Introduction to Cosmology

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Introduction

Cosmology

Cosmology is the study of the universe, or cosmos, regarded as a whole. The universe, after all, with structures on a vast range of scales; planets orbit stars, stars are collected into galaxies, galaxies are gravitationally bound into clusters, and even clusters of galaxies are found within larger superclusters.

Introduction

Inspiration

Cosmology deals with questions which are fundamental to the human existence. The questions which vex humanity are given in the title of a painting by Paul Gauguin-"What are we? Where do we come from? Where are we going?"

Metric System of the Cosmos

Scaling Units

Cosmology deals with distances that are very large, objects that are very big, and timescales that are very long. Cosmologists frequently find that the standard SI units are not convenient for their purposes

$$1\ LightYear = 9.461 \times 10^{15}\ m$$

$$1\ AU = 1.5 \times 10^{11}\ m$$

$$1\ pc = 3.1 \times 10^{16}\ m$$

$$1\ M_o = 2 \times 10^{30}\ Kg$$

Fundamental Observarions

Olbers' Paradox

In the night, you will see a dark sky, with roughly two thousand stars scattered across it. The fact that the night sky is dark at visible wavelengths, instead of being uniformly bright with starlight, is known as Olbers' Paradox, after the astronomer Heinrich Olbers.

Flux recieved on Earth from the star

$$F(r) = \frac{L}{4\pi r^2} \tag{1}$$

Intensity of the Incoming light

$$\frac{\mathrm{d}J(r)}{\mathrm{d}r} = \frac{L}{4\pi r^2} \times n \times r^2 \ = \frac{nL}{4\pi} \tag{2}$$

$$J(r) = \int_0^\infty \frac{\mathrm{d}J(r)}{\mathrm{d}r} = \frac{nL}{4\pi} \int_0^\infty dr = \infty \tag{3}$$

Fundamental Observarions

Assumptions based on the above result

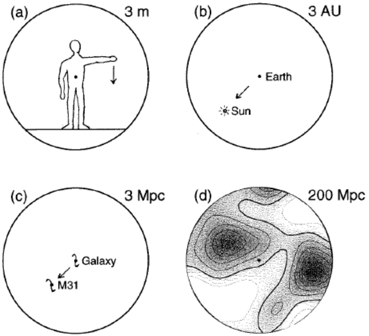
1. One assumption is that we have an unobstructed line of sight to every star in the universe.
2. Second assumption is that the number density n and mean luminosity L of stars are constant throughout the universe.
3. Third assumption is that the universe is infinitely large.
4. The fourth assumption is that the universe is infinitely old.
5. Fifth assumption is that the flux of light from a distant source is given by the inverse square law of equation.

Fundamental Observations

"Were the succession of stars endless, then the background of the sky would present us an uniform density, since there could be absolutely no point, in all that background, at which would not exist a star. The only mode, therefore, in which, under such a state of affairs, we could comprehend the voids which our telescopes find in innumerable directions, would be by supposing the distance of the invisible background so immense that no ray from it has yet been able to reach us at all."

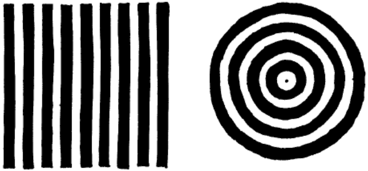
Fundamental Observations

Isotropy



Fundamental Observations

Homogenity



Cosmological Principle

The cosmological principle is the notion that the spatial distribution of matter in the universe is homogeneous and isotropic when viewed on a large enough scale.

Cosmological Redshift

Definiton

Cosmological redshift is a phenomenon where the wavelength at which the radiation is originally emitted is lengthened as it travels through space.Cosmological redshift results from the expansion of space itself and not from the motion of an individual body.

Redshift is given by

$$Z = \frac{\lambda_{ob} - \lambda_{em}}{\lambda_{em}} \tag{4}$$

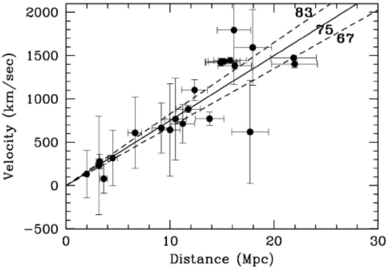
Redshift from Hubble's law

$$Z = \frac{H_0}{c} \times d \tag{5}$$

Where Ho is the Hubble's constant.

Cosmological Redshift

Redshift Graph



Under non-relativistic conditions

$$Z = \frac{v}{c} \tag{6}$$

$$v = H_0 \times r \tag{7}$$

Cosmological Redshift

Hubble's Constant

$$H_0 = 500 Km\, s^{-1} Mpc^{-1} \tag{8}$$

Revised Hubble's Constant

$$H_0 = 70 \pm 7 Km\, s^{-1} Mpc^{-1} \tag{9}$$

Hubble Time

$$T_o = \frac{r}{v} = \frac{r}{H_0\, r} = H_0^{-1} \tag{10}$$

Hubble's time the time that has elapsed since any two galaxies under observation, were in contact with each other. The Hubble's time for

$$H_0 = 70 \pm 7 Km\, s^{-1} Mpc^{-1} \tag{11}$$

is

$$H_0^{-1} = 14.0 \pm 1.4 GYr \tag{12}$$

Hubble Distance

Hubble's distance gives us the measure of the greatest distance a photon can travel since the beginning of the universe.

$$D_0 = \frac{c}{H_0} = 4300 \pm 400 Mpc$$

$$nL \approx 2 \times 10^8 L_0\ Mpc^{-3}$$

Steady State Universe

Hubble's Equation for Steady State Universe

$$\frac{\mathrm{d}r}{\mathrm{d}t} = H_0 \times r$$

$$r(t) \propto e^{H_0 t}$$

$$V = \frac{4\pi r^3}{3} \propto e^{3H_0 t}$$

$$M_{ss} = \rho_0 \times V = \rho_0 \times 3H_0 V$$

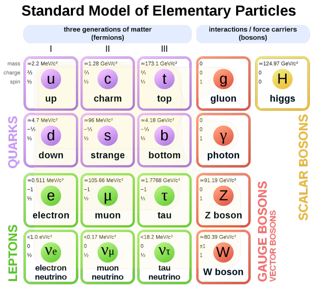
Density of the Steady State Universe will be,

$$\frac{M_{ss}}{V} = 3 \times H_0 \rho_0 \approx 6 \times 10^{-28}\ Kgm^{-3} Gyr^{-1}$$

Types of Particles in the Universe

Kinds of Particles

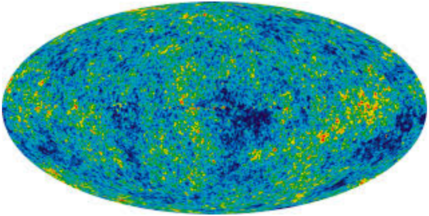
particle	symbol	rest energy (MeV)	charge
proton	p	938.3	+1
neutron	n	939.6	0
electron	e^-	0.511	-1
neutrino	ν_e, ν_μ, ν_τ	?	0
photon	γ	0	0
dark matter	?	?	0



Cosmic Microwave Background

Cosmic Microwave Background

Cosmic Microwave Background is the isotropic background of the universe observed using microwaves by Arno Penzias and Robert Wilson



Cosmic Microwave Background is exquisitely well fitted by a blackbody spectrum with a temperature,

$$T_0 = 2.275 \pm 0.0001 \ K$$

Cosmic Microwave Background

The number of photons in the CMB is, (From Black body Radiation Equation)

$$\eta_\gamma = \beta \times T^3$$

$$n_\gamma = 4.11 \times 10^8 \ m^3$$

The mean energy of the CMB photons is,

$$E_{mean} = 6.34 \times 10^{-4} eV$$

Cosmic Microwave Background

From Thermodynaics

$$dQ = dE + PdV \tag{13}$$

Since, in a homogeneous universe, there is no net flow of heat

$$dQ = 0 \tag{14}$$

Therefore,

$$\frac{\mathrm{d}E}{\mathrm{d}t} = -P(t)\frac{\mathrm{d}V}{\mathrm{d}t} \tag{15}$$

Since, for the photons of the CMB,

$$E = \epsilon_\gamma V = \alpha T^4 V \textit{ and } P = P_\gamma = \frac{\alpha T^4}{3} \tag{16}$$

(15) becomes,

$$\alpha(4T^3\frac{\mathrm{d}T}{\mathrm{d}t}V + T^4\frac{\mathrm{d}V}{\mathrm{d}t}) = \frac{-1}{3}\alpha T^4\frac{\mathrm{d}V}{\mathrm{d}t} \tag{17}$$

Cosmic Microwave Background

(17) can also be written as,

$$\frac{1}{T}\frac{\mathrm{d}T}{\mathrm{d}t} = \frac{-1}{3V}\frac{\mathrm{d}V}{\mathrm{d}t} \tag{18}$$

Since

$$V \propto \alpha T^4 \tag{19}$$

(18) can be written as,

$$\frac{\mathrm{d}\ln(T)}{\mathrm{d}t} = -\frac{\mathrm{d}\ln(a)}{\mathrm{d}t} \tag{20}$$

which implies,

$$T(t) \propto \alpha(t)^{-1} \tag{21}$$