Assignment A3

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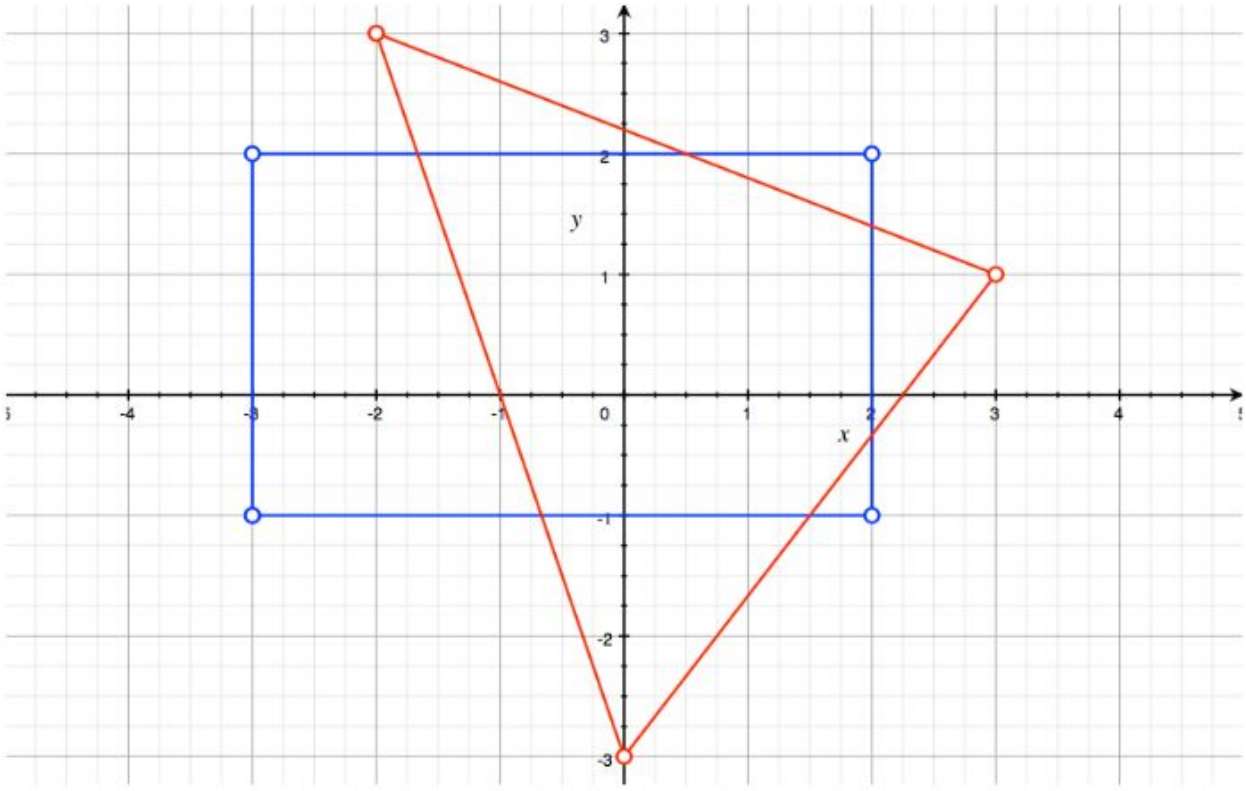
Exercise 1: *Hausdorff Distance*

There are two shapes in the figure below. The triangle in red has three vertices: A(-2,3), B(3,1), and C(0,-3). The rectangle in blue has four vertices: D(-3,2), E(2,2), F(2,-1), and G(-3, -1).



1. Consider two point sets *S1* and *S2*, where $S1 = \{A, B, C\}$ and $S2 = \{D, E, F, G\}$. Calculate the Hausdorff distance between these two point sets.

   $d_H(S1, S2) = max\{sup_{s1 \in S1} inf_{s2 \in S2} d(x, y),\ sup_{s2 \in S2} inf_{s1 \in S1}\ d(S1, S2)\}$

   $d_H(S1, S2) = inf\{\varepsilon \geq 0; S1 \subseteq S2_\varepsilon \text{ and } S2 \subseteq S1_\varepsilon\}$

   $A(-2, 3) \rightarrow D(-3, 2) : 1.414214$

   $A(-2, 3) \rightarrow E(2, 2) : 4.123106$

   $A(-2, 3) \rightarrow F(2, -1) : 5.656854$

   $A(-2, 3) \rightarrow G(-3, -1) : 4.123106$

   $B(3, 1) \rightarrow D(-3, 2) : 6.082763$

$B(3,1) \rightarrow E(2,2) : 1.414214$

$B(3,1) \rightarrow F(2,-1) : 2.236068$

$B(3,1) \rightarrow G(-3,-1) : 6.324555$

$C(0,-3) \rightarrow D(-3,2) : 5.830952$

$C(0,-3) \rightarrow E(2,2) : 5.385165$

$C(0,-3) \rightarrow F(2,-1) : 2.828427$

$C(0,-3) \rightarrow G(-3,-1) : 3.605551$

Hausdorff Distance: max (1.414214 , 2.828427 , 1.414214) = 2.828427

2. Now consider all the points forming these two polygons. What's the Hausdorff distance between the triangle and the rectangle?

   The Hausdorff Distance would be the same as shown above: 2.828427. The reason why it would be the same is because even if we consider every possible point from the vertices, which could be an infinite number of solutions, the result would be the same. Because we would have to choose the lowest distance but at the end choosing the maximum of these lowest distances would yield the same result.

Exercise 2: *Segmentation*

The following tables include all the local maxima and local minima of the grayscale histogram of an image.

Table 1: Local Maxima

Gray Values	52	54	103	231
# of Pixels	1000	1170	1750	1300

Table 2: Local Minima

Gray Values	0	53	75	157	255
# of Pixels	500	890	240	190	310

Calculate the highest peakiness and the corresponding threshold using Mode Method.

Mode Method: $[min\{H(g_i), H(g_j)\}]/H(g_k)$

$p(52, 54, 53) : min\{1000, 1170\}/890 = 1000/890 = 1.123595505617978$

$p(54, 231, 157) : min\{1170, 1750\}/240 = 1170/240 = 4.875$

$p(103, 231, 157)$: $min\{1750, 1300\}/190 = 1300/190 = 6.842$
$max(1.123595505617978, 4.875, 6.842) = 6.842$
Threshold $(g_k) = 157$