Microprocessors & Interfacing

Number Conversion

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Number Representation

- Any number can be represented in the form of

$$(a_n a_{n-1} \dots a_1 a_0 . a_{-1} \dots a_{-m})_r = a_n \times r^n + a_{n-1} \times r^{n-1} + \dots + a_1 \times r + a_0 + a_{-1} \times r^{-1} + \dots + a_{-m} \times r^{-m}$$

r : radix, base

$0 \le a_i < r$

Example

- Decimal

$$(3597)_{10} = 3 \times 10^3 + 5 \times 10^2 + 9 \times 10 + 7$$

  - The place values, from right to left, are 1, 10, 100, 1000
  - The base or radix is 10
  - All digits must be less than the base, namely, 0~9

Example

- Binary

$$(1011)_2 = 1 \times 2^3 + 0 \times 2^2 + 1 \times 2 + 1$$

  - The place values, from right to left, are 1, 2, 4, 8
  - The base or radix is 2
  - All digits must be less than the base, namely, 0~1

Example

- Hexadecimal

$$(F24B)_{16} = F \times 16^3 + 2 \times 16^2 + 4 \times 16 + B = 15 \times 16^3 + 2 \times 16^2 + 4 \times 16 + 11$$

  - The place values, from right to left, are 1, 16, 16^2, 16^3
  - The base or radix is 16
  - All digits must be less than the base, namely, 0~9,A,B,C,D,E,F

Number Conversion

- From base r to base 10
  - Using the formula below

$$(a_n a_{n-1} \dots a_1 a_0 . a_{-1} \dots a_{-m})_r = a_n \times r^n + a_{n-1} \times r^{n-1} + \dots + a_1 \times r + a_0 + a_{-1} \times r^{-1} + \dots + a_{-m} \times r^{-m}$$

- Examples are given next slide

Examples

- From base 2

$$(1011.1)_2 = 1\times 2^3 + 0\times 2^2 + 1\times 2 + 1 + 1\times 2^{-1} = 11.5$$

- From base 16

$$(10A)_{16} = 1\times 16^2 + 0\times 16 + 10 = 266$$

Number Conversion

- From base 10 to base r

  Based on Formula:

$$(a_n a_{n-1} \dots a_1 a_0 . a_{-1} \dots a_{-m})_r$$
$$= a_n \times r^n + a_{n-1} \times r^{n-1} + \dots + a_1 \times r + a_0 + a_{-1} \times r^{-1} + \dots + a_{-m} \times r^{-m}$$

  - For whole number
    - **Divide** the number/quotient repeatedly by **r** until the quotient is zero and the remainders are the digits of base r number, in reverse order
  - For fraction
    - **Multiply** the number/fraction repeatedly by **r**, the whole numbers of the products are the digits of the base r fraction number
- Examples are given in the next slides

Examples

- To base 2
  - To convert $(11.25)_{10}$ to binary
    - For whole number $(11)_{10}$ – repeated division **(by 2)**

11	1
5	1
2	0
1	1
0	

    - For fraction $(0.25)_{10}$ – repeated multiplication **(by 2)**

0.25	
0.5	0
0.0	1

$(11.25)_{10}$=$(1011.01)_2$

Examples

- To base 16
  - To convert $(99.25)_{10}$ to hexadecimal
    - For whole number $(99)_{10}$ – division **(by 16)**

99	3
6	6
0	

    - For fraction $(0.25)_{10}$ – multiplication **(by 16)**

0.25	
0.0	4

$(99.25)_{10}$=$(63.4)_{hex}$

Number Conversion (cont.)

- Between binary and octal
  - Direct mapping, based on the observation:

$$\begin{aligned}
&(abcdefgh.jklmn)_2 \\
&= (a\cdot 2 + b)\cdot 2^6 + (c\cdot 2^2 + d\cdot 2 + e)\cdot 2^3 + \\
&\quad (f\cdot 2^2 + g\cdot 2 + h) + (j\cdot 2^2 + k\cdot 2 + l)\cdot 2^{-3} + \\
&\quad (m\cdot 2^2 + n\cdot 2 + 0)\cdot 2^{-6} \\
&= (0ab_2)\cdot 8^2 + (cde_2)\cdot 8^1 + (fgh_2)\cdot 8^0 + \\
&\quad (jkl_2)\cdot 8^{-1} + (mn0_2)\cdot 8^{-2}
\end{aligned}$$

  - The expressions in parentheses, being less than 8, are the octal digits.

Number Conversion (cont.)

- Between binary and octal (cont.)
  - Binary to octal
    - The binary digits (“bits”) are grouped from the radix point, three digits a group. Each group corresponds to an octal digit.
  - Octal to binary
    - Each of octal digits is expanded to three binary digits

Examples

- Binary to octal
  - Convert $(10101100011010001000.10001)_2$ to octal :

  010 101 100 011 010 001 000 . 100 010 $_2$
  = 2 5 4 3 2 1 0 . 4 2 $_8$
  = 2543210.42_8 .
- Note:
  - Whole number parts are grouped from right to left. The leading 0 is optional
  - Fractional parts are grouped from left to right and padded with 0s

Examples

- Octal to binary
  - Convert 37425.62_8 to binary :

  3 7 4 2 5 . 6 2 $_8$
  = 011 111 100 010 101 . 110 010 $_2$
  = 11111100010101.11001_2
- Note:
  - For whole number parts, the leading 0s can be omitted.
  - For fractional parts, the trailing 0s can be omitted.

Number Conversion (cont.)

- Between binary and hexadecimal
  - Binary to hexadecimal
    - The binary digits ("bits") are grouped from the radix point, **four** binary digits a group. Each group corresponds to a hexadecimal digit.
  - Hexadecimal to binary
    - Each of hexadecimal digits is expanded to four binary digits

Examples

- Binary to hexadecimal
  - Convert $10101100011010001000.10001_2$ to hexadecimal :

  1010 1100 0110 1000 1000 . 1000 1000 $_2$
  = A C 6 8 8 . 8 8 $_{16}$
  = $AC688.88_{16}$.
- Note:
  - Whole number parts are grouped from right to left. The leading 0 is optional
  - Fractional parts are grouped from left to right and padded with 0s

Examples

- Hexadecimal to binary
  - Convert $2F6A.78_{16}$ to binary :

  2 F 6 A . 7 8 $_{16}$
  = 0010 1111 0110 1010 . 0111 1000 $_2$
  = 10111101101010.01111_2
- Note:
  - For whole number parts, the leading 0s can be omitted.
  - For fractional parts, the trailing 0s can be omitted.