WELCOME

STELLAR ASTROPHYSICS

(BASIC THEORETICAL IDEAS AND OBSERVATIONAL DATA)

STRUCTURE :

- Introduction
- Basic equations of stellar structure
  - Hydrostatic equilibrium in stars
  - virial theorem for stars
  - energy transport in stars
  - convection inside stars
- Some relations amongst stellar quantities

INTRODUCTION :

- Stellar Astronomy :

    - Stellar interior

    - stellar exterior/ stellar atmosphere

It appears from observational data (to be discussed in detail later) that various quantities pertaining to stars have some relations amongst each other.

BASIC EQUATIONS OF STELLAR STRUCTURE

<u>Hydrostatic equilibrium in stars</u> :

Let M_r be the mass inside the radius **r** of a star. Then the mass inside

radius **r+dr** should be M_r **+ d** M_r, which means that **d** M_r is the mass

of the spherical shell between radii **r** and **r+dr**. If the density is **ρ**, then the mass of this shell is $\rho \times 4\pi r^2\, dr$ i.e.,

$$dM_r = 4\pi r^2 \rho\, dr \;\Rightarrow\; \frac{dM}{dr} = 4\pi r^2 \rho \qquad \text{--------------- (1)}$$

- Let us now consider a small portion of the shell between **r and r + dr**.

If **dA** is the transverse area of this small element, the forces exerted by pressure acting on its inward and outward surfaces are **P dA** and **−(P + dP) dA.**

So, the net force due to pressure is : $-\boldsymbol{dP.dA}$

This should be balanced by gravity under equilibrium conditions.

$$-dP\,dA - \frac{GM}{r^2}\rho dr\,dA = 0$$

for which $$\frac{dP}{dr} = -\frac{GM_r}{r^2}\rho \quad \text{---------------} \quad \textbf{(2)}$$

VIRIAL THEOREM FOR STARS :

- In ordinary stars, we expect that the total thermal energy be the same order as the total gravitational energy.
- This established from the hydrostatic equilibrium equation **(2)**
- By multiplying on both sides of by $\boldsymbol{4\pi r^3}$ and then integrate from the centre of the star to its outer radius R. This gives :

$$\int_0^R \frac{dp}{dr} 4\pi r^3 \, \boldsymbol{dr} = \int_0^R \left(\frac{GM_r}{r^2}\rho\right) 4\pi r^3 \, \boldsymbol{dr}$$

$$\int_0^R \frac{dp}{dr} 4\pi r^3 \, dr = \int_0^R \left(\frac{GM_r}{r^2}\rho\right) 4\pi r^3 \, dr$$

The left-hand side can be easily integrated by parts, leading to :

$$-\int_0^R 3P \times 4\pi r^2 \, dr = \int_0^R \left(-\frac{GM_r}{r}\right) 4\pi r^2 \rho dr$$

The right-handed side is clearly the total gravitational energy $\mathbf{E_G}$ of the star, i.e.,

$$E_G = \int_0^R \left(-\frac{GM_r}{r}\right)^4 \pi\rho r^2 \, dr$$

- Since **(3/2)K_BT** is the mean energy of thermal motion per particle in a region of temperature T and hence **(3/2)nK_BT** is the thermal energy

per unit volume, the total thermal energy of the star is given by :

$$E_T = \int_0^R \frac{3}{2} nK_BT \times 4\pi r^2\, dr = \int_0^R \frac{3}{2} P \times 4\pi r^2\, dr$$

hence,

$$2E_T + E_G = 0$$

THIS ELEGANT AND FAMOUS RESULT IS KNOWN AS **"VIRIAL THEOREM"**

$$\mathrm{E_T} = -\frac{1}{2}\mathrm{E_G} = \frac{1}{2}|\mathrm{E_G}|$$

ENERGY TRANSPORT INSIDE STARS :

- Let $\mathbf{L_r}$ be the total amount of energy flux per unit time which flows outward across a spherical surface of radius r inside the star.
- We expect $\mathbf{L_r}$ to be equal to the luminosity **L** of the star at the outer radius **r = R** of the star.
- If we consider $\boldsymbol{\varepsilon}$ to be the rate of energy generated per unit mass per unit time, we will obtain :

$$\mathbf{dL_r = 4\pi r^2\, dr \times \rho\varepsilon}$$

for which :

$$\frac{dL_r}{dr} = 4\pi r^2\, \mathrm{d}r\rho\varepsilon \quad \text{---------- (3)}$$

CONVECTION INSIDE STARS :

- In radiative transfer, energy is transported without any material motion.
- Convection, on the other hand, involves up and down motions of the gas.
- Hot blobs of gas move upward and cold blobs of gas move downward, thereby transporting heat.

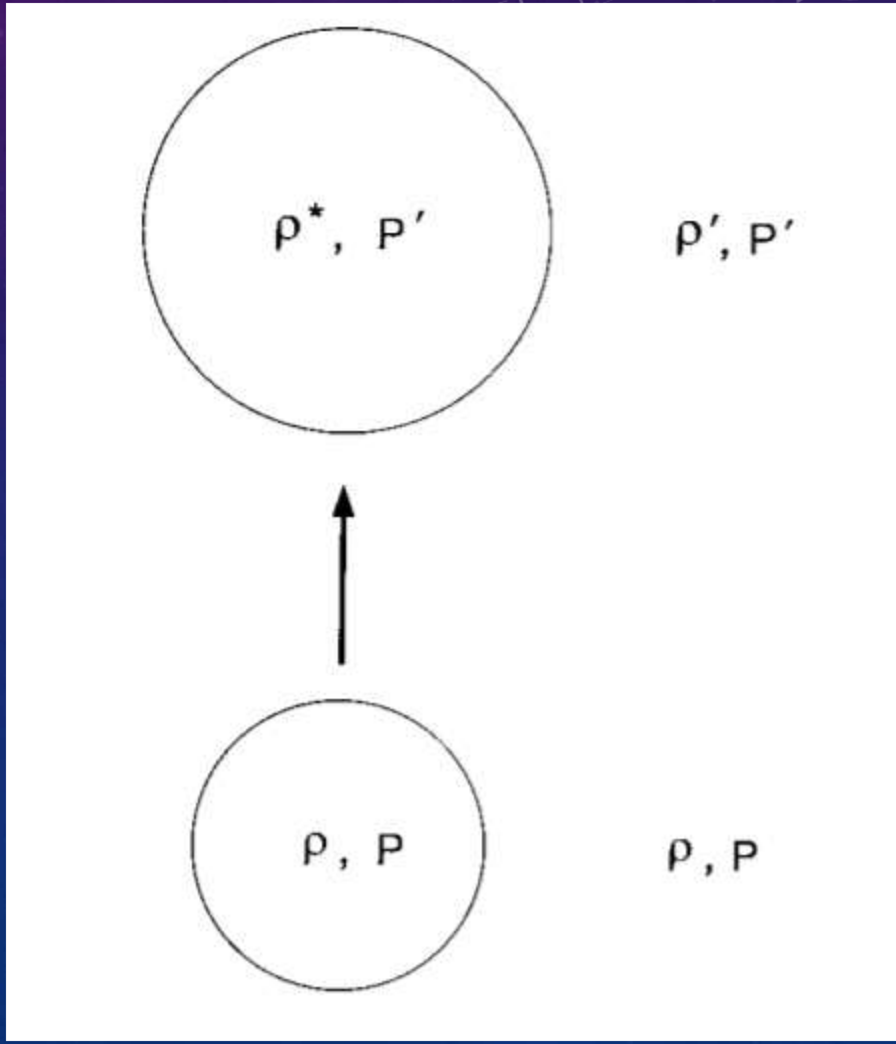
- Suppose we have a perfect gas in hydrostatic equilibrium inside a star.
- We now consider a blob of gas which has been displaced vertically upward.
- We know that pressure imbalances in a gas

are rather quickly removed by acoustic waves, but heat exchange between different parts of the gas takes more time.

Hence we consider the process to be adiabatic.

So **convection** is of the nature of an **instability in the system**.

To find the condition for convective instability, we have to determine whether $\rho*$ is greater than or less than the surrounding density ρ.



- From the assumption that the blob has been displaced adiabatically, it follows that :

$$\rho^* = \rho\left(\frac{P'}{P}\right)^{1/\gamma}$$

If dP/dr is the pressure gradient in the atmosphere, we can substitute :

$$\mathrm{P}' = \mathrm{P} + \frac{\mathrm{dP}}{\mathrm{dr}}\Delta\mathrm{r}$$

$$\Rightarrow \rho^* = \rho + \frac{\rho}{\gamma P}\frac{dP}{dr}\Delta r$$

using ρ=P/RT, we get :

$$\Rightarrow \rho^1 = \rho + \frac{\rho}{\mathrm{P}}\frac{\mathrm{dP}}{\mathrm{dr}}\Delta\mathrm{r} - \frac{\rho}{\mathrm{T}}\frac{\mathrm{dT}}{\mathrm{dr}}\Delta\mathrm{r}$$

Here, Here dρ/dr and dT/dr are the density and temperature gradients in the atmosphere.

$$\rho^* - \rho' = \left[-\left(1 - \frac{1}{\gamma}\right)\frac{\rho}{P}\frac{dP}{dr} + \frac{\rho}{T}\frac{dT}{dr}\right]\Delta r$$

Keeping in mind that dT/dr and dP/dr are both negative, the atmosphere is stable if :

$$\left|\frac{dT}{dr}\right| < \left(1 - \frac{1}{r}\right)\frac{T}{P}\left|\frac{dP}{dr}\right| \quad \text{------------------} \ \textbf{(4)}$$

This is the famous "**Schwarzschild stability condition**"

SOME RELATIONS BETWEEN AMONGST STELLAR QUANTITIES :

- From the stellar structure equations we shall try to extract some relations amongst various quantities pertaining to a star.

in equation **(2),** i.e. the hydrostatic equation, if we replace the L.H.S

by **–P/R**, and replacing $\frac{M}{r^2}$ by $\frac{M}{R^2}$, we get :

$$P \propto \frac{M}{R^2}\rho$$

by taking $\rho \propto \frac{M}{R^3} \cdot The\ equation\ of\ state$ $\mathbf{p \propto \rho T}$ *would imply :*

$$P \propto \frac{M}{R^3}T \qquad \Rightarrow \qquad T \propto \frac{M}{R}$$

after substituting this in the equation (2) :

$$\frac{T}{R} \propto \frac{M}{R^3 T^3} \frac{L}{R^2} \Rightarrow \quad L \propto \frac{(TR)^4}{M}$$

we have seen that TR should be proportional to M. Substituting this in the above equation, we get :

$$L \propto M^3$$ ------- the mass luminosity relation

THANK YOU…