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# Sparsified Distributed Adaptive Learning with Error Feedback

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## Abstract

1 To be completed...

## 2 1 Introduction

3 Most modern machine learning tasks can be casted as a large finite-sum optimization problem writ-  
4 ten as:

$$\min_{\theta \in \Theta} \frac{1}{n} \sum_{i=1}^n f_i(\theta) \quad (1)$$

5 where  $n$  denotes the number of workers,  $f_i$  represents the average loss for worker  $i$  and  $\theta$  the global  
6 model parameter taking value in  $\Theta$ , a subset of  $\mathbb{R}^d$ .

7 Some related work:

8 [18] develops variant of signSGD (as a biased compression schemes) for distributed optimization.  
9 Contributions are mainly on this error feedback variant. In [26], the authors provide theoretical  
10 results on the convergence of sparse Gradient SGD for distributed optimization (we want that for  
11 AMS here). [27] develops a variant of distributed SGD with sparse gradients too. Contributions  
12 include a memory term used while compressing the gradient (using top k for instance). Speeding up  
13 the convergence in  $\frac{1}{T^3}$ .

## 14 2 Preliminaries

### 15 Sparse Optimization Methods.

16 **Distributed Learning.** When a large number of compute engines is available, being able to  
17 train global machine learning models while mutualizing the available and *decentralized* source of  
18 computation has been a growing focus for the community.

19 Decentralized optimization methods include methods such as ADMM [6], Distributed Subgradient  
20 Descent [24], Dual Averaging [11], Prox-PDA [14], GNSD [21], and Choco-SGD [20].

21 A recent work [7], which focuses on adaptive gradient methods, namely the Adam [19] and the  
22 AMSGrad [25] optimization methods, develops a decentralized variant of gradient based and adap-  
23 tive methods in the context of gossip protocols. To date, very few contributions provided attempt  
24 to efficiently run adaptive gradient method in such a distributed setting. Apart from [7], (author?)  
25 [23] proposes a decentralized version of AMSGrad [25] which provably satisfies some non-standard  
26 regret. Though, no sparsified variants of them have been proposed for practical purposes nor been  
27 studied in the literature.

28 **Compression-Based Distributed Optimization.** While the capabilities of the compute powers  
 29 is exploding, the communication complexity between either the central server and the decentralized  
 30 workers or among workers is becoming ineffectively large [9, 22]. Gradient sparsification con-  
 31 stitutes one popular method to induce sparsity through the optimization procedure and reduce the  
 32 number of bits transmitted at each iteration. Extensive works have studied this technique to improve  
 33 the communication efficiency of SGD-based methods such as distributed SGD. This large class of  
 34 sparsification techniques include gradient quantization leveraging quantized vector of gradients in  
 35 the communication phase [2, 29, 16, 28, 13, 8, 15], gradient sparsification generally selection top  
 36  $k$  components of the vector to be communicated, see [27, 1], or variants of the particular SGD al-  
 37 gorithm such as low-precision SGD [4, 18] proposing a trade-off between communication cost and  
 38 precision, and signSGD [10, 30] where only the signs of the gradient vectors are communicated.  
 39 Most of these works apply to the SGD method [5] as a prototype where a novel method and some  
 40 convergence results are presented with a rate of  $\mathcal{O}(\frac{1}{\sqrt{T}})$  where  $T$  denotes the total number of itera-  
 41 tions, see [3], thus achieving the same rate as plain SGD, see [12, 17].

42 Yet these communication reduction techniques, still presents a negative dependence on the number  
 43 of workers, typically a linear dependence. Hence the need for even more efficient techniques which  
 44 constitutes the object of our paper.

### 45 3 Method

46 Consider standard synchronous distributed optimization setting. AMSGrad is used as the prototype,  
 47 and the local workers is only in charge of gradient computation.

#### 48 3.1 TopK AMSGrad with Error Feedback

49 The key difference (and interesting part) of our TopK AMSGrad compared with the following arxiv  
 50 paper “Quantized Adam” <https://arxiv.org/pdf/2004.14180.pdf> is that, in our model only  
 51 gradients are transmitted. In “QAdam”, each local worker keeps a local copy of moment estimator  
 52  $m$  and  $v$ , and compresses and transmits  $m/v$  as a whole. Thus, that method is very much like the  
 53 sparsified distributed SGD, except that  $g$  is changed into  $m/v$ . In our model, the moment estimates  
 54  $m$  and  $v$  are computed only at the central server, with the compressed gradients instead of the full  
 55 gradient. This would be the key (and difficulty) in convergence analysis.

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#### Algorithm 1 SPARS-AMS for Distributed Learning

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1: Input: parameter  $\beta_1, \beta_2$ , learning rate  $\eta_t$ .
2: Initialize: central server parameter  $\theta_0 \in \Theta \subseteq \mathbb{R}^d$ ,  $e_{t,i} = 0$  the error accumulator for each
   worker; sparsity parameter  $k$ ;  $N$  local workers;  $m_0 = 0, v_0 = 0, \hat{v}_0 = 0$ 
3: for  $t = 1$  to  $T$  do
4:   parallel for worker  $i \in [n]$  do:
5:     Receive model parameter  $\theta_{t-1}$  from central server
6:     Compute stochastic gradient  $g_{t,i}$  at  $\theta_t$ 
7:     Compute  $\tilde{g}_{t,i} = \text{TopK}(g_{t,i} + e_{t,i}, k)$ 
8:     Update the error  $e_{t+1,i} = e_{t,i} + g_{t,i} - \tilde{g}_{t,i}$ 
9:     Send  $\tilde{g}_{t,i}$  back to central server
10:  end parallel
11:  Central server do:
12:     $\bar{g}_t = \frac{1}{N} \sum_{i=1}^N \tilde{g}_{t,i}$ 
13:     $m_t = \beta_1 m_{t-1} + (1 - \beta_1) \bar{g}_t$ 
14:     $v_t = \beta_2 v_{t-1} + (1 - \beta_2) \bar{g}_t^2$ 
15:     $\hat{v}_t = \max(v_t, \hat{v}_{t-1})$ 
16:    Update global model  $\theta_t = \theta_{t-1} - \eta_t \frac{m_t}{\sqrt{\hat{v}_t + \epsilon}}$ 
17: end for

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## 56 3.2 Convergence Analysis

57 Several mild assumptions to make: Nonconvex and smooth loss function, unbiased stochastic gradi-  
 58 ent, bounded variance of the gradient, bounded norm of the gradient, control of the distance between  
 59 the true gradient and its sparse variant.

60 Check [7] starting with single machine and extending to distributed settings (several machines).

### 61 3.2.1 Single machine

62 Under the centralized setting, the goal is to derive an upper bound to the second order moment of  
 63 the gradient of the objective function at some iteration  $T_f \in [1, T]$ .

64 We begin by making the following assumptions.

65 **Assumption 1.** (Smoothness) For  $i \in \llbracket n \rrbracket$ ,  $f_i$  is  $L$ -smooth:  $\|\nabla f_i(\theta) - \nabla f_i(\vartheta)\| \leq L \|\theta - \vartheta\|$ .

66 **Assumption 2.** (Unbiased and Bounded gradient) For any iteration index  $t > 0$  and worker index  
 67  $i \in \llbracket n \rrbracket$ , the stochastic gradient is unbiased and bounded from above:  $\mathbb{E}[g_{t,i}] = \nabla f(\theta_t)$  and  
 68  $\|g_{t,i}\| \leq G$ .

69 **Assumption 3.** (Bounded variance) For any iteration index  $t > 0$  and worker index  $i \in \llbracket n \rrbracket$ , the  
 70 variance of the noisy gradient is bounded:  $\mathbb{E}[|g_{t,i} - \nabla f(\theta_t)|^2] < \sigma^2$ .

71 Denote by  $Q(\cdot)$  the quantization operator Line 7 of Algorithm 1, which takes as input a gradient  
 72 vector and returns a quantized version of it, and note  $\tilde{g} := Q(g)$ . Assume that

73 **Assumption 4.** (Bounded Quantization) For any iteration  $t > 0$ , there exists a constant  $q > 0$  such  
 74 that  $\|g_t - \tilde{g}_t\| \leq q \|g_t\|$ , where  $g_t$  is the stochastic gradient computed at iteration  $t$ .

75 We first define multiple auxiliary sequences. For the first moment, define

$$\begin{aligned}\bar{m}_t &= m_t + \mathcal{E}_t, \\ \mathcal{E}_t &= \beta_1 \mathcal{E}_{t-1} + (1 - \beta_1)(e_{t+1} - e_t),\end{aligned}$$

76 such that

$$\begin{aligned}\bar{m}_t &= \bar{m}_t + \mathcal{E}_t \\ &= \beta_1(m_t + \mathcal{E}_t) + (1 - \beta_1)(\tilde{g}_t + e_{t+1} - e_1) \\ &= \beta_1 \bar{m}_{t-1} + (1 - \beta_1)g_t.\end{aligned}$$

77 Denote the following auxiliary variables at iteration  $t + 1$

$$z_{t+1} = \theta_{t+1} + \frac{\beta_1}{1 - \beta_1}(\theta_{t+1} - \theta_t) \quad (2)$$

### 78 3.2.2 Multiple machine

## 79 4 Experiments

80 Our proposed TopK-EF with AMSGrad matches that of full AMSGrad, in distributed learning.  
 81 Number of local workers is 20. Error feedback fixes the convergence issue of using solely the  
 82 TopK gradient.

## 83 5 Conclusion

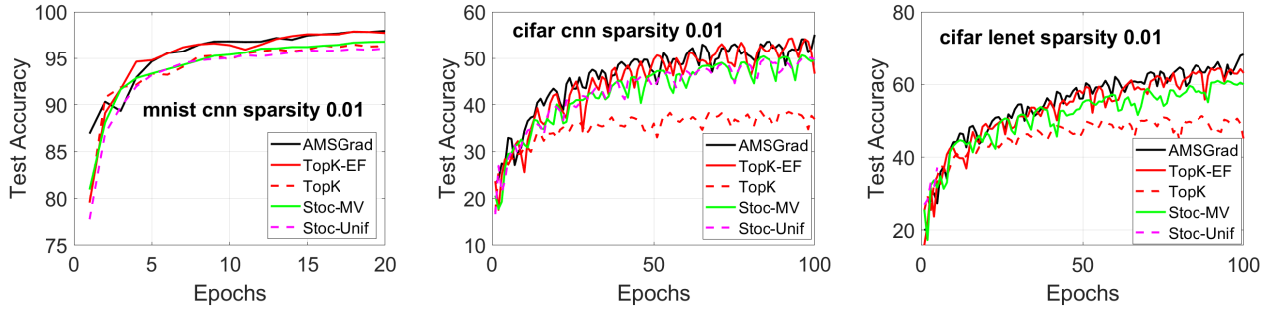


Figure 1: Test accuracy.

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