Feynman Diagrams

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October 31, 2020

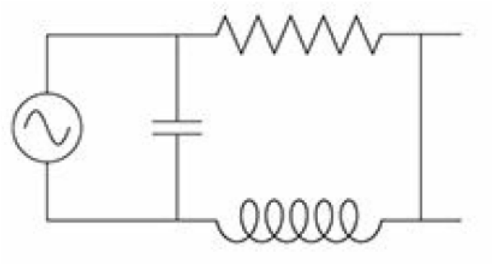
Overview

Abstract:
"In part 5 of the HEP lecture series we'll be going through the basics of electrical circuits to then help us learn about the feynman diagrams and a few toy models that exemplify it's effectiveness."

Motivation

Circuits

Consider the example circuit below(ONLY FOR ILLUSTRATION)



Clearly, we can see an A.C. Voltage source, a Resistor, an capacitor and an inductor.they represent different functions for the voltage across that element in terms of the charge flowing through the element.

Motivation

If Q is the total current in the system, Then the Voltage across different elemnts is:

$$V_C = \frac{Q}{C}$$

$$V_R = R\frac{\mathrm{d}Q}{\mathrm{d}t}$$

$$V_I = L\frac{d^2Q}{dt^2}$$

here, C is capacitance, R is resistance And L is inductance. So, we can compactly denote voltage/charge relationships with symbols in circuit diagrams.

Motivation

We can encode even more details into this like the energy and charge conservation. For this we have Kirchoff's rules.

- Kirchoff's first rule is that the charge flowing into a node (or vertex) is equal to the charge flowing out of a node.
- Kirchoff's second rule is the identical statement, but applied to energy. A consequence of this is that the net voltage around any closed loop in a circuit is zero.

Using these two, We can determine the Voltage Between the 2 open ends.

Motivation

Now We need to develop a similar Diagram for particle physics to figure out the probability that an initial collection of particles turns into a final collection of particles. it must contain :

- Distinguishable representation of different elements
- Relativistic Energy and Momentum conservation
- What happens at vertices.(in case of circuits, charge flowing into a vertex is equal to charge flowing out of it)

Note that in particle physics, we also need to specify what happens at vertices to conserve angular momentum.

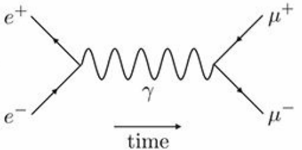
Motivation

Electron–Muon Scattering

let's consider one of the simplest processes in particle physics: the collision of an electron and a positron that annihilate and produce a muon and an anti-muon,

$$e^+e^- \to \mu^+\mu^-$$

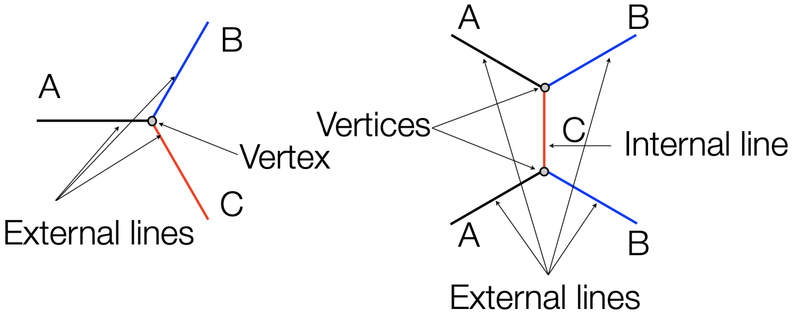
Since both of these are Electrically charged they interact through a Electromagnetism. Since photon is the force carrier for EM force, the simplest way they interact is through an exchange of photon. so the photon is called the **Propogater** and is represented by a wavy line. so the scattering process can be represented as:



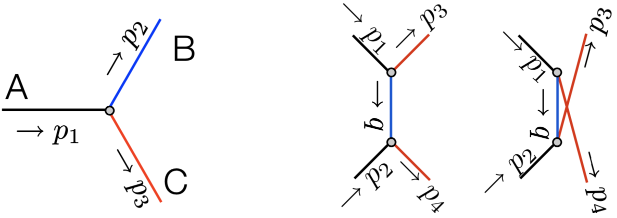
Components of a Feynman Diagram

- External Lines:(ingoing and outgoing momenta) Particles that come in and out in the initial and final state, respectively.
- Vertex factors:($-ig$, coupling constant) each vertex (i.e. where A, B, C meet) has a factor. determines "order" of diagram: order=number of vertices
- Internal lines: Factors for internal particles exchanged between vertices Only applies to particles "internal" to the diagram, not external lines
- Momentum Conservation at vertex: at each vertex enforcing 4-momentum conservation
- Integrals over internal momentum: Internal lines have any momentum consistent with 4-momentum conservation

Feynmann Diagrams



Feynman Rules



$-ig$ $\quad$ $(-ig)^2$ $\quad$ $(-ig)^2$

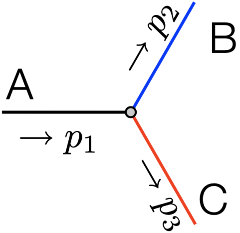
Step 1: Notation and coupling constant

Label the incoming and outgoing four-momenta. Label the internal momenta. Put an arrow on each line, to keep track of the "positive" direction. For each vertex,write down a factor of - *ig*, g is called the coupling constant.

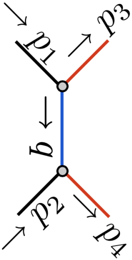
Feynman Rules

Step 2: Propagator

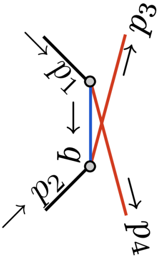
Introduce propagator for each internal line $\frac{i}{q^2 - m_B^2 c^2}$



$$(-ig)$$



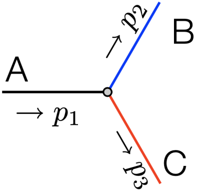
$$(-ig)^2 \frac{i}{q^2 - m_B^2 c^2}$$



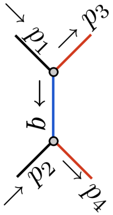
$$(-ig)^2 \frac{i}{q^2 - m_B^2 c^2}$$

Feynman Rules

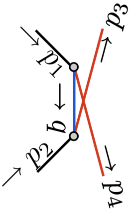
Step 3: Energy and Momentum Conservation
For each vertex,write a delta function of the form $(2\pi)^4\delta^4\left(k_1+k_2+k_3\right)$ where the k's are the three four-momenta coming into the vertex (if the arrow leads outward, then k is minus the four-momentum of that line).



$$(-ig)\times(2\pi)^4\,\delta^4(p_1-p_2-p_3)$$



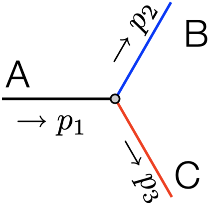
$$(-ig)^2\frac{i}{q^2-m_B^2c^2}$$
$$\times(2\pi)^4\delta^4(p_1-p_3-q)$$
$$\times(2\pi)^4\delta^4(p_2-p_4+q)$$

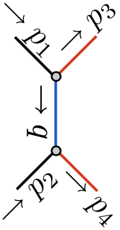


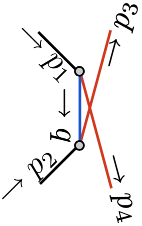
$$(-ig)^2\frac{i}{q^2-m_B^2c^2}$$
$$\times(2\pi)^4\delta^4(p_1-p_4-q)$$
$$\times(2\pi)^4\delta^4(p_2-p_3+q)$$

Feynman Rules

Step 4: Integrate over internal momenta. For each line write down a factor $\int \frac{1}{(2\pi)^4} d^4 q$







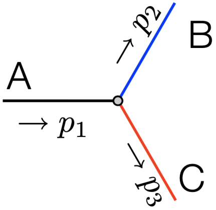
$$(-ig) \times (2\pi)^4 \ \delta^4(p_1 - p_2 - p_3)$$

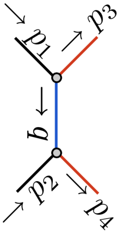
$$\int \frac{1}{(2\pi)^4} d^4 q (-ig)^2 \frac{i}{q^2 - m_B^2 c^2} \times (2\pi)^4 \delta^4(p_1 - p_3 - q) \times (2\pi)^4 \delta^4(p_2 - p_4 + q)$$

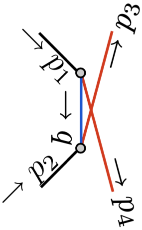
$$\int \frac{1}{(2\pi)^4} d^4 q (-ig)^2 \frac{i}{q^2 - m_B^2 c^2} \times (2\pi)^4 \delta^4(p_1 - p_4 - q) \times (2\pi)^4 \delta^4(p_2 - p_3 + q)$$

Feynman Rules

Step 5: Cancel the Delta Function and perform the integral







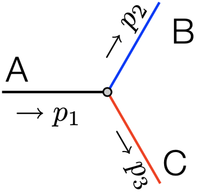
$$(-ig) \times (2\pi)^4 \; \delta^4(p_1 - p_2 - p_3)$$

$$(-ig)^2 \frac{i}{(p_4 - p_2)^2 - m_B^2 c^2} \times (2\pi)^4 \delta^4(p_1 - p_3 - p_4 + p_2)$$

$$(-ig)^2 \frac{i}{(p_3 - p_2)^2 - m_B^2 c^2} \times (2\pi)^4 \delta^4(p_1 - p_3 - p_4 + p_2)$$

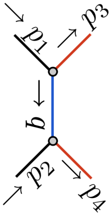
Feynman Rules

Step 6: The result will include a delta function $(2\pi)^4\delta^4\left(p_1+p_2+\cdots-p_n\right)$ Erase this factor, and what remains is $-iM$



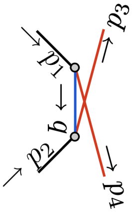
$$(-ig)$$

$$\mathcal{M}=g$$



$$(-ig)^2\frac{i}{(p_4-p_2)^2-m_B^2c^2}$$

$$\mathcal{M}=\frac{g^2}{(p_4-p_2)^2-m_B^2c^2}$$



$$(-ig)^2\frac{i}{(p_3-p_2)^2-m_B^2c^2}$$

$$\mathcal{M}=\frac{g^2}{(p_3-p_2)^2-m_B^2c^2}$$

Summary

Feynman diagrams and their construction from the feynman rules are a central tool for analyzing particle physics processes. here lets summarize the process:

- Identify the external and internal particles.
  - Draw arrows to represent positive direction
  - External particles - on shell solutions; real partices
  - Internal particles - off shell solution; virtual particles
- Connect the particles with a propagator
- Include a Delta function at each vertex to conserve energy and momentum
- Add the factor of $\int \frac{1}{2\pi} d^4 q$
- Cancel the delta function and solve

Conclusion

Caveat Emptor

Feynman diagrams are useful mathematical tools, but as physical descriptions of the scattering process, they should be interpreted with care. some limitations of the feynman diagram are:

- Quantum realm is probabalistic, 2 particles can interact in a number of ways. For more number of outcomes the feynman diagrams are not very accurate
- When there are constraints on the final state that restrict particle energies or angles, Feynman diagrams are no longer an accurate representation of the process.
- As the representation of the wavefunction overlap of an initial with a final state, Feynman diagrams do not exhibit any time evolution.

Conclusion

So using feynman diagrams is an elegant way to represent the equations involved but owing to the shear magnitude of possible outcomes, it is not very efficient. for higher order interactions the feynman diagrams give but an effective description. For such cases, an Amplitudehedron (shown below) is a step "ahead".

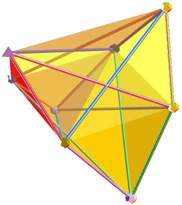


Figure: Notional amplituhedron visualization based on a theory by Nima Arkani-Hamed and Mathematica code from J. Bourjaily

Bibliography & Image Credits

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Cambrigde: Cambridge University Press

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