Homework 2 - Stat4ML Course, Chapter 1

Omid Safarzadeh

December 2021

***Notice: Deadline: 2 days after receiving the email!**

- Verify the following identities:
  - $A \setminus B = A \setminus (A \cap B) = A \cap B^c$
  - $B = (B \cap A) \cup (B \cap A^c)$
  - $B \setminus A = B \cap A^c$
  - $A \cup B = A \cup (B \cap A^c)$
- For events A and B, find formulas for the probabilities of the following events in terms of the quantities $P(A)$, $P(B)$, and $P(A \cap B)$.
  - either A or B or both
  - either A or B but not both
  - at least one of A or B
  - at most one of A or B
- Two pennies, one with $P(head) = u$ and one with $P(head) = w$, are to be tossed together independently. Define

$$p_0 = P(0 \text{ heads occur}),$$

$$p_1 = P(1 \text{ heads occur}),$$

$$p_2 = P(2 \text{ heads occur}),$$

  Can u and w be chosen such that $p_0 = p_1 = p_2$? Prove your answer.
- Prove the general version of DeMorgan's Laws. Let $\{A_\alpha : \alpha \in \Gamma\}$ be a (possibly uncountable) collection of sets. Prove that

$$(\cup_\alpha A_\alpha)^c = \cap_\alpha A_\alpha^c.$$

$$(\cap_\alpha A_\alpha)^c = \cup_\alpha A_\alpha^c.$$

- If $P(A) = \frac{1}{3}$ and $P(B^c) = \frac{1}{4}$, can A and B be disjoint? Explain.

- The Smiths have two children. At least one of them is a boy. What is the probability that both children are boys?

- Seven balls are distributed randomly into seven cells. Let $X_i =$ the number of cells containing exactly i balls. What is the probability distribution of X_3? (That is, find $P(X_3 = x)$ for every possible x.)

- When coded messages are sent, there are sometimes errors in transmission. In particular, Morse code uses "dots" and "dashes", which are known to occur in the proportion of 3:4.
  Suppose there is interference on the transmission line, and with probability $\frac{1}{8}$ a dot is mistakenly received as a dash, and vice versa. If we receive a dot, can we be sure that a dot was sent?

***Reference:** Casella, G., & Berger, R. (2002).Statistical inference. Cengage Learning.