

The background is a deep blue gradient filled with a field of small, distant stars. Overlaid on this are several faint, white geometric diagrams. On the left, a large circular scale with degree markings from 140 to 260 is visible, with concentric circles and arrows indicating celestial motion. Other smaller circular diagrams with arrows are scattered across the upper and lower portions of the frame.

# BASIC CONCEPTS OF POSITIONAL ASTRONOMY

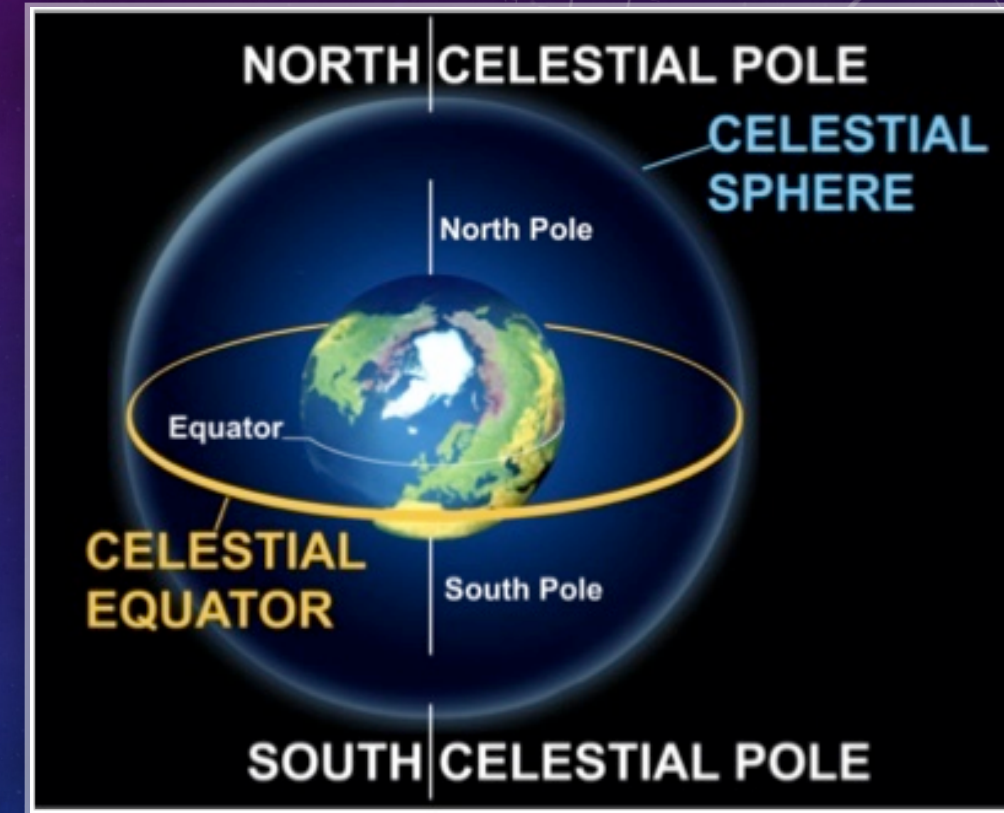
# INTRODUCTION...

- In this lecture we will learn the basic concepts of positional astronomy- various coordinate systems used in astronomy, the effect of earth's motion on the position of these objects. And finally we will see how time is measured in astronomy



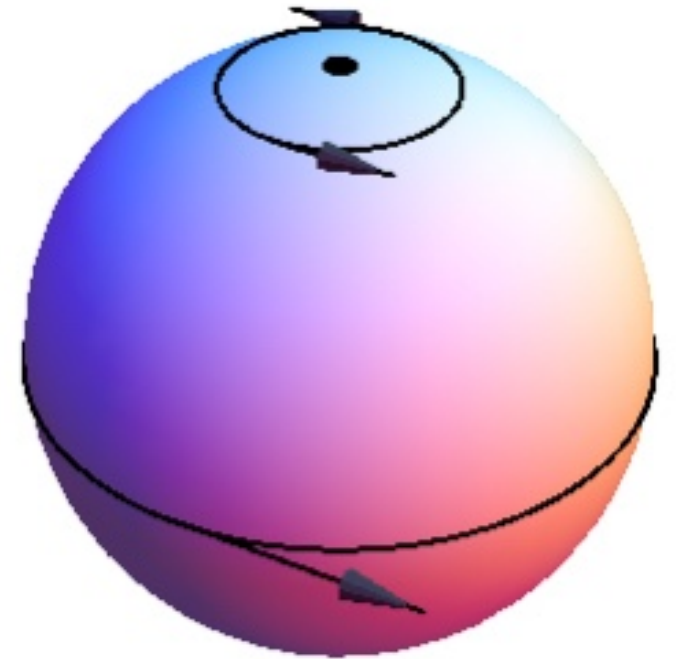
# CELESTIAL SPHERE:

- As we look at the clear night sky , stars appear to be distributed on the inner surface of a vast sphere. This sphere is called CELESTIAL SPHERE.
- As we know the stars are at very distances so the sufficient way is to define the direction of the stars on the sphere.
- Before determining the position of stars and celestial objects , we will learn the concept of geometry of sphere.
- 



# GEOMETRY OF A SPHERE:

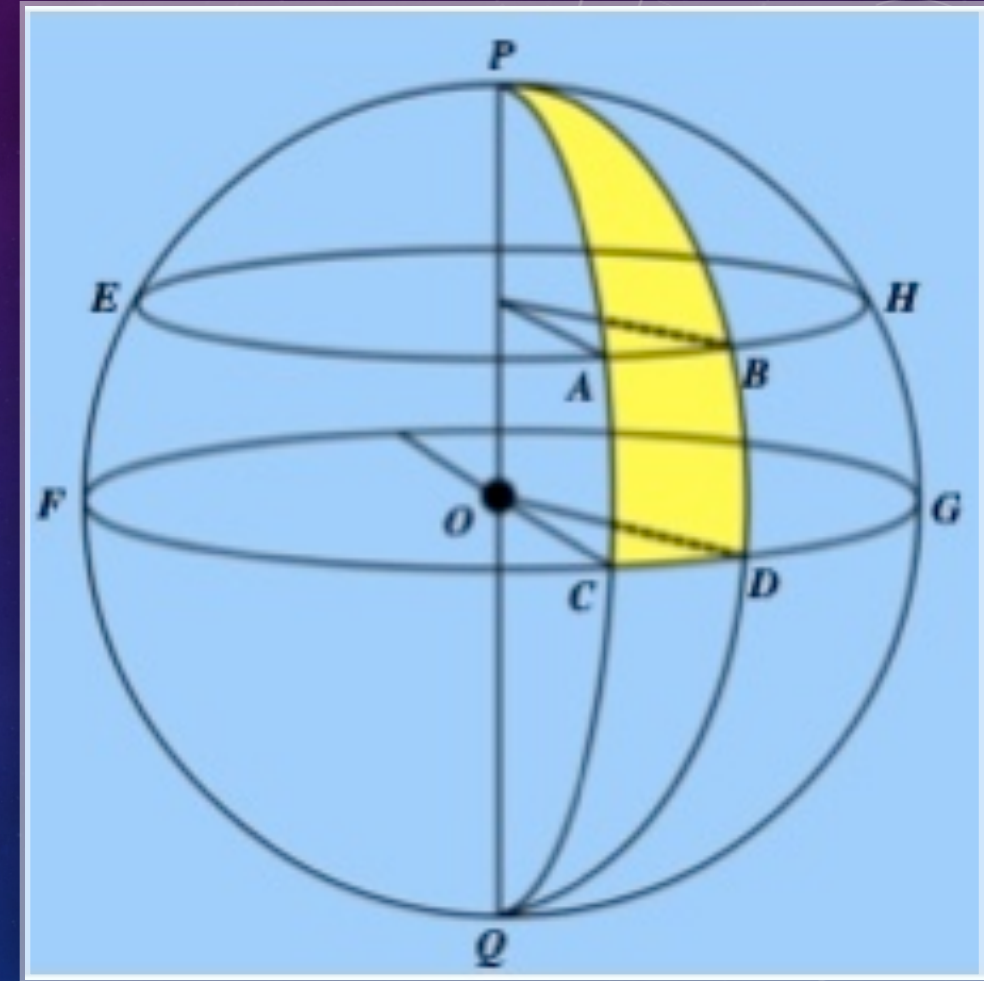
If we take a sphere of radius  $R=1$  unit. If a plane intersects the sphere it forms a circle. If that plane passes through the centre of the sphere we get a GREAT CIRCLE, and if it does not pass through the centre it forms a SMALL CIRCLE.





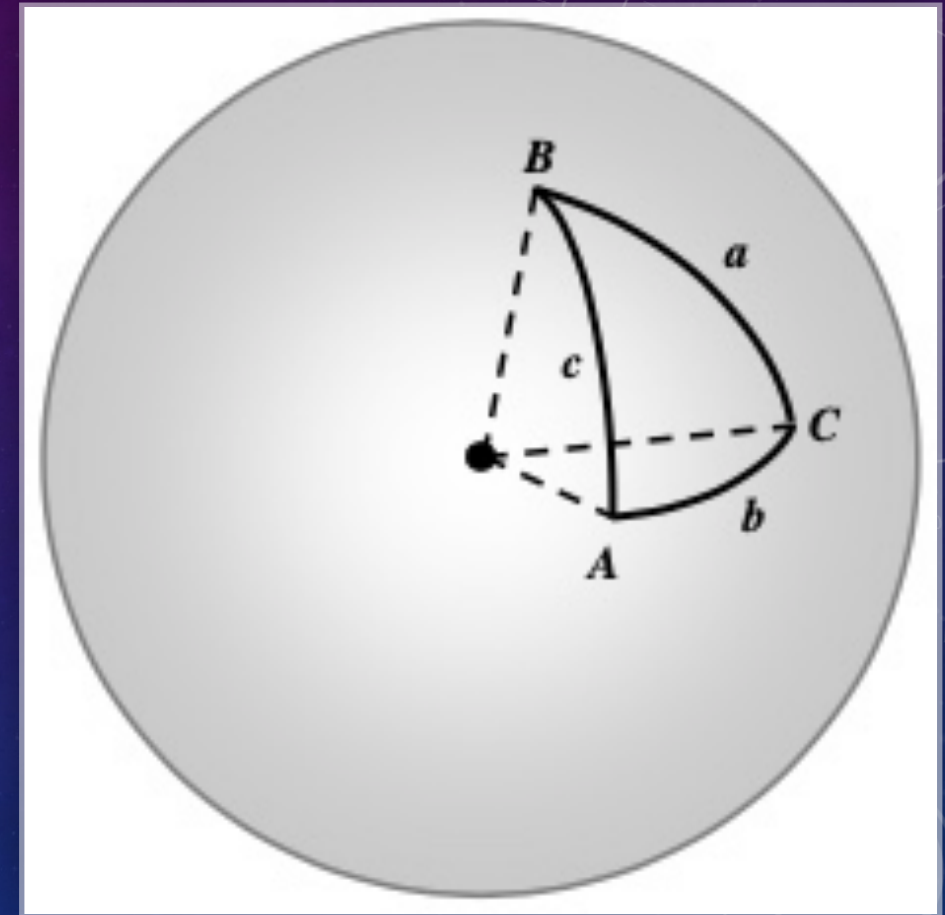
# GEOMETRY OF A SPHERE CONTINUED....

- In the given figure:
- FG=great circle passing through centre O
- EH=small circle
- The distance between two points on a sphere is measured by the length of the arc of the great circle passing through them. It is the shortest path on the surface of the sphere between these points. Thus, in figure , the distance between points C and D on the great circle FG is the arc length CD. It is equal to  $R$  (OD) times the  $\angle COD$  in radians. Since  $R = 1$ , arc  $CD = \angle COD$ .
- An angle between two great circles is called a SPHERICAL ANGLE. It is equal to the angle between the tangents to the great circles at the points of their intersection. It is also the angle between the planes of the two great circles. In the diagram, PACQ and PBDQ are two great circles intersecting at the points P and Q.  $\angle APB$  and  $\angle CPD$  are spherical angles between these great circles. These angles are equal. They are also equal to  $\angle COD$ , since the arc length CD is common to both great circles.



## SPHERICAL TRIANGLE:

- It is a closed figure that is formed when 3 great circles mutually intersect on a sphere. In the previous figure PCD is a spherical triangle. The elements of a spherical triangle ABC in the figure given alongside are 3 spherical angles denoted by  $A, B, C$  and 3 sides  $a, b, c$ .
- This is not merely a 3 sided triangle, the sides must be the arcs of great circles.
- One should keep in mind the sides are measured in degrees or radians.





# BASIC FORMULA RELATING ANGLE AND SIDE OF SPHERICAL TRIANGLE:

- $\frac{\sin A}{\sin a} = \frac{\sin B}{\sin b} = \frac{\sin C}{\sin c}$
- $\cos a = \cos b \cdot \cos c + \sin b \cdot \sin c \cdot \cos A$
- $\cos b = \cos c \cdot \cos a + \sin c \cdot \sin a \cdot \cos B$
- $\cos c = \cos a \cdot \cos b + \sin a \cdot \sin b \cdot \cos C$
- $\sin a \cdot \cos B = \cos b \cdot \sin c - \sin b \cdot \cos c \cdot \cos A$
- $\sin a \cdot \cos C = \cos c \cdot \sin b - \sin c \cdot \cos b \cdot \cos A$

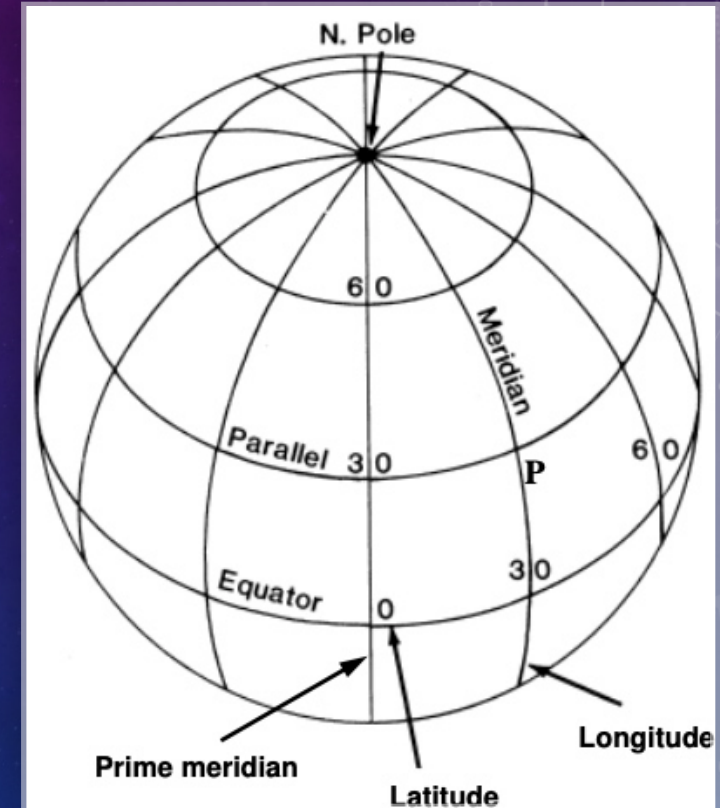
# ASTRONOMICAL COORDINATE SYSTEMS

- The method of fixing the position of a star or celestial object on the celestial sphere is the same as that of fixing the position of a place on the surface of the Earth. We shall, therefore, first consider geographical coordinates of a place on the surface of the Earth.



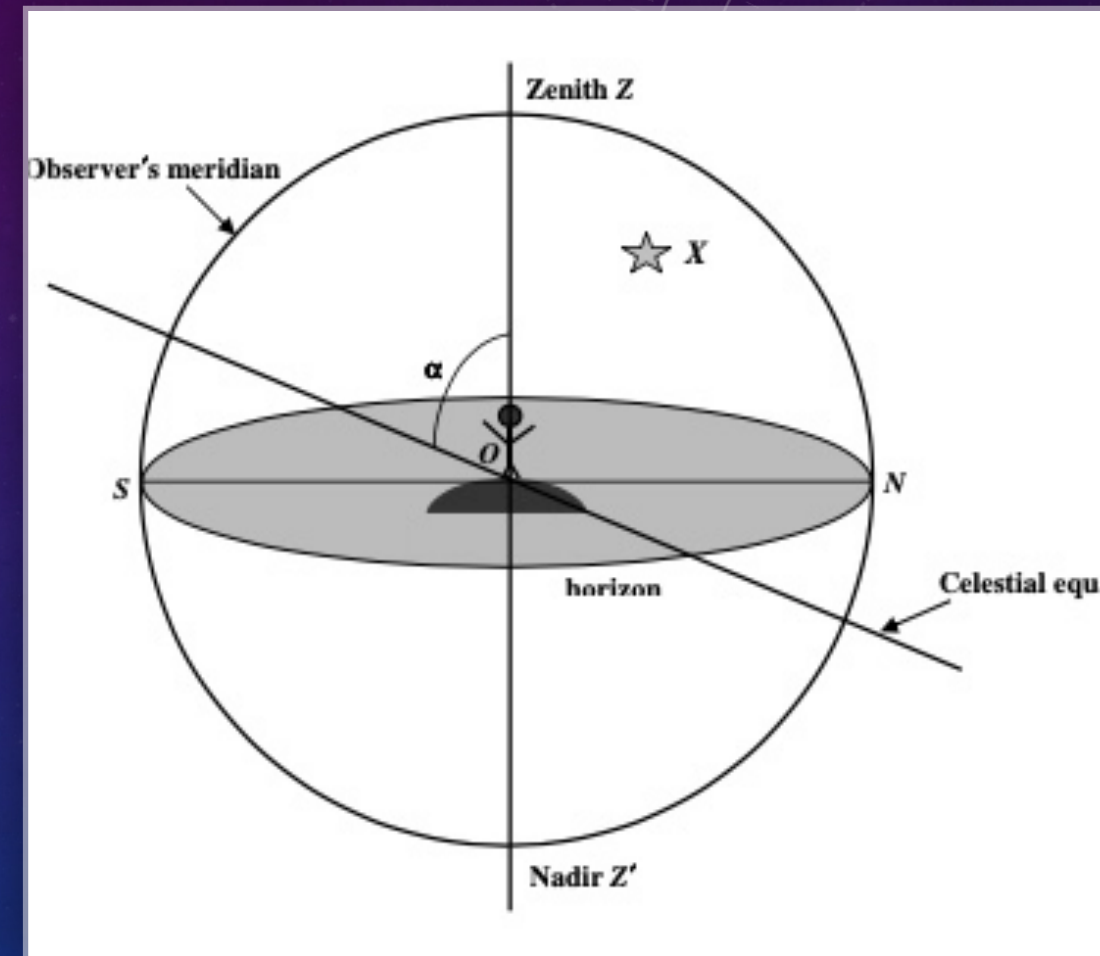
# GEOGRAPHICAL COORDINATES:

- The line joining the poles is always perpendicular to the equator.
- The circles parallel to the equator are the circles of latitude. The great circles drawn through the north and South Poles are called the circles of longitude.
- The great circle passing through P is called its meridian.
- By international agreement, the geographical coordinates (longitude and latitude) of the place P are specified with respect to the equator and the meridian through Greenwich, which is also called the prime meridian.



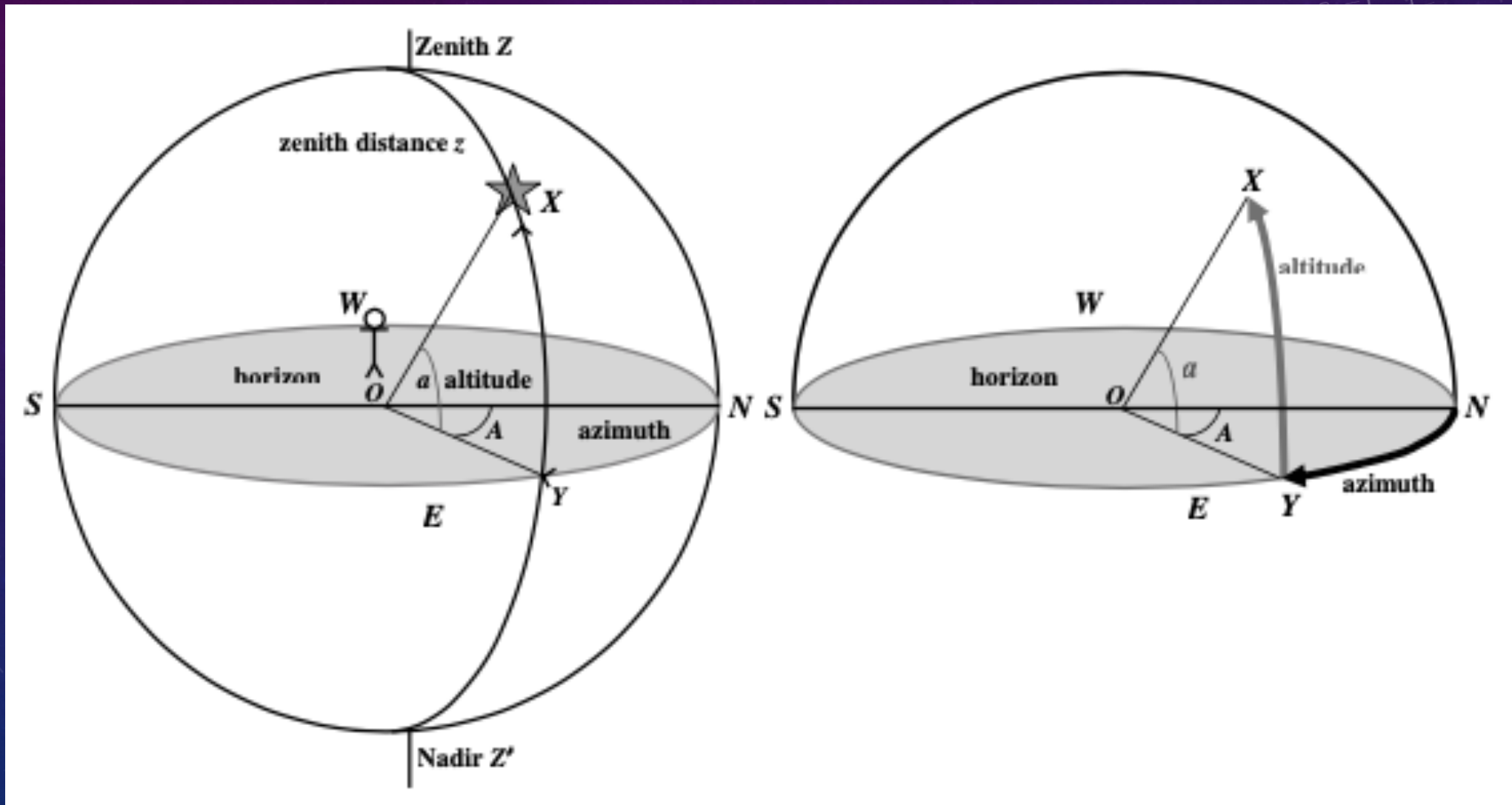
# HORIZON SYSTEM:

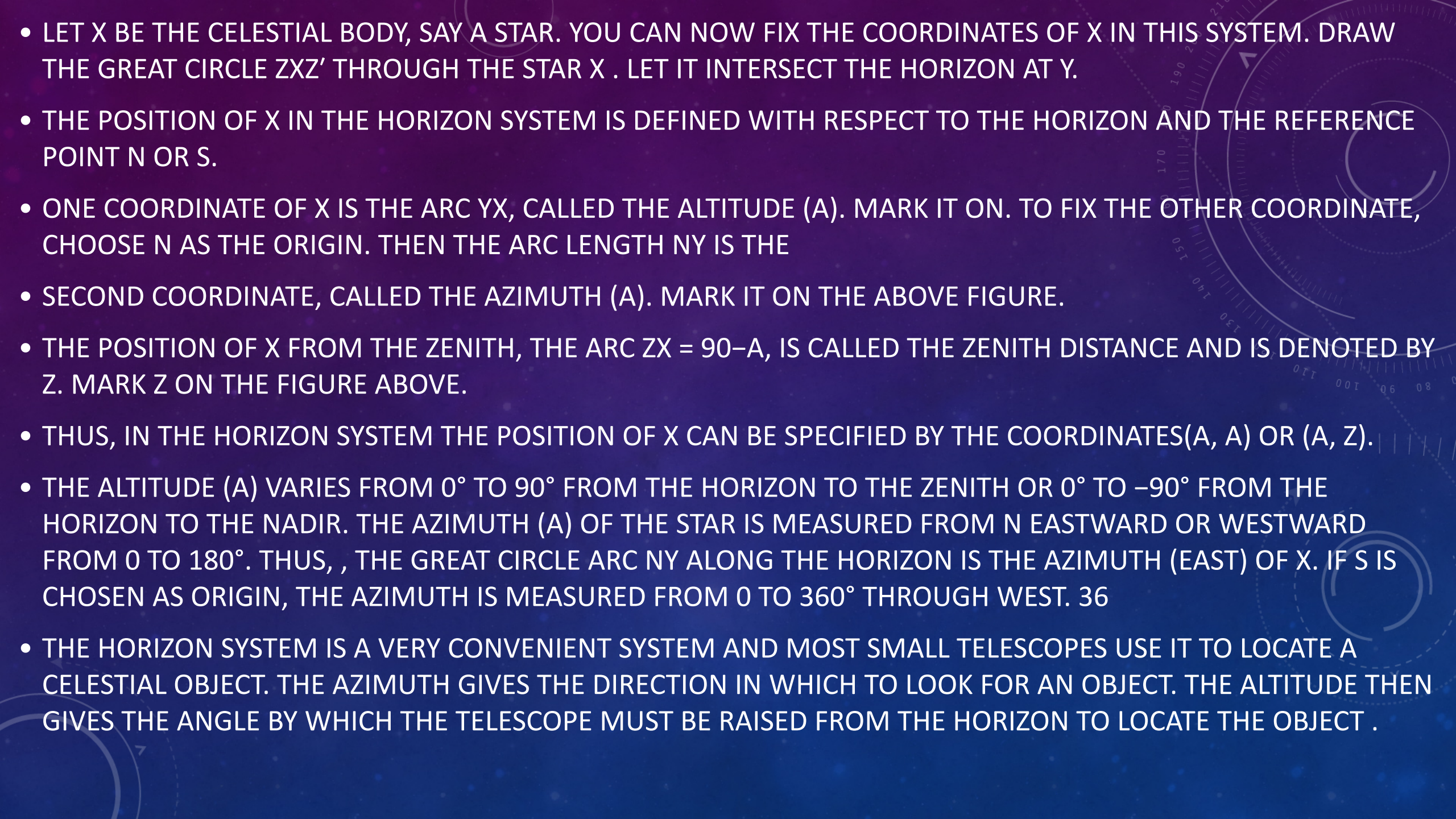
- Suppose the observer is at the point  $O$  at latitude  $\alpha$ . The point vertically overhead the observer is called the zenith. It is denoted by  $Z$  in . The point vertically below the observer is called the nadir. It is denoted by  $Z'$ .
- The great circle on the celestial sphere perpendicular to the vertical line  $ZOZ'$  is called the celestial horizon or just the horizon. This is the fundamental great circle chosen for reference in the horizon system.
- The great circle  $ZNZ'SZ$  containing the zenith of the observer is called the observer's meridian or the local meridian.  $N$  and  $S$  are the points of intersection of the horizon and the observer's meridian. These points indicate geographical north and south directions. Either  $N$  or  $S$  can be taken as the reference point in this system.





# HORIZON SYSTEM CONTINUED...



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- LET X BE THE CELESTIAL BODY, SAY A STAR. YOU CAN NOW FIX THE COORDINATES OF X IN THIS SYSTEM. DRAW THE GREAT CIRCLE ZXZ' THROUGH THE STAR X . LET IT INTERSECT THE HORIZON AT Y.
  - THE POSITION OF X IN THE HORIZON SYSTEM IS DEFINED WITH RESPECT TO THE HORIZON AND THE REFERENCE POINT N OR S.
  - ONE COORDINATE OF X IS THE ARC YX, CALLED THE ALTITUDE (A). MARK IT ON. TO FIX THE OTHER COORDINATE, CHOOSE N AS THE ORIGIN. THEN THE ARC LENGTH NY IS THE
  - SECOND COORDINATE, CALLED THE AZIMUTH (A). MARK IT ON THE ABOVE FIGURE.
  - THE POSITION OF X FROM THE ZENITH, THE ARC ZX =  $90 - A$ , IS CALLED THE ZENITH DISTANCE AND IS DENOTED BY Z. MARK Z ON THE FIGURE ABOVE.
  - THUS, IN THE HORIZON SYSTEM THE POSITION OF X CAN BE SPECIFIED BY THE COORDINATES(A, A) OR (A, Z).
  - THE ALTITUDE (A) VARIES FROM  $0^\circ$  TO  $90^\circ$  FROM THE HORIZON TO THE ZENITH OR  $0^\circ$  TO  $-90^\circ$  FROM THE HORIZON TO THE NADIR. THE AZIMUTH (A) OF THE STAR IS MEASURED FROM N EASTWARD OR WESTWARD FROM 0 TO  $180^\circ$ . THUS, , THE GREAT CIRCLE ARC NY ALONG THE HORIZON IS THE AZIMUTH (EAST) OF X. IF S IS CHOSEN AS ORIGIN, THE AZIMUTH IS MEASURED FROM 0 TO  $360^\circ$  THROUGH WEST. 36
  - THE HORIZON SYSTEM IS A VERY CONVENIENT SYSTEM AND MOST SMALL TELESCOPES USE IT TO LOCATE A CELESTIAL OBJECT. THE AZIMUTH GIVES THE DIRECTION IN WHICH TO LOOK FOR AN OBJECT. THE ALTITUDE THEN GIVES THE ANGLE BY WHICH THE TELESCOPE MUST BE RAISED FROM THE HORIZON TO LOCATE THE OBJECT .

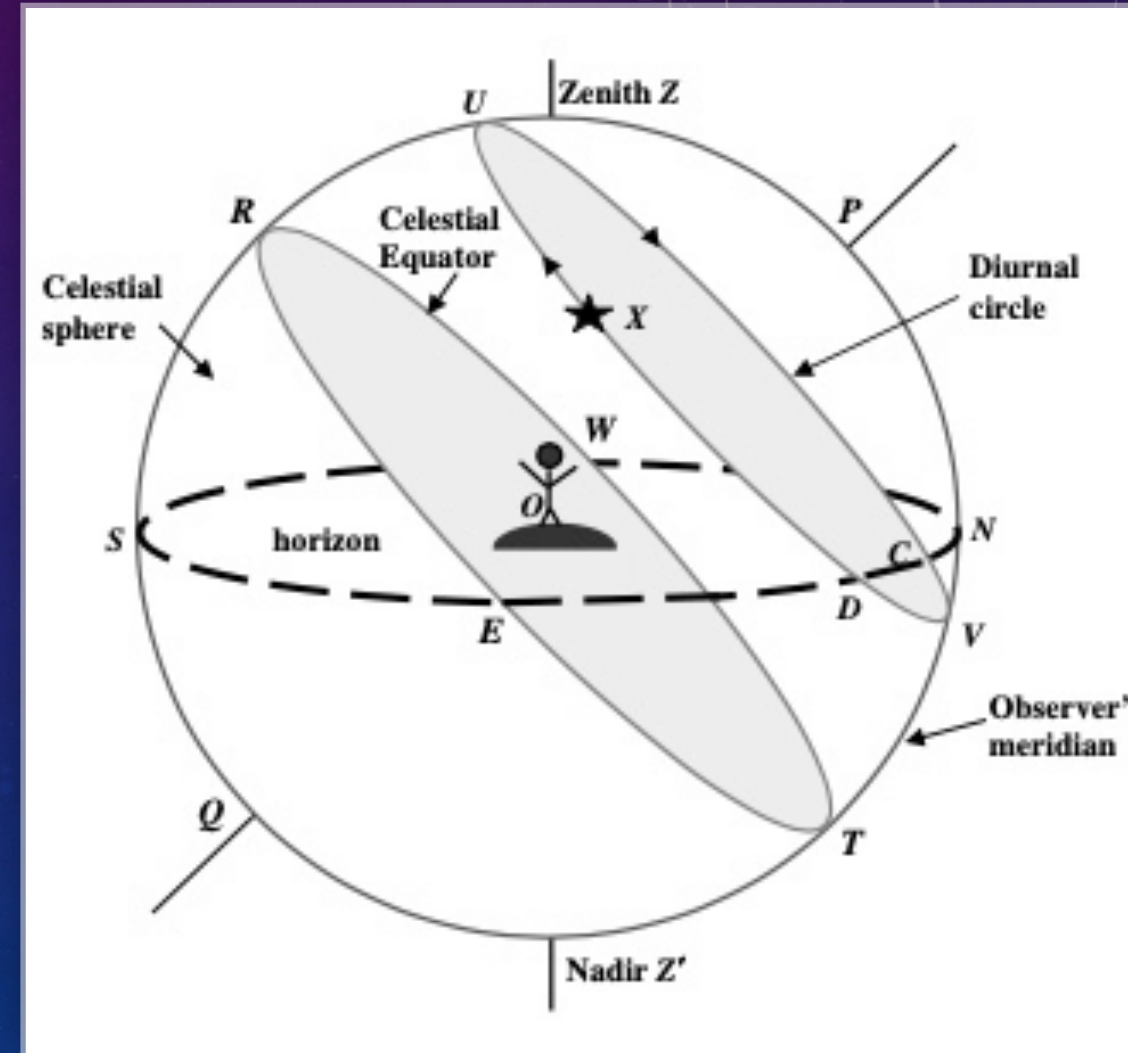


## “• Horizon coordinate system

- Horizon Coordinates:  $(A, a)$  or  $(A, z)$ :
- The azimuth,  $A$ , is the arc length  $NY$  along the horizon if  $N$  is taken as the origin.
- The altitude,  $a$ , is the arc length  $YX$  along the great circle  $ZXYZ'$  containing the zenith and the star.
- The zenith distance,  $z$ , is the arc length  $ZX$  from the zenith to the star on the great circle  $ZXYZ'$ .
- Fundamental great circle: Horizon
- Reference point: The points of intersection  $N$  or  $S$  of the horizon and the observer's meridian.

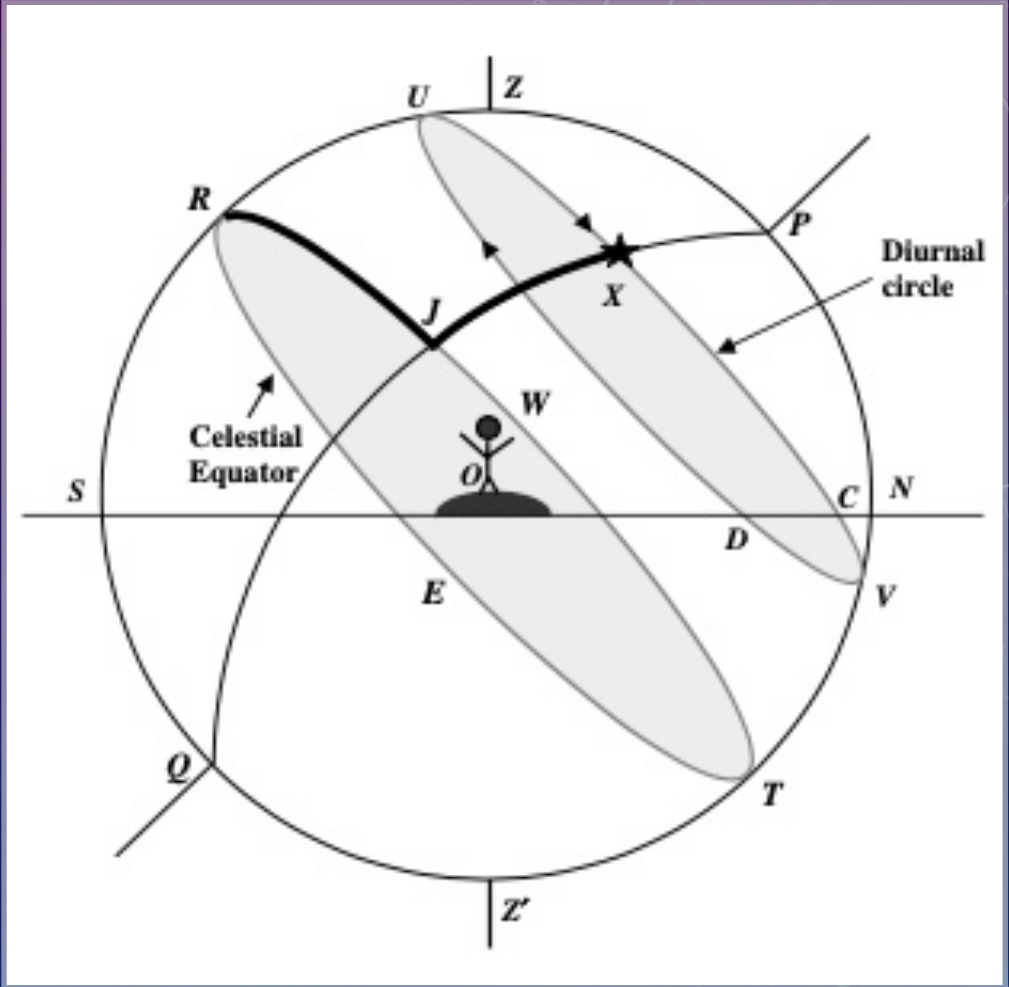
# EQUATORIAL SYSTEM :

- The figure shows the celestial sphere for an observer O. The great circle whose plane is parallel to the equatorial plane of the Earth, and contains the centre O of the celestial sphere is called the celestial equator. P and Q are the poles of the celestial equator: P is the north celestial pole and Q, the south celestial pole. These poles are directly above the north and south terrestrial poles. As you know, the point P also points to the pole star.
- In the diagram the great circle NESW is the observer's horizon with zenith Z as its pole and RWTE is the celestial equator for which P and Q are poles.
- The celestial equator and the observer's horizon intersect in two points, E and W, called the East and West points.
- Any semi-great circle through north and south celestial poles P and Q is called a meridian. But as we have learnt, the full great circle through the observer's zenith (PZRQT) is called the observer's meridian or the local meridian





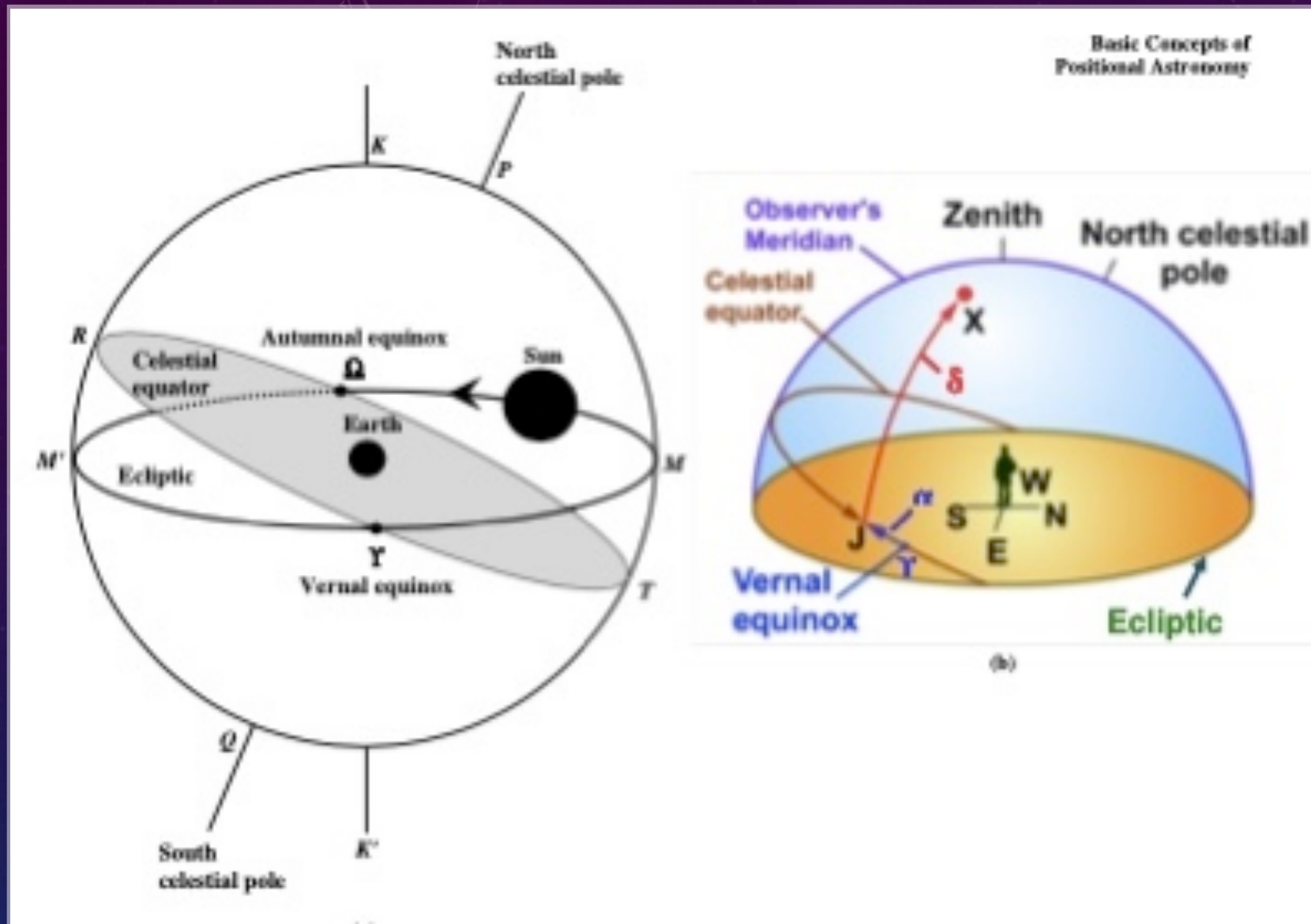
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- The diagram illustrates the celestial sphere with the following features:
- Celestial Equator:** A great circle passing through points  $S$  and  $N$  on the horizon.
  - Diurnal Circle:** A smaller circle representing the path of a star, passing through points  $U$  and  $Z$  at the top and  $E$  and  $T$  at the bottom.
  - Observer:** A stick figure labeled  $O$  is positioned on the horizon line between  $E$  and  $D$ . A point  $W$  is marked on the diurnal circle directly above the observer.
  - Star Path:** A thick black arc represents the star's path, starting at  $R$ , passing through  $J$  and  $X$ , and ending at  $P$ .
  - Other Points:**  $Q$  and  $V$  are on the celestial equator;  $C$  and  $D$  are on the horizon;  $Z'$  is the nadir.



# LOCAL EQUATORIAL SYSTEM:

- Local Equatorial Coordinates:  $(H, \delta)$
- The hour angle :  $H$ , is the arc length  $RJ$  along the celestial equator.
- The declination :  $\delta$ , is the arc length  $JX$  along the star's hour circle.
- Fundamental great circle : Celestial equator.
- Reference point : Intersection of observer's meridian and celestial equator.





VERNAL EQUINOX:

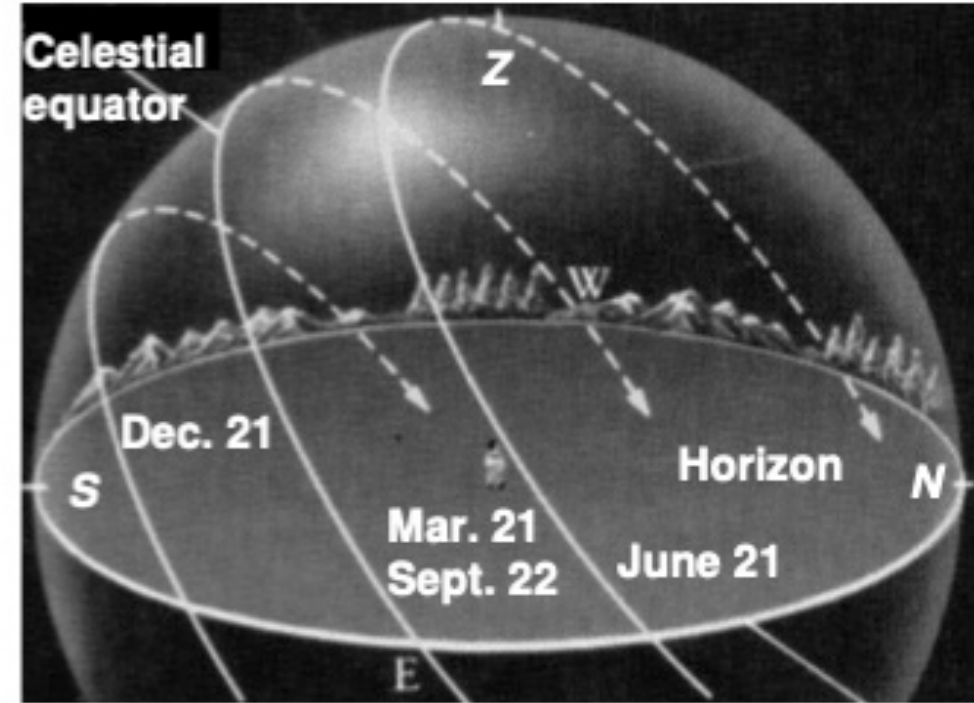
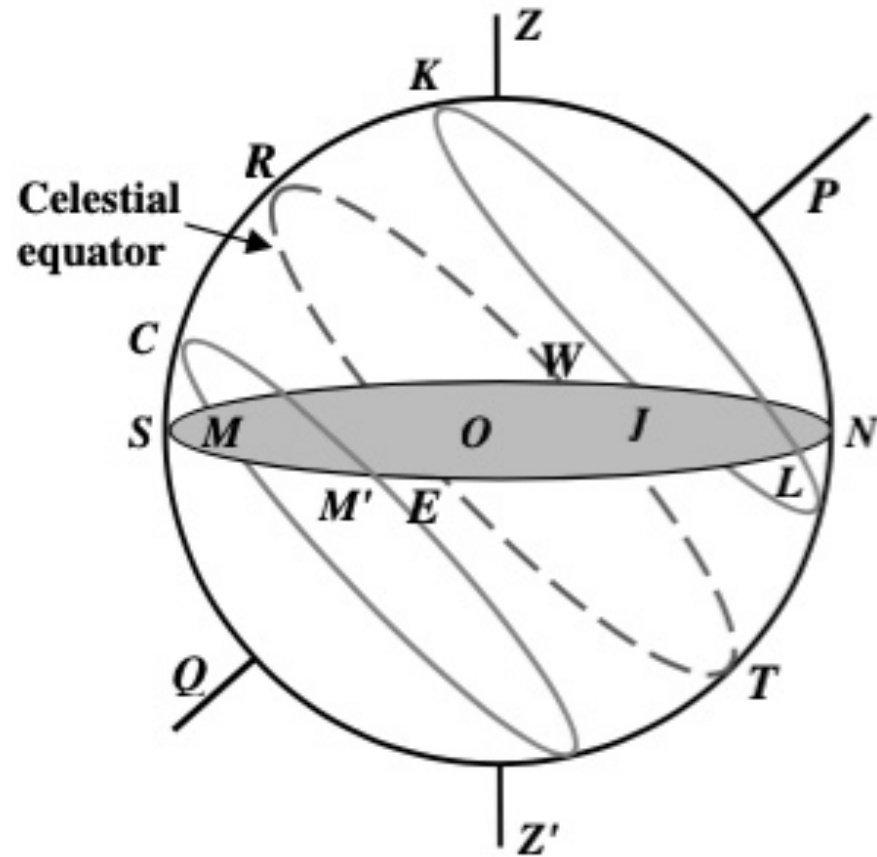
- WE KNOW THAT THE EARTH MOVES AROUND THE SUN IN A NEARLY CIRCULAR ORBIT. THE PLANE OF THE EARTH'S MOTION AROUND THE SUN INTERSECTS THE CELESTIAL SPHERE IN A GREAT CIRCLE CALLED THE ECLIPTIC
- AS SEEN FROM THE EARTH, THE SUN APPEARS TO MOVE AROUND THE EARTH IN THIS PLANE. THUS THE ECLIPTIC IS THE ANNUAL PATH OF THE SUN AGAINST THE BACKGROUND STARS. , R Y T IS THE CELESTIAL EQUATOR AND M Y M' IS THE ECLIPTIC. THE ECLIPTIC IS INCLINED TO THE EQUATOR AT AN ANGLE OF  $23^{\circ}27'$ . POINTS K AND K' ARE THE POLES OF THE ECLIPTIC.
- THE ECLIPTIC AND THE CELESTIAL EQUATOR INTERSECT AT TWO POINTS CALLED VERNAL EQUINOX (Y) AND AUTUMNAL EQUINOX ( $\Omega$ ).
- DURING THE ANNUAL APPARENT MOTION, THE SUN IS NORTH OF THE EQUATOR ON Y M  $\Omega$  AND SOUTH OF THE EQUATOR ON  $\Omega$  M' Y. THUS, THE SUN'S DECLINATION CHANGES FROM SOUTH TO NORTH AT VERNAL EQUINOX (Y) AND FROM NORTH TO SOUTH AT AUTUMNAL EQUINOX ( $\Omega$ ).
- WE CAN DEFINE THE POSITION OF A STAR X WITH RESPECT TO THE CELESTIAL EQUATOR AND THE VERNAL EQUINOX . LET THE HOUR CIRCLE OF X INTERSECT THE CELESTIAL EQUATOR AT J.
- ONE COORDINATE OF X IS THE ARC LENGTH YJ ALONG THE CELESTIAL EQUATOR MEASURED FROM VERNAL EQUINOX EASTWARD. IT IS KNOWN AS THE RIGHT ASCENSION OF THE STAR AND IS DENOTED BY A.
- THE OTHER COORDINATE IS THE DECLINATION  $\Delta$  (ARC JX ).
- AS THE EARTH ROTATES, THE POINTS Y AND J ON THE CELESTIAL EQUATOR ROTATE TOGETHER. THUS, THE SEPARATION BETWEEN Y AND J DOES NOT CHANGE AND THE RIGHT ASCENSION OF THE STAR REMAINS FIXED.
- THIS SYSTEM OF COORDINATES (A,  $\Delta$ ) IS CALLED THE UNIVERSAL EQUATORIAL SYSTEM. THE A AND  $\Delta$  OF STARS DO NOT CHANGE APPRECIABLY FOR CENTURIES.



- Universal Equatorial System
- Universal Equatorial Coordinates : ( $\alpha$ ,  $\delta$ )
- The right ascension,  $\alpha$  : is the arc length YJ eastward along the celestial equator.
- The declination,  $\delta$  : is the arc JX along the star's hour circle.
- Fundamental great circle : Celestial equator.
- Reference point: Vernal equinox.

Object	Universal equatorial coordinates	Coordinates
Crab Nebula	RA 5:35 (h,m)	Declination 22°:01'm
Andromeda	RA 00:42.7 (h,m)	Declination 41°:16'm
Ring Nebula	RA 18:53:35.16 (h,m,s)	Declination 33°:01'm
Orion Nebula	RA 05:35.4 (h,m)	Declination -05°:27'm

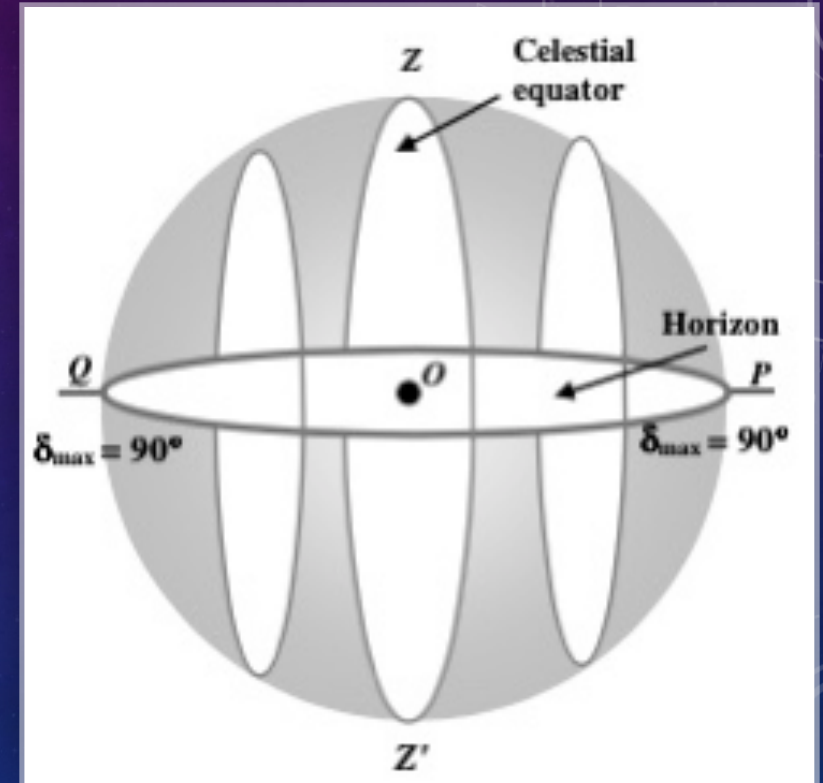




## DIURNAL MOTION OF THE STARS:

In the above figures , drawn for an observer in northern latitudes. The diurnal circles of stars north of the equator are more than half above the horizon (arc  $JKL$ ). The diurnal circles for stars south of the equator are less than half above the horizon (arc  $MCM'$ ).

- An observer in northern latitudes will see more northern stars for longer duration above the horizon compared to the southern stars. For an observer on the equator, the zenith is on the celestial equator. Hence, both northern and southern stars are equally visible for 12 hours as their diurnal circles are perpendicular to the horizon.



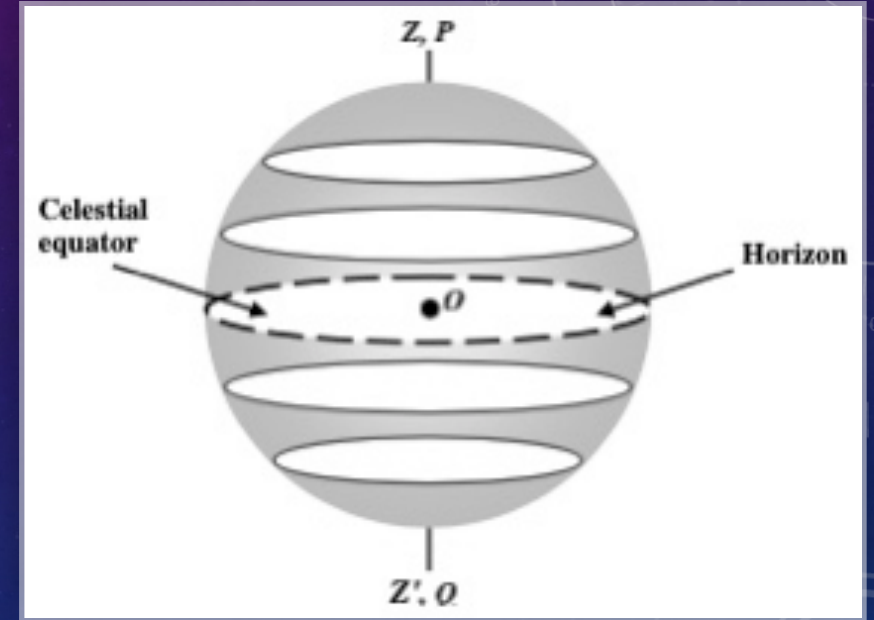


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- The diagram illustrates the celestial sphere as seen from an observer's perspective. Key features include:
- Observer:** A stick figure labeled *O* is at the center of the sphere.
  - Horizon:** A horizontal circle passing through *O*, with points *S* (South) and *N* (North) on its circumference.
  - Celestial Equator:** A great circle passing through *O*, tilted relative to the horizon. It has points *R* and *T* at its top and bottom extremes.
  - Observer's Meridian:** A vertical dashed line passing through *O*, with points *Z* (Zenith) at the top and *Z'* (Nadir) at the bottom.
  - Star Paths:**
    - Star *X* is in the upper right quadrant, with a dashed arc and arrow indicating it moves from right to left, never crossing the horizon. Labeled "Star never sets".
    - Star *Y* is in the lower left quadrant, with a dashed arc and arrow indicating it moves from left to right, never crossing the horizon. Labeled "Star never rises".
  - Other Labels:** Points *E* and *D* are on the horizon; points *C* and *P* are on the meridian; point *Q* is on the celestial equator.

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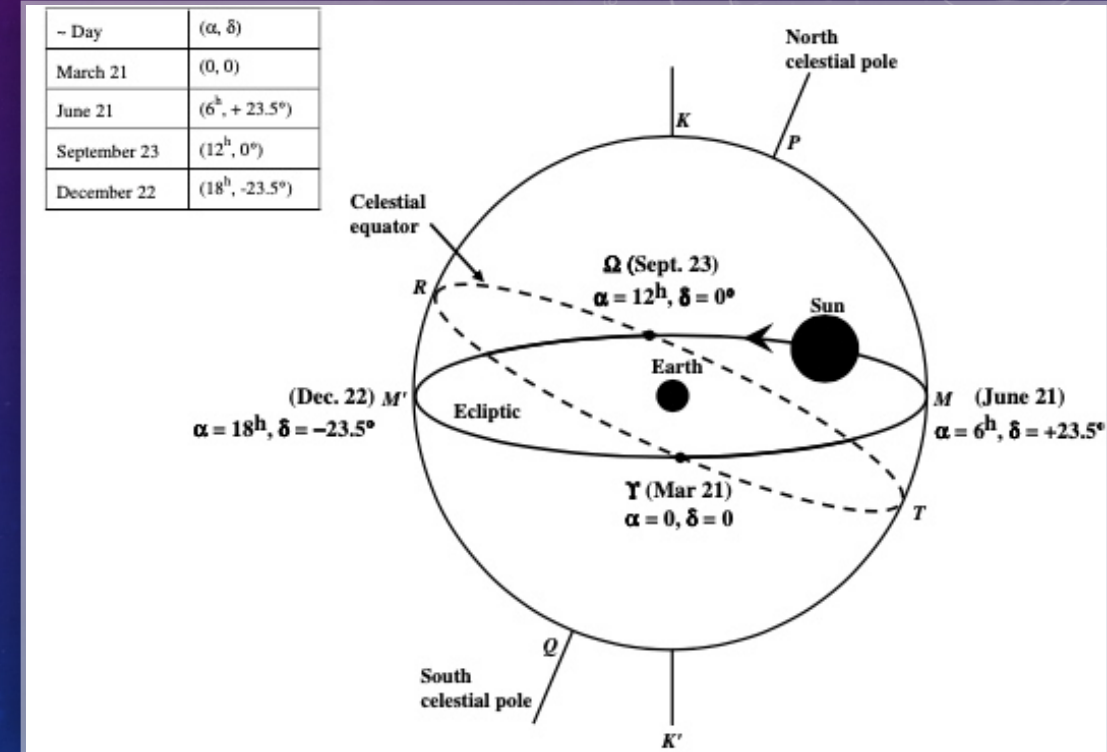


- At North Pole ( $\phi = 90^\circ$ ), all the stars lying north of the equator, that is,  $\delta \geq 0$  are circumpolar, while the southern stars ( $\delta < 0$ ) are below the horizon. These will be invisible.



# MOTION OF STARS:

- Previously we learnt that because of the yearly revolution of the Earth round the Sun, the Sun appears to move around the Earth along the ecliptic. It moves at a rate of about  $1^\circ$  per day. As a result, the right ascension and declination of the Sun change continuously. The Sun is at  $\Upsilon$  on or around March 21 every year. At that time its coordinates are  $\alpha = 0$ ,  $\delta = 0$ .
- At  $M$ , around June 21, the Sun is at  $\alpha = 6^h$ ,  $\delta = +23.5^\circ$ . At  $\Omega$  around September 23,  $\alpha = 12^h$ ,  $\delta = 0^\circ$  for the Sun. At  $M'$  around December 22, the Sun is at  $\alpha = 18^h$ ,  $\delta = -23.5^\circ$ .
- $\delta = -23.5^\circ$
- The point  $M$  with  $\delta = \delta_{\max} = +23.5^\circ$  is the summer solstice which marks the beginning of Dakshinayan, the south-ward motion of the Sun. The opposite point  $M'$
- with  $\delta = \delta_{\min} = -23.5^\circ$  is the winter solstice which is the beginning of Uttarayan, the north-ward journey of the Sun. Note that the arc lengths  $YM$  and  $YM'$  are equal even if it may not seem so.





# CONVERSION OF COORDINATES:

- We relevant formulae for conversion of coordinates from local equatorial system to horizon system:
- Conversion of  $(H, \delta)$  to  $(A, z)$  :
- $\sin z \sin A = \cos \delta \sin H$
- $\sin z \cos A = \sin \delta \cos \phi - \cos \delta \sin \phi \cos H$
- $\cos z = \sin \delta \sin \phi + \cos \delta \cos H \cos \phi$
- Conversion of  $(A, z)$  to  $(H, \delta)$ :
- $\cos \delta \sin H = \sin z \sin A$
- $\cos \delta \cos H = \cos z \cos \phi - \sin z \sin \phi \cos A$
- $\sin \delta = \cos z \sin \phi + \sin z \cos \phi \cos A$

## MEASUREMENT OF TIME:

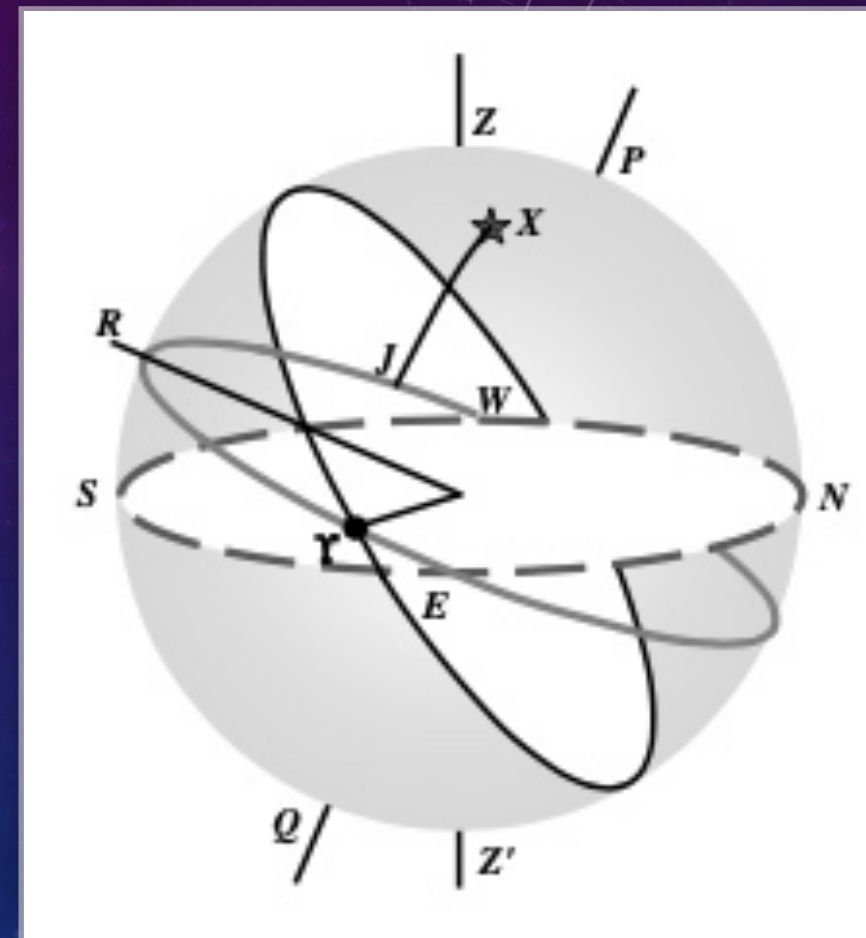
- The basic unit of time, the day, is defined as the time required for the Earth to rotate once on its axis. If this rotation is considered with respect to the Sun then the time for one rotation relative to the Sun is the solar day, that is, the time from one Sunrise to the next Sunrise. On the other hand, rotation period of the Earth relative to the stars is called a sidereal day
- The Earth has to rotate by about  $1^\circ$  more to bring the Sun overhead and complete the solar day. It takes roughly 4 minutes for the Earth to rotate through  $1^\circ$  since it rotates through  $360^\circ$  in 24 hours. Thus, the sidereal day is shorter than the average solar day by about 4 minutes. In a month (30 days), the sidereal time falls behind the mean solar time by two hours, and in six months the difference becomes 12 hours. Then at solar midnight we have sidereal noon.
- Since the Sun regulates human activity on the Earth, we use solar time in our daily life. The sidereal time is not useful for civil purposes but is used in astronomical observations.





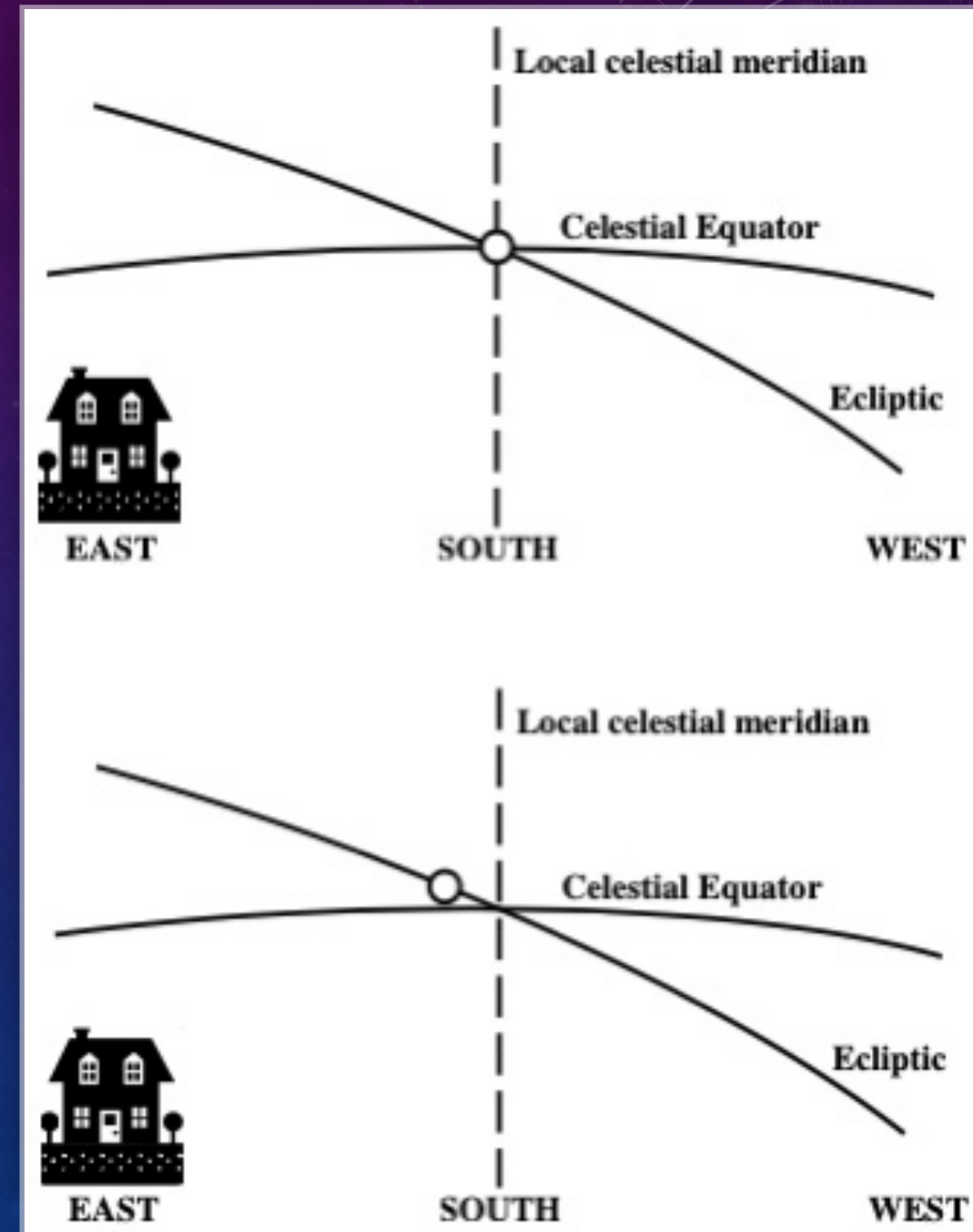
# SIDEREAL TIME:

- Due to the rotation of the Earth, the vernal equinox  $\Upsilon$ , moves along the equator once in 24 hours like any other star. The hour angle of  $\Upsilon$  for an observer is called the sidereal time for that observer or the local sidereal time (LST).
- NWSE is the horizon,  $P$  and  $Z$  are the north celestial pole and the zenith, respectively. The hour angle (HA) of vernal equinox is the great circle arc  $RY$  measured towards the west from the observer's meridian. When  $\Upsilon$  is at  $R$ , at upper transit on the observer's meridian, its HA is  $0^h$  and consequently the local sidereal time (LST) is  $0^h$  at this instant.
- Let  $X$  be the position of a star. The arc  $RJ$  (towards west) is the HA of  $X$  (HAX) and the arc  $YJ$  (towards east) is the right ascension of  $X$  (RAX). We have
- $RY = RJ + YJ$ .
- Hence, this is an important relation in the measurement of time. If  $X$  is the Sun, denoted by  $\odot$
- Then we have the relation:
- $LST = HA \odot + RA \odot$
- 



# APPARENT SOLAR TIME:

- The hour angle of the Sun at any instant is the apparent solar time. The interval between two consecutive transits of the Sun over the same meridian is called one apparent solar day.
- In the figure given alongside, we show the Sun at noon on March 21, when the Sun and the vernal equinox are on the local meridian. The celestial sphere rotates and some time during the next day, the vernal equinox is again on the meridian. That interval of time is one sidereal day; a fixed point on the celestial sphere has gone around once.
- But in that time the Sun has moved eastward along the ecliptic. But the Sun is not yet on the meridian and it is not yet noon. The celestial sphere has to rotate a bit more to bring the Sun up to the meridian. This time interval represents the apparent solar day.

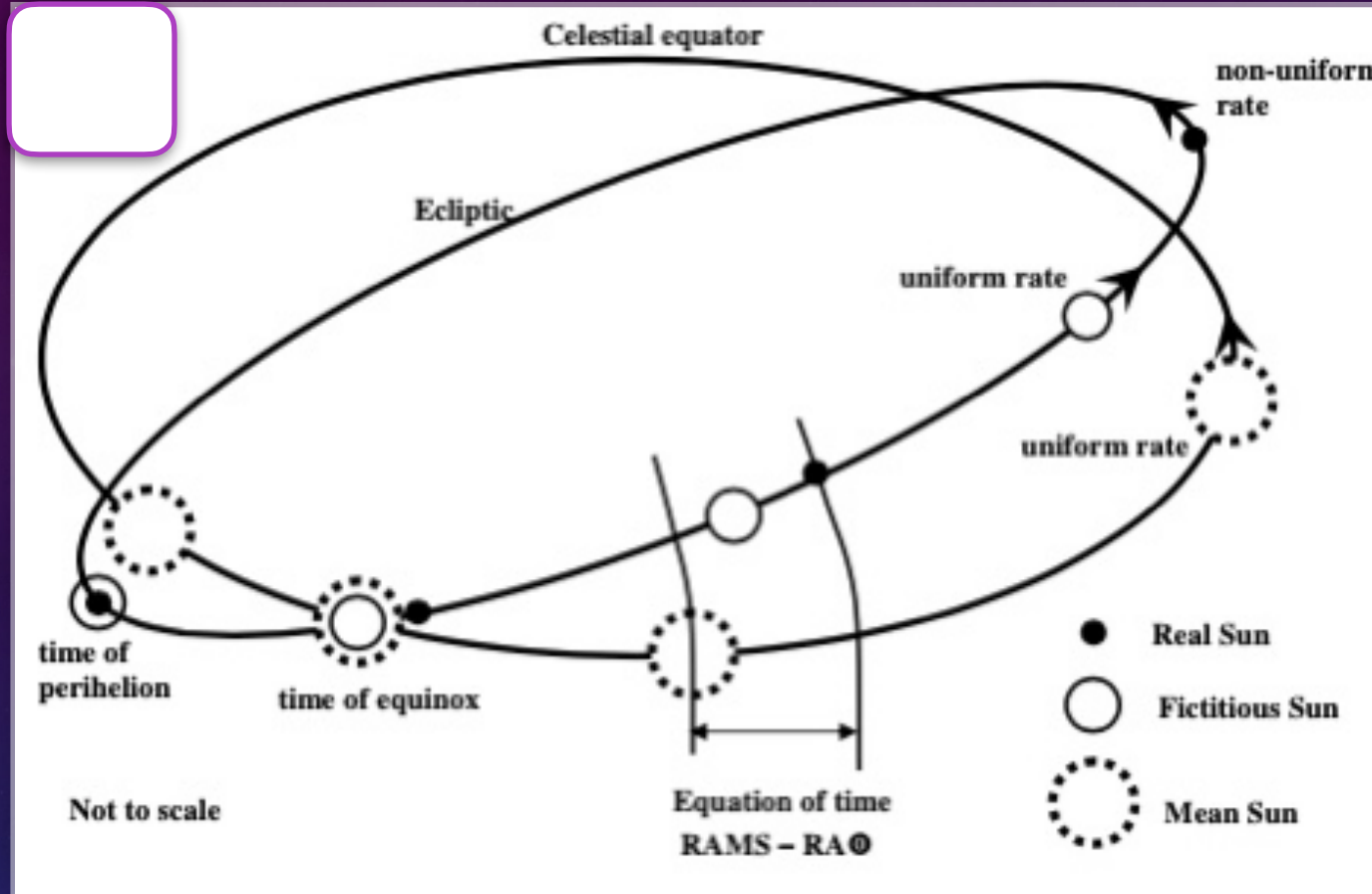




# MEAN SOLAR TIME:

- In order to have a uniform solar time, a fictitious body called the mean Sun is introduced. The mean Sun is supposed to move uniformly (that is, in a circular path) along the equator completing one revolution in the same time as the actual Sun does round the ecliptic. The hour angle of the mean Sun at any instant is defined as the Mean Solar Time (MST), or simply, the Mean Time.
- The time interval between the successive transits of the mean Sun over the same meridian is defined as the mean solar day.
- The mean solar day is subdivided into hours, minutes and seconds. The civil day begins at mean midnight for reasons of convenience. Our clocks are designed to display the mean solar time.
- By international agreement, the time of the Greenwich meridian is called the Greenwich mean time (GMT). It can be related to the mean time of any other place through the longitude of that place.
- Since the Earth completes one rotation on its axis, that is 360 degrees of longitude in 24 hours, each degree of longitude corresponds to 4 minutes of time. Therefore, GMT at a given instant is related to the mean time of any other place by a simple relation:
- $\text{Local Mean Time} = \text{GMT} + \lambda$  (in units of time)
- where  $\lambda$  is the geographical longitude of the place expressed in units of time. In the above relation,  $\lambda$  is to be taken as positive if the longitude of the place is east of Greenwich, and negative if it is west of Greenwich. The above relationship holds good for both the sidereal as well as the solar time.
- It is obvious that if each town and city in a country were to use its own local time, the life of citizens would become very difficult. To overcome this problem, a whole country uses the local time of a place as its standard time.

# EQUATION OF TIME(RELATION BETWEEN MEAN & APPARENT SOLAR TIME):



As we have seen in before:

$$LST = HA + RA$$

for the Sun and the mean Sun, we can write:

$$RAMS - RA_{☉} = HA_{☉} - HAMS$$

The difference in right ascensions of the mean Sun and true Sun at a given instant called, Equation of Time(ET).

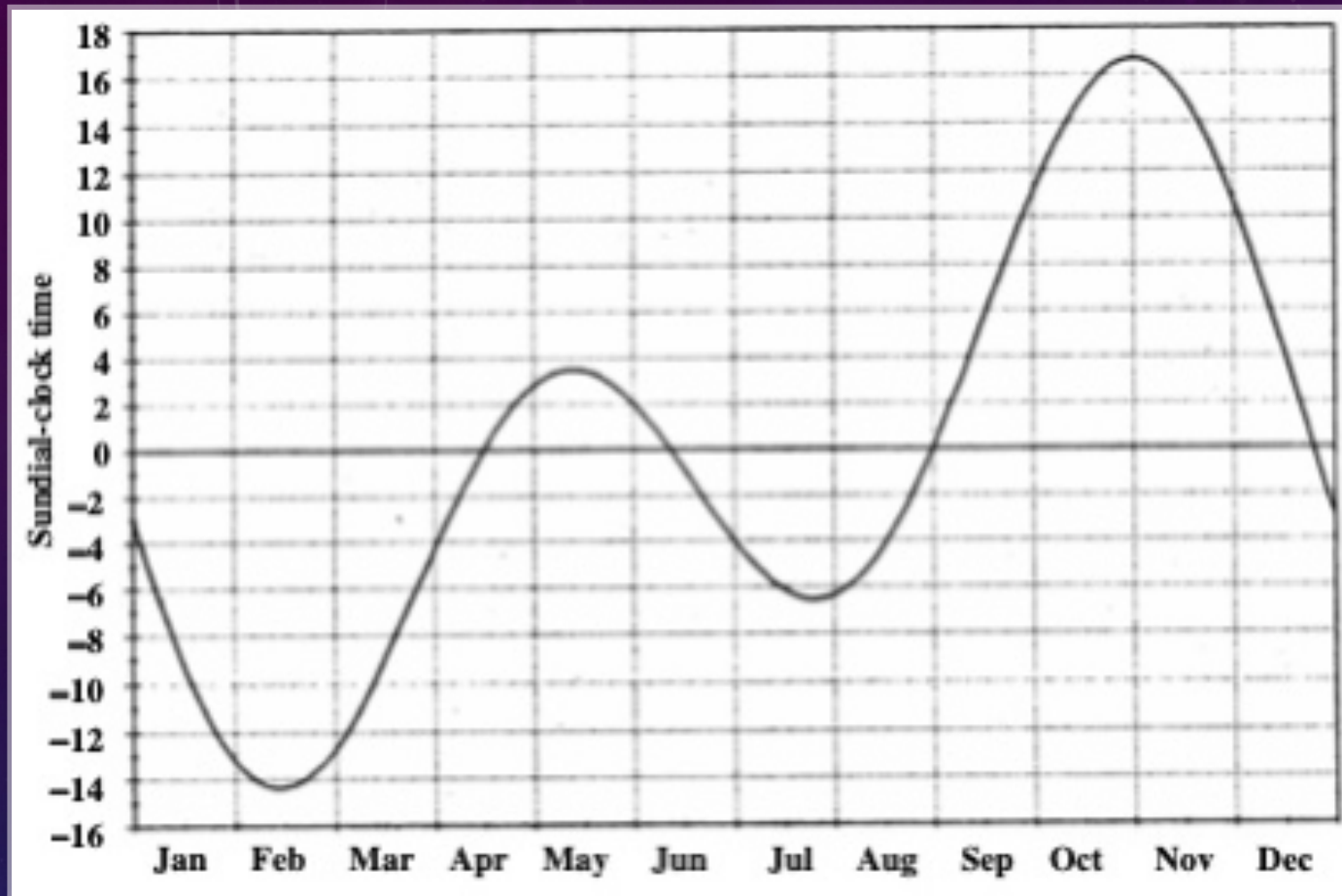
HA<sub>☉</sub> : is the symbol of hour angle for the Sun.

RA<sub>☉</sub> : is the symbol of right ascension of the Sun.

HAMS : is the symbol of hour angle for the mean Sun.

RAMS : is the symbol of right ascension of the mean Sun.





Therefore from above equation we have:

$$ET = HA - HAMS$$

Hence , we conclude that the ET at any instant is the difference between the apparent solar time given by HA and the mean solar time given by HAMS.

## CALENDARS:

As we all know any discussion on Time in Astronomy is insufficient without Calendars , so →

- The civil year contains 365 mean solar days. As the actual period of Earth's revolution, called a tropical year contains 365.2422 mean solar days, a fraction 0.2422 of a day is omitted each year. This resulted in the loss of a number of days over several centuries and the civil year would get out of step with the seasons.
- To overcome this confusion, Julius Caesar introduced Julian calendar, named after him, in which the year was made of 365.25 mean solar days. Consequently, every 4th year was made to contain 366 mean solar days. This was called a leap year. The extra day was added in February. Thus, in this calendar, a year, which is divisible by 4, is a leap year in which the month of February has 29 days.
- In Julian calendar, the tropical year was assumed to be of 365.25 mean solar days, but its actual length is 365.2422 mean solar days. The small difference between assumed and actual lengths of the year, created serious problem by the 16th century and the civil year was out of step in relation to the seasons. To overcome this problem, Pope Gregory, introduced in 1582 the calendar known as Gregorian calendar, which we are using now.
- In this calendar, a leap year is defined as before, with the exception that when a year ends in two zeros, it will be a leap year only if it is divisible by 400. Thus in a cycle of 400 years, there are 100 leap years according to Julian Calendar while there are only 97 in Gregorian Calendar, since years 100, 200 & 300 are not leap years. This makes the average civil year to consist of 365.2425 mean solar days, which is very close to the true length of the tropical year. No serious discrepancy can arise in the Gregorian calendar for many centuries.



Thank You...