Group Theory Notes

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Chapter 1

The First Chapter

1.1 Representation

A representation R of a group element g is a one to one map to an element of a vector space i.e. it is homeomorphic.

$$g \rightarrow R(g)$$

The following properties are preserved:

- $R(e) = I$
- $R(g^{-1}) = (R(g))^{-1}$
- $R(g) \circ R(h) = R(gh)$

A representation identifies with each point (abstract group eement) of the group manifold (the abstract group) a linear transformation of a vector space. Ggenerally if one accepts arbitrary (not linear) transformations of an arbitrary (not necessarily a vector) space. Such a map is called a realization.

1.1.1 Similartiy Transform

$$R \rightarrow R^{'} := S^{-1} R S$$

This means that if we have a representation, we can transform its elements wildly with literally any non-singular matrix S [1] to get nicer matrices.

[1] $det(s) \neq 0$

1.1.2 Invariant Subspaces

This means, if we have a vector in the subspace V' and we act on it with arbitrary group elements, the transformed vector will always be again part of the subspace V'. If we find such an invariant subspace we can define a representation R' of G on V', called a subrepresentation of R, by

$$R'(g)v = R'(g)v$$

This means that the representation R is actually composed of

1.1.3 Irreducible Representation

Chapter 2

The Second Chapter

2.1 Quantum Mechanics

Everytim the aciton of a generator leaves the Lagrangian invariant it leads to the conservation of a quantity. The Canonical commutation algebra is a realtionship between the position and momentum opearators that

2.2 Quantum Field Theory

Quantum Field Theory is about the dance between various quantizable fields i.e. functions of space and time $\phi(\vec{x}, t)$. We will therefore be dealing with points in spacetime and thus it natural to talk about the densities of our dynamical variables such as conjugate momentum $\pi = \pi(x)$ rather than the total quantities we get by integrating them over spacetime i.e. s.

Earlier we discovered that invariance under displacements of the field itself is a new conserved quantity called conjugate momentum. We now identify it with the corresponding generator: