need those exact values. From a communication cost point of view, for strongly convex objective and compared to [Ivkin et al., 2019], we improve the total communication per worker from $RB = O\left(\frac{d}{\epsilon}\log\left(\frac{d}{\delta\sqrt{\epsilon}}\max\left(\frac{d}{m}, \frac{1}{\sqrt{\epsilon}}\right)\right)\right)$ to

$$RB = O\left(\kappa(d-m+\tfrac{m}{k})\log\tfrac{1}{\epsilon}\log\left(\tfrac{\kappa d}{\delta}(\tfrac{d-m}{m}+1/k)\log\tfrac{1}{\epsilon}\right)\right) \ .$$

We note that while reducing communication cost, our scheme requires $\tau = O(d/m((\frac{d-m}{m}+1/k)\epsilon)) > 1$, which scales down with the number of sampled devices k. Moreover, unlike [Ivkin et al., 2019], we do not use bounded gradient assumption. Therefore, we obtain stronger results with weaker assumptions. Regarding general non-convex objectives, our result improves the total communication cost per worker displayed in [Ivkin et al., 2019] from $RB = O\left(\max(\frac{1}{\epsilon^2}, \frac{d^2}{k^2\epsilon}\right)\log(\frac{d}{\delta}\max(\frac{1}{\epsilon^2}, \frac{d^2}{k^2\epsilon})))$ for *only one device* to $RB = O(\frac{m}{\epsilon}\log(\frac{d}{\epsilon\delta}))$. We also highlight that we can obtain similar rates for Algorithm 3 in heterogeneous environment if we make the additional assumption of uniformly bounded gradient.

Note: Such improved communication cost over prior works is due to the joint exploitation of *sketching*, reducing the dimension of communicated messages, and the use of *local updates*, reducing the number of communication rounds reaching a specific convergence error.

4.2 Convergence of `FedSKETCHGATE`

We start with a bounded local variance assumption:

Assumption 3 (Bounded Local Variance). *For all $j \in [p]$, we can sample an independent mini-batch Ξ_j of size $|\xi_j| = b$ and compute an unbiased stochastic gradient $\tilde{\mathbf{g}}_j = \nabla f_j(\boldsymbol{x}; \xi_j)$ with $\mathbb{E}_\xi[\tilde{\mathbf{g}}_j] = \nabla f_j(\boldsymbol{x}) = \mathbf{g}_j$. Moreover, the variance of local stochastic gradients is bounded such that $\mathbb{E}_\xi\left[\|\tilde{\mathbf{g}}_j - \mathbf{g}_j\|^2\right] \le \sigma^2$.*

Theorem 2. *Suppose Assumptions 1 and 3 hold. Given $0 < m \le d$, and considering FedSKETCHGATE in Algorithm 4 with sketch size $B = O\left(m\log\left(\frac{dR}{\delta}\right)\right)$ and $\gamma \ge p$ with probability $1-\delta$ we have*

*In the **non-convex** case, $\eta = \frac{1}{L\gamma}\sqrt{\frac{mp}{R\tau(cd)}}$, $\{\boldsymbol{x}^{(r)}\}_{r=>0}$ satisfies $\frac{1}{R}\sum_{r=0}^{R-1}\mathbb{E}\left[\left\|\nabla f(\boldsymbol{x}^{(r)})\right\|_2^2\right] \le \epsilon$ if:*

- *FS-PRIVIX:*

$$R = O((d+m)/m\epsilon) \quad \textit{and} \quad \tau = O(1/(p\epsilon)) \ .$$

- *FS-HEAPRIX: $R = O(d/m\epsilon)$ and $\tau = O(1/(p\epsilon))$.*

*In the **PL or Strongly convex** case, $\{\boldsymbol{x}^{(r)}\}_{r=>0}$ satisfies $\mathbb{E}\Big[f(\boldsymbol{x}^{(R-1)}) - f(\boldsymbol{x}^{(*)})\Big] \le \epsilon$ if:*

- *FS-PRIVIX, for $\eta = 1/(2L(\frac{cd}{m}+1)\tau\gamma)$: $R = O\left((\frac{d}{m}+1)\kappa\log(1/\epsilon)\right)$ and $\tau = O\left(1/(p\epsilon)\right)$*
- *FS-HEAPRIX, for $\eta = m/(2cLd\tau\gamma)$: $R = O\left((\frac{d}{m})\kappa\log(1/\epsilon)\right)$ and $\tau = O\left(1/(p\epsilon)\right)$.*

Theorem 2 implies that the number of communication rounds and local updates are similar to the corresponding quantities in homogeneous setting except for the non-convex case where the number of rounds also depends on the compression rate (summarized Table 2-3 of the Appendix). For the convergence result of convex objectives please see Section 3 in appendix.

We note that the convergence analysis of FS-`PRIVIX` provided in [Li et al., 2019] for convex objectives is further tightened in our contribution. Moreover, FS-`HEAPRIX` improves the communication complexity of FS-`PRIVIX` for both PL and non-convex objectives which is empirically validated in Figures 1 and 2.

4.3 Comparison with Prior Methods

We stress that privacy is another purpose of using unbiased sketching in addition to communication efficiency. Therefore, our main competing schemes are distributed algorithms based on sketching. Nonetheless, we also compare with prior non-sketching based distributed algorithms ([Karimireddy et al., 2019, Basu et al., 2019, Reisizadeh et al., 2020, Haddadpour et al., 2020]) in Section C of the Appendix.

[Li et al., 2019]. Note that our convergence analysis does not rely on the bounded gradient assumption. We also improve both the number of communication rounds R and the size of transmitted bits B per communication round (please see Table 3 of Section C in appendix). Additionally, we highlight that, while [Li et al., 2019] provides a convergence analysis for convex objectives, our analysis holds for PL (thus strongly convex case), general convex and general non-convex objectives.

[Rothchild et al., 2020]. Due to gradient tracking, our algorithm tackles data heterogeneity issue, while algorithms in [Rothchild et al., 2020] do not particularly. As a consequence, in `FedSKETCHGATE` each device has to store an additional state vector compared to [Rothchild et al., 2020]. Yet, as our method is built upon an unbiased compressor, server does not need to store any additional error correction vector. The convergence results for both `FetchSGD` variants in [Rothchild et al., 2020] rely on the uniform bounded gradient assumption which may not be applicable with L-smoothness assumption when data distribution is highly heterogeneous, as in FL, see [Khaled et al., 2020], while our bounds do not assume such boundedness. Besides, Theorem 1 [Rothchild et al., 2020] assumes that *Contraction Holds* for the sequence of gradients which may not hold in practice, yet based on this strong assumption, their total communication cost (RB) in order to achieve ϵ error is $RB = O\left(m\max(\frac{1}{\epsilon^2}, \frac{d^2-dm}{m^2\epsilon})\log\left(\frac{d}{\delta}\max(\frac{1}{\epsilon^2}, \frac{d^2-dm}{m^2\epsilon})\right)\right)$. For the sake of comparison, we let the compression ratio in [Rothchild et al., 2020] to be $\frac{m}{d}$. In contrast, without any extra assumptions, our results in Theorem 2 for `PRIVIX` and `HEAPRIX` are respectively $RB = O(\frac{(d+m)}{\epsilon}\log(\frac{(\frac{d^2}{m})+d}{\epsilon\delta}))$ and $RB = O(\frac{d}{\epsilon}\log(\frac{d^2}{\epsilon m\delta}))$ which improves the total communication cost of Theorem 1 in [Rothchild et al., 2020] under regimes such that $\frac{1}{\epsilon} \geq d$ or $d \gg m$. Theorem 2 in [Rothchild et al., 2020] is based the *Sliding Window Heavy Hitters* assumption, which is similar to the gradient diversity assumption in [Li et al., 2020b, Haddadpour and Mahdavi, 2019]. Under that assumption, the total communication cost is shown to be $RB = O\left(\frac{m\max(I^{2/3},2-\alpha)}{\epsilon^3\alpha}\log\left(\frac{d\max(I^{2/3},2-\alpha)}{\epsilon^3\delta}\right)\right)$ where I is a constant related to the window of gradients. We improve this bound under weaker assumptions in a regime where $\frac{I^{2/3}}{\epsilon^2} \geq d$. We also provide bounds for PL, convex and non-convex objectives contrary to [Rothchild et al., 2020].

5 Numerical Study

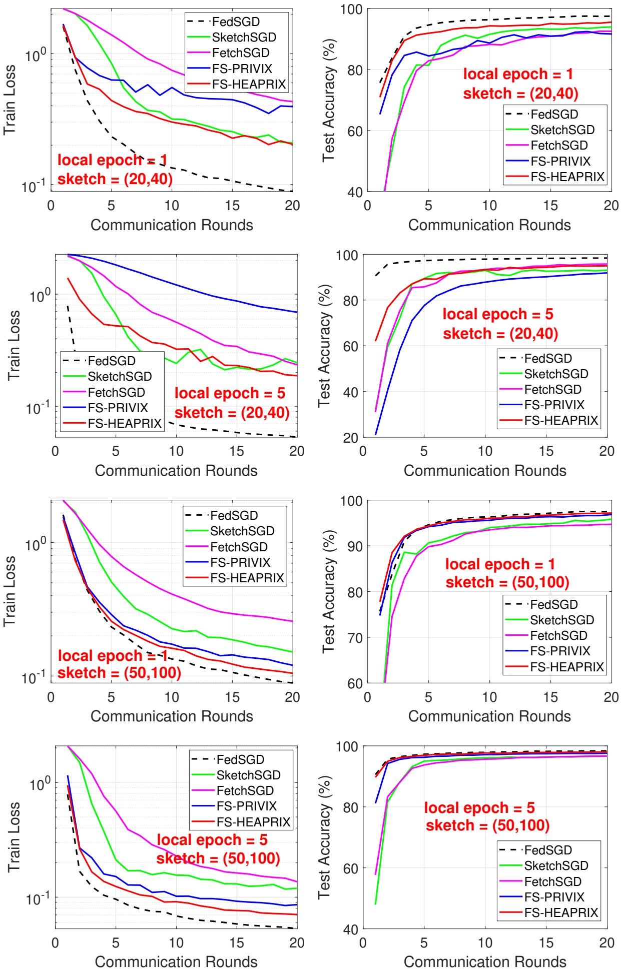

Figure 1: Homogeneous case: Comparison of compressed optimization methods on LeNet CNN.

In this section, we provide empirical results on MNIST benchmark dataset to demonstrate the effectiveness of our proposed algorithms. We train LeNet-5 Convolutional Neural Network (CNN) architecture introduced in [LeCun et al., 1998], with 60 000 parameters. We compare Federated SGD (`FedSGD`) as the full-precision baseline, along with four sketching methods `SketchSGD` [Ivkin et al., 2019], `FetchSGD` [Rothchild et al., 2020], and two FedSketch variants `FS-PRIVIX` and `FS-HEAPRIX`. Note that in Algorithm 3, `FS-PRIVIX` with global learning rate $\gamma = 1$ is equivalent to the `DiffSketch` algorithm proposed in [Li et al., 2020b]. Also, `SketchSGD` is slightly modified to compress the change in local weights (instead of local gradient in every iteration), and `FetchSGD` is implemented with second round of communication for fairness. (The original proposal does not include second round of communication, which performs worse with small sketch size.) As suggested in [Rothchild et al., 2020], the momentum factor of `FetchSGD` is set to 0.9, and we also follow some recommended implementation tricks to improve its performance, which are detailed in the Appendix. The number of workers is set to 50 and we report the results for 1 and 5 local epochs. A local epoch is finished when all workers go through their local data samples once. The local batch size is 30. In each round, we randomly choose half of the devices to be active. We tune the learning rates (η and γ, if applicable) over log-scale and report the best results, for both *homogeneous* and *heterogeneous* setting. In the former case, each device receives uniformly drawn data samples, and in the latter, it only receives samples from one or two classes among ten.

Homogeneous case. In Figure 1, we provide the training loss and test accuracy with different number of local epochs and sketch size, $(t, k) = (20, 40)$ and $(50, 100)$. Note that, these two choices of sketch size correspond to a 75× and 12× compression ratio, respectively. We conclude that

- In general, increasing the compression ratio sacrifices the learning performance. In all cases, `FS-HEAPRIX` performs the best in terms of both training objective and test accuracy, among all compressed methods.
- `FS-HEAPRIX` is better than `FS-PRIVIX`, especially with small sketches (high compression ratio). `FS-HEAPRIX` yields acceptable extra test error compared to full-precision `FedSGD`, particularly when considering the high compression ratio (e.g., 75×).
- The training performance of `FS-HEAPRIX` improves when the number of local updates increases. *That is, the proposed method is able to further reduce the communication cost by reducing the number of rounds required for communication.* This is also consistent with our theoretical findings.

In general, `FS-HEAPRIX` outperforms all competing methods, and a sketch size of $(50, 100)$ is sufficient to approach the accuracy of full-precision `FedSGD`.

Heterogeneous case. We plot similar set of results in Figure 2 for non-i.i.d. data distribution, which leads to more twists and turns in the training curves. We see that `SketchSGD` performs very poorly in the heterogeneous case, which is improved by error tracking and momentum in `FetchSGD`, as expected. However, both of these methods are worse than our proposed `FedSketchGATE` methods, which can achieve similar generalization accuracy as full-precision `FedSGD`, even with small sketch size (i.e., 75× compression). Note that, slower convergence and worse generalization of `FedSGD` in non-i.i.d. data distribution case is also reported in e.g. [McMahan et al., 2017, Chen et al., 2020].

We also notice in Figure 2 the edge of `FS-HEAPRIX` over `FS-PRIVIX` in terms of training loss and test accuracy. However, we see that in the heterogeneous setting, more local updates tend to undermine the learning performance, especially with small sketch size. Nevertheless, when the sketch size is not too small, i.e., $(50, 100)$, `FS-HEAPRIX` can still provide comparable test accuracy as `FedSGD` in both cases. Our empirical study demonstrates that `FedSketch` (and `FedSketchGATE`) frameworks are able to perform well in homogeneous (resp. heterogeneous) settings, with high compression rate. In particular, `FedSketch` methods are beneficial over `SketchedSGD` [Ivkin et al., 2019] and `FetchSGD` [Rothchild et al., 2020] in all cases. `FS-HEAPRIX` performs the best among all the tested compressed algorithms, which in many cases achieves similar generalization accuracy as full-precision FedSGD with small sketch size.

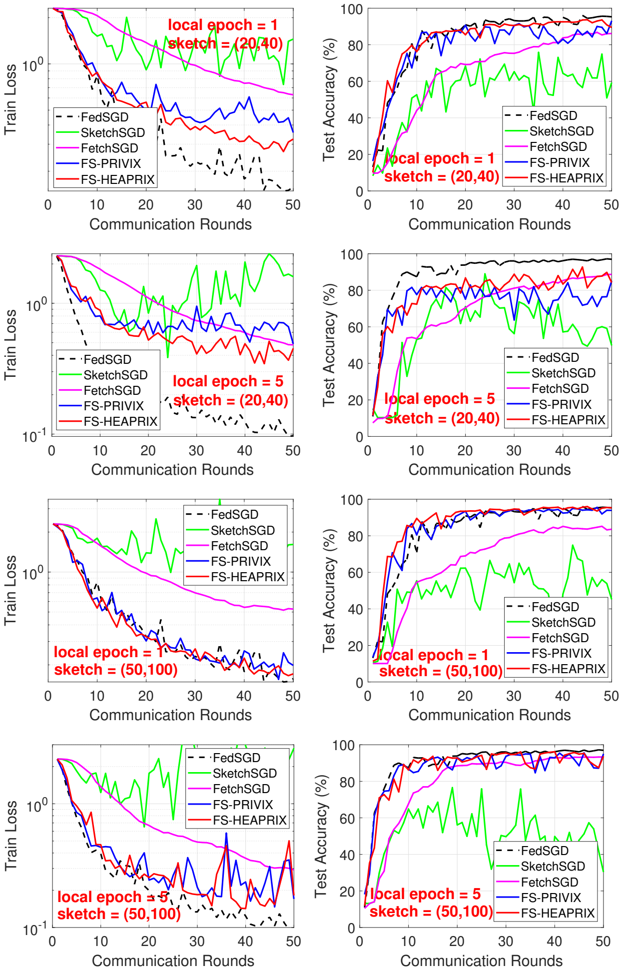

Figure 2: Heterogeneous case: Comparison of compressed optimization algorithms on LeNet CNN.

6 Conclusion

In this paper, we introduced `FedSKETCH` and `FedSKETCHGATE` algorithms for homogeneous and heterogeneous data distribution setting respectively for Federated Learning wherein communication between server and devices is only performed using count sketch. Our algorithms, thus, provide communication-efficiency and privacy, through random hashes based sketches. We analyze the convergence error for *non-convex*, *PL* and *general convex* objective functions in the scope of Federated Optimization. We provide insightful numerical experiments showcasing the advantages of our `FedSKETCH` and `FedSKETCHGATE` methods over current federated optimization algorithm. The proposed algorithms outperform competing compression method and can achieve comparable test accuracy as Federated SGD, with high compression ratio.

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