Introduction to Cosmology

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Dark Matter

Dark Matter in Clusters

The acceleration of the i^{th} galaxy in the coma cluster is given by ,

$$\ddot{\vec{x_i}} = G \sum_{j \neq 1} m_j \frac{\vec{x_j} - \vec{x_i}}{(\vec{x_j} - \vec{x_i})^3} \tag{1}$$

The gravitational potential energy of the system of N galaxies is

$$W = -\frac{G}{2} \sum_{j \neq 1} \frac{m_i m_j}{\vec{x_j} - \vec{x_i}} \tag{2}$$

The potential energy of the cluster can also be written as,

$$W = -\alpha \frac{GM^2}{r_h} \tag{3}$$

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The kinetic energy associated with the relative motion of the galaxies in the cluster is,

$$K = \frac{1}{2} \sum_i m_i |\dot{\vec{x}}_i|^2 \tag{4}$$

The kinetic energy K can also be written in the form

$$K = \frac{1}{2} M \langle v^2 \rangle \tag{5}$$

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The moment of inertia is also an important factor to understand when we try to understand the radial velocities of galaxies. Hence the moment of inertia can be written as,

$$I = \sum_i m_i |\dot{\vec{x_i}}|^2 \tag{6}$$

The moment of inertia in terms of the potential and kinetic energy can be written as,

$$\ddot{I} = 2\sum_i m_i (\vec{x_i} \times \ddot{\vec{x_i}} + \dot{\vec{x_i}} \times \dot{\vec{x_i}}) \tag{7}$$

(7) can be written as,

$$\ddot{I} = 2\sum_i m_i (\vec{x_i} \times \ddot{\vec{x_i}}) + 4K \tag{8}$$

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From equation (8) we can formulate the expression for potential energy.

$$\sum_i m_i(x_i \times \ddot{\vec{x_i}}) = G\sum_i m_i m_j \frac{\dot{\vec{x_i}} \times (\vec{x_j} - \vec{x_i})}{|\vec{x_i} - \vec{x_j}|^3} \tag{9}$$

$$\sum_i m_i(x_i \times \ddot{\vec{x_i}}) = \sum_j m_j(x_j \times \ddot{\vec{x_j}}) \tag{10}$$

Using equations (9) and (10),

$$\sum_i m_i(x_i \times \ddot{\vec{x_i}}) = \frac{1}{2}[\sum_i m_i(x_i \times \ddot{\vec{x_i}}) + \sum_j m_j(x_j \times \ddot{\vec{x_j}})] \tag{11}$$

$$\sum_i m_i(x_i \times \ddot{\vec{x_i}}) = -\frac{G}{2}\sum_{ij} \frac{m_i m_j}{|\vec{x_i} - \vec{x_j}|} = W \tag{12}$$

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We can substitute this relation in equation (8),

$$\ddot{I} = 2W + 4K \tag{13}$$

This relation is known as the virial theorem
If I = constant, then the steady-state virial theorem is,

$$0 = W + 2K \tag{14}$$

$$K = -\frac{W}{2} \tag{15}$$

Using (3) and (5) in (15),

$$\frac{1}{2}M\langle v^2\rangle = \frac{\alpha G M^2}{2r_h} \tag{16}$$

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Using this we can calculate the mass of the cluster with the help of virial theorem,

$$M = \frac{\langle v^2 \rangle r_h}{\alpha G} \tag{17}$$

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About the coma Cluster.......

$$z = 0.0232 \tag{18}$$

$$d_{coma} = (\frac{C}{H_0})\langle z \rangle = 99Mpc \tag{19}$$

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$$v_r = cz = 6960 km\, s^{-1} \tag{20}$$

$$r_h = 1.5 Mpc = 4.6 \times 10^{22} \tag{21}$$

$$M_{coma} = \frac{\langle v^2 \rangle r_h}{\alpha G} \approx 2 \times 10^{15} M_0 \tag{22}$$

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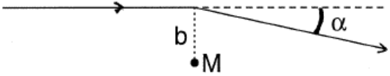
Gravitational Lensing



From Einstein's theory of General Relativity,If a photon passes such a compact massive object at an impact parameter b,the local curvature of space-time will cause the photon to be deflected by an angle

$$\alpha = \frac{4GM}{c^2 b} \tag{23}$$

If the MACHO is exactly along the line of sight between the observer and the lensed star, the angular radius of the image produced is given by,

$$\theta_E = \left(\frac{4GM}{c^2 b}\frac{1-x}{x}\right)^{\frac{1}{2}} \tag{24}$$

Dark Matter

Dark Matter - Probably the matter which matters the most.
What is it? Why do we care so much about it? What is its significance in our universe?