Physically-Based Simulation Proseminar

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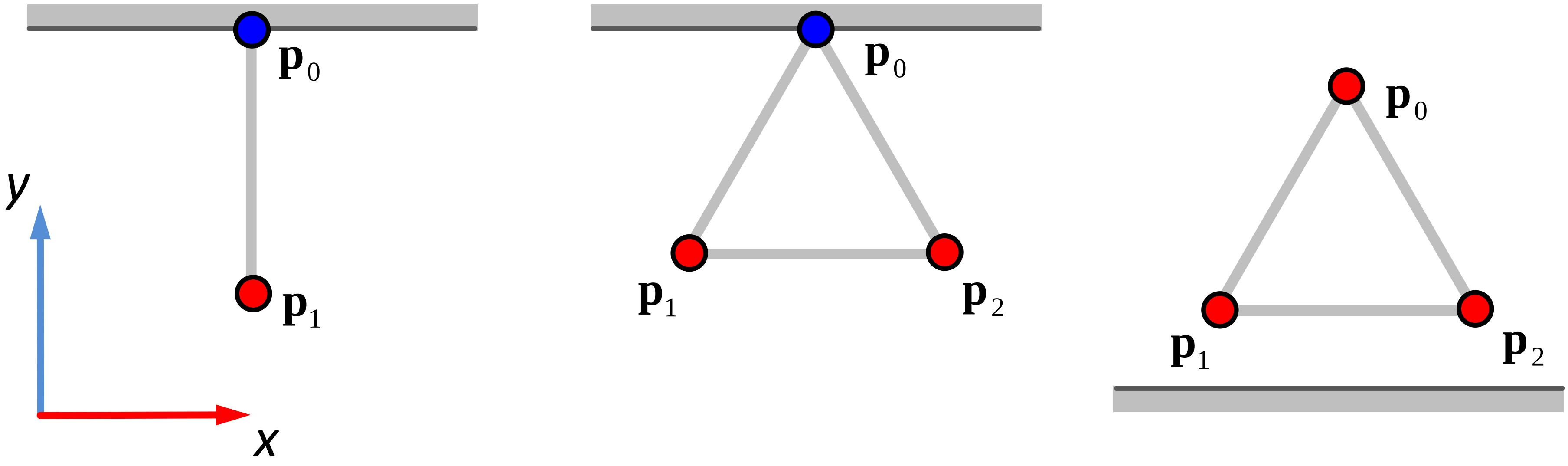
Winter semester 2017

First Programming Assignment

- Simulation of simple mass-spring examples



Hanging mass | Hanging triangle | Falling triangle

First Programming Assignment

Main tasks:

1. Implement basic numerical solvers for equations of motion
2. Test behavior for different settings and test scene
3. Compare numerical with exact solution

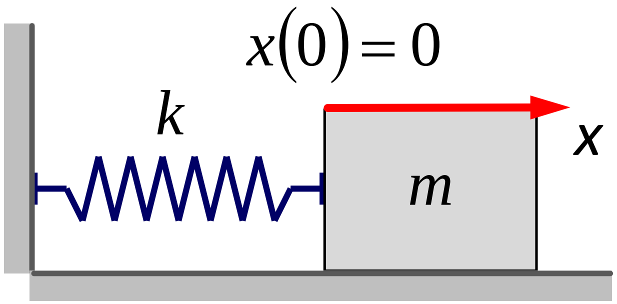
Analytic Solution

- In simple cases analytic solution available for ODEs
- Equation of motion with damping

$$m\frac{d^2\mathbf{x}(t)}{dt^2}+\gamma\frac{d\mathbf{x}(t)}{dt}+k\mathbf{x}(t)=0$$

- In 1D case equivalent to harmonic oscillator

$$m\ddot{x}(t)+\gamma\dot{x}(t)+kx(t)=0$$



Simple Harmonic Oscillator

- Disregard damping

$$m\ddot{x}(t) + kx(t) = 0$$

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$$m\ddot{x}(t) + kx(t) = 0$$

$$\ddot{x}(t) + \frac{k}{m}x(t) = 0$$

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$$m\ddot{x}(t)+kx(t)=0$$

$$\ddot{x}(t)+\omega^2 x(t)=0 \qquad \omega=\sqrt{\frac{k}{m}}$$

Simple Harmonic Oscillator

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$$m\ddot{x}(t) + kx(t) = 0$$

$$\ddot{x}(t) + \omega^2 x(t) = 0 \qquad\qquad \omega = \sqrt{\frac{k}{m}}$$

- Possible solution

$$x(t) = \sin \omega t$$

Simple Harmonic Oscillator

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$$m\ddot{x}(t) + kx(t) = 0$$

$$\ddot{x}(t) + \omega^2 x(t) = 0 \qquad\qquad \omega = \sqrt{\frac{k}{m}}$$

- Possible solution

$$x(t) = \sin \omega t$$

$$\dot{x}(t) = \omega \cdot \cos \omega t$$

Simple Harmonic Oscillator

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$$\ddot{x}(t) + \omega^2 x(t) = 0 \qquad\qquad \omega = \sqrt{\frac{k}{m}}$$

- Possible solution

$$x(t) = \sin \omega t$$

$$\dot{x}(t) = \omega \cdot \cos \omega t$$

$$\ddot{x}(t) = -\omega^2 \cdot \sin \omega t$$

Simple Harmonic Oscillator

- Disregard damping

$$m\ddot{x}(t)+kx(t)=0$$

$$\ddot{x}(t)+\omega^2 x(t)=0 \qquad \omega=\sqrt{\frac{k}{m}}$$

- Possible solution

$$x(t)=\cos\omega t$$

$$\dot{x}(t)=-\omega\cdot\sin\omega t$$

$$\ddot{x}(t)=-\omega^2\cdot\cos\omega t$$

Simple Harmonic Oscillator

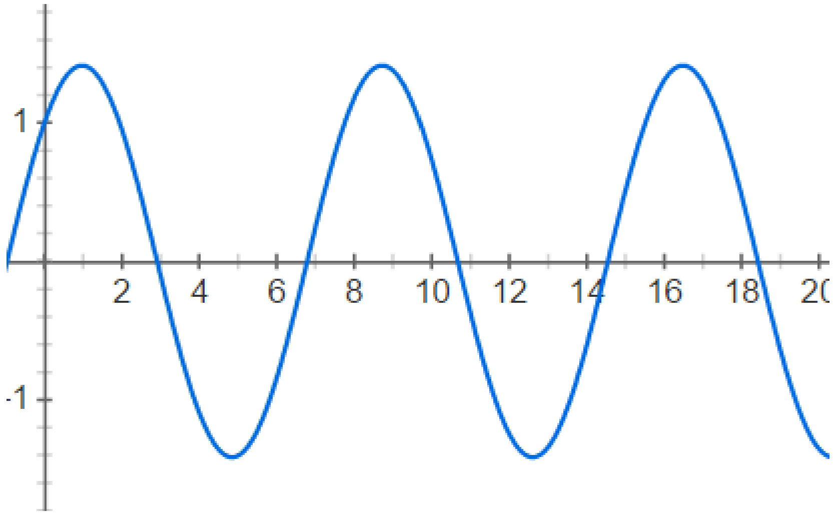
- Disregard damping

$$m\ddot{x}(t) + kx(t) = 0$$

$$\ddot{x}(t) + \omega^2 x(t) = 0 \qquad \omega = \sqrt{\frac{k}{m}}$$

- General solution

$$x(t) = A\cos\omega t + B\sin\omega t$$



Damped Harmonic Oscillator

- Consider damping in equation

$$m\ddot{x}(t) + \gamma\dot{x}(t) + kx(t) = 0$$

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- General solution (underdamped system)

$$x(t)=e^{-\omega_r t}\left(A\cos\overline{\omega}t+B\sin\overline{\omega}t\right)$$

$$\omega_r=\frac{\gamma}{2m}$$

$$\overline{\omega}=\sqrt{\omega^2-\omega_r^2}$$

$$\omega=\sqrt{\frac{k}{m}}$$

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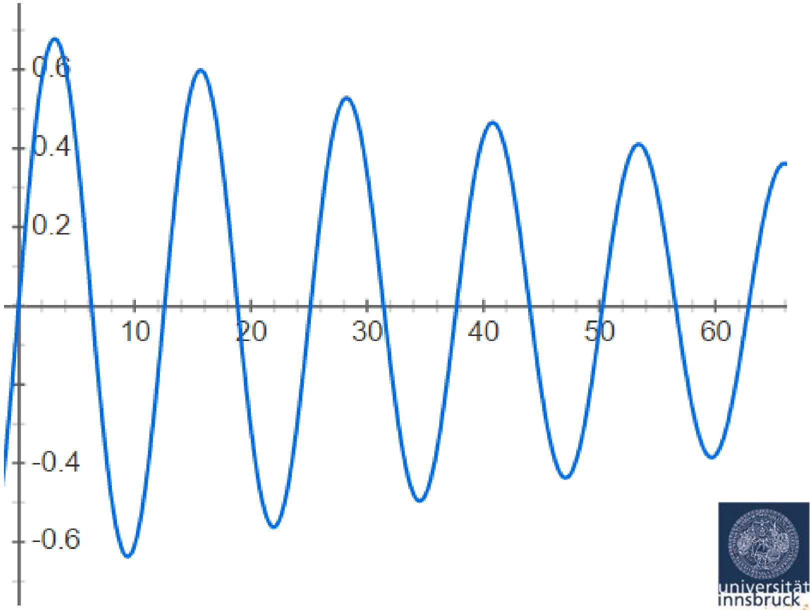

Damped Harmonic Oscillator

- Consider damping in equation

$$m\ddot{x}(t) + \gamma\dot{x}(t) + kx(t) = 0$$

- General solution (underdamped system) $\gamma < 2\sqrt{mk}$

$$x(t) = e^{-\omega_r t}\left(A\cos\overline{\omega}t + B\sin\overline{\omega}t\right)$$

$$\omega_r = \frac{\gamma}{2m}$$

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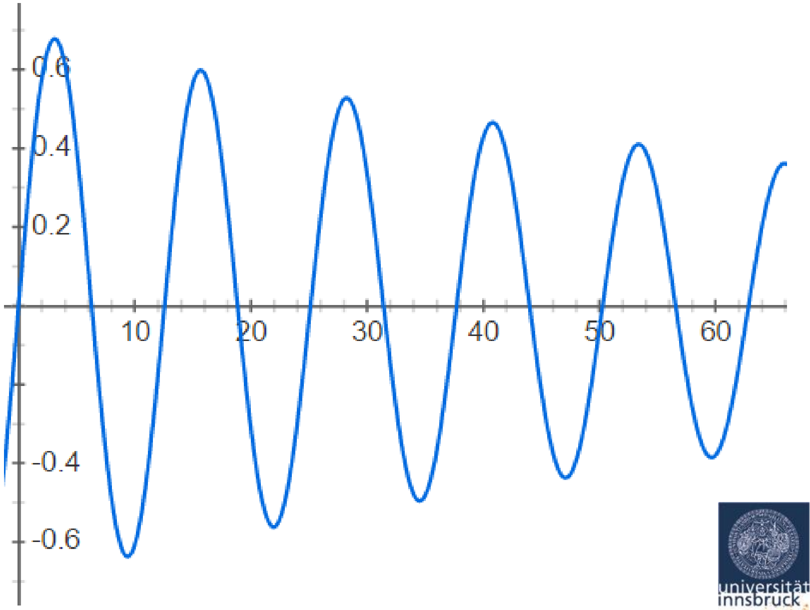

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Damped Harmonic Oscillator

- Consider damping in equation

$$m\ddot{x}(t) + \gamma\dot{x}(t) + kx(t) = 0$$

$$\ddot{x}(t) + 2\omega_r \dot{x}(t) + \omega^2 x(t) = 0 \qquad \omega = \sqrt{\frac{k}{m}} \qquad \omega_r = \frac{\gamma}{2m}$$

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- Assume solution

$$x(t)=e^{\alpha t}$$

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- Assume solution

$$x(t)=e^{\alpha t}$$

$$\dot{x}(t)=\alpha e^{\alpha t}$$

$$\ddot{x}(t)=\alpha^2e^{\alpha t}$$

Damped Harmonic Oscillator

- Insert into equation

$$\ddot{x}(t) + 2\omega_r \dot{x}(t) + \omega^2 x(t) = 0 \qquad \omega = \sqrt{\frac{k}{m}} \qquad \omega_r = \frac{\gamma}{2m}$$

Damped Harmonic Oscillator

- Insert into equation

$$\ddot{x}(t)+2\omega_r\dot{x}(t)+\omega^2 x(t)=0 \qquad \omega=\sqrt{\frac{k}{m}} \qquad \omega_r=\frac{\gamma}{2m}$$

$$\alpha^2 e^{\alpha t}+2\omega_r\alpha e^{\alpha t}+\omega^2 e^{\alpha t}=0$$

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$$e^{\alpha t}\left(\alpha^2 + 2\omega_r \alpha + \omega^2\right) = 0$$

$$\Leftrightarrow \alpha^2 + 2\omega_r \alpha + \omega^2 = 0$$

$$x^2 + px + q = 0$$

$$x = -\frac{p}{2} \pm \sqrt{\frac{p^2}{4} - q}$$

Damped Harmonic Oscillator

- Insert into equation

$$\ddot{x}(t) + 2\omega_r \dot{x}(t) + \omega^2 x(t) = 0 \qquad \omega = \sqrt{\frac{k}{m}} \qquad \omega_r = \frac{\gamma}{2m}$$

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$$\alpha=\omega_r\pm\sqrt{\omega_r^2-\omega^2}$$

- Underdamped case

$$\omega_r^2<\omega^2$$

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- Insert into equation

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- Underdamped case

$$\omega_r^2 < \omega^2 \qquad \left(\frac{\gamma}{2m}\right)^2 < \frac{k}{m}$$

Damped Harmonic Oscillator

- Insert into equation

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- Underdamped case

$$\omega_r^2<\omega^2 \qquad \gamma^2<\frac{4m^2k}{m}$$

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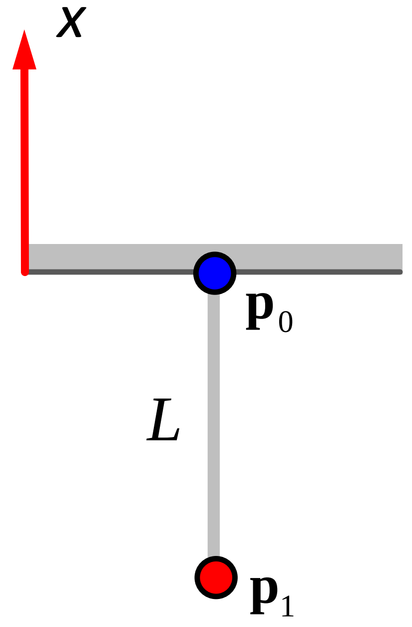
Oscillator With Hanging Mass

- Extend solution with gravity forces and initial length

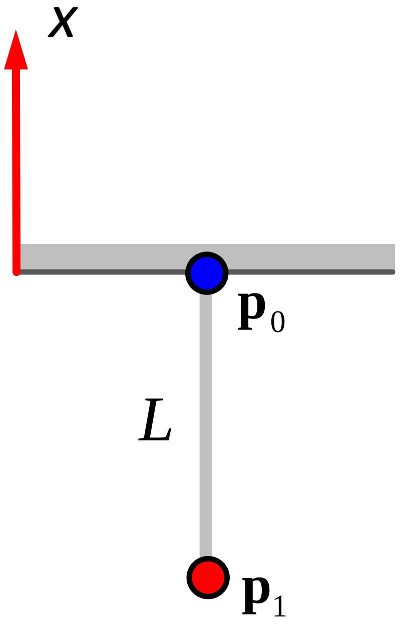


$$x(0) = -L$$

Oscillator With Hanging Mass

- Extend solution with gravity forces and initial length

$$x(t) = e^{-\omega_r t}\left(A\cos\overline{\omega}t + B\sin\overline{\omega}t\right) - \frac{mg}{k} - L$$



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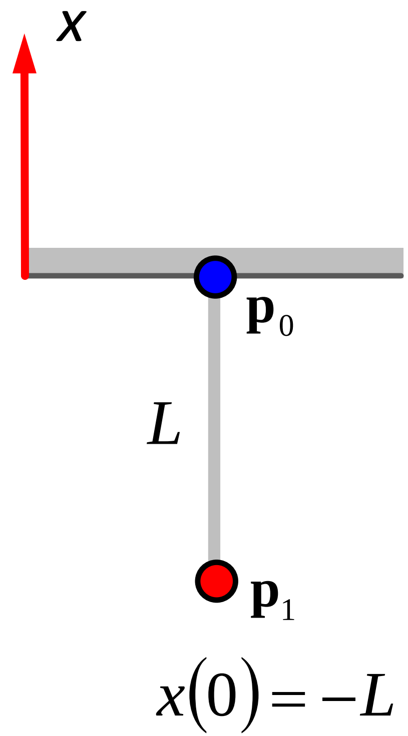

- Determine parameters A and B based on initial values

$$x(0) = -L$$

$$\dot{x}(0) = 0$$

Oscillator With Hanging Mass

- Extend solution with gravity forces and initial length

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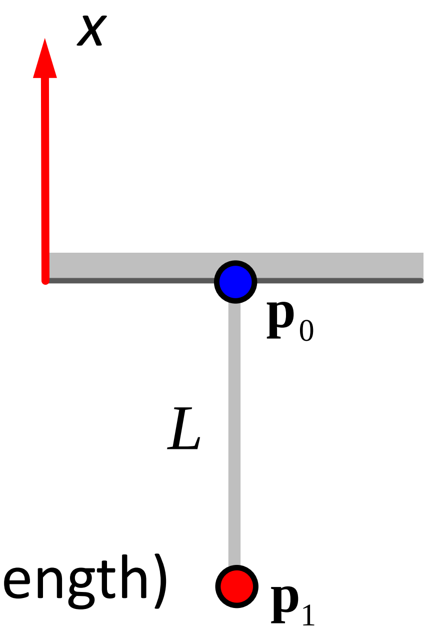

- Determine parameters A and B based on initial values

$x(0) = -L$ No elastic energy (spring at rest length)

$\dot{x}(0) = 0$ No kinetic energy (mass at rest)

$$x(0) = -L$$

Proseminar Schedule

Date	Topic	Remark
5.10.	Introduction	Programming assignment 1
12.10.	Programming support and advice	
19.10.	Programming support and advice	
26.10	*(No Proseminar)*	*Hand-in PA1*
2.11	*(No Proseminar)*	
9.11.	Presentation of solutions	Programming assignment 2
16.11.	Programming support and advice	
23.11.	Programming support and advice	
30.11.	Presentation of solutions	Programming assignment 3, *Hand-in PA2*
7.12.	Programming support and advice	
14.12.	Programming support and advice	
	Christmas break	
11.1.	Presentation of solutions	*Project proposal, Hand-in PA3*
18.1.	Programming support and advice	
25.1.	Programming support and advice	
1.2.	Project presentations	*Submission final project*