

# Homework 5

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## Question 1

Suppose a given website receives an average of 30 visitors per hour. Using the Poisson distribution, calculate the probability that the website receives more than a certain number of visitors in a given hour.

### Solution:

$E(X) = \lambda = 30$ . So

- $P(X > 35 \text{ visitors}) = 0.15738$
- $P(X > 40 \text{ visitors}) = 0.03231$
- $P(X > 45 \text{ visitors}) = 0.00396$

And so on.

This gives hosting companies an idea of how much bandwidth to provide to different websites to ensure that they'll be able to handle a certain number of visitors each hour.

## Question 2

suppose a given restaurant receives an average of 500 customers per day. Find the probability that the restaurant receives more than a certain number of customers via Poisson distribution.

### Solution:

$E(X) = \lambda = 500$ . So

- $P(X > 520 \text{ customers}) = 0.17930$
- $P(X > 540 \text{ customers}) = 0.03633$
- $P(X > 560 \text{ customers}) = 0.00390$

And so on.

This gives restaurant managers an idea of the likelihood that they'll receive more than a certain number of customers in a given day.

## Question 3

suppose it is known that 7% of adults who take a certain medication experience negative side effects. Use a Binomial Distribution Calculator to find the probability that more than a certain number of patients in a random sample of 150 will experience negative side effects.

### Solution:

$E(X) = np = 150 * 0.07 = 10.5$ . So

- $P(X > 10 \text{ patients experience side effects}) = 0.48149$
- $P(X > 15 \text{ patients experience side effects}) = 0.06127$
- $P(X > 20 \text{ patients experience side effects}) = 0.00188$

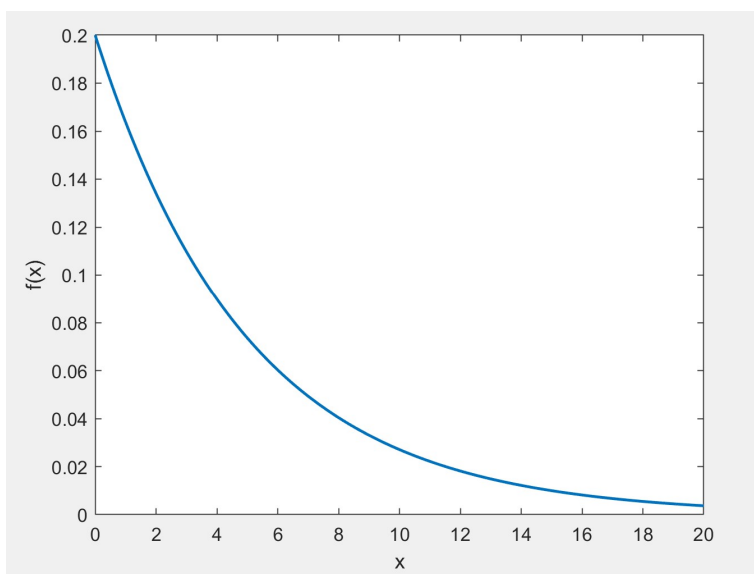
This gives medical professionals an idea of how likely it is that more than a certain number of patients will experience negative side effects.

## Question 4

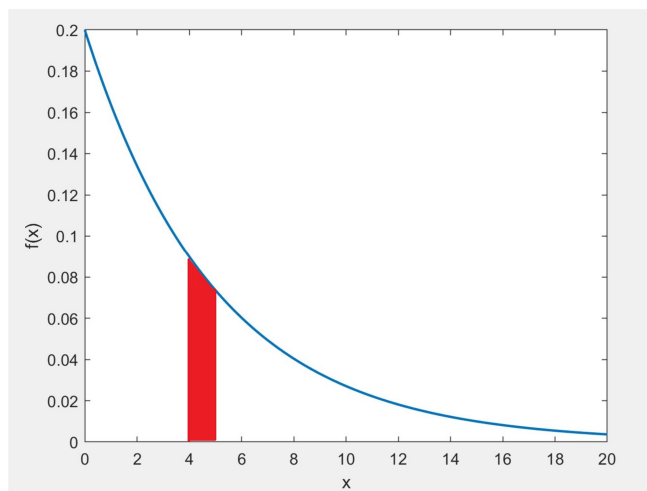
Let  $X$  = amount of time (in minutes) a postal clerk spends with his or her customer. The time is known to have an exponential distribution with the average amount of time equal to five minutes. ( $\mu = \sigma$ )  
find the probability that a clerk spends four to five minutes with a randomly selected customer.

**Solution:**

$$f(x|\beta) = \frac{1}{\beta}e^{-x/\beta}, \quad \beta = \mu = 5 \Rightarrow f(x|\beta) = 0.2e^{-0.2x}$$



Now, we should find the the probability that a clerk spends four to five minutes or  $P(4 < X < 5)$ .



As we have in previous session (Chapter 3: Distributions), we know that

$$P(X > x) = e^{-\frac{x}{\beta}}$$

To find  $P(4 < X < 5)$ , we have:

$$P(4 < X < 5) = P(X < 5) - P(X < 4)$$

So:

$$P(X < 5) = 1 - P(X > 5) = 1 - e^{-\frac{5}{5}} = 0.6321 \quad \text{and} \quad P(X < 4) = 1 - e^{-\frac{4}{5}} = 0.5507$$

So the final answer is:

$$P(4 < X < 5) = P(X < 5) - P(X < 4) = 0.6321 - 0.5507 = 0.0814.$$