Group Theory and Particle Physics

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Abstract

1 Transformations

What are transformations and why do physicists care about it? Well there are four distinct types of transformations

Physicists care about transformations for purely physical reasons that follow from the second postulate of Special Relativity: "The Laws of Physics remains the same for all observers" So. This however is a very non-trivial activity that will require us to build new mathematical structures and use their strengths whilst still gluing them to Ontology i.e. not just utilizing them as objects that ease away calculation or throw complexity under the rug but as objects whose relations provide us an understanding of reality.

2 What is a Group?

What is a Group? In Physics speak, a group is a set of transformations which are related to one another. Let's look at a very naive model of this: rotations of a triangle. We first start by labelling the vertices of a triangle

3 Rotation Groups

The simplest example of a Group is present in rotations. This is also the easiest one to picture. So let's try to imagine a rotation in a two-dimensional plane (the Euclidian plane). We have a picture similar to this,

3.1 Rotations in 3-dimensions

4 Lie Theory

4.1 Generators

4.2 Abstract Defintion of Lie Algebra

4.3 Abstract Defintion of Lie Groups

5 A Deeper Look at $SU(2)$

6 The Lorentz Group

7 A Little Quantum Mechanics

8 title

A Casimir is defined as the sum of the operators squared in a particular representation. For example, let's look at Spin:

$$\hat{S}^2 = \tag{1}$$

It is rotation invariant and measures the magnitude of spin

9 Spin

One may ask what is spin but not why spin? To answer this question in a physical sense, spin is simply the internal angular monetum of a quantum mechanical object. It is yet another entity in quantum mechanics that does not have a classical analog. What do we know of it? We know for a fact that it is discete. However, s can in fact be a "half-integer" as well as an integer, which is to say:

$$s = 0, \frac{1}{2}, 1, \frac{3}{2}, 2, \frac{5}{2}, 3, ...$$

9.1 Addition of Angular Momenta

Another interesting question that we can pose is, how do we add Angular momenta?

9.2 Spin $\frac{1}{2}$

10 A Short tale about Quarks

It all begins in Heisenberg's Hypothesis

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