JOINT REFLECTION DESIGN AND CHANNEL ESTIMATION FOR INTELLIGENT REFLECTIVE SURFACES

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ABSTRACT

In this report, joint design of IRS reflection patterns and channel estimation is explored via two popular methods, namely the ON/OFF method [1] and DFT-based method [2]. The latter method is also viewed as the optimal design of reflection patterns. Statistical performances of estimation under these two methods are derived theoretically based on CRLB. Simulations and numerical results are presented to validate the theoretical performances.

Keywords Intelligent reflective surfaces · channel estimation · CRLB · reflection pattern

1 Introduction

In 5G beyond and 6G communications, intelligent reflective surface (IRS) has emerged as a promising technology for improving the wireless communication coverage, throughput, and energy efficiency [3–5]. It enables the reconfiguration of wireless propagation environment by smart control of the signal reflections via massive low-cost passive elements [6]. Through adjusting the amplitude and phase shift at each passive element on IRS, the signals reflected by IRS and propagate through direct(non-reflection) links are combined at the receiver side to enhance signal powers. Due to the passive relay and full-duplex mode, IRS has much lower energy consumption and cost than traditional active relays.

However, to realize the above benefits in practice, accurate channel state information (CSI) is required [7, 8]. Many critical techniques in wireless systems, such as beamforming design [9], power allocation [10], and multi-user scheduling [11] rely on the channel estimation results, i.e., CSI. It is very challenging to conduct channel estimation for IRS-aided systems for the following reasons [2, 12, 13],. First, IRS can contain a large number of elements, which increases the number of channel coefficients to be estimated. Second, since IRS does not have signal processing capabilities, the channels on both reflection and non-reflection links can only be estimated at the receiver instead of IRS. Third, the amplitudes and phase shifts at IRS elements (i.e., reflection pattern) needs to be configured appropriately for channel estimation. Therefore, for IRS-aided systems, joint design of reflection patterns and channel estimation is essential.

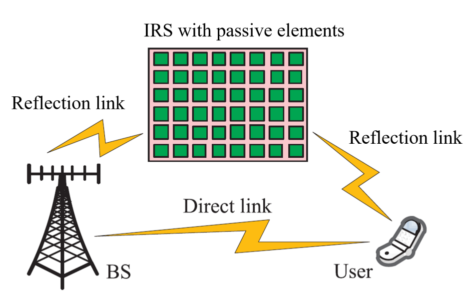


Figure 1: A IRS-aided MIMO system.

In recent work, several channel estimation techniques have been developed for IRS-aided communication systems. Some approaches equip IRS with a few additional active elements [7] to reduce the complexity of channel estimation. Assuming IRS with fully passive elements, an ON/OFF method is adopted in [1, 12], where only one element is turned "ON" at each stage to estimate the links one by one. Note that the state "OFF" means that there is only structure-mode reflection as if the element is a conducting object, whereas the state "ON" means that there are both structure-mode reflection and antenna-mode reflection [7]. The structure-mode can be absorbed into the direct link in channel modeling. In [7], a random reflection pattern is proposed where the elements are turned on randomly. Then a two-stage channel estimation algorithm is conducted based on sparse matrix factorization completion.

In the above mentioned work, reflection patterns of IRS are designed as sparse matrices with many zeros. Dense matrices based on discrete fourier transform (DFT) are adopted in [10, 13], but no optimality analysis is provided for this kind of design. In [2], the ON/OFF method is proved to be sub-optimal, and it is shown that an optimal IRS reflection pattern follows the form of a DFT matrix theoretically by exploiting the Cramer-Rao lower bound (CRLB). It should be noted that channel estimation for IRS-aided systems falls into two categories in general. The reflection links include transmitter-to-IRS links and IRS-to-receiver links. The first category is to estimate each part of the reflection links separately [7], while another category is to estimate the reflection links as a whole [2], i.e., cascaded channel. Since the cascaded channel information is sufficient for many communication techniques, we only focus on estimating the reflection links as a whole.

In this report, we explore joint design of IRS reflection patterns and channel estimation via two popular methods, namely the ON/OFF method [1] and DFT-based method [2]. The latter method is also viewed as the optimal design of reflection patterns [2]. First we establish the link model for IRS-aided point-to-point communication and demonstrate the channel parameters to be estimated. Then least squares estimation and the CRLB are presented as the theoretical background. Next, we elaborate on the ON/OFF method and DFT-based method and compare their statistical performance. Finally, we conduct numerical simulations to validate theoretical results. Some further discussion on computational complexity, Bayesian estimators, and pilot signals are provided, which can be left for future work.

2 System Model and Problem Statement

For a point-to-point uplink transmission, there are M antennas deployed at the BS and one antenna deployed at the user. An IRS with K passive elements is utilized to reflect the uplink signal from the user to the BS. Let the transmitted uplink pilot signal be x_t and received signal be $\mathbf{s}_t$. Let $\mathbf{h}_\mathrm{d}$ be the direct channel coefficients from the user to the base station, $\mathbf{G}$ be the channel matrix from IRS to the BS, and $\mathbf{h}$ be the channel coefficients from the user to IRS. The parameters of IRS passive elements (i.e., reflection patterns) are denoted by vector $\boldsymbol{\phi}_t$. Note that we assume the channel matrix and vectors remain constant during the estimation period, i.e., over T pilot signals (samples), which is also known as pilot training period. However, the reflection patterns $\boldsymbol{\phi}_t$ can be designed and tuned for each sample t. We also assume there is non-colored additive white Gaussian noise (AWGN) $\mathbf{n}_t$.

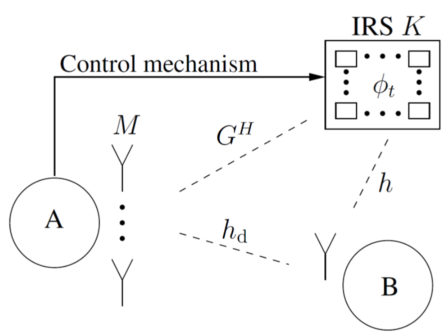


Figure 2: System model and channel representations.

Considering frequency-flat channels in a narrow-band system, the signal model is represented as

$$\mathbf{s}_t = \left(\mathbf{h}_\mathrm{d} + \mathbf{G}^H \operatorname{diag}(\boldsymbol{\phi}_t)\, \mathbf{h}\right) x_t + \mathbf{n}_t, \tag{1}$$

where $\mathbf{s}_t \in \mathbf{C}^M$, $x_t \in C$, $\mathbf{n}_t \in \mathbf{C}^M$, $\mathbf{h}_\mathrm{d} \in \mathbf{C}^M$, $\mathbf{h} \in \mathbf{C}^K$, $\mathbf{G} \in \mathbf{C}^{K\times M}$ and $\boldsymbol{\phi}_t \in \mathbf{C}^K$. Since we only focus on estimating $\mathbf{G}$ and $\mathbf{h}$ as a whole, we define the cascaded channel $\mathbf{V}$ as

$$\mathbf{G}^H \operatorname{diag}(\mathbf{h}) = \mathbf{V} = \left[\begin{array}{ccc} \mathbf{v}_1 & \cdots & \mathbf{v}_K \end{array}\right], \tag{2}$$

where $\mathbf{V} \in \mathbf{C}^{M\times K}$, $\mathbf{v}_1, \cdots, \mathbf{v}_K$ is the column vectors of cascaded channel $\mathbf{V}$, and

$$\mathrm{diag}(\mathbf{h}) = \begin{bmatrix} h_1 & & \\ & \ddots & \\ & & h_K \end{bmatrix}. \tag{3}$$

Then the equation (1) is transformed into

$$\mathbf{s}_t = (\mathbf{h}_\mathrm{d} + \mathbf{V}\boldsymbol{\phi}_t)\, x_t + \mathbf{n}_t, t = 1, ..., T. \tag{4}$$

The channel coefficients $\mathbf{h}_\mathrm{d}$ and $\mathbf{V}$ are estimated based on the received signal $\mathbf{s}_t$, the known pilot signals x_t, and the reflection patterns $\boldsymbol{\phi}_t$. The next step is to represent the equation (4) in the standard linear data model form.

Let $\boldsymbol{\theta}$ be all the channel coefficients to be estimated such that

$$\boldsymbol{\theta} = \begin{bmatrix} \mathbf{h}_\mathrm{d} \\ \mathbf{v}_1 \\ \vdots \\ \mathbf{v}_K \end{bmatrix} \in \mathbf{C}^{M(K+1)}. \tag{5}$$

For a specific sample t, we have

$$\mathbf{s}_t = x_t \begin{bmatrix} \mathbf{I}_M & \boldsymbol{\phi}_{t,1}\mathbf{I}_M & \cdots & \boldsymbol{\phi}_{t,K}\mathbf{I}_M \end{bmatrix} \boldsymbol{\theta} + \mathbf{n}_t, t = 1, 2, ...T. \tag{6}$$

Then all the T data samples are stacked together as $\mathbf{S}$, $\mathbf{N}$, and $\mathbf{X}$

$$\mathbf{S} = \begin{bmatrix} \mathbf{s}_1 \\ \vdots \\ \mathbf{s}_T \end{bmatrix}, \mathbf{N} = \begin{bmatrix} \mathbf{n}_1 \\ \vdots \\ \mathbf{n}_T \end{bmatrix}, \mathbf{X} = \mathrm{diag}\left(\begin{bmatrix} x_1 \mathbb{1}_M \\ \vdots \\ x_T \mathbb{1}_M \end{bmatrix} \right), \tag{7}$$

where $\mathbf{S} \in \mathbf{C}^{TM}$, $\mathbf{N} \in \mathbf{C}^{TM}$, $\mathbf{X} \in \mathbf{C}^{TM\times TM}$.

We further define a reflection pattern matrix $\boldsymbol{\Phi}$ that absorbs the direct channel, that is,

$$\boldsymbol{\Phi} = \begin{bmatrix} 1 & \phi_{1,1} & \cdots & \phi_{1,K} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & \phi_{T,1} & \cdots & \phi_{T,K} \end{bmatrix} \in \mathbf{C}^{T\times(K+1)}, \boldsymbol{\Psi} = \boldsymbol{\Phi} \otimes \mathbf{I}_M. \tag{8}$$

Based on the above definitions, the equation (4) is represented in the standard linear form

$$\begin{aligned} \mathbf{S} &= \mathbf{X}\boldsymbol{\Psi}\boldsymbol{\theta} + \mathbf{N} \\ &= \mathbf{H}\boldsymbol{\theta} + \mathbf{N}, \end{aligned} \tag{9}$$

where $\mathbf{H} = \mathbf{X}\boldsymbol{\Psi}$. Note that the parameters $\boldsymbol{\theta}$ are kept in the vector form, and thus the observation matrix $\mathbf{H} \in \mathbf{C}^{TM\times M(K+1)}$. The noise in the MISO and MIMO channels is assumed to be AWGN with the circle complex Gaussian distribution

$$\mathbf{N} \sim \mathcal{CN}\left(0, \sigma^2 \mathbf{I}_{TM}\right). \tag{10}$$

3 Theoretical Background

Assume we do not have any prior information on $\boldsymbol{\theta}$. For such a linear model with Gaussian assumption, the least squares (LS) estimator is the minimum variance unbiased (MVU) estimator and also attains the CRLB. Note that we assume the observation matrix $\mathbf{H}$ has full column rank, so that $\mathbf{H}^H\mathbf{H}$ is invertible. The LS estimator is

$$\hat{\boldsymbol{\theta}} = \begin{bmatrix} \hat{\mathbf{h}}_\mathrm{d} \\ \hat{\mathbf{v}}_1 \\ \vdots \\ \hat{\mathbf{v}}_K \end{bmatrix} = \mathrm{argmin} \left\| \mathbf{H}\boldsymbol{\theta} - \mathbf{S} \right\|_2^2 = \left(\mathbf{H}^H\mathbf{H}\right)^{-1} \mathbf{H}^H \mathbf{S}. \tag{11}$$

According to the estimation theory, the CRLB is computed as

$$\mathbf{C}_{\hat{\theta}} = \sigma^2 \left(\mathbf{H}^H \mathbf{H}\right)^{-1}. \tag{12}$$

If we further assume equal power allocation across the estimation period T, that is, $x_i^2 = x_j^2, \forall i, j = 1, ..., T$, and the power of each pilot signal x_t is normalized to one. Thus, we have $\mathbf{X}^H \mathbf{X} = \mathbf{I}_{TM}$. Now the CRLB is

$$\begin{aligned} \mathbf{C}_{\hat{\theta}} &= \sigma^2 \left(\left(\mathbf{\Phi} \otimes \mathbf{I}_M\right)^H \mathbf{X}^H \mathbf{X} \left(\mathbf{\Phi} \otimes \mathbf{I}_M\right) \right)^{-1} \\ &= \sigma^2 \left(\left(\mathbf{\Phi}^H \mathbf{\Phi} \otimes \mathbf{I}_M\right) \right)^{-1} \\ &= \sigma^2 \left(\mathbf{\Phi}^H \mathbf{\Phi}\right)^{-1} \otimes \mathbf{I}_M. \end{aligned} \tag{13}$$

The CRLB attained by the MVU estimator is the inverse of Fisher information matrix $I(\boldsymbol{\theta})$. For the diagonal elements of Fisher information matrix, the following lower bound is derived:

$$\left[\mathbf{C}_{\hat{\boldsymbol{\theta}}}\right]_{i,i} = \left[\mathcal{I}^{-1}(\boldsymbol{\theta})\right]_{i,i} \geq \frac{1}{[\mathcal{I}(\boldsymbol{\theta})]_{i,i}}. \tag{14}$$

When $I^{-1}(\boldsymbol{\theta})$ is diagonal, the lower bound of CRLB can be achieved. This conclusion provides critical insights for the DFT-based method, which will be elaborated later.

The estimation performance, i.e., covariance matrix, depends on the design of reflection pattern matrix $\mathbf{\Phi}$. Thus, in the following part of this report, we elaborate on two methods for the design of reflection pattern, namely ON/OFF method and DFT-based method.

4 ON/OFF Method

To achieve an observation matrix $\mathbf{H}$ with full column rank, one common approach in the existing work [1, 12] is to switch on each IRS element one by one, such that the reflection pattern $\mathbf{\Phi}$ that has full column rank. Note that the first column of $\mathbf{\Phi}$ is the forced selection to incorporate the direct channel in the signal model.

The ON/OFF method is shown as follows. First, all the IRS elements are switched off to estimate the direct channel $\mathbf{h}_d$. Then, each IRS element is switched on one by one for each t to estimate each column of the cascaded channel $\mathbf{V}$. This idea is even more straightforward if the equation (9) is expressed as follows:

$$\begin{bmatrix} \mathbf{s}_1 \\ \vdots \\ \mathbf{s}_T \end{bmatrix} = \begin{bmatrix} x_t \left[\begin{array}{cccc} \mathbf{I}_M & \phi_{t,1}\mathbf{I}_M & \cdots & \phi_{t,K}\mathbf{I}_M \end{array} \right] \\ \vdots \\ x_t \left[\begin{array}{cccc} \mathbf{I}_M & \phi_{t,1}\mathbf{I}_M & \cdots & \phi_{t,K}\mathbf{I}_M \end{array} \right] \end{bmatrix} \begin{bmatrix} \mathbf{h}_\mathrm{d} \\ \mathbf{v}_1 \\ \vdots \\ \mathbf{v}_K \end{bmatrix} + \begin{bmatrix} \mathbf{n}_1 \\ \vdots \\ \mathbf{n}_T \end{bmatrix} \tag{15}$$

The reflection pattern $\mathbf{\Phi}$ is constructed as a $T \times (K+1)$ square matrix with $T = K + 1$,

$$\mathbf{\Phi} = \begin{bmatrix} 1 & \mathbb{0}_K^T \\ \mathbb{1}_K & I_K. \end{bmatrix} \tag{16}$$

Note that T is set as $K + 1$ to estimate each column of the cascaded channel $\mathbf{V}$. The reflection pattern $\mathbf{\Phi}$ has full column rank. Obviously, if the estimation period T is set to be larger than $K + 1$, the reflection pattern still has full column rank. In practice, however, T should be configured as small as possible to reduce the overheads of pilot training. Based on the ON/OFF design, the covariance matrix of the LS estimator is

$$\begin{aligned} \mathbf{C}_{\hat{\theta}} &= \sigma^2 \left(\mathbf{\Phi}^H \mathbf{\Phi}\right)^{-1} \otimes \mathbf{I}_M \\ &= \sigma^2 \begin{bmatrix} 1 & -\mathbb{1}_K^T \\ -\mathbb{1}_K & E_K + I_K \end{bmatrix} \otimes \mathbf{I}_M. \end{aligned} \tag{17}$$

The variance of each estimated parameter (channel coefficient) is taken from the diagonal elements of the covariance matrix,

$$\begin{aligned} &\mathrm{var}\left(\left[\hat{\mathbf{h}}_\mathrm{d}\right]_m\right) = \sigma^2, m = 1, \ldots, M, \\ &\mathrm{var}\left(\left[\hat{\mathbf{v}}_k\right]_m\right) = 2\sigma^2, m = 1, \ldots, M, k = 1, \ldots, K. \end{aligned} \tag{18}$$

The ON/OFF method provides a simple way to design reflection patterns and the corresponding LS estimators. Several insights are drawn as follows:

- The estimation error of the direct channel coefficients $[\mathbf{h}_\mathrm{d}]_m$ propagates to the estimation of cascaded channel coefficients $[\mathbf{v}_k]_m$. Thus, the variance of $[\hat{\mathbf{v}}_k]_m$ is twice of the variance of $\left[\hat{\mathbf{h}}_\mathrm{d}\right]_m$.
- When the estimation period or pilot training period T is extended, the accuracy of estimation will not be improved (the variances do not depend on T). The variances depend on T.
- Since the LS estimator is adopted, the estimator performance does not depend on any statistical information of unknown parameters, such as spatial correlation of MIMO and MISO channels. Due to the sparsity of reflection pattern $\mathbf{\Phi}$, the LS estimator can be computed with $\mathcal{O}(TM)$ operations.

5 Optimal Design of Reflection Pattern

As shown in the equation (13), the performance of the estimator depends on the design of reflection pattern $\mathbf{\Phi}$. Although the ON/OFF design is simple and easy to implement, it cannot achieve the optimal performance (the lowest CRLB). In this section, the optimal design of reflection pattern $\mathbf{\Phi}$ is explored under the guidance of the CRLB.

5.1 Problem Formulation

According to [2], it is desirable to minimize the sum of estimation variances for all the parameters (i.e., the trace of the estimation covariance matrix $\mathbf{C}_{\hat{\boldsymbol{\theta}}}$). Let $\mathcal{L}$ be the set of phase quantization levels of IRS elements. In practice, uniform phase quantization is applied, that is $\mathcal{L} = 0, 2\pi/T, ..., 2\pi(T+1)/T$.

Problem 1 (P1):

$$\min_{\mathbf{\Phi}} \quad \mathrm{tr}\left(\mathbf{C}_{\hat{\boldsymbol{\theta}}}\right), \tag{19}$$

$$\textbf{s.t.}: \quad \mathbf{C}_{\hat{\boldsymbol{\theta}}} = \sigma^2\left(\mathbf{\Phi}^H\mathbf{\Phi}\right)^{-1} \otimes \mathbf{I}_M, \mathbf{\Phi} \in \mathbf{C}^{T\times(K+1)} \tag{20}$$

$$[\mathbf{\Phi}]_{t,1} = 1, t = 1, \ldots, T \tag{21}$$

$$\Phi_{t,k} = \beta_{t,k}\exp\left(jp_{t,k}\right), t = 1, \ldots, T; k = 1, \ldots, K \tag{22}$$

$$\beta_{t,k} \in [0,1], t = 1, \ldots, T; k = 1, \ldots, K \tag{23}$$

$$p_{t,k} \in \mathbf{L}, t = 1, \ldots, T; k = 1, \ldots, K, \tag{24}$$

where the objective function (19) is minimizing the sum of estimation variances; the constraint (20) gives the expression of estimation covariance matrix since the LS estimator attains the CRLB; the constraint (21) sets the first column of $\mathbf{\Phi}$ to one to incorporate the direct channel; the constraint (22) means that the reflection pattern is composed of amplitude change and phase shifts; according to constraints (23,24), the IRS is assumed not to amplify the signal and it only support uniform phase quantization.

Problem 1 is generally difficult to solve. However, an elegant method is composed to obtain the optimal solution to problem 1 based on the property of Fisher information matrix.

5.2 Problem Transformation

Recall that for the diagonal elements of Fisher information matrix, the following lower bound is derived:

$$\left[\mathbf{C}_{\hat{\boldsymbol{\theta}}}\right]_{i,i} = \left[\mathcal{I}^{-1}(\boldsymbol{\theta})\right]_{i,i} \geq \frac{1}{[\mathcal{I}(\boldsymbol{\theta})]_{i,i}}, \tag{25}$$

where the lower bound of CRLB can be achieved if $I^{-1}(\boldsymbol{\theta})$ is diagonal. Guided by this critical insight, we first assume that there is a feasible design such that $\mathbf{\Phi}^H\mathbf{\Phi}$ is diagonal (i.e., $I^{-1}(\boldsymbol{\theta})$ is diagonal). Based on this assumption, the optimization objective is transformed into maximizing $\mathrm{tr}\left(\mathbf{\Phi}^H\mathbf{\Phi}\right)$. Thus, problem 1 can be transformed into

Problem 2 (P2):

$$\max_{\mathbf{\Phi}} \quad \mathrm{tr}\left(\mathbf{\Phi}^H\mathbf{\Phi}\right), \tag{26}$$

$$\textbf{s.t.}: \quad (20) - (24), \tag{27}$$

$$\mathbf{\Phi}^H\mathbf{\Phi} = \mathrm{diag}(\alpha_1, ..., \alpha_{TM}), \tag{28}$$

where the objective (26)is still minimizing the sum of estimation variances, and the constraint (28) gives the restriction on the reflection pattern such that $\mathbf{\Phi}^H\mathbf{\Phi}$ is diagonal.

For the previous ON/OFF method, $\mathbf{\Phi}^H\mathbf{\Phi}$ is not diagonal, and thus it cannot be the solution to problem 2. Next, we elaborate on how to find the optimal solution to problem 2, i.e., the optimal reflection pattern $\mathbf{\Phi}$.

5.3 DFT-based Method

Given that $\mathbf{\Phi}^H\mathbf{\Phi}$ is diagonal, we first given a thought on unitary matrix $\mathbf{U}$ such that $\mathbf{U}^H\mathbf{U} = \mathbf{I}$. Combined with the form of elements in $\mathbf{\Phi}$ as shown in the constraint (22), it is natural to come up with DFT matrix $\mathbf{W}$,

$$W = \left(\frac{e^{j\frac{2\pi}{N}mn}}{\sqrt{N}}\right)_{m,n=0,\dots,N-1}, \tag{29}$$

and it can be shown easily that the DFT matrix is a unitary matrix,

$$[W^H W]_{mn} = \sum_{k=0}^{N-1} e^{j\frac{2\pi}{N}(m-n)k} = \frac{1-e^{j\frac{2\pi}{N}(m-n)N}}{1-e^{j\frac{2\pi}{N}(m-n)}} = 0, \forall m \neq n, \tag{30}$$

$$WW^H = W^H W = I. \tag{31}$$

Inspired by DFT matrix, we design the reflection pattern matrix in the following way:

$$[\mathbf{\Phi}]_{t,k} = [F_{T,K+1}]_{t,k} = \exp(-j2\pi(t-1)(k-1)/T), \tag{32}$$

where $t = 1, \dots, T, k = 1, \dots, K+1$. We can also derive that

$$\mathbf{\Phi}^H\mathbf{\Phi} = F_{T,K+1}^H F_{T,K+1} = TI_{K+1}, \quad \forall T \geq K+1, \tag{33}$$

and thus this design must satisfy the constraints in problem 2. Further, this design is proved to be optimal since the objective of problem 2 has the following upper bound:

$$\operatorname{tr}\left(\mathbf{\Phi}^H\mathbf{\Phi}\right) = \sum_{t=1}^{T}\sum_{k=1}^{K+1} \left|[\mathbf{\Phi}]_{t,k}\right|^2 \leq (K+1)T. \tag{34}$$

where $t = 1, \dots, T, k = 1, \dots, K+1$, and the designed matrix $F_{T,K+1}$ obviously achieves this upper bound,

$$\operatorname{tr}\left(\mathbf{\Phi}^H\mathbf{\Phi}\right) = \operatorname{tr}\left(F_{T,K+1}^H F_{T,K+1}\right) = \operatorname{tr}\left(TI_{K+1}\right) = T(K+1). \tag{35}$$

Therefore, it is validated that $F_{T,K+1}$ is indeed the optimal solution to problem 2, and also the optimal solution to problem 1. Based on the optimal design of IRS reflection pattern, the covariance matrix of the LS estimator is derived as

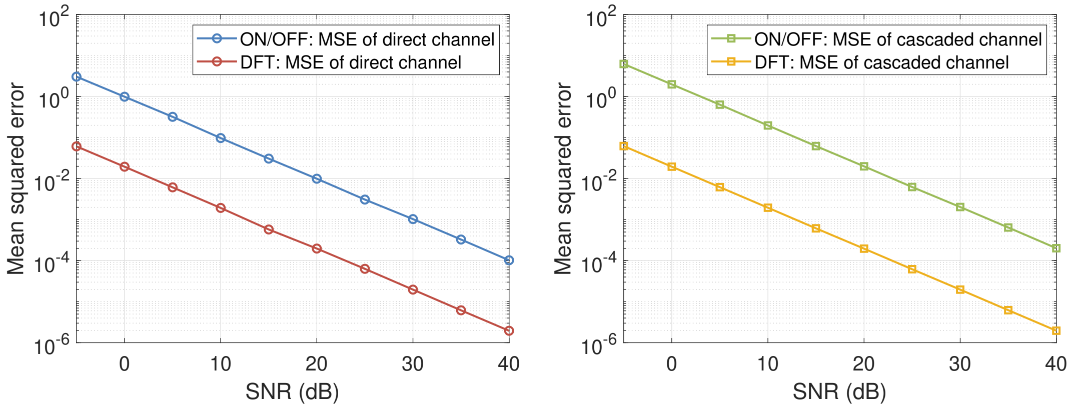
$$\begin{aligned} \mathbf{C}_{\hat{\boldsymbol{\theta}}} &= \sigma^2\left(\mathbf{\Phi}^H\mathbf{\Phi}\right)^{-1} \otimes \mathbf{I}_M \\ &= \sigma^2\left(F_{T,K+1}^H F_{T,K+1}\right)^{-1} \otimes \mathbf{I}_M \\ &= \frac{\sigma^2}{T} I_{M(K+1)}. \end{aligned} \tag{36}$$

The variance of each estimated parameter (channel coefficient) is taken from the diagonal elements of the covariance matrix,

$$\operatorname{var}\left(\left[\hat{h}_{\mathrm{d}}\right]_m\right) = \operatorname{var}\left([\hat{v}_k]_m\right) = \frac{\sigma^2}{T}, m = 1, \dots, M, k = 1, \dots, K \tag{37}$$

DFT-based method is proved to be an ingenious way to construct the optimal IRS reflection pattern. Several insights are drawn as follows:

- The estimation error of the direct channel coefficients $[\mathbf{h}_{\mathrm{d}}]_m$ will not propagate to the estimation of cascaded channel coefficients $[\mathbf{v}_k]_m$. Actually, the variance of $[\hat{\mathbf{v}}_k]_m$ is equal to the variance of $\left[\hat{\mathbf{h}}_{\mathrm{d}}\right]_m$.
- When the estimation period or pilot training period T is extended (larger than $K+1$), the performance of the estimator will be improved.
- Due to the choice of dense matrix $\mathbf{\Phi}$, the LS estimator can be computed with $\mathcal{O}(TM\log(T))$ operations, which is more complex than the ON/OFF method by $\log(T)$.



(a) Direct channel (b) Cascaded channel

Figure 3: MSE for direct channel and cascaded channel with varying SNR.

6 Simulation and Numerical Results

6.1 Simulation setup

We assume a block-fading Rayleigh channel with coherence time T, i.e., the channel remains unchanged within each transmission block of length T and varies from block to block. T is viewed as the estimation period with the smallest value $K+1$. Per-column model is used to simulate a spatial correlation for SIMO channels, where $\boldsymbol{\theta}^{\text{corr}} = \mathbf{R}^{\frac{1}{2}}\boldsymbol{\theta}$, where $\mathbf{R} \in \mathbf{R}^{M(K+1)\times M(K+1)}$. The correlation matrix $\mathbf{R}$ is given as $[\mathbf{R}]_{i,j} = r^{|i-j|}$, where r is the correlation coefficient within $[0,1]$. The pilot signals $x_1, ..., x_T$ are randomly generated and then modulated via QPSK/QAM modulation. Monte-Carlo simulations are conducted for each configuration with $N = 1000$, which means 1000 independent channel and AWGN realizations are generated. The performance of estimators is measured by mean squared errors (MSE) $e = \sum_{n=1}^{1000}(\hat{\boldsymbol{\theta}}_m - \boldsymbol{\theta}_m)^2, m = 1, ..., M(K+1)$. Then the MSE is averaged over different scalar parameters. For example, the MSE of direct channel estimation is presented as $e_d = \sum_{n=1}^{1000}\sum_{m=1}^{M}(\hat{\boldsymbol{\theta}}_m - \boldsymbol{\theta}_m)^2$.

6.2 Estimation for Direct and Cascaded Channels

Let $M = 10$, $K = 50$, and $T = K+1 = 51$. The variance of noise varies from 10^{-4} to $10^0.5$, such that the MSEs under various received Signal noise ratio (SNR) can be evaluated. In figure (3), we compare the estimation performances under ON/OFF method and DFT-based method. For the direct channel parameters, the MSE based on DFT-based method are approximately $\frac{1}{T}$ of that based on ON/OFF method. For the cascaded channel parameters, the MSE based on DFT-based method are approximately $\frac{2}{T}$ of that based on ON/OFF method. Revealed by figure (4), one drawback of ON/OFF method is that the MSE of cascaded channel parameters is twice larger than that of direct channel parameters. By utilizing DFT-based method, the error of direct channel estimation will not propagate to the cascaded channel estimation.

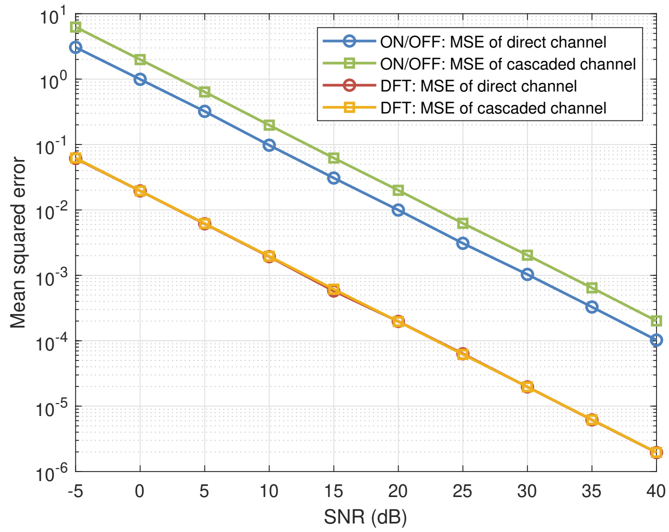


Figure 4: MSE based on two different methods with varying SNR.

6.3 Various numbers of IRS elements

When the number of IRS elements increases (K is usually large for massive IRS) and the estimation period $T = K + 1$, one critical insight we draw from the theory is that the estimation performance based on DFT-based method will be improved. To validate this point, the noise variance is fixed as 10^{-2}, and K varies from 20 to 100. In figure (5), as K increases and thus T also increases, the MSE based on ON/OFF method remains unchanged, while the MSE based on DFT-based method decreases.

6.4 Extended estimation periods

Finally, the number of IRS elements is fixed as 50 and the noise variance is fixed as 10^{-2}. Only the estimation period T is extended from 51 to 100. As shown in figure (6) ,as T increases, the MSE based on DFT-based method decreases. Although extending T incurs more overheads, it can effectively improve the estimation accuracy.

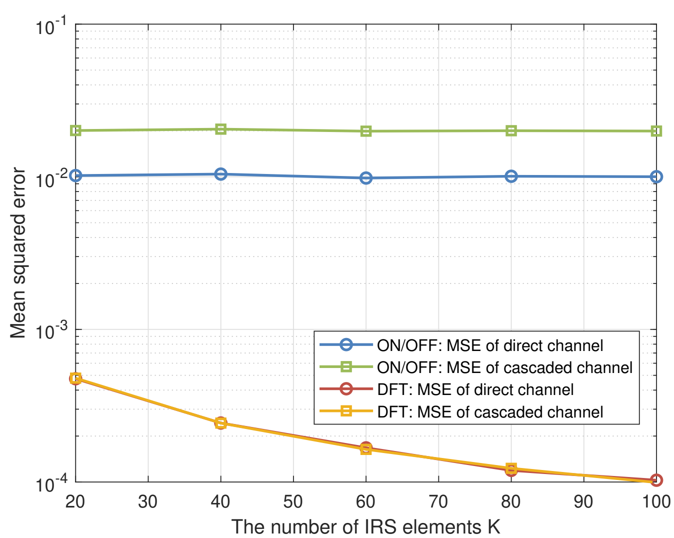


Figure 5: MSE with various numbers of IRS elements.

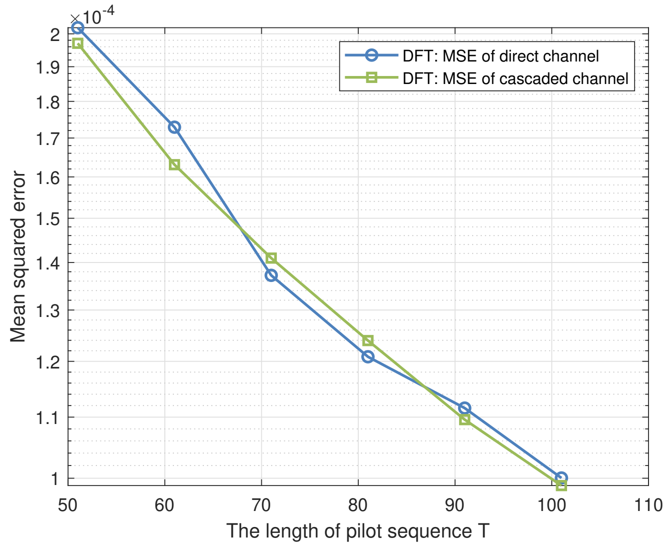


Figure 6: MSE with various estimation periods.

7 Summary and Further Discussion

7.1 Summary

In this report, joint design of IRS reflection patterns and channel estimation is explored via two popular methods, namely the ON/OFF method and DFT-based method. At the sacrifice of computational complexity, the DFT-based method achieves much higher estimation accuracy than the ON/OFF method. The estimation performance of DFT-based method can be further improved by extending the estimation period, i.e., adding extra pilot signals.

7.2 LMMSE Estimator

For the previous sections, we focus on least square estimators that do not include any prior information. In this section, we study the performance of a Bayesian estimator, i.e., LMMSE estimator [10]. Assume that we already know the mean and correlation matrix of the unknown channels $\boldsymbol{\theta}$. The noise in the MISO and MIMO channels is assumed to be AWGN with the circle complex Gaussian distribution.

$$\mathbb{E}[\boldsymbol{\theta}] = \mathbf{0}, \quad \mathbb{E}[\boldsymbol{\theta}\boldsymbol{\theta}^H] = \mathbf{R}_\theta. \tag{38}$$

$$\mathbf{N} \sim \mathcal{CN}\left(0, \sigma^2 \mathbf{I}_{TM}\right). \tag{39}$$

Then the LMMSE estimator is given by

$$\hat{\boldsymbol{\theta}} = \left(\mathbf{R}_\theta^{-1} + \frac{1}{\sigma^2}\mathbf{H}^H\mathbf{H}\right)^{-1} \frac{1}{\sigma^2}\mathbf{H}^H\mathbf{S}. \tag{40}$$

The error covariance matrix is

$$\mathbf{C}_\epsilon = \left(\mathbf{R}_\theta^{-1} + \frac{1}{\sigma^2}\mathbf{H}^H\mathbf{H}\right)^{-1} = \left(\mathbf{R}_\theta^{-1} + \frac{1}{\sigma^2}\boldsymbol{\Phi}^H\boldsymbol{\Phi} \otimes \mathbf{I}_M\right)^{-1}, \tag{41}$$

and the Bayesian mean squared error is

$$\mathrm{Bmse}(\hat{\boldsymbol{\theta}}_i) = [\mathbf{C}_\epsilon]_{ii}. \tag{42}$$

Since the channel spatial correlation matrix $\mathbf{R}_\theta$ is given as constant, the optimal reflection pattern design that minimizes the sum of Bayesian errors is the same as the non-bayesian case.

7.3 Joint design of pilot signals and reflection patterns

One critical assumption is that we assume equal power allocation and normalization on pilot signals $x_1, ..., x_T$ such that $\mathbf{X}^H\mathbf{X}$ is an identity matrix. This assumption significantly simplifies the optimization problem. However, joint design of pilot signals $\mathbf{X}$ and reflection patterns $\mathbf{\Phi}$ is an interesting topic left for future work, which can further improve the estimator performance. In this report, we focus on the uplink SIMO channel of a single user with only one antenna. The pilot signal design for MIMO channel estimation will be more complicated, as the orthogonality among multiple pilot signals should be carefully considered [10].

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