

GROUPNAME – Title of project for the 2024-2025 assignment

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(Dated: March 30, 2025)

[illegible]

## RULES

- Compile the latex file (found in the google folder from moodle) with the format of this template, without changing the parameters (page size etc.).
- The text, figures, and references should be of about four pages.
- There is a deadline for submission is written in moodle.
- Write a statement in which the contribution of each member of the group is specified.

- Do not change the time of verbs; it is simpler to speak in simple present, however also writing always in past tense is fine.
- In figures, use fonts that match the size of the main text fonts (tiny fonts should be avoided). Use lines with different dashing, color, and symbol as appropriate for better distinguishing the curves. Use log scale when it is better for highlighting smaller scales or flattening larger scales.
- Remember the grid explained in the intro video of the course, which will be used for evaluations. It contains suggestions for improving the text.

**Latex** – A modern online tool to handle and share latex files is *Overleaf*. The other option is a standard latex installation on the computer. Locally, this text is compiled with the command `pdflatex` and is based on `revtex`. Packages (of which, maybe not all are needed) in Arch Linux may be installed via

```
sudo pacman -S texlive-core
texlive-bibtexextra texlive-fontsextra
texlive-formatsextra texlive-latexextra
texlive-pictures texlive-pstricks
texlive-publishers texlive-science
```

In Ubuntu there is a similar installation with `sudo apt install`, maybe `sudo apt install texlive-full` if you want to lose less time to pick the right packages. Similar tools should be available in Windows and via e.g. macports on Mac OS.

## INTRODUCTION

The main purpose of this assignment is to simulate the writing of a short paper, to train the writing skills and the capability to explain a subject with effective simple sentences and a logic chain.

The topic of your assignment is specified at the lesson. It requires you to describe your findings in one of the exercises.

In this introduction, you should describe the main topic in general terms, introducing what you want to discover, why, and which methods you use do perform this study. There could also be citations like this [1] to papers, websites, etc. forming the list of references that other people could be interested in consulting for a better understanding your points.

## Tips

- In English use sentences shorter than what you might normally be using in Italian, German, etc...
- Possibly, Explain concepts at a level which is accessible to everybody.
- Do not use colloquial forms in scientific writing, thus avoid it's, aren't, don't, etc.

## METHODS

**Log-likelihood** To solve a problem using Bayesian method, as we do, we have to define

- the *likelihood function*,  $p(X|\theta)$ , that describes the probability of observing a dataset  $X$  given the value of parameters  $\theta$
- the *prior distribution*,  $p(\theta)$ , that describes the *a-priori* knowledge we have about the parameters.

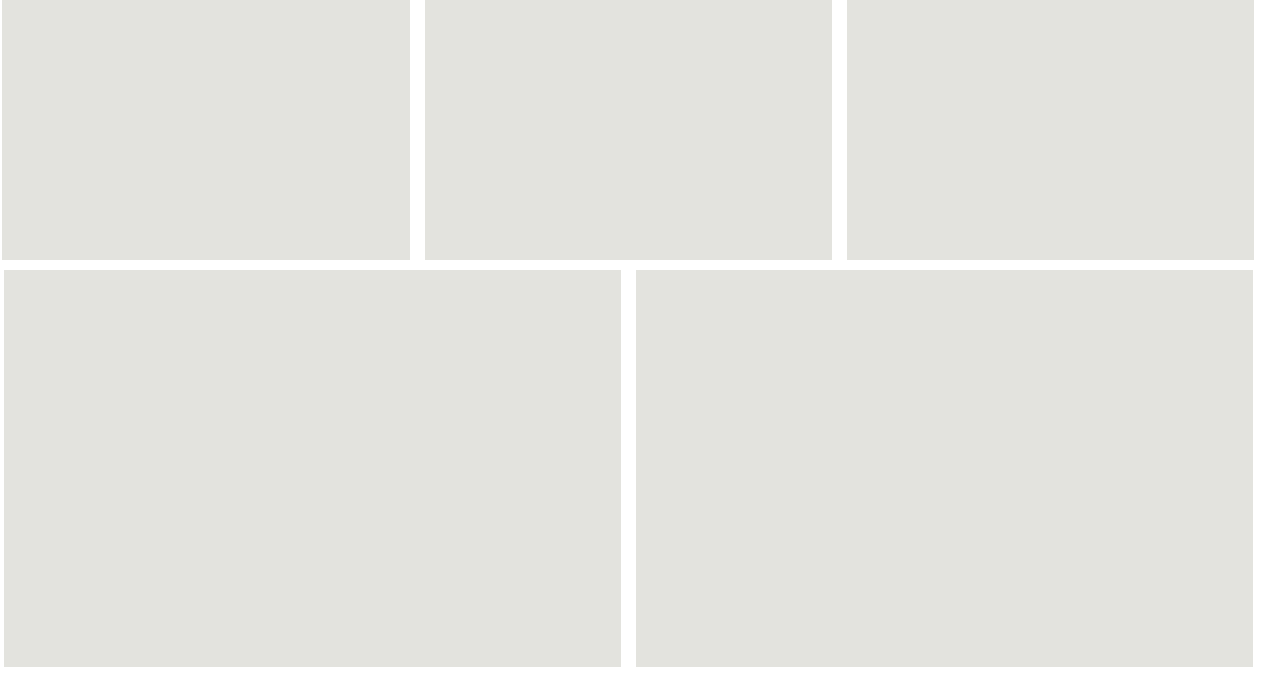


FIG. 1. Description of the panels: (a)..... (b)... etc. This caption should give enough info on the content of figures to make them mostly readable without consulting the main text. However, repetitions with the main text should be avoided if possible. **If this format is difficult to frame in the page you want, just break it into multiple single figures.**

These two are used to compute the *posterior distribution*

$$p(\theta|X) = \frac{p(X|\theta)p(\theta)}{\int d\theta' p(X|\theta')p(\theta')} \quad (1)$$

that describes the knowledge we have about parameters  $\theta$  after observing the data  $X$ . As we will see the denominator of the posterior distribution in many cases is not possible to compute analytically. Markov Chain Monte Carlo methods are required to draw random samples of  $p(\theta|X)$ . The likelihood function is determined by the model and the measurement noise. Many generative models follow a *Maximum Likelihood Estimation* (MLE). In MLE parameters  $\hat{\theta}$  that maximize the likelihood of generating observed data are chosen. Equivalently, the log-likelihood since log is monotonic.

$$\hat{\theta} = \arg_{\theta} \max \log p(X|\theta) \quad (2)$$

The most common approach used for training a generative model is to maximize the log-likelihood of the training dataset. By choosing the negative log-likelihood as the cost function, the learning procedure tries to find parameters that maximize the probability of the data. The log-likelihood  $\ell_{\theta}(x)$  per data point  $x$ , averaged over  $M$  data points, gives the log-likelihood of data

$$\mathcal{L} = \frac{1}{M} \sum_{m \leq M} \ell_{\theta}(x^{(m)}) \quad (3)$$

In the case of RBMs, we have

$$\ell_{\theta}(x) = \ln \sum_z e^{-E(x,z)} - \ln \sum_{x'} \sum_z e^{-E(x',z)} \quad (4)$$

where the second term is the partition function  $Z$ . The hard part resides in summing up the Boltzmann weights of all possible configurations in  $Z$ , with  $D$  visible units and  $L$  hidden units, there are  $2^{D+L}$  possible configurations. We followed instead the procedure suggested by Baiesi **CITA**, that takes advantage of the energy function.

$$H_i(z) = a_i + \sum_{\mu} w_{i\mu} z_{\mu} \quad (5)$$

$$E(x, z) = - \sum_i H_i(z) x_i - \sum_{\mu} b_{\mu} z_{\mu} \quad (6)$$

$$e^{-E(x,z)} = \prod_{\mu} e^{b_{\mu} z_{\mu}} \prod_i e^{H_i(z) x_i} \quad (7)$$

in eq. 7 the first factor is defined as  $G(z)$ . With this we can reach a reduced partition function  $Z(z)$  defined as

$$Z(z) = G(z) \prod_i \left(1 + e^{H_i(z)}\right) \quad (8)$$

This is easy to compute and becomes numerically stable limiting the argument to avoid overflow. Since we used low value of  $L$  in our RBM we can compute the partition function at the start of the training

$$\ln Z = \ln \left[ \sum_z G(z) \prod_{i=1}^D \left(1 + e^{H_i(z)}\right) \right] \quad (9)$$

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TABLE I. Description of the table.

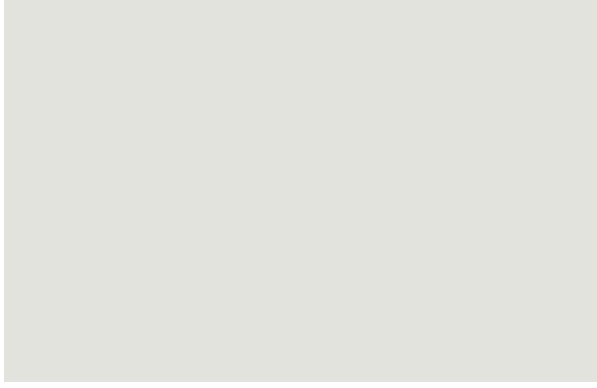


FIG. 3. Description...

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## CONCLUSIONS

Discuss the key aspects that we can take home from this work.

Check if your text is light, swift, and correct in exposing its passages.

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[1] B. Franklin, J. Here There **10**, 20–40 (1800).

[2] A. Einstein, Int. J. There Here **20**, 125–133 (1910).

# Assignment score grid

<b>Structure:</b> the exposition follow a logic order	8
<b>Clarity:</b> the text is brief enough, avoids complicated sentences and specifies all concepts and links	8
<b>Depth:</b> the text is not a shallow repetition of notions, there emerges a good understanding	8
<b>Rigor:</b> the analysis of the results is precise, quantitatively, and convincing	8
<b>Innovation:</b> new methods/ideas are introduced; conclusions beyond what introduced in the class	4