

# Human Involvement in the Ecosystem

Logan Basara  
Shirley Chen  
Harjot Grewal  
Belina Jang

University of Victoria  
Department of Mathematics and Statistics

# Table of Contents

<b>TABLE OF CONTENTS</b>	<b>2</b>
<b>ABSTRACT</b>	<b>3</b>
<b>1 INTRODUCTION</b>	<b>3</b>
<b>2 BACKGROUND</b>	<b>3</b>
<b>3 MODEL DERIVATION</b>	<b>4</b>
3.1 ASSUMPTIONS	4
3.2 SYSTEM OF DIFFERENTIAL EQUATIONS	4
3.3 EXPLANATION OF SYSTEM	5
<b>4 MODEL ANALYSIS</b>	<b>5</b>
4.1 MODEL CONSTRUCTIONS	5
4.2 DISCUSSION	6
4.2.1 <i>Scenario 1</i>	6
4.2.2 <i>Scenario 2</i>	7
4.2.3 <i>Scenario3</i>	7
4.2.4 <i>Scenario 4</i>	9
4.3 ASSESSMENT	12
4.3.1 <i>Strengths</i>	12
4.3.2 <i>Weaknesses</i>	12
<b>5 CONCLUSION</b>	<b>12</b>
<b>6 BIBLIOGRAPHY</b>	<b>15</b>
<b>7 APPENDIX – R CODE</b>	<b>15</b>

## Abstract

In most ecosystems in nature, multiple predators and prey can usually be found. Predators are species that feed on prey species that are typically smaller and weaker. The interactions of being eaten and hunting between these predators and prey derive changes in population distribution of the ecosystem. Ecological balance in this population distribution plays an important role in keeping the ecosystem stable so that no species overpopulate or die out. However, ecological imbalance occurs when there is severe apparent competition between species, or some change occurs. We will explore the effect of human intervention by using a system of differential equations in 4 different scenarios to model the ecosystem with different levels of human intervention for 2 predators and 2 preys' ecosystem to analyze the effect of human intervention in ecosystems then explain the need for human interventions in various ecosystems. We concluded human involvement in ecosystem is essential to keep the ecosystem balanced.

## 1 Introduction

Ecological balance is an important condition for the whole biosphere to maintain a normal life support system. It also provides suitable environmental conditions and stable natural resources for human beings. However, some species may surge or die out because of a broken biological chain or apparent competition between species. Therefore, we build this model to determine if human involvements are needed to protect/control species and to keep the balance of the ecosystem.

We derive a two predators- two prey model inspired by an article by Brodie Farquhar entitled "Wolf Reintroduction Changes Ecosystem in Yellowstone". This model allows us to predict how the ecosystem will behave with or without human involvement such as hunting or adding or taking species out of the population. The ecosystem in our model consists of predators (wolves, bears), prey (elk, bison) and other food sources (grass). They interact with each other in the way of eating/ being eaten and competition.

Our goal is to use ordinary differential equations to show whether human involvement is important to maintain an ecosystem or not. We change the variables in those ODEs to modify the existing predator-prey model. And we plot them in R to observe the population growth differences and to make comparisons between species interaction/ no interaction, as well as human intervention/ no human intervention. For the sake of applying this model with credibility, real data in Yellowstone National Park is used.

## 2 Background

The predator-prey model is a useful mathematical model describing how predators and prey interact dynamically. There are a few assumptions in this model. First, if the predator doesn't exist, the number of preys will increase exponentially. Secondly, the predators will starve and die when they cannot find enough prey. We also introduced the concept of carrying capacity in our model, which refers to the maximum populations of predators and prey that can live in an ecosystem.

In the Yellowstone Ecosystem, the wolves play an important role not only as the predator but also as the food distributor. Back to the 1930s, there was a rapid increase in elk populations after the wolves were killed off in Yellowstone (Brodie, 2021). And the decrease in the population of wolves put severe stress on other species in the ecosystem, such as beaver. To keep the ecosystem balanced, the wolves were reintroduced in Yellowstone by human beings, which not only contributed to the increase in beaver populations but also benefited other scavengers like bears. The reintroduction of wolves inspires us to realize how important human involvement is to maintain an ecosystem.

### 3 Model Derivation

#### 3.1 Assumptions

- Assumes only 2 predator and 2 prey animals in ecosystem -main assumption in model
- Limited interaction between prey and predators -only interactions are as defined
- No encounter rate, density, or rate of depletion
- Perfect Constants -No fluctuations, faults, or failures

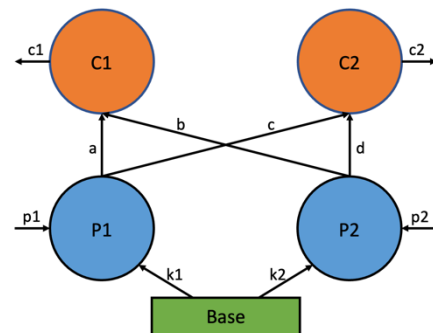
#### 3.2 System of Differential Equations

$$P1' = p1 * P1 (1 - (P1/k1)) - (a * C1 + c * C2) - h1 * P1$$

$$P2' = p2 * P2 (1 - P2/k2) - (b * C1 + d * C2) - h2 * P2$$

$$C1' = -C1 * c1 + (a * P1 + c * P2) - h3 * C1$$

$$C2' = -C2 * c2 + (b * P1 + d * P2) - h4 * C2$$



Variable	Description
P1	Initial population of prey – Elk
P2	Initial population of prey – Bison
C1	Initial population of predator – Wolf
C2	Initial population of predator – Bear
p1	Growth rate of Elk
p2	Growth rate of Bison
c1	Death rate of Wolf
c2	Death rate of Bear
k1	Carrying capacity of Elk
k2	Carrying capacity of Bison
a	Interaction term between Wolf and Elk
b	Interaction term between Wolf and Bison
c	Interaction term between Bear and Elk

d	Interaction term between Bear and Bison
h1	Human interaction with Elk
h2	Human interaction with Bison
h3	Human interaction with Wolf
h4	Human interaction with Bear

### 3.3 Explanation of System

The population of the prey animals being dependant on the population of the previous time interval less the carrying capacity, rate at which they are eaten by the predators, and potential human hunting/culling. The population of the predators is likewise, dependant on the population in the previous time interval, the rate at which prey animals are eaten, the death rate, and human hunting/culling.

## 4 Model Analysis

### 4.1 Model constructions

We begin by defining the relevant parameters and variables to ease our subsequent exposition:

Variable	Description	Parameter (These numbers are from the Yellowstone census website)
P1	Initial population of prey – Elk	5349
P2	Initial population of prey – Bison	5500
C1	Initial population of predator – Wolf	108
C2	Initial population of predator – Bear	690
p1	Growth rate of Elk	0.24
p2	Growth rate of Bison	0.23
c1	Death rate of Wolf	0.13
c2	Death rate of Bear	0.10
k1	Carrying capacity of Elk	1000
k2	Carrying capacity of Bison	2000
a	Interaction term between Wolf and Elk	0.075
b	Interaction term between Wolf and Bison	0.035
c	Interaction term between Bear and Elk	0.01
d	Interaction term between Bear and Bison	0.07
h1	Human interaction with Elk	Varying (independent variable)
h2	Human interaction with Bison	Varying (independent variable)
h3	Human interaction with Wolf	Varying (independent variable)
h4	Human interaction with Bear	Varying (independent variable)

## 4.2 Discussion

To reiterate for the readers convenience, our paper aims to capture the impact of human involvement and interaction on an ecosystem with multiple predator-prey relationship.

### 4.2.1 Scenario 1

We start by mentioning the importance of relationship between predator and prey to maintain an ecosystem is multifold as it limits certain species to grow out of bound and inhibits extinction of another species within the ecosystem.

From a plot below (Fig 1 and 2), we can see if there is no relationship between preys (Elk and Bison) and predators (Wolf and Bear) i.e., if no Elks and Bison are being eaten by Wolf and Bear, the prey population results in exponential growth and predator population results in exponential decline.

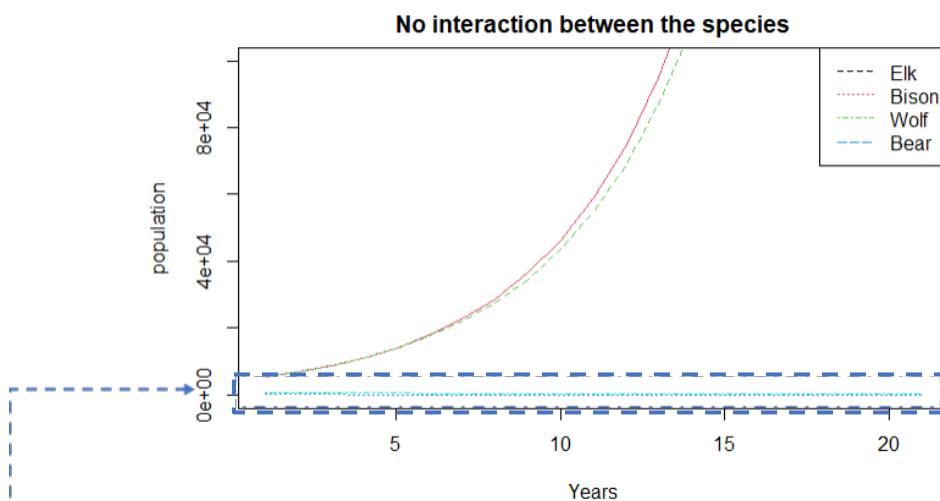


Figure 1: Plot showing no relationship between predator and prey

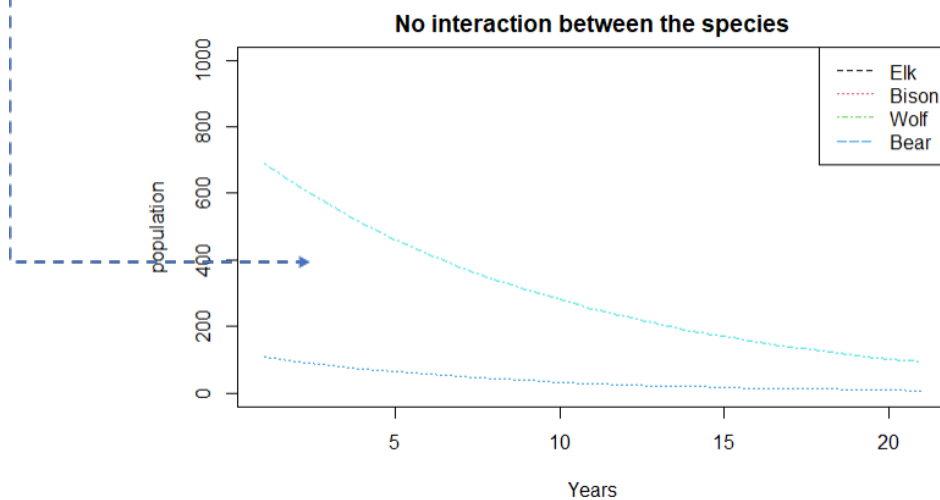


Figure 2: Plot zooming on figure 1 to show exponential decline in predator population

#### 4.2.2 Scenario 2

To control our prey population, we introduced the concept of carrying capacity of  $k_1$  and  $k_2$  respectively which leads to logistic growth of prey populations (Figure 3), however since there is no interaction between our prey and predator population, predator population will keep on declining exponentially. Increasing prey population and decline predator population could have negative impact on the ecosystem. It might become hard for humans to manage the needs of the remaining prey species.

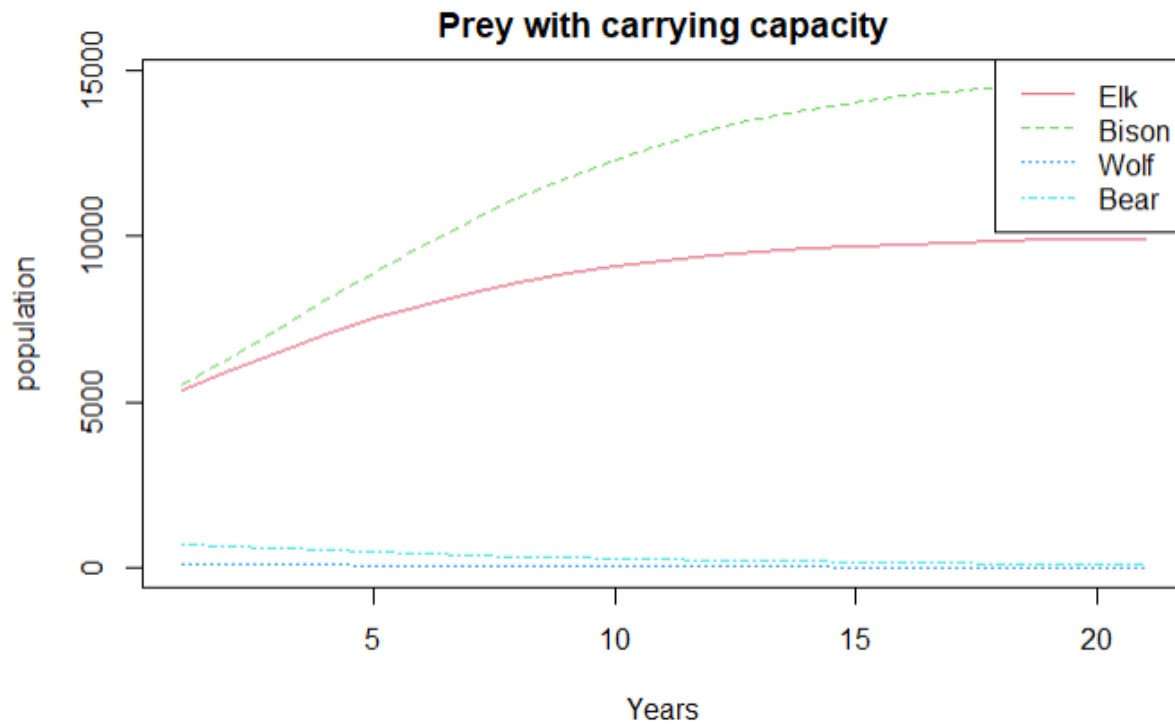


Figure 3: Plot showing effect of carrying capacity on prey populations.

#### 4.2.3 Scenario3

From the above plots, we can deduce that predator-prey relationship is very important in the survival of predator species and maintaining an ecosystem as the prey population might not increasing as rapidly but still increasing substantially. In order to study the impact of the relationship on population, we introduced our model where each predator feeds on each prey at different rate. We look at the change in population over the following years (10,20, 30). From the plots below, we can see with current growth rate of each species and interaction rate, the population is increasing for all the predators and preys for first 10 years. However, between 10 – 20 years, the Elk and Bear populations is interchanged as the elk population starts to decline while the Bear population starts to increase. If we look the following 10 years (20-30 years), we can see that the Bison population also starts to decline, and the predator population is increasing. This type of ecosystem might eventually collapse as the predator population overpowers the prey population. No prey population means eventually predator population will start to decline leading to ecosystem to cease to exist. It might also become hard for humans to manage the needs of the remaining population.

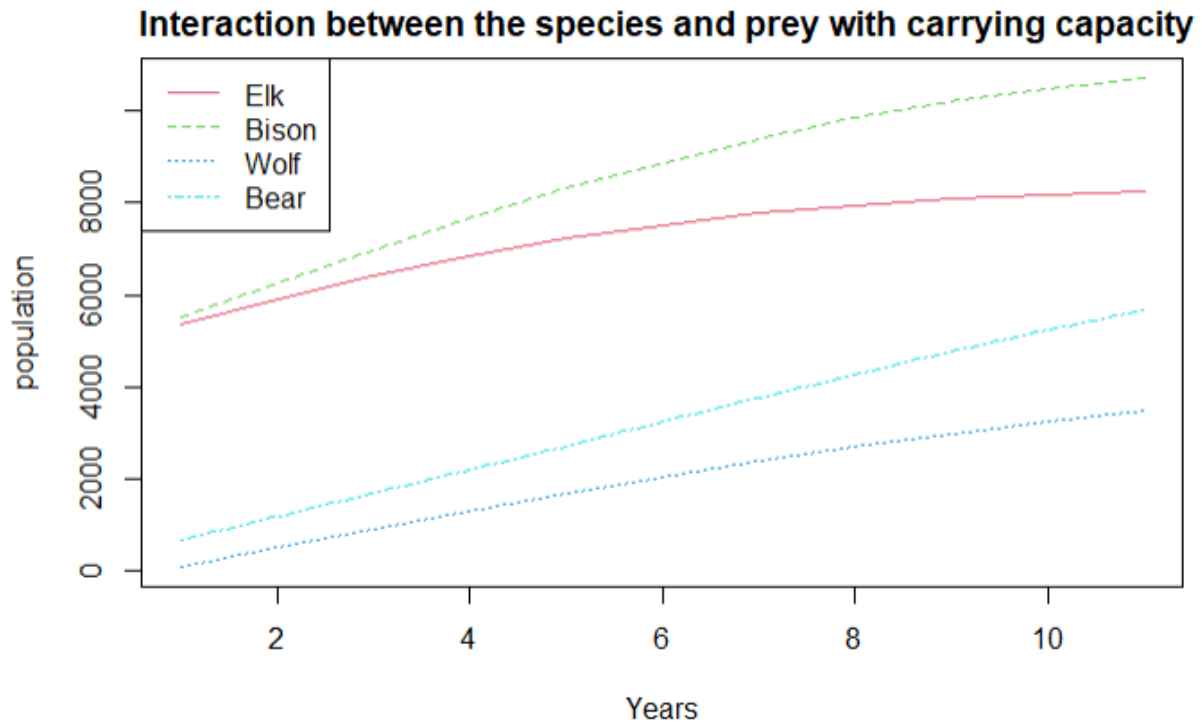


Figure 4: Effect of interaction between the prey and predator population in 10 years.

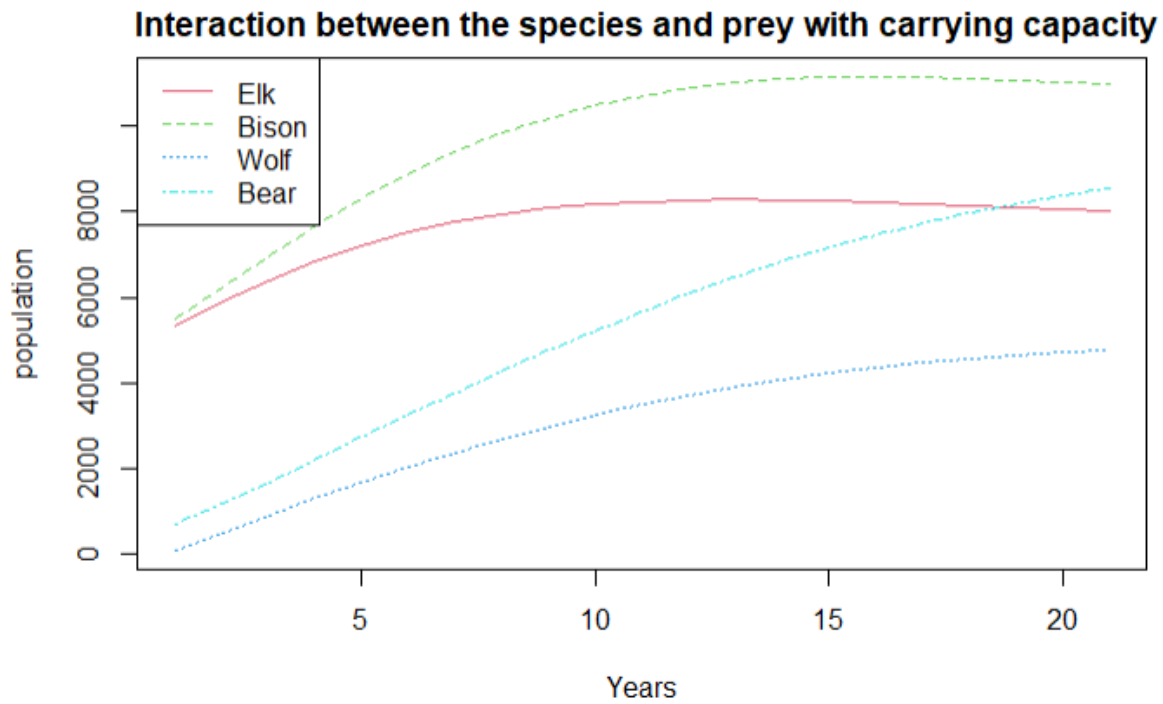
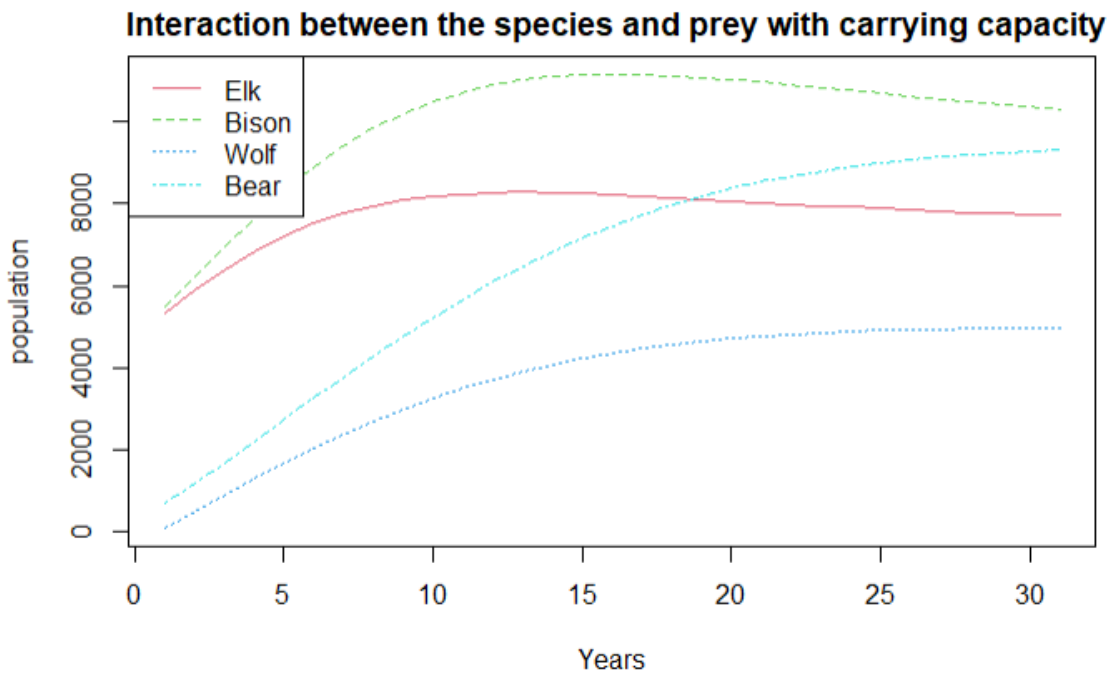


Figure 5: Effect of interaction between the prey and predator population in 20 years.





**Figure 6: Effect of interaction between the prey and predator population in 30 years.**

#### 4.2.4 Scenario 4

Leaving the ecosystem to function on its own could lead to eventual collapse as we studied in above scenario. Even if the population somehow manages to rebound, any external factor such as migration, weather changes, food shortage could lead to the ecosystem spiral out of control. As mentioned in introduction, the wolf's re-introduction to Yellowstone Park in 1995, helped the ecosystem rebalance. Our model so far validates the need for human involvement in maintaining the ecosystem whether its humans introducing a new species, adding more of the existing species, reducing the numbers of the existing species. We model one such scenario here where introducing control hunting could lead to the balanced ecosystem. From the plots below, we can see that different rate of hunting for each species could lead to different types of population changes into the ecosystem. This could help control the changes in ecosystem due to unseen or uncontrollable circumstances. We can see if we introduce hunting at multiple of different rates, we can get a balanced ecosystem with desired numbers of population density. However, if the rate of hunting is not properly controlled, the ecosystem could collapse.

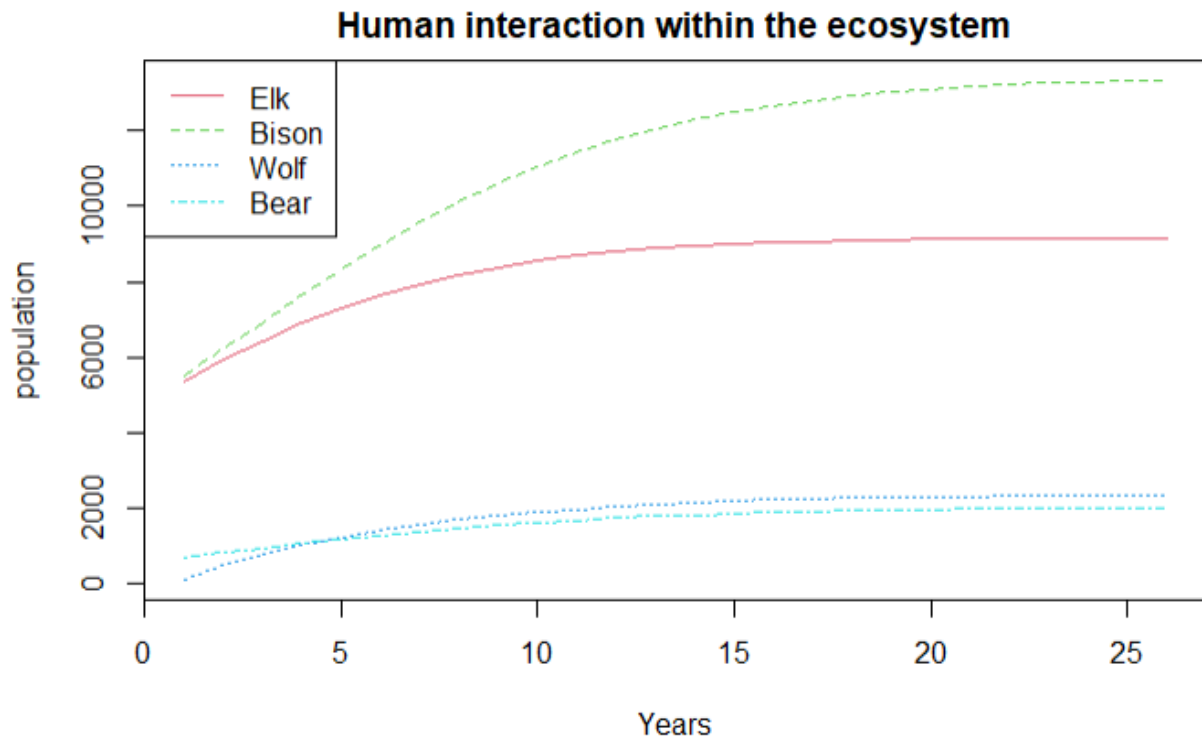


Figure 7: Effect of human involvement in the ecosystem (rate of interaction:  $h_1 = 0$ ,  $h_2 = 0.008$ ,  $h_3 = 0.2$ ,  $h_4 = 0.5$ )

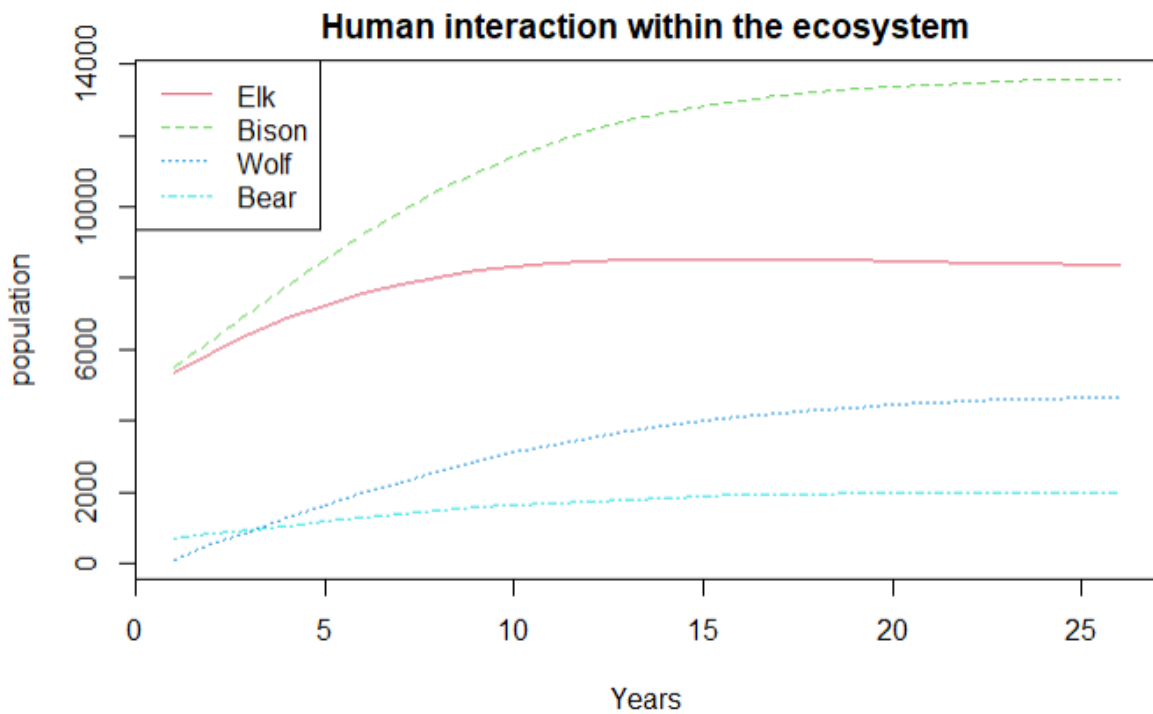


Figure 8: Effect of human involvement in the ecosystem (rate of interaction:  $h_1 = 0$ ,  $h_2 = 0$ ,  $h_3 = 0.02$ ,  $h_4 = 0.5$ )

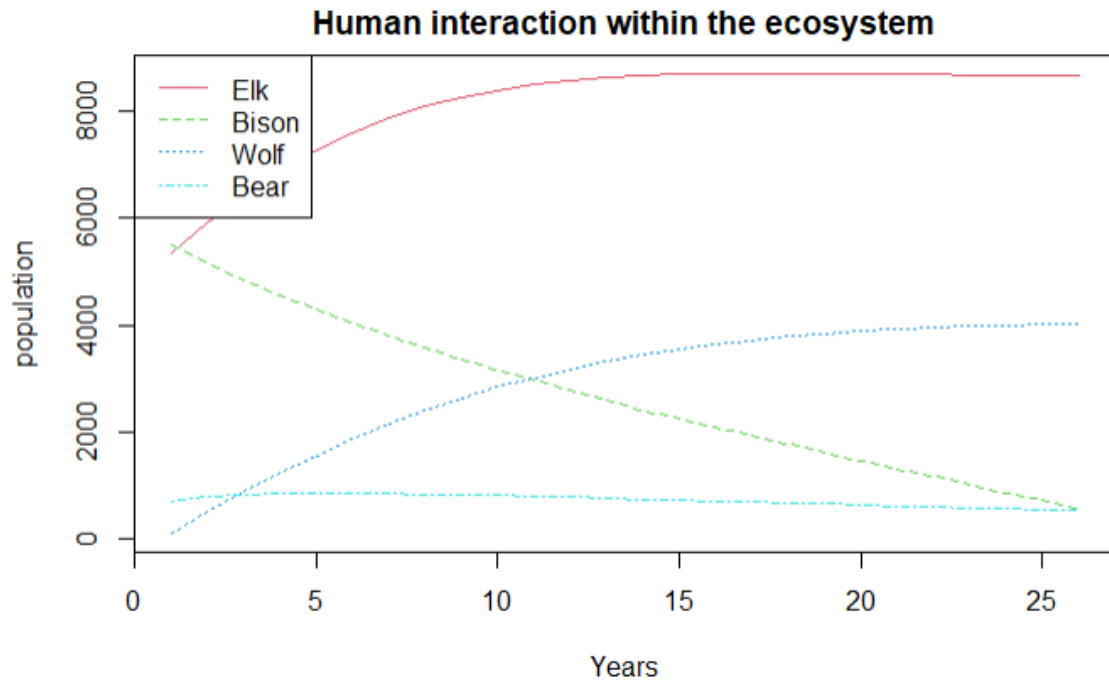


Figure 9: Effect of human involvement in the ecosystem (rate of interaction:  $h_1 = 0$ ,  $h_2 = 0.2$ ,  $h_3 = 0.02$ ,  $h_4 = 0.5$ )

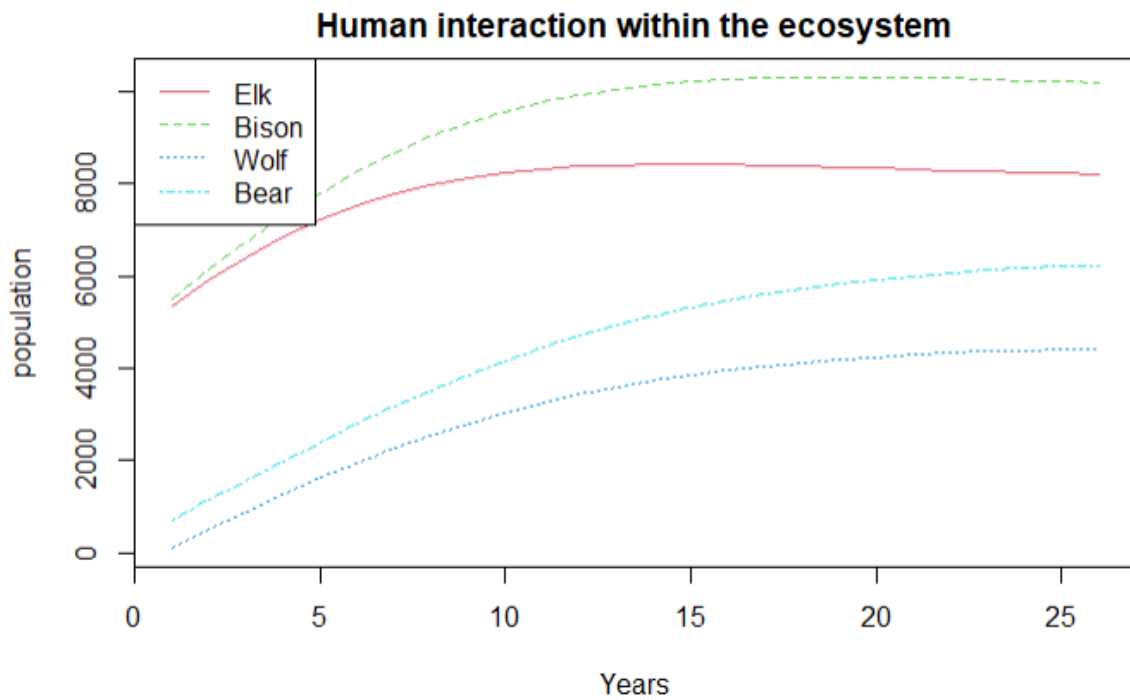


Figure 10: Effect of human involvement in the ecosystem (rate of interaction:  $h_1 = 0$ ,  $h_2 = 0.02$ ,  $h_3 = 0.02$ ,  $h_4 = 0.05$ )

### 4.3 Assessment

This model will predict and show when there is a need for human action in maintenance of the ecosystem's equilibrium. If a certain population is decreasing/increasing adjustments, such as culling and hunting, or introducing new animals/species, can be implemented to reduce/increase apparent competition and keep the ecosystem in a healthy and balanced manor at equilibrium. For example, if too many elk are dying than a potential solution could be the culling of wolves and restricting the hunting of elk. There are several scenarios in the analysis portion. This model can be used to optimize human involvement in the ecosystem to maintain equilibrium.

#### 4.3.1 Strengths

- Intuitive, Easy to understand/Explain, visually appealing, and great graphical results
- Easily changeable/flexible model; model can be adjusted to match a new reality in an ecosystem
- Stable equilibrium
- Complex enough model to accurately represent real ecosystem, yet still manageable enough to yield correct results
- Can accurately show and predict population of predators, and prey. And when human action is required to maintain equilibrium in ecosystem.
- Can be easily increased in scope to add more predators and prey into the model

#### 4.3.2 Weaknesses

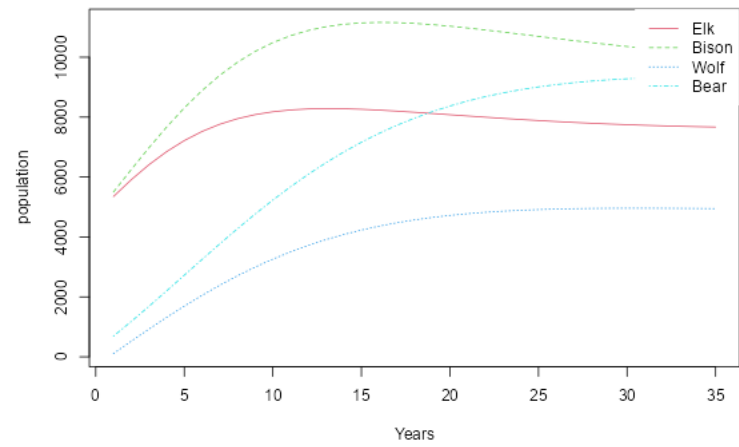
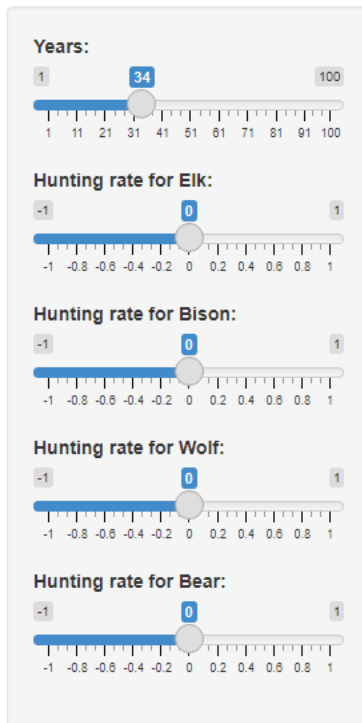
- Only 2 predators, 2 prey. This is the main weakness of this model. A necessary simplifying assumption that inherently limits the scope of the model. However, scaled up/ down version of the model can be created to suite needs of specific ecosystems.
- Dependant on data that can be hard to collect
- Limited interaction between species
- Can result in exponential growth/decay if certain parameters are null
- Future improvement of the model can correct Weaknesses listed above

## 5 Conclusion

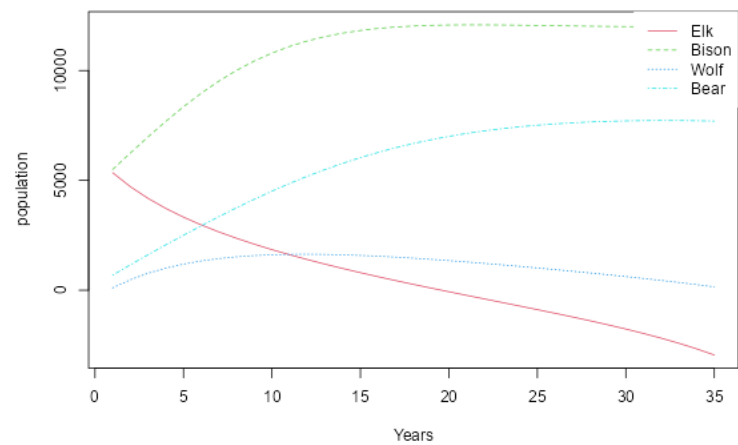
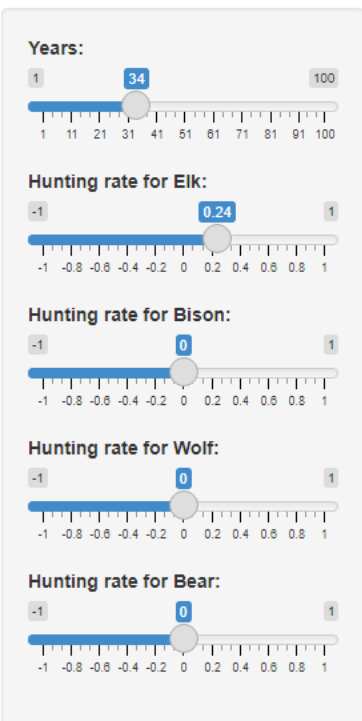
Our model justifies the need of human involvement in maintenance of an ecosystem. As we have seen that the ecosystem is so fragile that any unexpected changes could lead to a drastic impact. However, a minor unplanned, unexamined change could help cause an entire ecosystem to be disrupted. Our model could help predict the upcoming changes into the ecosystem and ways to rebalance it. By changing the human interaction terms ( $h_1$ ,  $h_2$ ,  $h_3$ ,  $h_4$ ), we could predict how the ecosystem will behave in the upcoming years. Based on the desired outcome, we could make changes to the ecosystem using different human interaction terms.

We have also created a dashboard/app to show study these changes and determining the right rate of change. Link to the app: [https://hkgrewal.shinyapps.io/MATH377\\_PROJECT\\_APP/](https://hkgrewal.shinyapps.io/MATH377_PROJECT_APP/)

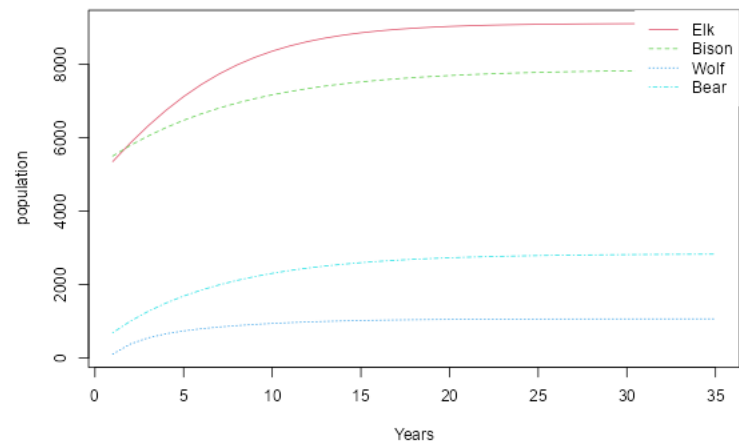
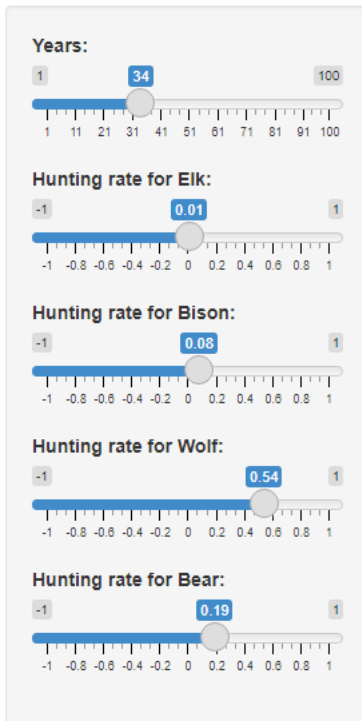
## Hunting impact in Yellowstone Park



## Hunting impact in Yellowstone Park



## Hunting impact in Yellowstone Park



## 6 Bibliography

Serrouya, Robert, et al. "Using predator-prey theory to predict outcomes of broadscale experiments to reduce apparent competition." *The American Naturalist* 185.5 (2015): 665-679.

Farquhar, Brodie. "Wolf reintroduction changes ecosystem in Yellowstone." *Retrieved from July 1* (2020): 2020.

Yellowstone Center for Resources. 2018. The State of Yellowstone Vital Signs and Select Park Resources, 2017. YCR–2018–01. Yellowstone Center for Resources, Yellowstone National Park, Wyoming, USA. [https://www.nps.gov/yell/learn/management/upload/Vital-Signs\\_Report\\_2017\\_508\\_Reduced.pdf](https://www.nps.gov/yell/learn/management/upload/Vital-Signs_Report_2017_508_Reduced.pdf)

## 7 Appendix – R Code

```
library(deSolve)

years <- seq(0,20, by = 1)

Initial <- c(P1 = 5349, P2 = 5500, C1 = 108, C2 = 690 )

parms <- c(c1 <- 0.13, c2 <- 0.10, p1 <- 0.24, p2 <- 0.23, k1 <- 10000, k2 <- 15000, a= 0.07, b = 0.03, c= 0.01, d = 0.07, h1 = 0.05, h2 = 0.1 , h3= 0.1, h4 = 0.05 )
```

```
constant <-function(years , parms, Initial){
  with(as.list(c(parms, Initial)),{
    dP1 = 0
    dP2 = 0
    dC1 = 0
    dC2 = 0

    return(list(c(dP1, dP2, dC1, dC2)))

  })
}
```

```

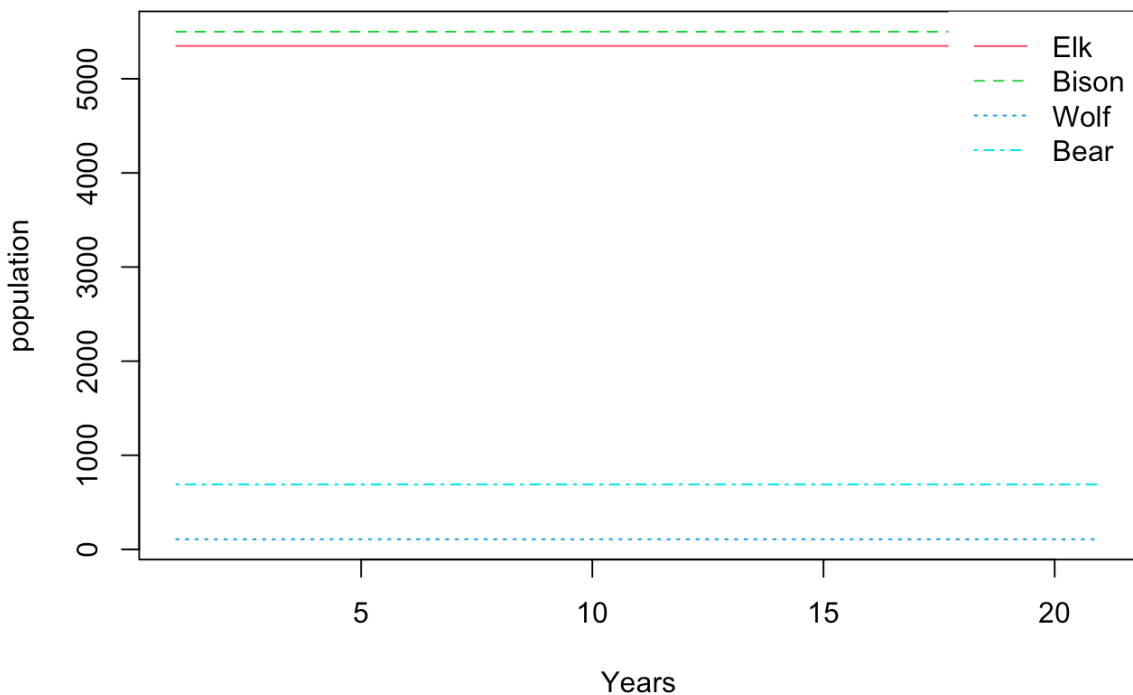
result0 <- as.data.frame(ode(fun = constant, y = Initial, parms = parms, time
s = years))

matplot(result0[,-1], t = "l", col = c(2,3,4,5), xlab = "Years", ylab = "pop
ulation", main = "No rate of change")

legend("topright", c("Elk", "Bison", "Wolf", "Bear"), lty = c(1,2,3,4), col =
c(2,3,4,5), box.lwd = 0)

```

### No rate of change



```

#Initial- no interaction - no carrying capacity
no_carrying_Capacity <- function(years , parms, Initial){
  with(as.list(c(parms, Initial)),{
    dP1 = P1*p1
    dP2 = P2*p2
    dC1 = -C1*c1
    dC2 = -C2*c2

    return(list(c(dP1, dP2, dC1, dC2)))
  }
}

```



```

}))

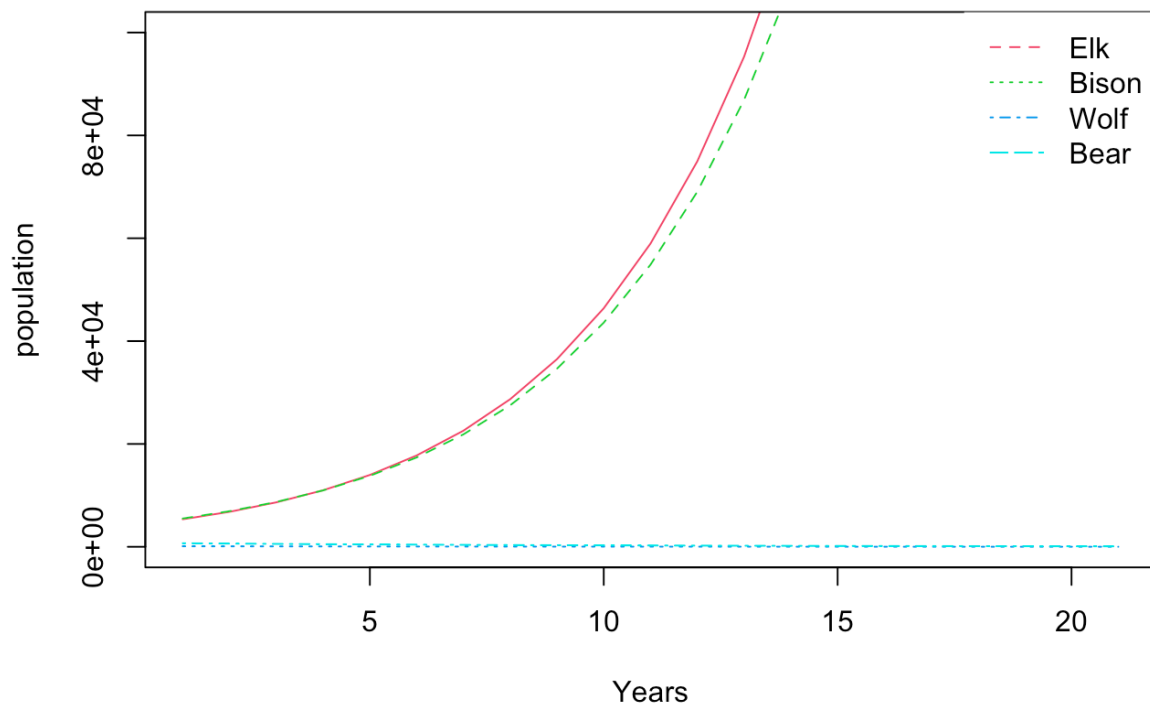
}

result1 <- as.data.frame(ode(fun = no_carrying_Capacity, y = Initial, parms =
parms, times = years))

matplot(result1[,-1], type = "l", col = c(2,3,4,5), xlab = "Years", ylab = "
population" ,ylim=c(0,100000), main = "No interaction between the species")
legend("topright", c("Elk", "Bison", "Wolf", "Bear"), lty = c(2,3,4,5), col =
c(2,3,4,5), box.lwd = 0)

```

### No interaction between the species

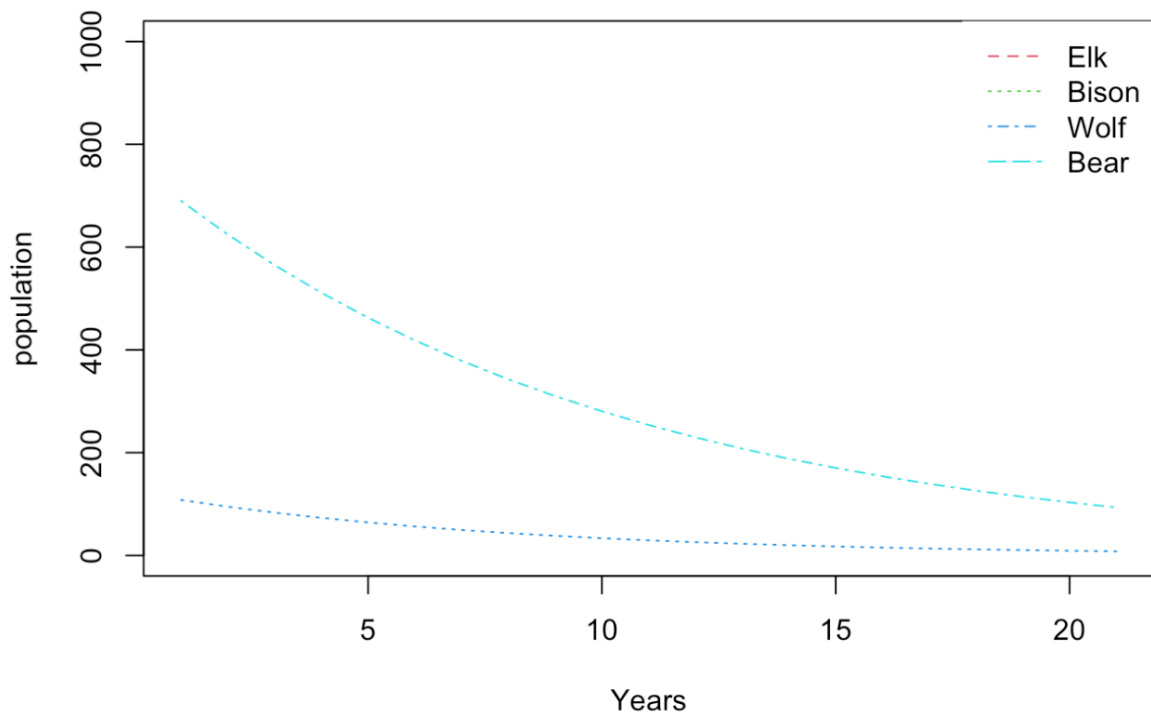


```

matplot(result1[,-1], type = "l", col = c(2,3,4,5), xlab = "Years", ylab = "
population" ,ylim=c(0,1000), main = "No interaction between the species")
legend("topright", c("Elk", "Bison", "Wolf", "Bear"), lty = c(2,3,4,5), col =
c(2,3,4,5), box.lwd = 0)

```

### No interaction between the species



The prey are not being eaten by the predator, which means population of prey will grow exponentially as Future population = Initial + change in population Since Predators are not feeding on the prey in this model, we can see the decline in the population as the change in the predator population is negative due to unavailability of food as Future population = Initial - change in population.

```
no_int <-function(years , parms, Initial){
  with(as.list(c(parms, Initial)),{
    dP1 = p1*P1*(1-(P1/k1))
    dP2 = p2*P2*(1-(P2/k2))
    dC1 = -C1*c1
    dC2 = -C2*c2

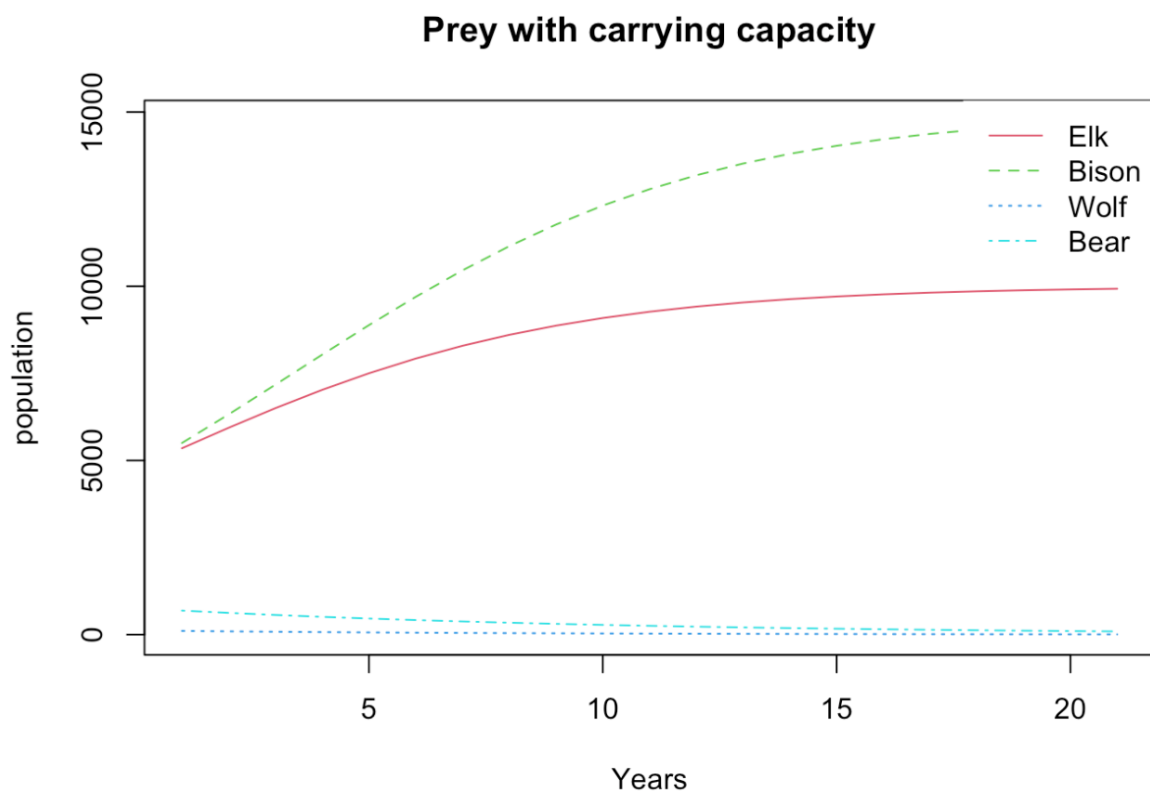
    return(list(c(dP1, dP2, dC1, dC2)))

  })
}
```

```
result2 <- as.data.frame(ode(fun = no_int, y = Initial, parms = parms, times
= years))

matplot(result2[,-1], type = "l", xlab = "Years", ylab = "population", col =
c(2,3,4,5) , main = "Prey with carrying capacity")

legend("topright", c("Elk", "Bison", "Wolf", "Bear"), lty = c(1,2,3,4), col =
c(2,3,4,5), box.lwd = 0)
```



We introduced carrying capacity to control the prey population from growing out of bounds. The prey are not being eaten by the predator, which means population of prey will grow exponentially as Future population = Initial + change in population, however with the carrying capacity, the growth rate is slower. The predator population follows the previous model.

```
years <- seq(0,30, by = 1)

parms <- c(c1 <- 0.13, c2 <- 0.10, p1 <- 0.24, p2 <- 0.23, k1 <- 10000, k2 <-
15000, a= 0.07, b = 0.03, c= 0.01, d = 0.07, h1 = 0.05, h2 = 0.1 , h3= 0.1,
h4 = 0.05 )

int <-function(years , parms, Initial){
  with(as.list(c(parms, Initial)),{
    dP1 = p1*P1*(1-(P1/k1)) - (a*C1+ c*C2)
```

```

dP2 = p2*P2*(1-(P2/k2)) - (b*C1 + d*C2)
dC1 = -C1*c1 + (a*P1+ c*P2)
dC2 = -C2*c2 + (b*P1 + d*P2)

return(list(c(dP1, dP2, dC1, dC2)))

})

}

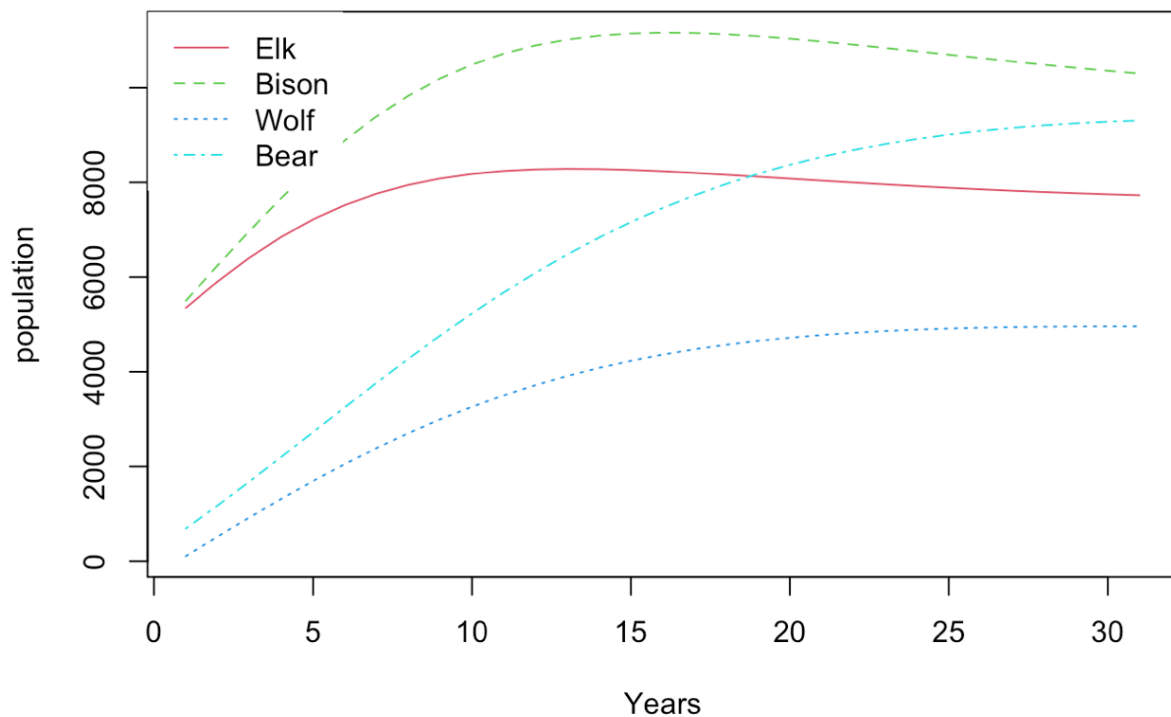
result3 <- as.data.frame(ode(fun = int, y = Initial, parms = parms, times = years))

matplot(result3[,-1], type = "l", xlab = "Years", col = c(2,3,4,5), ylab = "population", main = "Interaction between the species and prey with carrying capacity")

legend("topleft", c("Elk", "Bison", "Wolf", "Bear"), lty = c(1,2,3,4), col = c(2,3,4,5), box.lwd = 0)

```

### Interaction between the species and prey with carrying capacity



```

parms <- c(c1 <- 0.13, c2 <- 0.10, p1 <- 0.24, p2 <- 0.23, k1 <- 10000, k2 <-
15000, a= 0.07, b = 0.03, c= 0.01, d = 0.07, h1 = 0.02, h2 = 0.02 , h3= 0.0
2, h4 = 0.005 )

years <- seq(0,25, by = 1)

human_int <-function(years , parms, Initial){
  with(as.list(c(parms, Initial)),{
    dP1 = p1*P1*(1-(P1/k1)) - (a*C1+ c*C2) - h1*P1
    dP2 = p2*P2*(1-(P2/k2)) - (b*C1 + d*C2) - h2*P2
    dC1 = -C1*c1 + (a*P1+ c*P2) - h3*C1
    dC2 = -C2*c2 + (b*P1 + d*P2) - h4*C2

    return(list(c(dP1, dP2, dC1, dC2)))

  })
}

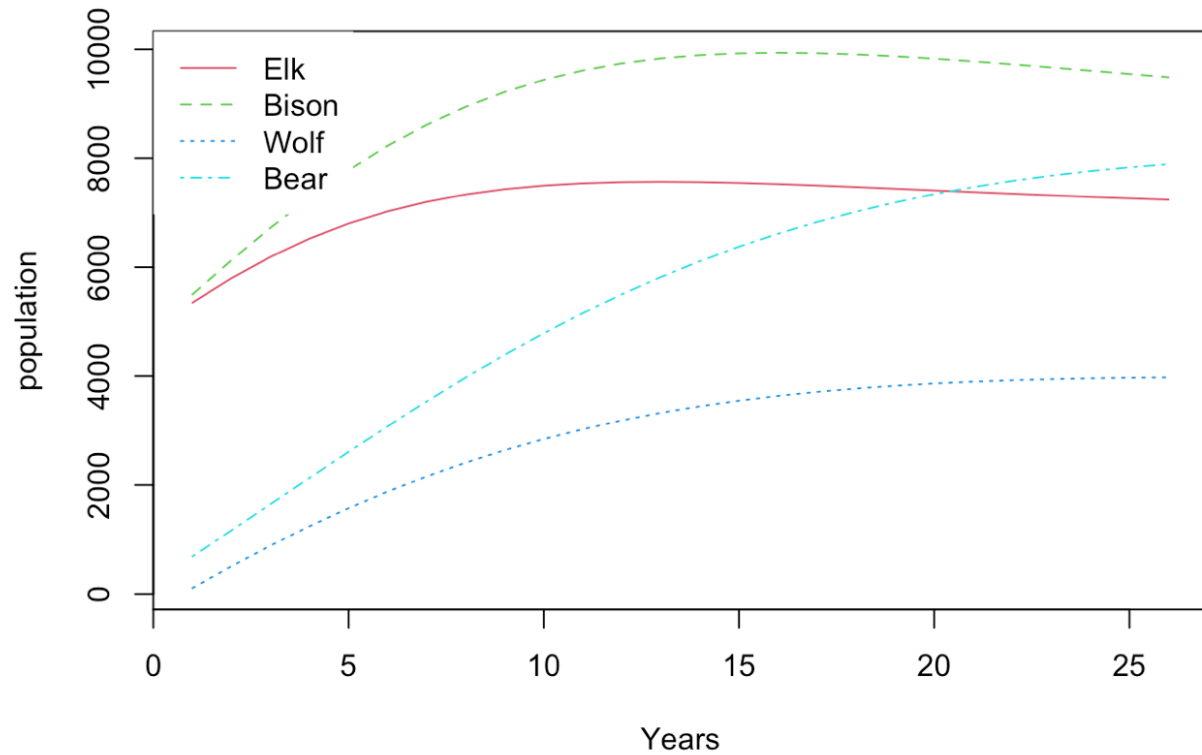
result4 <- as.data.frame(ode(fun = human_int, y = Initial, parms = parms, tim
es = years))

plot <- matplot(result4[,-1], type = "l", xlab = "Years", ylab = "population
" , col = c(2,3,4,5), main = "Human interaction within the ecosystem")

legend("topleft", c("Elk", "Bison", "Wolf", "Bear"), lty = c(1,2,3,4), col =
c(2,3,4,5), box.lwd = 0)

```

## Human interaction within the ecosystem



```
library(shiny)
library(shinydashboard)
library(leaflet)
ui <- fluidPage(

  # App title ----
  titlePanel("Hunting impact in Yellowstone Park"),

  # Sidebar layout with input and output definitions ----
  sidebarLayout(

    # Sidebar to demonstrate various slider options ----
    sidebarPanel(

      sliderInput("y", "Years:",
```

```

min = 1, max = 100, value = 0, step = 1),

  sliderInput("h1", "Hunting rate for Elk:",
min = -1, max = 1, value = 0, step = 0.01),

  sliderInput("h2", "Hunting rate for Bison:",
min = -1, max = 1, value = 0, step = 0.01
),

  sliderInput("h3", "Hunting rate for Wolf:",
min = -1, max = 1, value = 0, step = 0.01),

  sliderInput("h4", "Hunting rate for Bear:",
min = -1, max = 1, value = 0, step = 0.01),

),

# Main panel for displaying outputs ----
mainPanel(

  # Output: Plot
  plotOutput("Hunting")

)
)
)
server <- function(input, output){
  sliderValues <- reactive({

data.frame(
  Name = c("Years",
    "Hunting rate for Elk",
    "Hunting rate for Bison",

```

```

        "Hunting rate for Wolf",
        "Hunting rate for Bear"
    ),
    Value = as.numeric(c(input$y,
        input$h1,
        input$h2,
        input$h3,
        input$h4)),
    stringsAsFactors = FALSE)

})

# Show the values in an HTML table ----
output$Hunting <- renderPlot({

    years <- seq(0,input$y, by = 1)

    Initial <- c(P1 = 5349, P2 = 5500, C1 = 108, C2 = 690 )

    parms <- c(c1 <- 0.13, c2 <- 0.10, p1 <- 0.24, p2 <- 0.23, k1 <- 10000, k
2 <- 15000, a= 0.07, b = 0.03, c= 0.01, d = 0.07, h1 = input$h1, h2 = input$h
2 , h3= input$h3, h4 = input$h4 )

    human_int <-function(years , parms, Initial){
        with(as.list(c(parms, Initial)),{
            dP1 = p1*P1*(1-(P1/k1)) - (a*C1+ c*C2) - h1*P1
            dP2 = p2*P2*(1-(P2/k2)) - (b*C1 + d*C2) - h2*P2
            dC1 = -C1*c1 + (a*P1+ c*P2) - h3*C1
            dC2 = -C2*c2 + (b*P1 + d*P2) - h4*C2

            return(list(c(dP1, dP2, dC1, dC2)))
        }
    }

```



```
  })  
  
}  
  
result4 <- as.data.frame(ode(fun = human_int, y = Initial, parms = parms, times = years))  
  
plot <- matplot(result4[,-1], type = "l", xlab = "Years", ylab = "population", col = c(2,3,4,5),)  
legend("topright", c("Elk", "Bison", "Wolf", "Bear"), lty = c(1,2,3,4), col = c(2,3,4,5), box.lwd = 0)  
  
  })  
}  
  
shiny::shinyApp(ui = ui, server = server)
```