

Mathematical Physics

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July 2020

Preface

This document is simply the collective summary of all of the fundamental mathematics that I have learnt over the years. This assumes a little mathematical maturity. More niche subjects such as analysis, topology and Lie theory are relegated in separate documents. This should only be treated as a revision/recall document and not as a learning material since it is devoid of lengthy explanations and often assumes you have at least briefly read through the cited texts or similar material.

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Chapter 1

Rings

1.1 Definition

1.2 Formal Construction of Integers and Polynomials

Chapter 2

Galois Theory

2.1 Heuristics

2.1.1 The big question

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Chapter 3

Unconstrained Optimization & Linear Programming

3.1 Basic Concepts

3.2 Linear Programming

3.3 Simplex Method

3.3.1 Simplex Method: Difficulties

Chapter 4

Graphs & Combinatorial Optimization

4.1 Combinatorial Optimization

Combinatorial Optimization concerns optimization problems of a discrete or combinatorial structure. It uses graphs and digraphs as basic tools.

4.2 Graphs

4.3 Digraphs

4.4 Shortest Path Problems

4.4.1 Complexities

4.5 Bellman's Principle

4.6 Dijkstra's Algorithm

4.7 Shortest Spanning Trees

4.7.1 Greedy's Algorithm

4.7.2 Prim's Algorithm

4.8 Flows in Networks

4.8.1 Maximum Flow: Ford-Fulkerson Algorithm

4.9 Bipartite Graphs

Chapter 5

Probability

Chapter 6

Statistics

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