Noether's Theorem: #1 Introduction

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Questions that we'll answer

- Analytical Mech
  - Lagr.
  - Ham.
  - Ham-Jacobi
- Abstract Alg
  - Vec. spaces
  - Groups
- Field
- Gauge
- Examples
  - CM
  - QM
  - CFT

A Brief Glance of topics

Pedagogy

↳ Comments/Qs

↳ EOCs

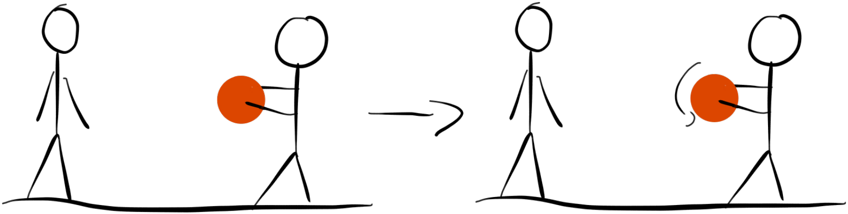
  ↳ Reflective
  ↳ Problems

$\left.\begin{array}{l} \text{Reflective} \\ \text{Problems} \end{array}\right\}$ Appendices

This Discussion

- Invariance
- Symmetry
- Passive and Active Transformations
- Global and Local Transformations
- Conservation
- What is Noether's Theorem?
- Functionals

Invariance

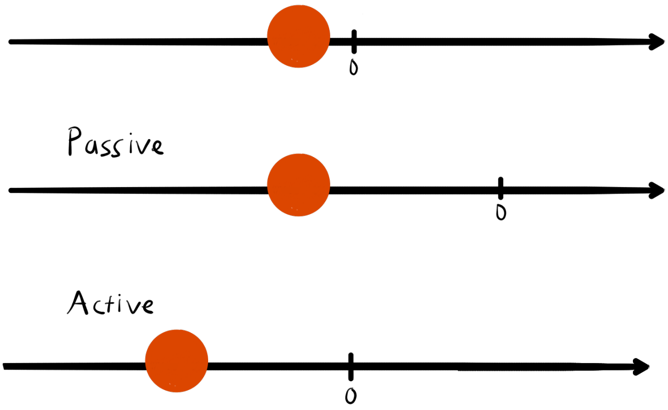


Symmetry

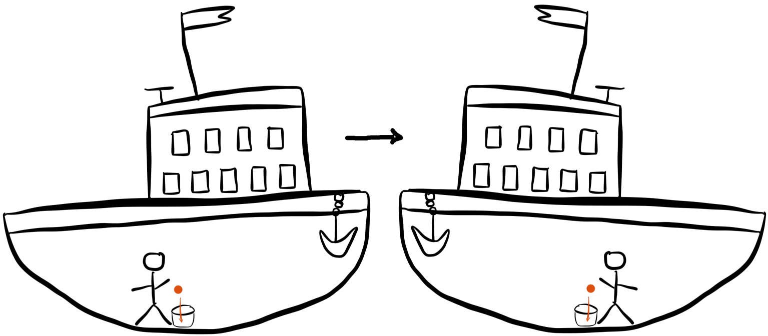
Definition of Symmetry

The invariance of an object under a particular transformation is termed a symmetry.

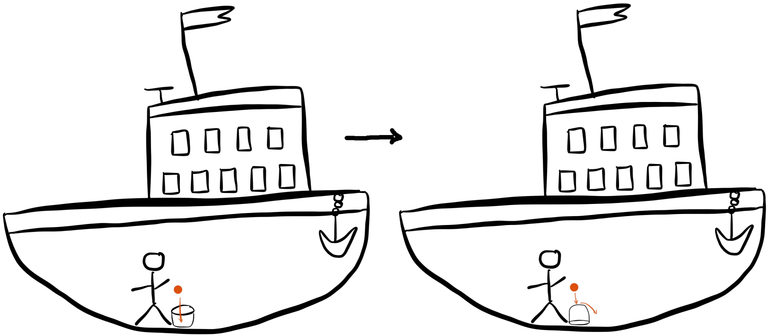
Passive and Active Transformations



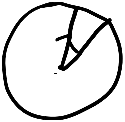
Global Transformations



Local Transformations



Continuous vs Discete Transformations



$$g(\theta) = I + \underbrace{\varepsilon}_{<1} X \;\Big\}\text{Generator}$$

$$g(\theta) = I + \frac{\theta}{N} X$$

Continuous iff $\prod \sum ds$

$$h(\theta) = [g(\theta)]^N = \left[I + \frac{\theta}{N}X\right]^N = e^{\theta X}$$

$\lim_{N \to \infty}$

$= $ Taylor Expand

Conserved "Current"

$$\rho = \psi\psi^*$$

$$\frac{\partial \rho}{\partial t} = 0$$

$$\frac{\partial \rho}{\partial t} = -\nabla \cdot \vec{J}$$

$$\vec{J} = \frac{\hbar}{2mi}\left(\psi^* \nabla \psi - \psi \nabla \psi^*\right)$$

A conserved current is an object that follows a continuity equation

What is Noether's Theorem?

Symmetry $\leftrightarrow$ Conserved

Transformations = Continuous

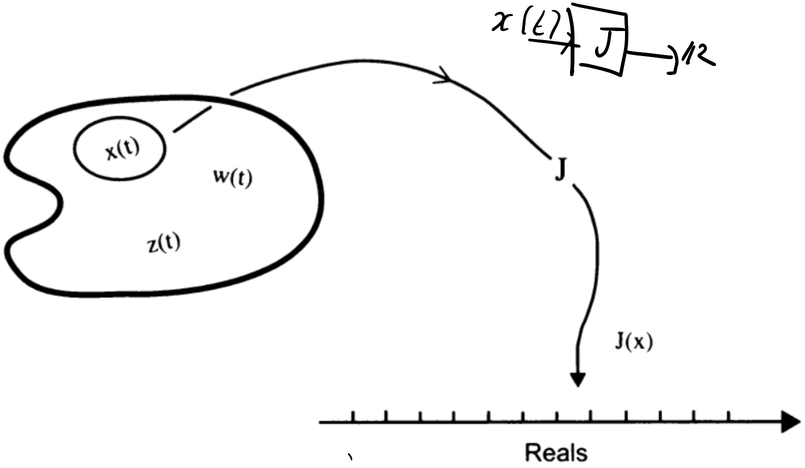
Every differentiable symmetry of the action of a physical system has a corresponding conservation law. = Conserved

"Functional"

$S = \int L\,dt = \int (T - V)\,dt$

"Extremize"

Functionals



A functional J is a mapping from a set of well defined functions to the real number line.

Examples

↳ Length / Distance $= \int ds$

$$= \int \sqrt{dx^2 + dy^2} = \int \sqrt{1+y'^2}\, dx$$

↳ Fermat's Principle

$$\int n(x)\, L(x)\, dx = 0$$

↳ Hamilton's principle

$$\int_a^b \underbrace{(T-V)}_{=L}\, dt$$

$L(q, \dot{q}, t)$ Ostrog:

Lagrangian
$J(x) = \int_a^b \underline{\underline{L}}\, dx$
Functional
$J: V \to \mathbb{R} \ \forall_{v \in V}$

A Word About Mathematical Structures

Set + Operations + Rules

$\{2, 1, \ldots\}$

$\in$

C

Binary

$(\, , \,) \to (\,)$

Associative: $A(BC) = A(BC)$

$A \cdot B = B \cdot A$

Vector Spaces

$+ = v, w \in \mathbb{V}$ $\quad \vec{U}, \vec{V}$

$\cdot =$

Axioms

A linear vector space or simply a vector space $\mathbb{V}$ is a set along with the multiplication (.) and addition (+) operations over $\mathbb{R}$ or $\mathbb{C}$, such that the following axioms hold:

Field

- **Commutativity:** $|U\rangle + |V\rangle = |V\rangle + |U\rangle$
- **Associativity:** $(|U\rangle + |V\rangle) + |W\rangle = |V\rangle + (|U\rangle + |W\rangle)$
- **Additive Identity:** $\exists\ |0\rangle \in \mathbb{V} \mid\ |V\rangle + |0\rangle = |0\rangle + |V\rangle = |V\rangle$
- **Additive Inverse:** $\forall\ |V\rangle\ \exists\ \left|V^{-1}\right\rangle \mid\ |V\rangle + \left|V^{-1}\right\rangle = |0\rangle$
- **Multiplicative identity:** $\exists\ 1 \in \mathbb{V} \mid 1.|V\rangle = |V\rangle$
- **Multiplicative Associativity:** $(\alpha\beta)|V\rangle = \alpha(\beta|V\rangle)$
- **Distributive Properties:**
  - $(\alpha + \beta)|U\rangle = \alpha|U\rangle + \beta|U\rangle$
  - $\alpha(|U\rangle + |V\rangle) = \alpha|U\rangle + \alpha|V\rangle$

Kets

Here, $\alpha, \beta \in \mathbb{R}$ or $\mathbb{C}$ and $|U\rangle, |V\rangle$ and $|W\rangle \in \mathbb{V}$

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