Assignment 7

Stefano Migneco

1 Numerical Differentiation

There are several methods to calculating numerical derivatives. The first of these is forward differencing, where you start with the first-order Taylor series for (x+h), a step forward:

$$f(x+h) = f(x) + \frac{f'(x)h}{1!} \tag{1}$$

$$f'(x) = \frac{f(x+h) - f(x)}{h} \tag{2}$$

This same process can be repeated for backwards differencing, giving the result of $f'(x) = \frac{f(x)-f(x-h)}{h}$
By taking the difference of both a step forward and backward, and including the second order term, you can arrive at the central differencing scheme:

$$f(x+h) = f(x) + \frac{f'(x)h}{1!} + \frac{f''(x)h^2}{2!} \tag{3}$$

$$f(x-h) = f(x) - \frac{f'(x)h}{1!} + \frac{f''(x)h^2}{2!} \tag{4}$$

$$f(x+h) - f(x-h) = f'x(h) + f'(x)h \tag{5}$$

$$f'(x) = \frac{f(x+h) - f(x-h)}{2h} \tag{6}$$

This can be further expanded to a higher-order scheme, such as the five-point stencil approximation, where you take two points on either side of the point you are evaluating:

$$f(x+h) = f(x) + \frac{f'(x)h}{1!} + \frac{f''(x)h^2}{2!} + \frac{f'''(x)h^3}{3!} \tag{7}$$

$$f(x-h) = f(x) - \frac{f'(x)h}{1!} + \frac{f''(x)h^2}{2!} - \frac{f'''(x)h^3}{3!} \tag{8}$$

$$f(x+2h) = f(x) + \frac{2f'(x)h}{1!} + \frac{4f''(x)h^2}{2!} + \frac{8f'''(x)h^3}{3!} \tag{9}$$

$$f(x-2h) = f(x) - \frac{2f'(x)h}{1!} + \frac{4f''(x)h^2}{2!} - \frac{8f'''(x)h^3}{3!} \tag{10}$$

$$E1 = f(x+h) - f(x-h) = \frac{2f'(x)h}{1!} + \frac{f'''(x)h}{3} \tag{11}$$

$$E2 = f(x+2h) - f(x-2h) = \frac{4f'(x)h}{1!} + \frac{8f'''(x)h^3}{3} \tag{12}$$

$$8E1 - E2 = \frac{12f'(x)h}{1!} \tag{13}$$

$$f'(x) = \frac{8f(x+h) - 8f(x-h) - f(x+2h) + f(x-2h)}{12h} \tag{14}$$

2 Electric Potential

This was the most difficult part of the assignment. In this, a dict of 50 random strength point charges at 50 random locations was created. I extracted the dict into two separate arrays, and then summed each 2D array of voltages created by looping through the dict to create a map. Then, I used a quiver plot to create a display of the electric field.

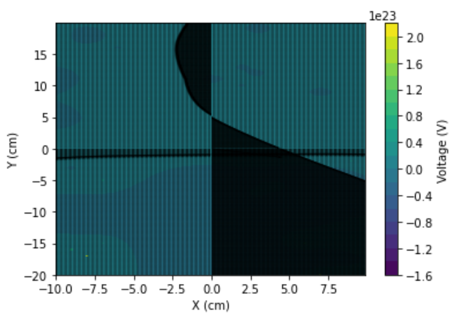

Figure 1: A sample plot of the charges and electric field

3 Harmonic Oscillator

In this part of the assignment, I programmed two ODEs to represent damped oscillation. There were 4 cases: 0 damping, critical damping at exactly $\frac{4k}{m}$, under-damping at less than $\frac{4k}{m}$, and over-damping at greater than $\frac{4k}{m}$.

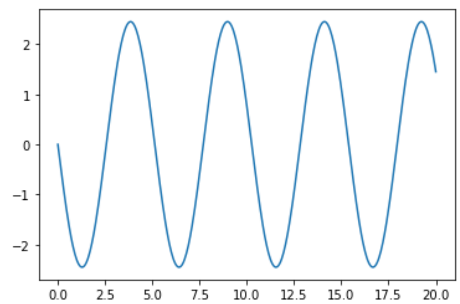

Figure 2: Oscillation with 0 damping

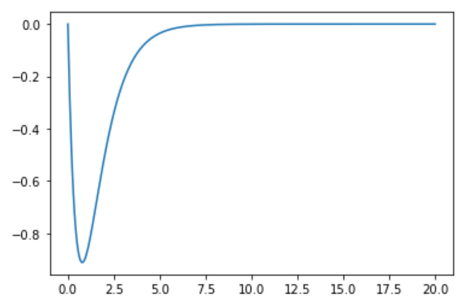

Figure 3: No oscillation due to critical damping

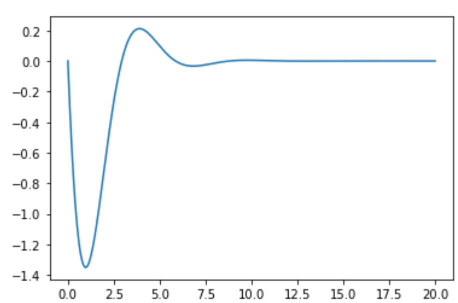

Figure 4: Oscillation with under-damping

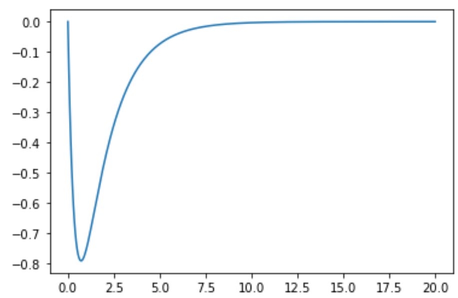

Figure 5: No oscillation due to over-damping

4 Cycling

In this question, we were given an ODE representing acceleration as a function of power, mass, and velocity. We were then tasked with solving the ODE over a span of 200 seconds, given initial velocity of 4 m/s, mass of 70kg, and power of 400W.