

Notes on Electrodynamics

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Chapter 1

Vector Analysis

1.1 Vector Algebra

1.2 Differential Calculus

1.2.1 "Ordinary Derivatives"s

1.2.2 Gradient

1.2.3 The Del Operator

1.2.4 The Divergence

1.2.5 The Curl

1.2.6 Product Rules

1.2.7 Second Derivatives

1.3 Integral Calculus

1.4 Curvilinear Coordinates

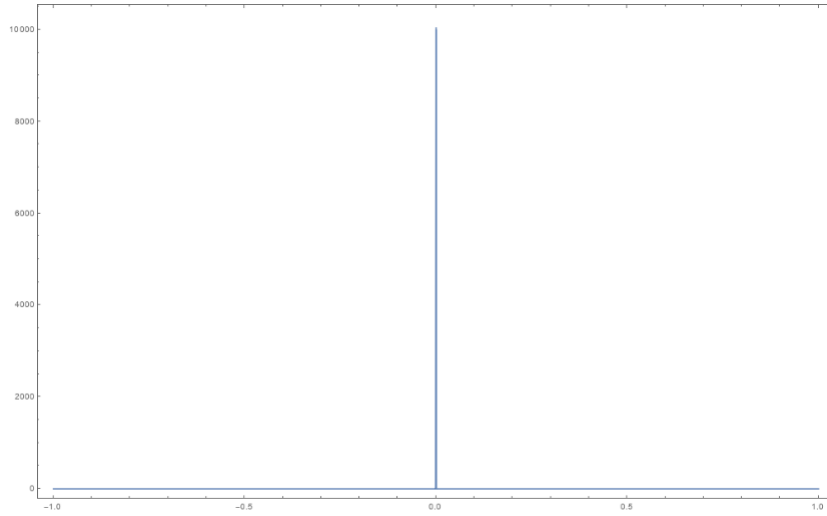
1.4.1 Spherical Coordinates

1.4.2 Cylindrical Coordinates

1.5 The Dirac Delta Function

1.5.1 The Divergence of $\frac{\hat{r}}{r^2}$

$$\nabla \cdot \frac{\hat{r}}{r^2} = 0 \tag{1.1}$$

Figure 1.1: A Plot of $\delta(x)$

1.5.2 The One-Dimensional Dirac Delta Function

The Dirac Delta is a functional ¹ which we define as,

$$\delta(x - a) = \begin{cases} 0, & \text{if } x \neq a \\ \infty, & \text{if } x = a \end{cases} \quad (1.2)$$

$$\int_{-\infty}^{+\infty} \delta(x - a) dx = 1 \quad (1.3)$$

$\forall a \in \mathbb{R}$ We can visualize it as a sharp peak at a , We can interpret 1.3 as saying "the area of the delta distribution is always 1".

$$f(x)\delta(x - a) = f(a) \quad (1.4)$$

We can combine these to get,

$$\int_{-\infty}^{+\infty} \delta(x - a) f(x) dx = f(a) \quad (1.5)$$

A few interesting properties

$$\delta(kx) = \frac{1}{|k|} \delta(x) \quad (1.6)$$

$$\frac{d}{dx}(\delta(x)) = -\delta(x) \quad (1.7)$$

¹An object that is a map between functions

where k is a constant

$$\frac{d\theta}{dx} = \delta(x) \quad (1.8)$$

Where θ is the step function defined as,

$$\theta(x) = \begin{cases} 1, & \text{if } x > 0 \\ 0, & \text{if } x \leq 0 \end{cases} \quad (1.9)$$

1.5.3 The Three-Dimensional Dirac Delta Function

We generalize (1.8) to three dimensions,

$$\delta^3(\vec{r} - \vec{a}) = \delta(x - a_x)\delta(y - a_y)\delta(z - a_z) \quad (1.10)$$

$$\int_{-\infty}^{+\infty} \delta^3(\vec{r} - \vec{a}) dV = 1 \quad (1.11)$$

We can also define the three-dimensional delta function as

$$\mathbf{r} \quad (1.12)$$

$$\mathbf{r} =$$

1.6 The Theory of Vector Fields

1.6.1 The Helmholtz Theorem

This does cause an issue with "Ontology" ²

1.6.2 Potentials

²What's real and what isn't?

Chapter 2

Potentials And Fields

Chapter 3

Electromagnetic Waves

Chapter 4

Electromagnetic Radiation

Chapter 5

Tensors and Lagrangians

We can write the Faraday tensor as,

$$\frac{\partial J^\mu}{\partial x^\mu} = 0 \tag{5.1}$$

$$\frac{\partial F^{\mu\nu}}{\partial x^\nu} = \mu_0 J^\mu \tag{5.2}$$

$$\frac{\partial G^{\mu\nu}}{\partial x^\nu} = 0 \tag{5.3}$$

Chapter 6

Helmholtz Theorem

