A Summary of Graph Theory

Pugazharasu A D

November 30, 2020

Contents

Chapter 1

Basics

- A Graph is a set of objects and the relationships between pairs of objects
- A Graph $G(V, E)$, is a set of V **Vertices/nodes** and E **Edges**

Figure 1.1: A visual representation of a simple Graph

- For the above figure we say that:
  - e **Connects** u and v
  - u and v are **End Points** of e
  - u and e are **Incident**
  - u and v are **Adjacent**
  - u and v are **Neighbors**
- Or in set theory lingo as $G(\{u, v\}, \{e\})$
- There also exist **directed Edges/Arcs** i.e. , they describe asymmetric relations
- Adding 1.2.1 to another directed graph with the same vertices but the edge pointing in the other direction results in a non-directed graph

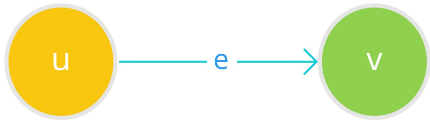

Figure 1.2: A visual representation of a simple directed Graph. Here u is called the **tail** and v the **head**

- **Degree** of a vertex is the number of its incident edges i.e. neighbours denoted by $deg(v)$
- The degree of a graph is the maximum degree of its vertices
- A **Regular graph** is a graph where each vertex has the same degree
- A regular graph of n degrees is called $n-$Regular
- The Complement of a graph $G = (V, E)$ is a graph $\bar{G} = (V, \bar{E})$ on the same set of vertices V and the following set of edges:
  - Two vertices are connected in $\bar{G}$ iff they are not connected in G i.e. $(u, v) \in \bar{E}\ iff\ (u, v) \notin E$
  - A **Path** is a continuous sequence of edges that connect two vertices
  - A **Walk** in a graph is a sequence of edges, such that each edge except for the first one starts with a vertex where the previous edge ended
  - The **Length** of a walk is the number of edges in it
  - A **Path** (rigorously) is a walk where all edges are distinct
  - A **Simple Path** is a walk where all vertices are distinct
- A **Cycle** in a graph is a path whose first vertex is the same as the last one; In particular, *all the edges in a Cycle are distinct*
- A **Simple Cycle** is a cycle where all vertices except for the first one are distinct and there first vertex is taken twice
- A graph is called **Connected** if there is a path between every pair of its vertices
- A **Connected Component** of a graph G is a maximal connected subgraph of G i.e., a connected subgraph of G which is not contained in a larger connected subgraph of G

- The **Indegree** of a vertex v is the number of edges ending at v
- The **Outdegree** of a vertex v is the number of edges leaving v
- A **Weighted Graph** associates a *weight* with every edge
- The **Weight** of a path is the sum of the weights of its edges
- A **Shortest Path** between two vertices is a path of the minimum weight
- The **Distance** between two vertices is the length of a shortest path between them
- A **Path Graph** $P_n \; \forall \; n \geq 2$, has n vertices labeled v_n and $n-1$ edges $\{v_{n-1}, v_n\}$
- A **Cycle Graph** $P_n \; \forall \; n \geq 2$, has n vertices labeled v_n and $n-1$ edges $\{v_{n-1}, v_n\}$

1.1 Trees

1.1.1 How to make a tree?

1.2 Bipartite Graphs

- A graph G is **Bipartite** if its vertices can be partitioned into two disjoint sets (sets with no common elements) L and R such that every edge of G connects a vertex in L to a vertex in R i.e., no edge connects two vertices from the same part
- L and R are called the parts of G

1.2.1 Examples and counter examples

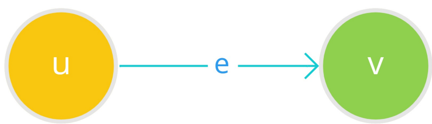

Figure 1.3: C_5 is not bipartite. In general, for odd $n > 2$, C_n is not bipartite

Chapter 2

Optimization