

Mathematical Physics

Pugazharasu

July 2020

Preface

This document is simply the collective summary of all of the fundamental mathematics that I have learnt over the years. This assumes a little mathematical maturity. More niche subjects such as analysis, topology and Lie theory are relegated in separate documents. This should only be treated as a revision/recall document and not as a learning material since it is devoid of lengthy explanations and often assumes you have at least briefly read through the cited texts or similar material.

Contents

Preface	vi
1 Rings	1
1.1 Definition	1
1.2 Formal Construction of Integers and Polynomials	1
2 Galois Theory	3
2.1 Heuristics	4
2.1.1 The big question	4
2.1.2 More big questions	4
2.1.3 Visualizing field extensions	4
2.1.4 Irreducible polynomials	4
2.1.5 Galois groups	4
2.1.6 The heart of Galois theory	4
2.1.7 Unsolvability	4
2.2 The Main Theorem of Galois Theory	4
2.3 Cubic Equations	4
2.4 Symmetric Functions	4
2.5 Primitive Elements	4
2.6 Proof of the Main Theorem	4
2.7 Quartic Equations	4
2.8 Kummer Extensions	4
2.9 Cyclotomic Extensions	4
2.10 Quintic Equations	4
3 Unconstrained Optimization & Linear Programming	5
3.1 Basic Concepts	5
3.2 Linear Programming	5
3.3 Simplex Method	5
3.3.1 Simplex Method: Difficulties	5

4	Graphs & Combinatorial Optimization	7
4.1	Combinatorial Optimization	7
4.2	Graphs	8
4.3	Digraphs	8
4.4	Shortest Path Problems	8
4.4.1	Complexities	8
4.5	Bellman's Principle	8
4.6	Dijkstra's Algorithm	8
4.7	Shortest Spanning Trees	8
4.7.1	Greedy's Algorithm	8
4.7.2	Prim's Algorithm	8
4.8	Flows in Networks	8
4.8.1	Maximum Flow: Ford-Fulkerson Algorithm	8
4.9	Bipartite Graphs	8
5	Probability	9
5.1	Venn Diagrams	10
5.2	Probability	10
5.3	Permutations and Combinations	10
5.4	Random Variables and Distributions	10
5.5	Properties of Distributions	10
5.6	Functions of Random Variables	10
5.7	Generating Functions	10
5.8	Important Discrete Distributions	10
5.9	Important Continuous Distributions	10
5.10	The Central Limit Theorem	10
5.11	Joint Distributions	10
5.12	Properties of Joint Distributions	10
5.13	Generating Functions for Joint Distributions	10
5.14	Important Joint Distributions	10
6	Statistics	11

Chapter 1

Rings

1.1 Definition

1.2 Formal Construction of Integers and Polynomials

Chapter 2

Galois Theory

2.1 Heuristics

2.1.1 The big question

2.1.2 More big questions

2.1.3 Visualizing field extensions

2.1.4 Irreducible polynomials

2.1.5 Galois groups

2.1.6 The heart of Galois theory

2.1.7 Unsolvability

2.2 The Main Theorem of Galois Theory

2.3 Cubic Equations

2.4 Symmetric Functions

2.5 Primitive Elements

2.6 Proof of the Main Theorem

2.7 Quartic Equations

2.8 Kummer Extensions

2.9 Cyclotomic Extensions

2.10 Quintic Equations

Chapter 3

Unconstrained Optimization & Linear Programming

3.1 Basic Concepts

3.2 Linear Programming

3.3 Simplex Method

3.3.1 Simplex Method: Difficulties

Chapter 4

Graphs & Combinatorial Optimization

4.1 Combinatorial Optimization

Combinatorial Optimization concerns optimization problems of a discrete or combinatorial structure. It uses graphs and digraphs as basic tools.

4.2 Graphs

4.3 Digraphs

4.4 Shortest Path Problems

4.4.1 Complexities

4.5 Bellman's Principle

4.6 Dijkstra's Algorithm

4.7 Shortest Spanning Trees

4.7.1 Greedy's Algorithm

4.7.2 Prim's Algorithm

4.8 Flows in Networks

4.8.1 Maximum Flow: Ford-Fulkerson Algorithm

4.9 Bipartite Graphs

Chapter 5

Probability

- 5.1 Venn Diagrams
- 5.2 Probability
- 5.3 Permutations and Combinations
- 5.4 Random Variables and Distributions
- 5.5 Properties of Distributions
- 5.6 Functions of Random Variables
- 5.7 Generating Functions
- 5.8 Important Discrete Distributions
- 5.9 Important Continuous Distributions
- 5.10 The Central Limit Theorem
- 5.11 Joint Distributions
- 5.12 Properties of Joint Distributions
- 5.13 Generating Functions for Joint Distributions
- 5.14 Important Joint Distributions

Chapter 6

Statistics

- 6.1 Experiments, samples and populations
- 6.2 Sample Statistics
- 6.3 Estimators and Sampling Distributions
- 6.4 Some Basic Estimators
- 6.5 Maximum-Likelihood Method
- 6.6 The Method of Least Squares
- 6.7 Hypothesis Testing

Bibliography

- [1] Jean Claude Dutailly. *Mathematics for theoretical physics*.
arXiv:1209.5665v2
- [2] Michel Goossens, Frank Mittelbach, and Alexander Samarin. *The L^AT_EX Companion*. Addison-Wesley, Reading, Massachusetts, 1993.
- [3] Nathan Carter *Visual Group Theory*. Annalen der Physik, 322(10):891–921, 1905.
- [4] James R. Munkres *Topology*. Annalen der Physik, 322(10):891–921, 1905.
- [5] John Stillwell *Naive Lie Theory*. Annalen der Physik, 322(10):891–921, 1905.
- [6] Michael Artin *Algebra* Computers and Typesetting,
<http://www-cs-faculty.stanford.edu/~uno/abcde.html>