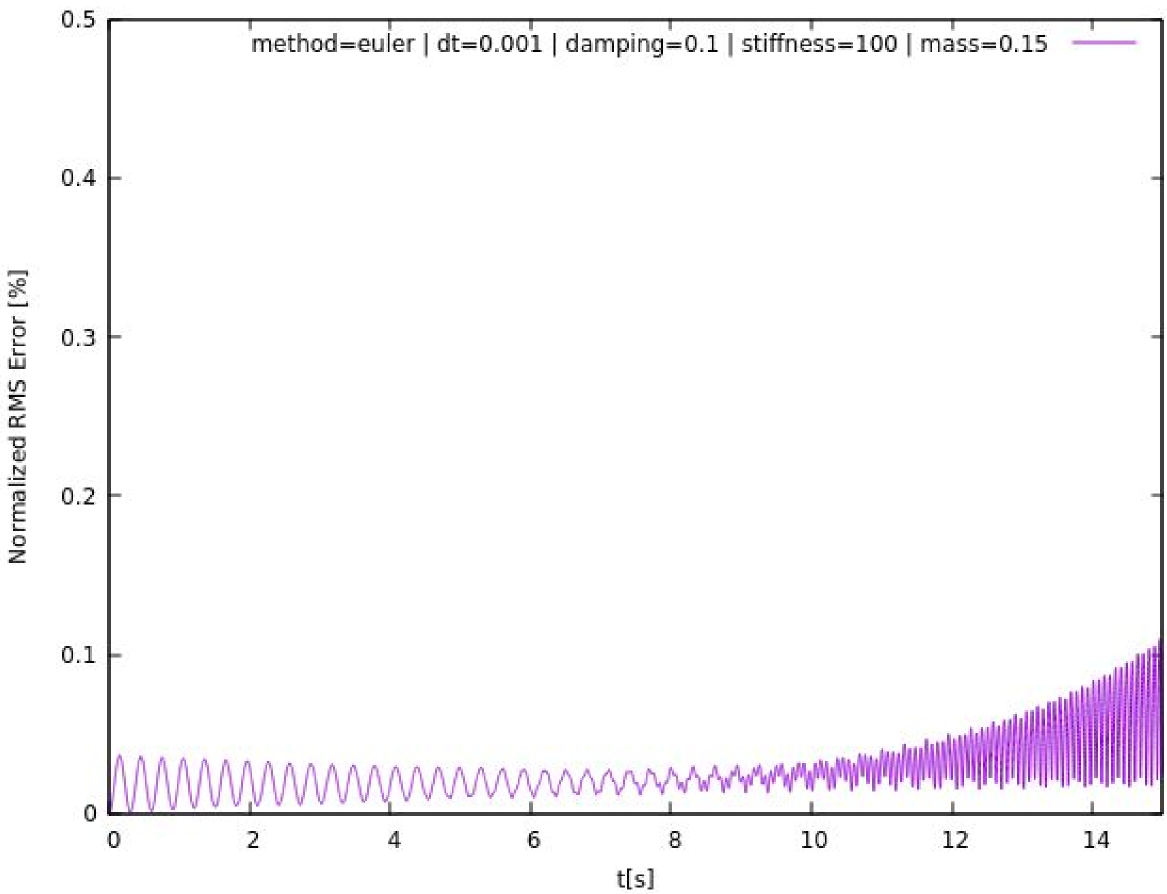
Report

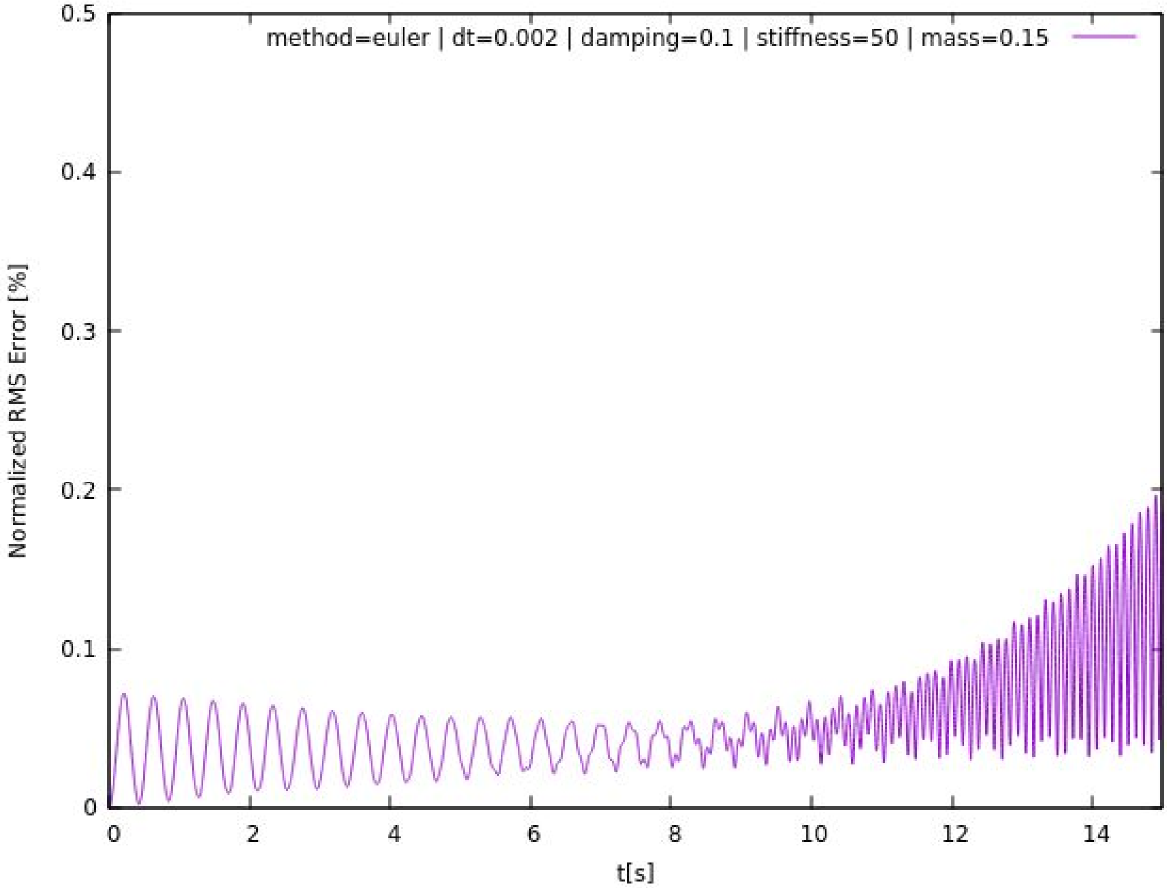
Comparison numerical solution to exact analytical solution.
We figured out that the stiffness and the step size are the most crucial parameter for the behavior of the spring. The experiments containing the borders of instability for each Method.

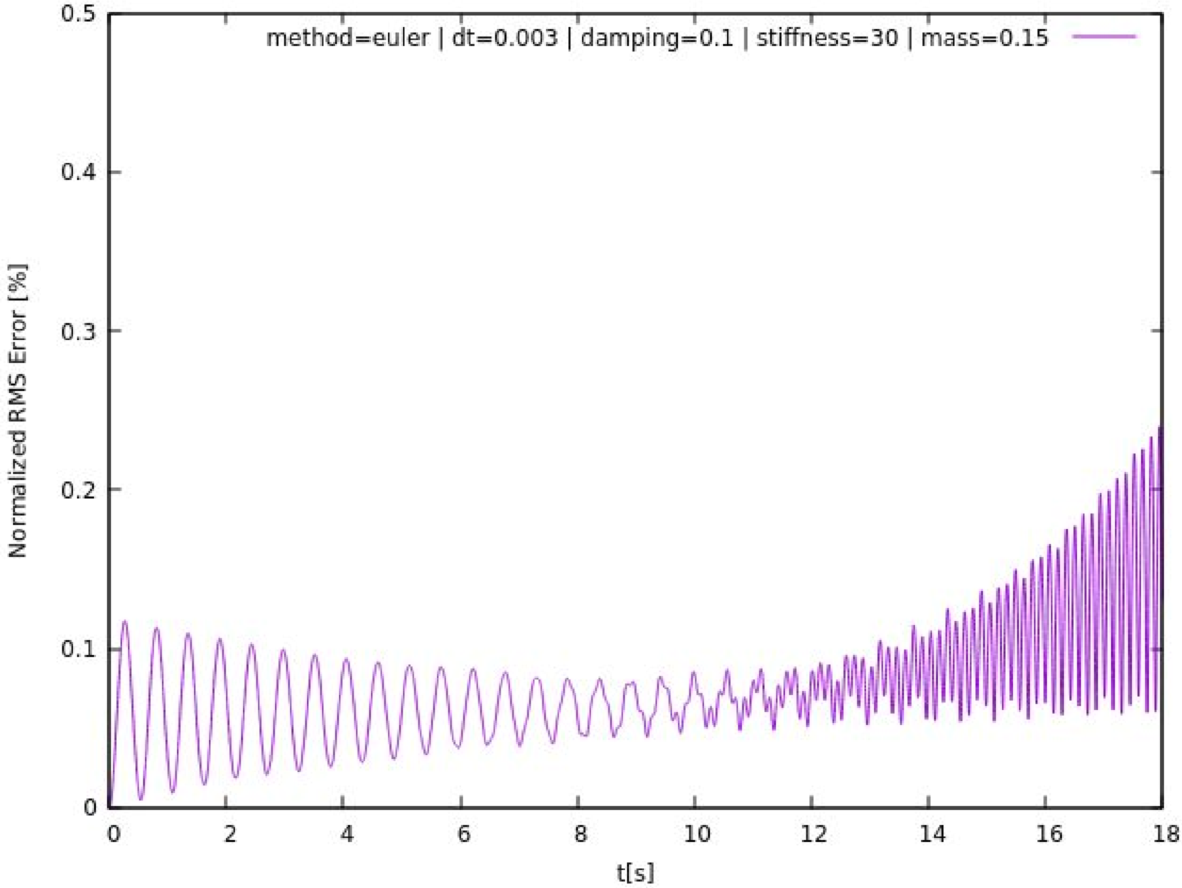
Euler:

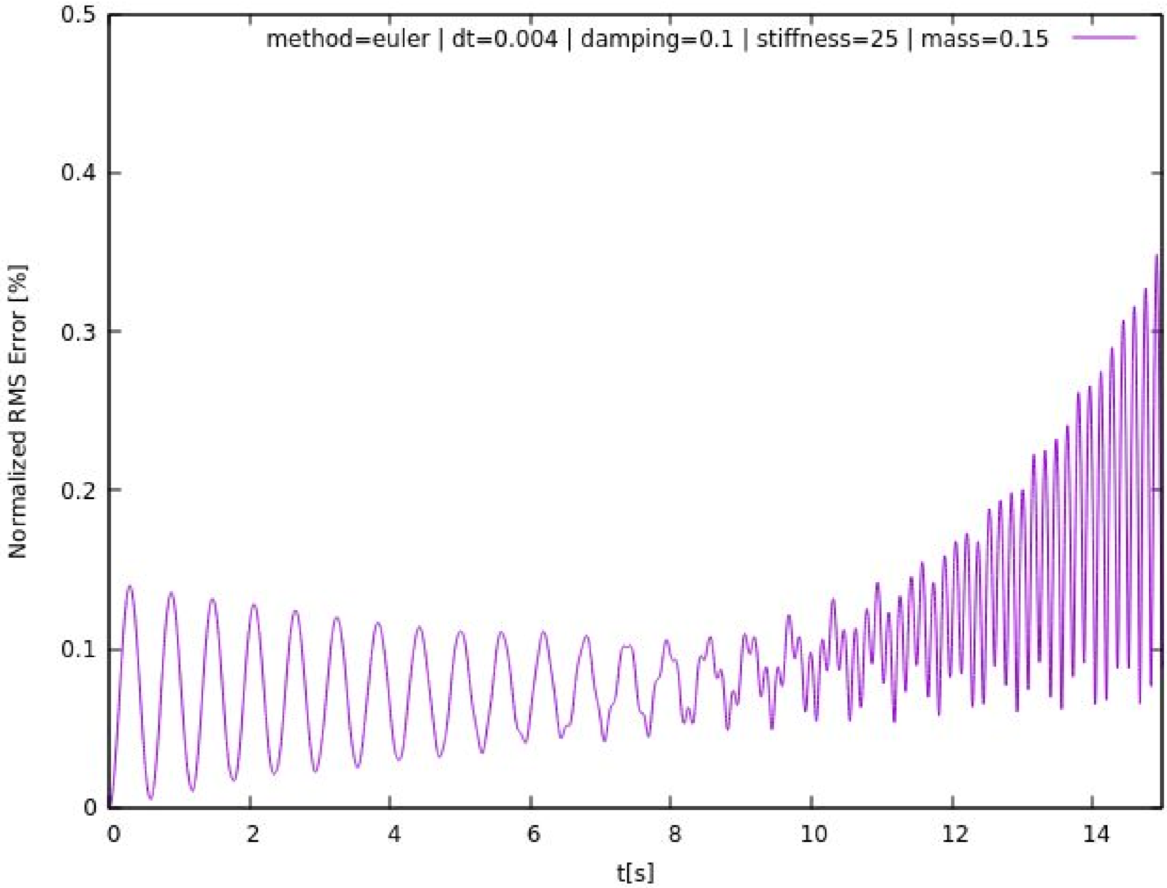
For the Euler method the parameters have to be very small to have a stable spring-mass-system, especially the step size.

	100	50	30	25
0.001	unstable	stable	stable	stable
0.002	unstable	unstable	stable	stable
0.003	unstable	unstable	unstable	stable
0.004	unstable	unstable	unstable	unstable




method=euler | dt=0.002 | damping=0.1 | stiffness=50 | mass=0.15
Normalized RMS Error [%]
t[s]
0
0.1
0.2
0.3
0.4
0.5
0
2
4
6
8
10
12
14

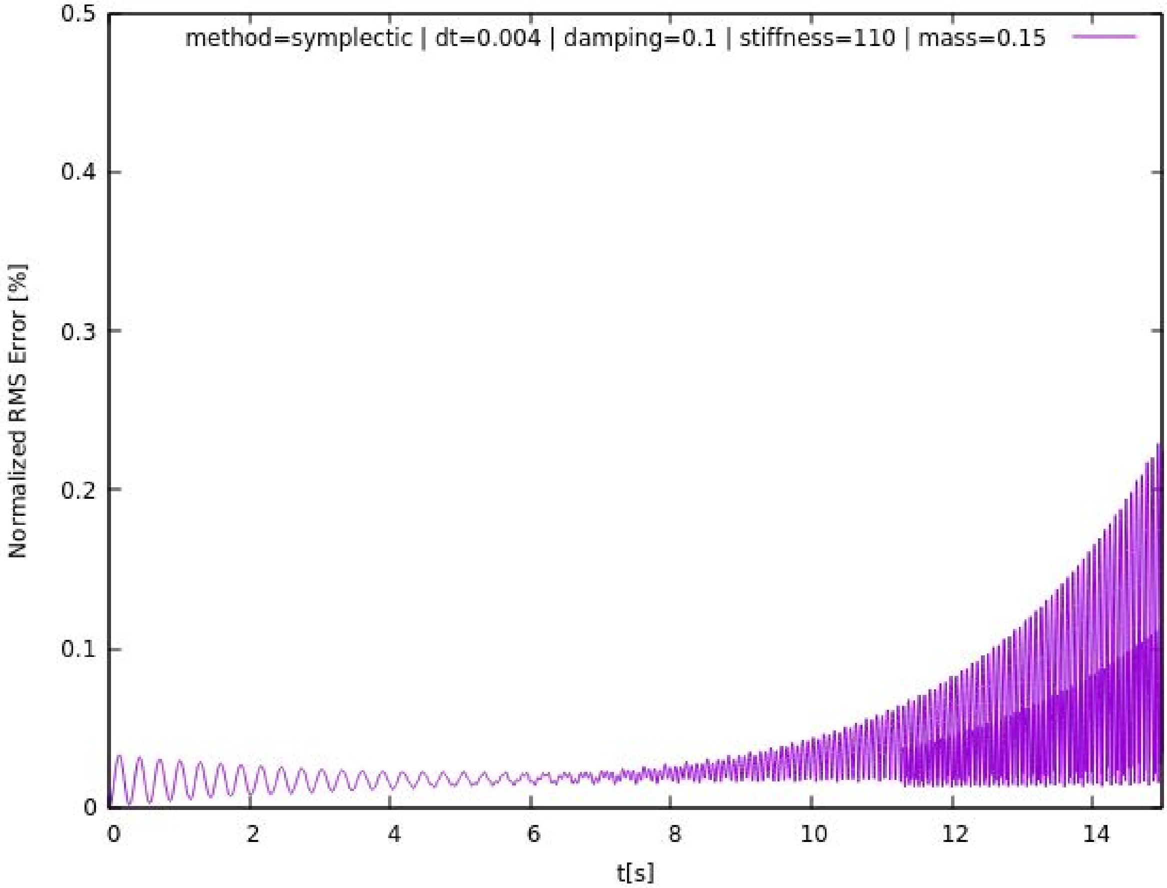

method=euler | dt=0.003 | damping=0.1 | stiffness=30 | mass=0.15
Normalized RMS Error [%]
t[s]
0
0.1
0.2
0.3
0.4
0.5
0
2
4
6
8
10
12
14
16
18

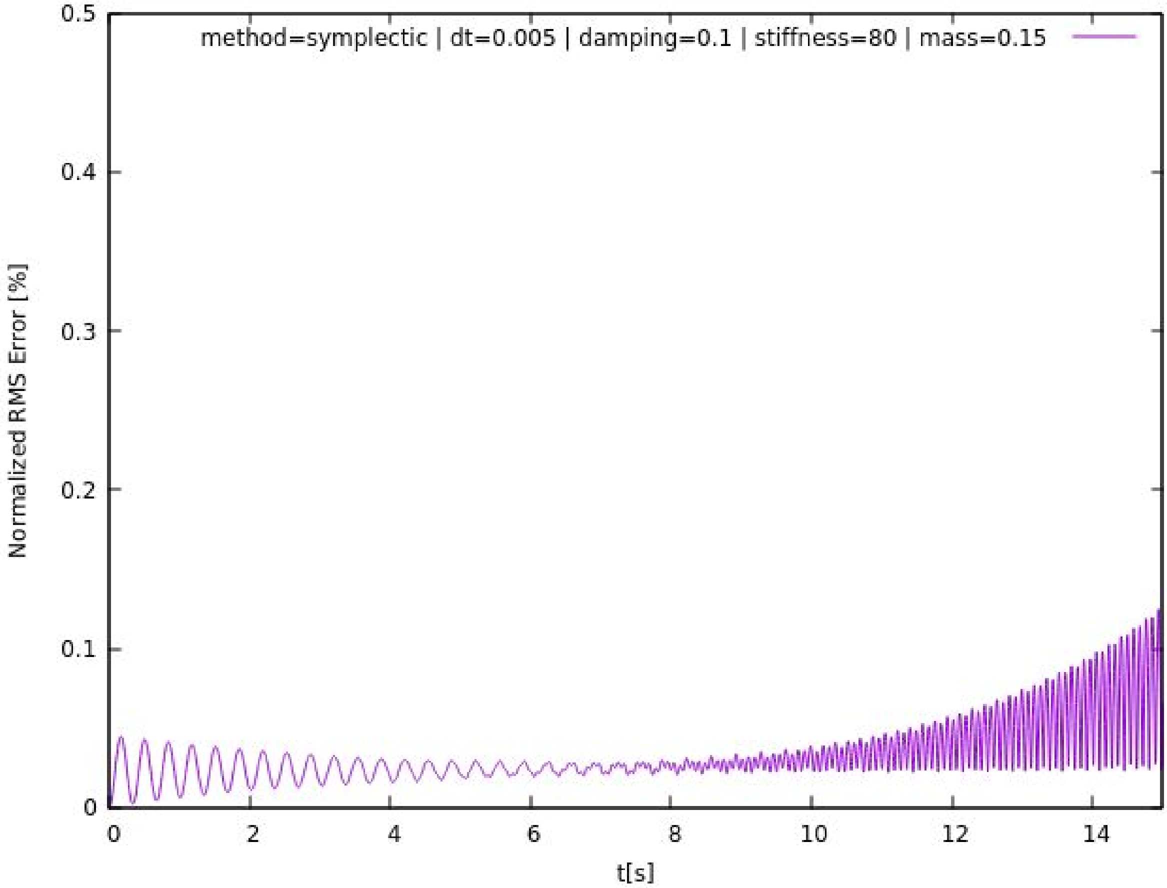


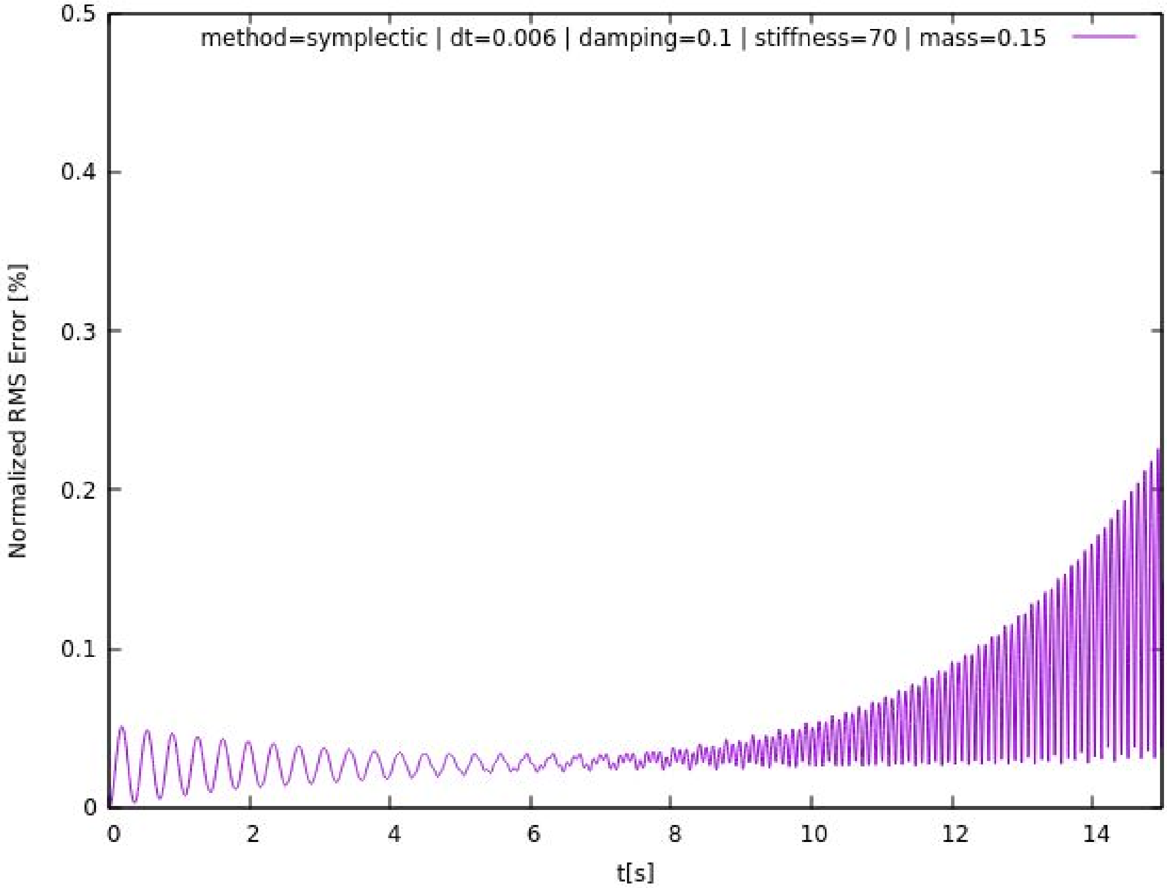
Symplectic:

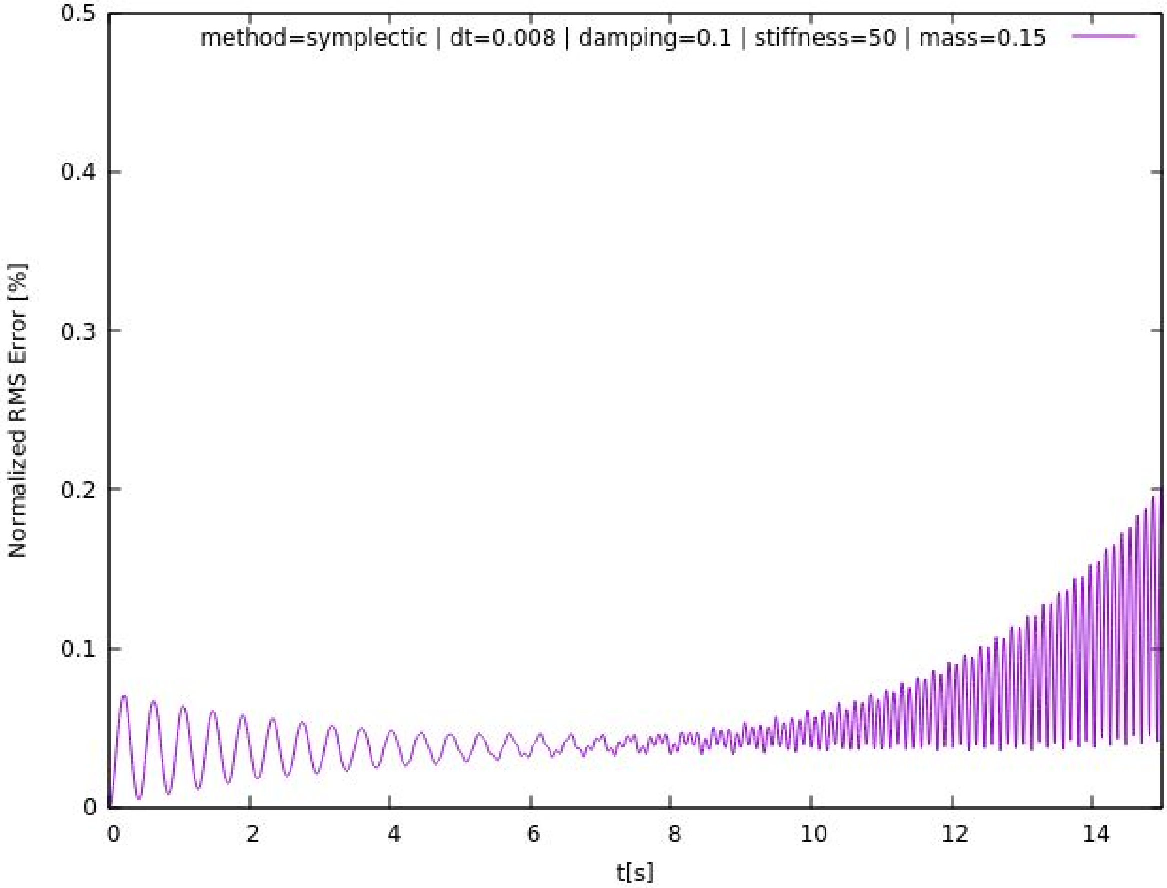
Similar to the Euler method the symplectic method is unstable for relatively small parameters.

	110	80	70	50
0.004	unstable	stable	stable	stable
0.005	unstable	unstable	stable	stable
0.006	unstable	unstable	unstable	stable
0.008	unstable	unstable	unstable	unstable


method=symplectic | dt=0.004 | damping=0.1 | stiffness=110 | mass=0.15
Normalized RMS Error [%]
0.5
0.4
0.3
0.2
0.1
0
0
2
4
6
8
10
12
14
t[s]


method=symplectic | dt=0.005 | damping=0.1 | stiffness=80 | mass=0.15
Normalized RMS Error [%]
0.5
0.4
0.3
0.2
0.1
0
0
2
4
6
8
10
12
14
t[s]

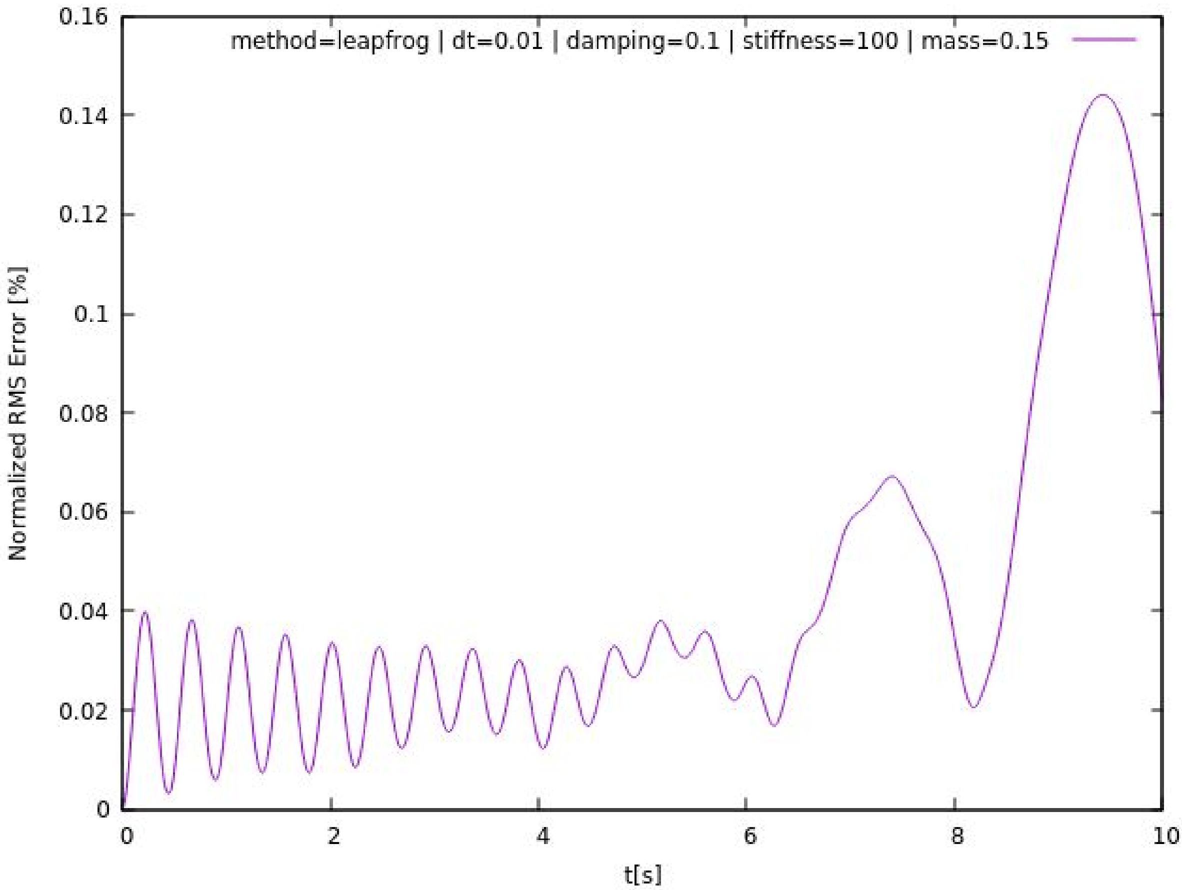

method=symplectic | dt=0.006 | damping=0.1 | stiffness=70 | mass=0.15
Normalized RMS Error [%]
0.5
0.4
0.3
0.2
0.1
0
0
2
4
6
8
10
12
14
t[s]

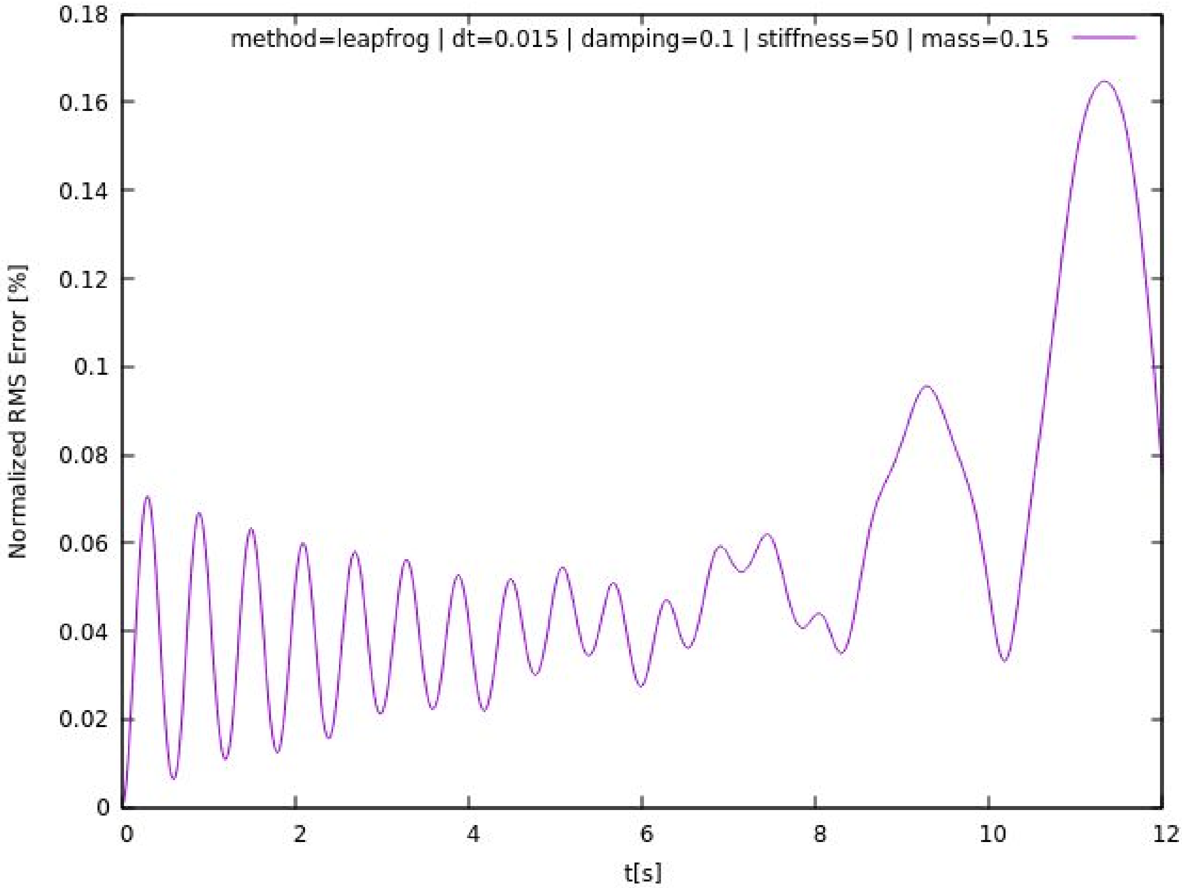

method=symplectic | dt=0.008 | damping=0.1 | stiffness=50 | mass=0.15
Normalized RMS Error [%]
0.5
0.4
0.3
0.2
0.1
0
0
2
4
6
8
10
12
14
t[s]

Leapfrog:

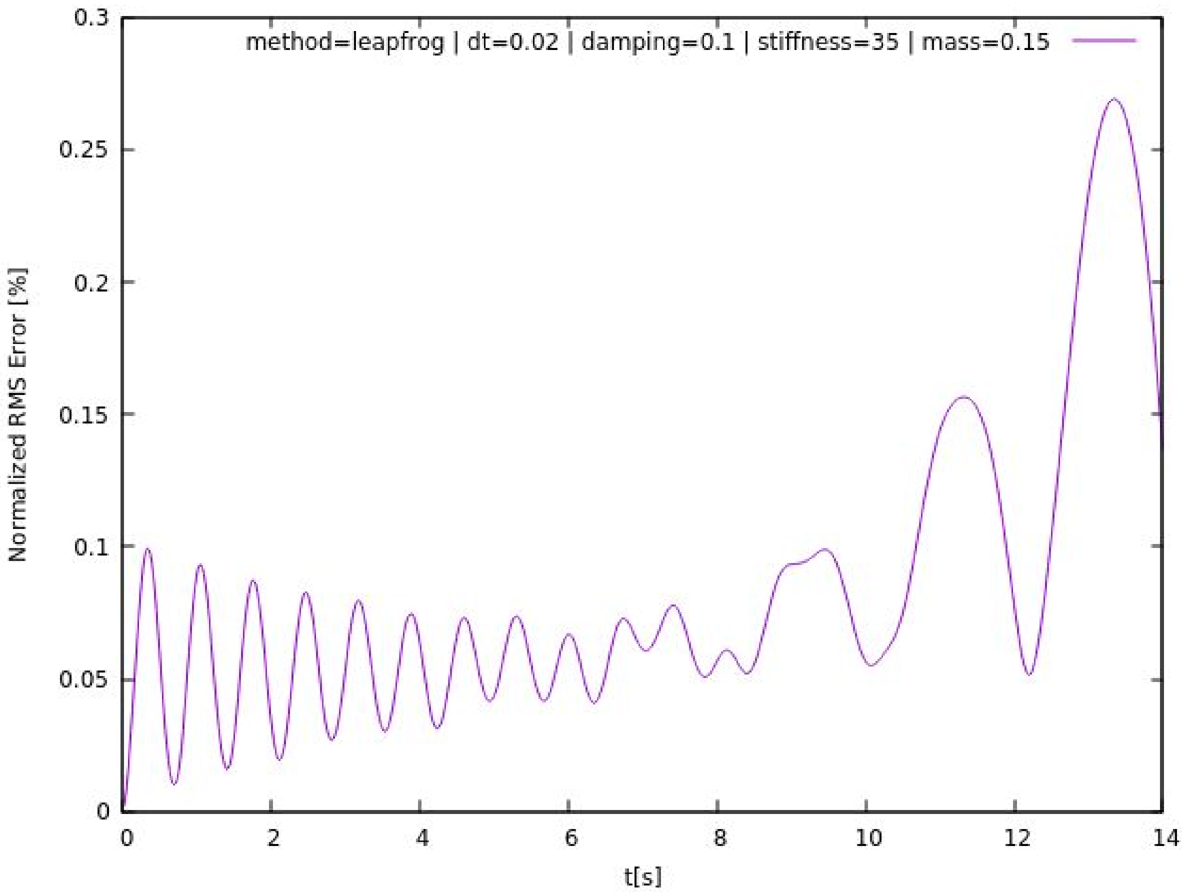
The Leapfrog is pretty stable also at high parameters, interesting is the way the spring behaves when it gets instable. It's also visible in the plot of the RMS Error.

	100	50	35	25
0.01	unstable	stable	stable	stable
0.015	unstable	unstable	stable	stable
0.02	unstable	unstable	unstable	stable





method=leapfrog | dt=0.015 | damping=0.1 | stiffness=50 | mass=0.15
Normalized RMS Error [%]
t[s]
0
0.02
0.04
0.06
0.08
0.1
0.12
0.14
0.16
0.18
0
2
4
6
8
10
12

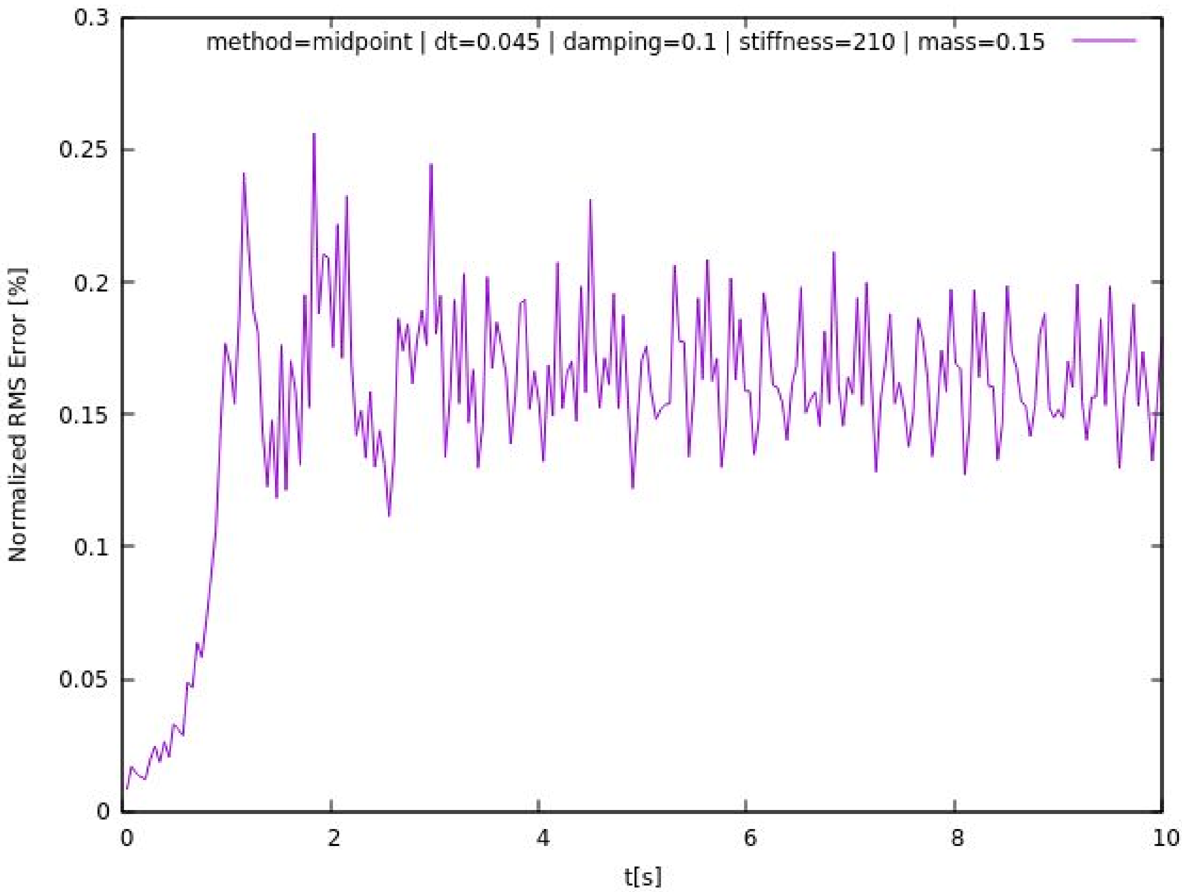


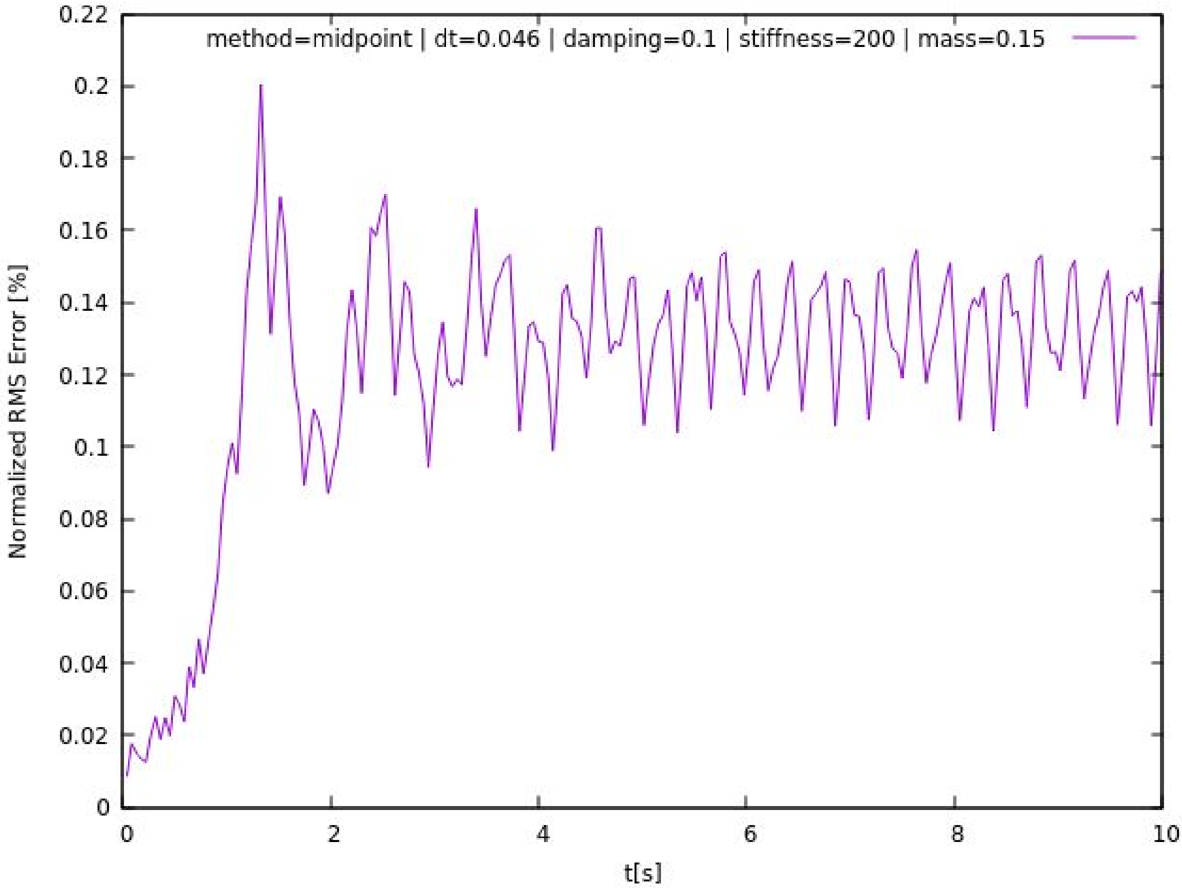
method=leapfrog | dt=0.02 | damping=0.1 | stiffness=35 | mass=0.15
Normalized RMS Error [%]
t[s]
0
0.05
0.1
0.15
0.2
0.25
0.3
0
2
4
6
8
10
12
14

Midpoint:

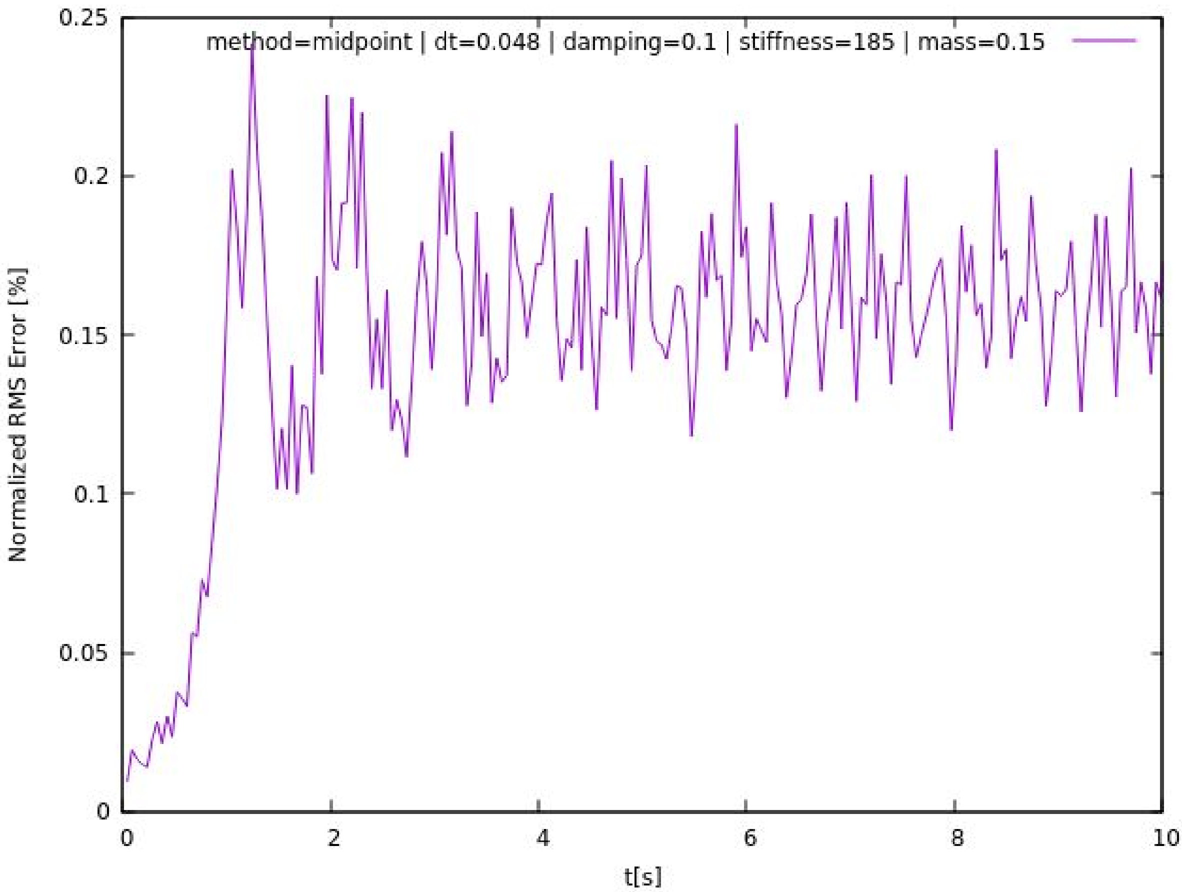
The midpoint method had the best results and it was pretty hard to find the instability threshold. Until the step size of 0.045 this method was almost “unbreakable”. The midpoint method don’t crash like the other methods after a few seconds, it crashes immediately after 1 second. With a step size of 0.048 and a stiffness of 190 the spring mass system goes totally crazy just after 1 second.

	210	200	185	100	90
0.045	unstable	stable	stable	stable	stable
0.046	unstable	unstable	stable	stable	stable
0.048	unstable	unstable	unstable	stable	stable
0.065	unstable	unstable	unstable	unstable	stable

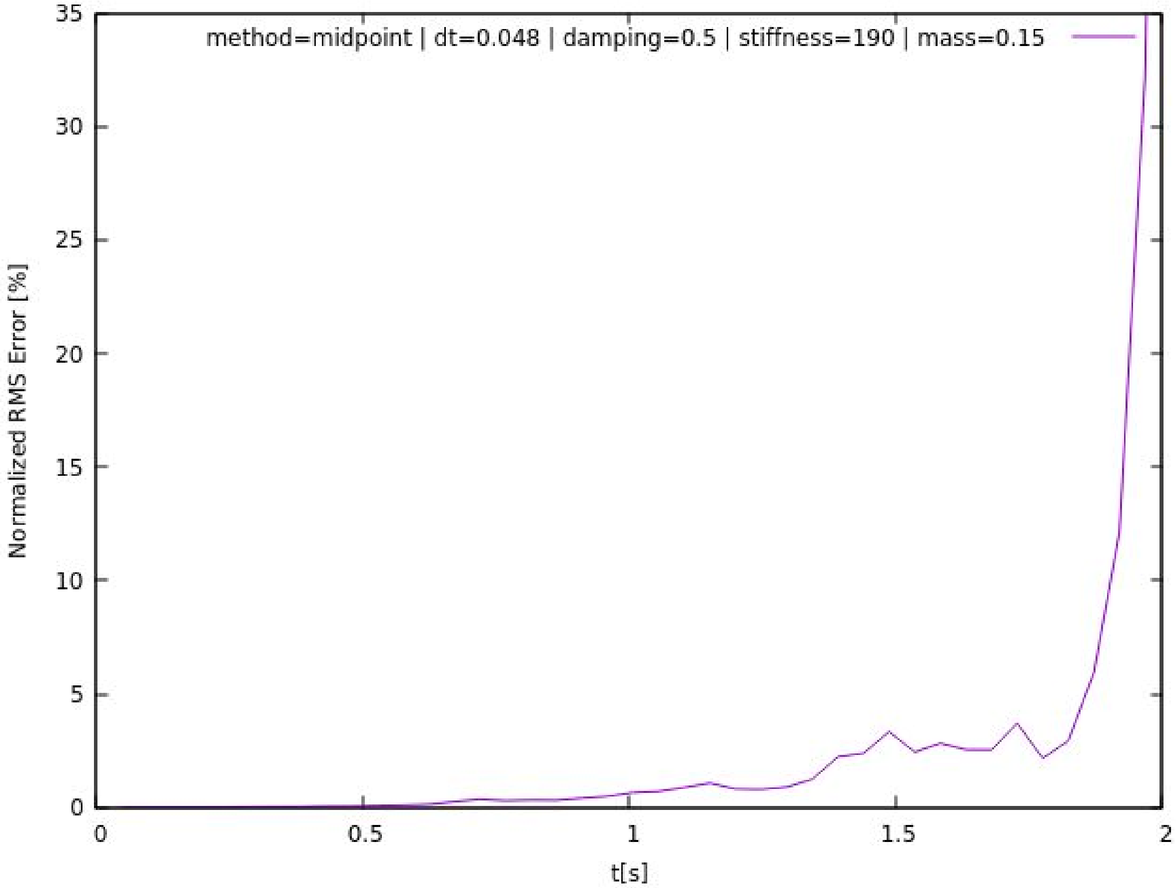


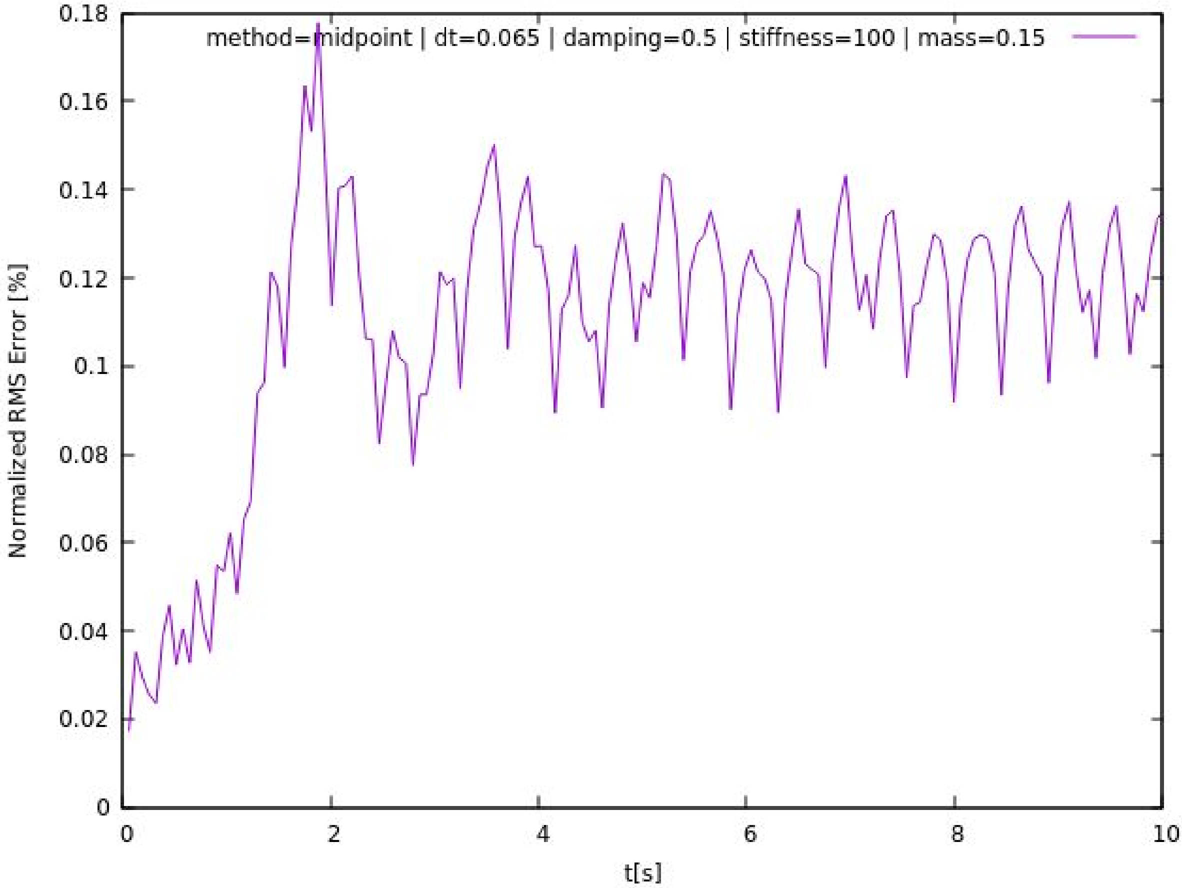


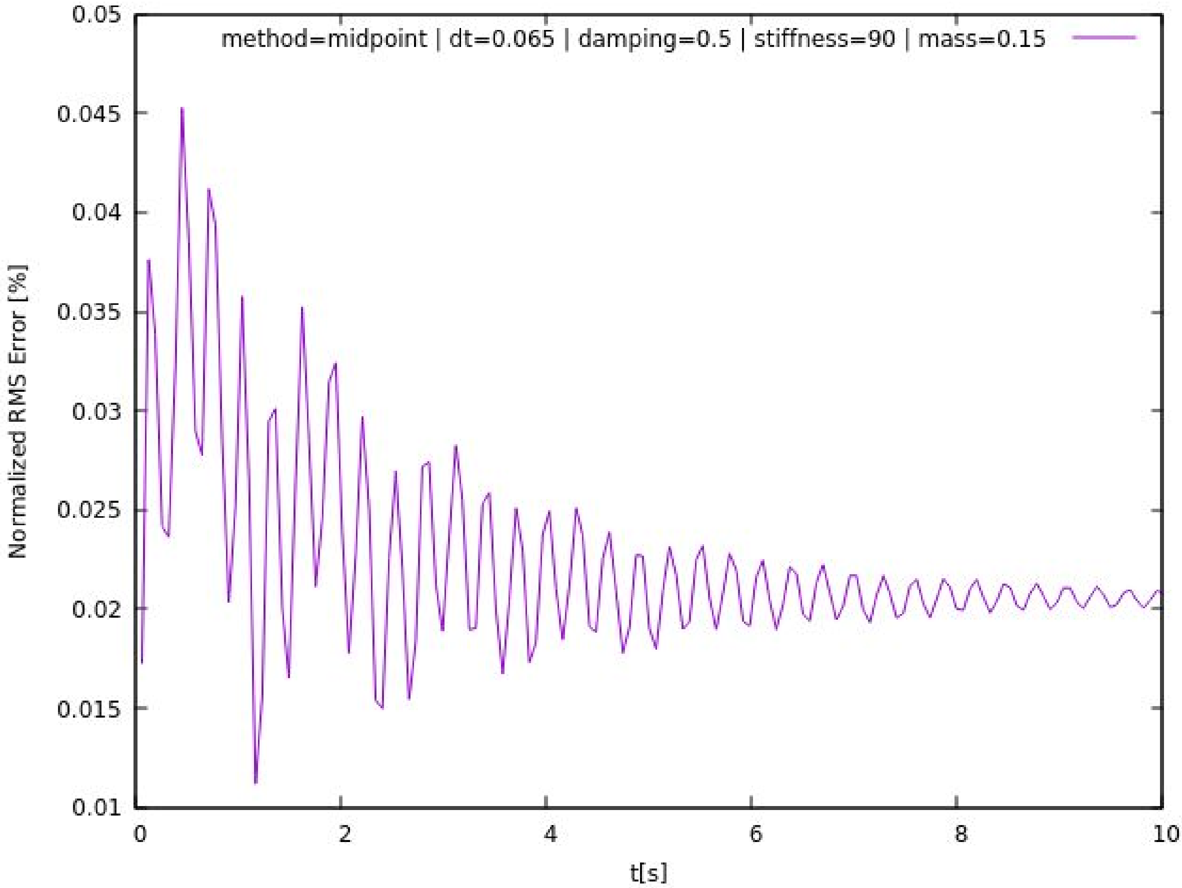
method=midpoint | dt=0.046 | damping=0.1 | stiffness=200 | mass=0.15
Normalized RMS Error [%]
t[s]
0.22
0.2
0.18
0.16
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
2
4
6
8
10



method=midpoint | dt=0.048 | damping=0.1 | stiffness=185 | mass=0.15
Normalized RMS Error [%]
t[s]
0.25
0.2
0.15
0.1
0.05
0
2
4
6
8
10


method=midpoint | dt=0.048 | damping=0.5 | stiffness=190 | mass=0.15
Normalized RMS Error [%]
t[s]
0
5
10
15
20
25
30
35
0.5
1
1.5
2


method=midpoint | dt=0.065 | damping=0.5 | stiffness=100 | mass=0.15
Normalized RMS Error [%]
t[s]
0
0.02
0.04
0.06
0.08
0.1
0.12
0.14
0.16
0.18
2
4
6
8
10



method=midpoint | dt=0.065 | damping=0.5 | stiffness=90 | mass=0.15
Normalized RMS Error [%]
0.05
0.045
0.04
0.035
0.03
0.025
0.02
0.015
0.01
0
2
4
6
8
10
t[s]