Introduction to Cosmology

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Dark Matter

Observable Matter

Stars primarily emit light in the infrared, visible, and ultraviolet range of the electromagnetic spectrum. The Sun's luminosity in the B band is $L_{0,B} = 4.7 \times 10^{25} W$.

In the B band, the total luminosity density of stars within a few hundred megaparsecs of our Galaxy is

$$J_{\star,B} = 1.2 \times 10^{8}\, L_{0,B}\, Mpc^{-3} \tag{1}$$

To convert a luminosity density $J_{\star,B}$ into a mass density ρ , we need to know the mass-to-light ratio for the stars

We can say that there is one solar mass of stars for each solar luminosity of output power, or

$$\langle M/L_{0,B} \rangle = 1 M_0 / L_{0,B} \tag{2}$$

For main sequence stars, powered by hydrogen fusion in their centers, the mass-to-light ratio ranges from $M/L_B \approx 10^{-3} M_0/L_{0,B}$ for the brightest, most massive stars to $M/L_B \approx 10^{3} M_0/L_{0,B}$.

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Thus, the mass-to-light ratio of the stars in a galaxy will depend on the mix of stars which it contains. Within 1 kiloparsec of the Sun, the mass-to-light ratio of the stars works out to be,

$$\langle M/L_B \rangle = 4M_0/L_{0,B} \approx 170000\ Kg\ W^{-2} \tag{3}$$

If the mass-to-light ratio of the stars within a kiloparsec of us is not unusually high or low, then the mass density of stars in the universe is

$$\rho_{\star,0} = \langle M/L_B \rangle_{j_{\star,0}} \approx 5 \times 10^8 M_0\ Mpc^{-3} \tag{4}$$

Since the current critical density of the universe is equivalent to a mass density of $\rho_{c,0} = \frac{\epsilon_{c,0}}{c^2} = 1.4 \times 10^{11} M_0\ Mpc^{-3}$

Dark Matter

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The current density parameter of stars is

$$\Omega_{\star,0} = \frac{\rho_{\star,0}}{\rho_{c,0}} \approx \frac{5 \times 10^{8} M_0}{1.4 \times 10^{11} M_0} \approx 0.004 \tag{5}$$

Stars make up less than 0.5 percent of the density necessary to flatten the universe. In truth, the number $\Omega_{\star,0} \approx 0.004$ is not a precisely determined one, largely because of the uncertainty in the number of low-mass, low-luminosity stars in galaxies.

In our Galaxy, for instance, ≈ 95 percent of the stellar luminosity comes from stars more luminous than the Sun, but ≈ 80 percent of the stellar mass comes from stars less luminous than the Sun.

The density parameter in stars will be further increased if you include in the category of stars stellar remnants and brown dwarfs. A brown dwarf is a self-gravitating ball of gas which is too low in mass to sustain nuclear fusion in its interior. Because brown dwarfs and isolated cool stellar remnants are difficult to detect, their number density is not well known

Dark Matter

Observable Matter

Galaxies also contain baryonic matter which is not in the form of stars,stellar remnants, or brown dwarfs. The interstellar medium contains significant amounts of gas. In our Galaxy and in M31, for instance, the mass of interstellar gas is roughly equal to 10 percent of the mass of stars. In irregular galaxies such as the Magellanic Clouds, the ratio of gas to stars is even higher. In addition, there is a significant amount of gas between galaxies.

The Coma cluster contains thousands of galaxies, their summed luminosity in the B band comes to $L_{coma,B} = 8 \times 10^{12}\ L_{0,B}$. If the mass-to-light ratio of the stars in the Coma cluster is $\langle M/L_B \rangle \approx 4 M_0/L_{0,B}$, then the total mass of stars in the Coma cluster is $M_{coma,\star} \approx 3 \times 10^{13}\ M_0$

Although 30 trillion solar masses represents a lot of stars, the stellar mass in the Coma cluster is small compared to the mass of the hot, intracluster gas between the galaxies in the cluster.

Dark Matter

Visible Matter

The best current limits on the baryon density of the universe come from the predictions of primordial nucleosynthesis. The amounts of deuterium and other elements present in primordial gas clouds indicate that the density parameter of baryonic matter must be

$$\Omega_{bary,0} = 0.04 \pm 0.01 \tag{6}$$

an order of magnitude larger than the density parameter for stars

Dark Matter

Dark Matter in galaxies

The majority of the matter in the universe is nonbaryonic dark matter, which doesn't absorb, emit, or scatter light of any wavelength. One way of detecting dark matter is to look for its gravitational influence on visible matter.

A classic method of detecting dark matter involves looking at the orbital speeds of stars in spiral galaxies such as our own milky way and M31.

If the radius of the orbit is R and the orbital speed is v, then the star experiences an acceleration

$$a = \frac{v^2}{R} \tag{7}$$

directed toward the center of the galaxy. If the acceleration is provided by the gravitational attraction of the galaxy, then

$$a = \frac{GM(R)}{R^2} \tag{8}$$

Dark Matter

Dark matter in glaaxies

Equating (7) and (8) we get,

$$\frac{v^2}{R} = \frac{GM(R)}{R^2} \tag{9}$$

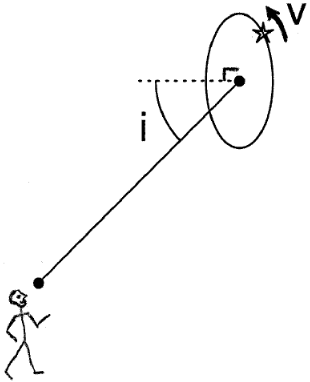
The surface brightness I of the disk of a spiral galaxy typically falls off exponentially with distance from the center by,

$$I(R) = I_0 e^{\frac{-R}{R_s}} \tag{10}$$

Once we are a few scale lengths from the center of the spiral galaxy, the mass of stars becomes essentially constant. Thus, if stars contributed all, or most, of the mass in a galaxy, the velocity would fall as $v \propto \frac{1}{\sqrt{R}}$. This is called as **Keplerian rotation**.

Dark Matter

Dark Matter in galaxies



The disk we see in projection will be elliptical, not circular, with an axis ratio,

$$\frac{b}{a} = \cos(i) \tag{11}$$

Dark Matter

Dark Matter in galaxies

Since the redshift contains only the component of the stars' orbital velocity which lies along the line of sight, the radial velocity which we measure will be,

$$v(R) = v_{gal} + v(R)\sin(i) \tag{12}$$

where v_{gal} is the radial velocity of the galaxy as a whole, resulting from the expansion of the universe, and v(R) is the orbital speed at a distance R from the center of the disk.

$$v(R) = \frac{v_r(R) - v_{gal}}{\sqrt{1 - \frac{b^2}{a^2}}} \tag{13}$$

Dark Matter

Dark matter in galaxies

If we approximate the orbital speed v as being constant with radius, the mass of a spiral galaxy, including both the luminous disk and the dark halo can be found,

$$M(R) = \frac{v^2}{G} = 9.6 \times 10^{10}\ M_0(\frac{v}{220}^2)(\frac{R}{8.5Kpc}) \tag{14}$$

Using our sun as an example, $L_{gal} = 2.3 \times 10^{10} L_{0,B}$, the mass-to-light ratio of our Galaxy, taken as a whole, is,

$$\langle M\ \ L_B \rangle_{Gal} \approx 50 M_0 / L_{0,B}(\frac{R_{halo}}{100Kpc}) \tag{15}$$