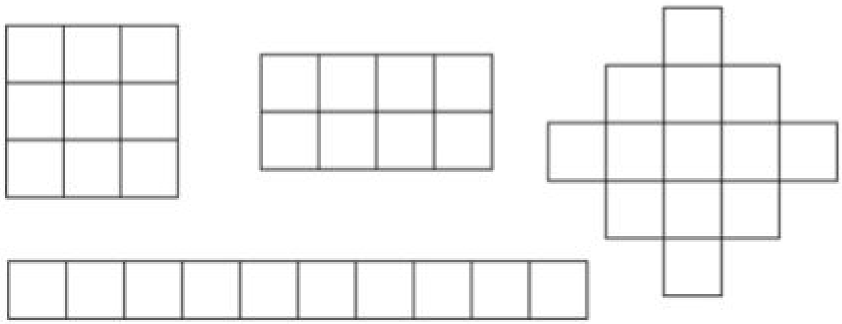
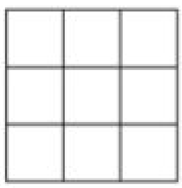
Assignment A2

Part 1: paper-and-pencil

Assignment: 2nd Moments NCC, Evaluating Binary Classifiers of Videos

1. Using 2nd moments for binary image analysis (Use material from lectures on Feb. 2 and
    1. Given four objects with different shapes, please calculate the following values respectively for each object.



a. 

i. Location:

Centroid: measurement of location $(\overline{X}, \overline{Y})$ assuming each shape starts in the origin (0,0).

$\overline{x_i} = 0 + \frac{1}{2}base = \frac{1}{2}(3-0) = \frac{1}{2}(3) = \frac{3}{2} = 1.5$

$\overline{y_i} = 0 + \frac{1}{2}base = \frac{1}{2}(3-0) = \frac{1}{2}(3) = \frac{3}{2} = 1.5$

$A_i = base \times height = 3 \times 3 = 9$

$\overline{x_i}A_i = 1.5 \times 9 = 13.5$

$\overline{y_i}A_i = 1.5 \times 9 = 13.5$

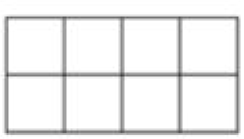
$\overline{X} = \frac{\Sigma \overline{x_i} A_i}{\Sigma A_i} = \frac{13.5}{9} = 1.5$

$\overline{Y} = \frac{\Sigma \overline{y_i} A_i}{\Sigma A_i} = \frac{13.5}{9} = 1.5$

ii. Circularity (E_{min}/E_{max}): P^2/A

$P^2/A = (8)^2/(9) = 64/9 = 7.1$

iii. Orientation:

b. 

i. Location:

Centroid: measurement of location $(\overline{X}, \overline{Y})$ assuming each shape starts in the origin.

$\overline{x_i} = 0 + \frac{1}{2}base = \frac{1}{2}(4 - 0) = \frac{1}{2}(4) = \frac{4}{2} = 2$

$\overline{y_i} = 0 + \frac{1}{2}base = \frac{1}{2}(2 - 0) = \frac{1}{2}(2) = \frac{2}{2} = 1$

$A_i = base \times height = 4 \times 2 = 8$

$\overline{x_i}A_i = 2 \times 8 = 16$

$\overline{y_i}A_i = 1 \times 8 = 8$

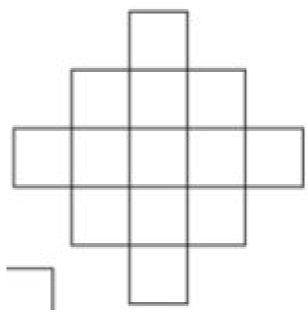
$\overline{X} = \frac{\Sigma\overline{x_i}A_i}{\Sigma A_i} = \frac{16}{8} = 2$

$\overline{Y} = \frac{\Sigma\overline{x_i}A_i}{\Sigma A_i} = \frac{1}{8} = 0.125$

ii. Circularity (E_{min}/E_{max}): P^2/A

$P^2/A = (8)^2/(8) = 64/8 = 8$

iii. Orientation:

c. 

i. Location:

Centroid: measurement of location $(\overline{X}, \overline{Y})$ assuming each shape starts in the origin.

$\overline{x_i} = 0 + \frac{1}{2}base = \frac{1}{2}(5 - 0) = \frac{1}{2}(5) = \frac{5}{2} = 2.5$

$\overline{y_i} = 0 + \frac{1}{2}base = \frac{1}{2}(5 - 0) = \frac{1}{2}(5) = \frac{5}{2} = 2.5$

$A_i = base \times height = 3 \times 3 + 4 = 13$

$\overline{x_i}A_i = 2.5 \times 13 = 32.5$

$\overline{y_i}A_i = 2.5 \times 13 = 32.5$

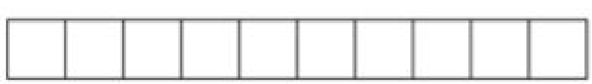
$\overline{X} = \frac{\Sigma\overline{x_i}A_i}{\Sigma A_i} = \frac{32.5}{13} = 2.5$

$\overline{Y} = \frac{\Sigma\overline{x_i}A_i}{\Sigma A_i} = \frac{32.5}{13} = 2.5$

ii. Circularity (E_{min}/E_{max}): P^2/A

$P^2/A = (8)^2/13 = 64/13 = 4.92$

iii. Orientation:

d. 

i. Location:

Centroid: measurement of location $(\overline{X}, \overline{Y})$ assuming each shape starts in the origin.

$\overline{x_i} = 0 + \frac{1}{2}base = \frac{1}{2}(10 - 0) = \frac{1}{2}(10) = \frac{10}{2} = 5$

$\overline{y_i} = 0 + \frac{1}{2}base = \frac{1}{2}(1 - 0) = \frac{1}{2}(1) = \frac{1}{2} = 0.5$

$A_i = base \times height = 10 \times 1 = 10$

$\overline{x_i}A_i = 5 \times 10 = 50$

$\overline{y_i}A_i = 0.5 \times 10 = 5$

$\overline{X} = \frac{\Sigma\overline{x_i}A_i}{\Sigma A_i} = \frac{50}{10} = 5$

$\overline{Y} = \frac{\Sigma\overline{x_i}A_i}{\Sigma A_i} = \frac{0.5}{10} = 0.05$

ii. Circularity (E_{min}/E_{max}): P^2/A

$P^2/A = (10)^2/(10) = 100/10 = 10$

iii. Orientation:

If the centroid were to fall in between pixels I would take the average of the pixel before it and the pixel after it.

2. Yes. This would change my answers from above because it would change the center location of the object. For example, for the images, my values for $\overline{x_i}$ and $\overline{y_i}$ would be different. This would in turn also change the centroid values.
3. No. If the image is larger it should still have the same values as above because the objects are still the same ratio.

2. *NCC*
   1. Show that the normalized correlation coefficient r is invariant to linear brightness changes in the scene s (or template m). This means you need to prove $r(m, s) = r(m, s + b)$ for images m and s, and some constants a and b.
   We defined the normalized correlation coefficient in class as
   $r = \frac{1}{n}\Sigma_i((s_i - mean\ (s)) \times (m_i - mean(m)))\ /\ (\sigma_s\sigma_m)$,
   where s_i and m_i are the respective brightness values of the ith pixel, mean(m) and σ_m are mean and standard deviation of all pixels in the template and mean(s) and σ_s are mean and standard deviation of all pixels in the sub-image of the scene.

$$r(m, as + b) = \frac{1}{n}\sum_i \frac{((as_i+b)-\overline{(as+b)})(m_i-\overline{m})}{\sigma_{as+b}\sigma_m}$$

$$\overline{(as + b)} = \frac{1}{n}\sum_i(as_i + b) = a \times \frac{1}{n}\sum_i s_i + b = a\overline{s} + b$$

$$\sigma_{as+b} = \sqrt{\frac{1}{n}\sum_i((as_i + b) - \overline{(as + b)})^2}$$

$$= \sqrt{\frac{1}{n}\sum_i((as_i + b) - (a\overline{s} + b))^2}$$

$$= \sqrt{\frac{1}{n}\sum_i(a(s_i - \overline{s}))^2}$$

$$= a\sqrt{\frac{1}{n}\sum_i((s_i - \overline{s}))^2}$$

$$= \frac{1}{n}\sum_i \frac{(s_i - \overline{s})(m_i - \overline{m})}{\sigma_s\sigma_m}$$

$$r(m, as + b) = r(m, s\ )$$

2. Explain why the linear invariance property of the normalized correlation coefficient, shown in part (a), could be useful for image analysis.
    a. This could be useful in image analysis because it allows for template matches with different brightness levels.
3. The range of the *NCC* is between -1 and 1: -1 <= r <= 1. Explain why this property could be useful for image analysis.
    a. This property is useful for image processing because if we visualize image analysis with unit vectors then their dot product would have an output that the range of the *NCC* would grasp: [-1 . . .1]. 1 giving us back the feedback of a perfect match, and -1 giving us the feedback that it's not correlated. Bright pixels match with dark pixels and dark pixels match with bright pixels.
4. The expected value of the *NCC* is $0 : E[r] = 0$. Explain why this property could be useful for image analysis
    a. *NCC* being 0 means that the images being compared don't have a mutual relationship (they don't match).

3. Evaluating Binary Classifiers of Videos

   Suppose that we have a test dataset of ultrasound videos of the lungs of 200 babies -- 120 have pneumonia and 80 do not have pneumonia. We use it to test the performance of a 2-class video classification model that predicts which baby has pneumonia. Our experiment produces the following confusion matrix.

Confusion Matrix		True Class	
		Pneumonia	Healthy
Hypothesized Class	Pneumonia	64	24
	Healthy	X	Y

TP	*FP*
FN	*TN*

1. $X = 36$
2. $Y = 56$
3. Recall: $\frac{TP}{TP+FN} = \frac{64}{64+36} = \frac{64}{100} = 0.64$
4. Precision: $\frac{TP}{TP+FP} = \frac{64}{64+24} = \frac{64}{88} = 0.73$
5. Accuracy: $\frac{TP+TN}{TP+TN+FP+FN} = \frac{64+56}{64+56+24+36} = \frac{120}{180} = 0.67$
6. Specific: $\frac{TN}{TN+FP} = \frac{56}{56+24} = \frac{56}{80} = 0.7$ aka. 70% specificity
7. Sensitivity: $\frac{TP}{TP+FN} = \frac{64}{64+36} = \frac{64}{100} = 0.64$ aka 64% sensitive
8. The most important to improve would be the false negative (*FN*) because this is crucial data for the next steps of the baby's health. If a baby is diagnosed with having no more pneumonia then the treatment would be slowed down if not stopped. As mentioned, when a baby has a false alarm of being healthy further precautions are put into place, by having X Ray ordered etc. Even Though they are double checking that the baby is healthy (which is a good approach) this will still slow down the treatment process because the patient is not being treated as if they have pneumonia.