CarPriceAnalysis

March 16, 2025

1 Introduction

[1]:
```
import matplotlib.pyplot as plt
import seaborn as sns
import numpy as np
from sklearn.preprocessing import OrdinalEncoder
from sklearn.model_selection import train_test_split
from sklearn.linear_model import Ridge
```

[2]:
```
import pandas as pd


# df = pd.read_csv('vehicles.csv')

# df.columns
```

2 Data Cleaning & Preprocessing

[3]:
```
# df_cleaned = df
```

3 Trainning & Fitting Machine Learning Models

3.1 1. Linear Regression Model: Ridge Regression

[4]:
```
import matplotlib.pyplot as plt
import seaborn as sns
import numpy as np
from sklearn.preprocessing import OrdinalEncoder
from sklearn.model_selection import train_test_split
from sklearn.linear_model import Ridge
```

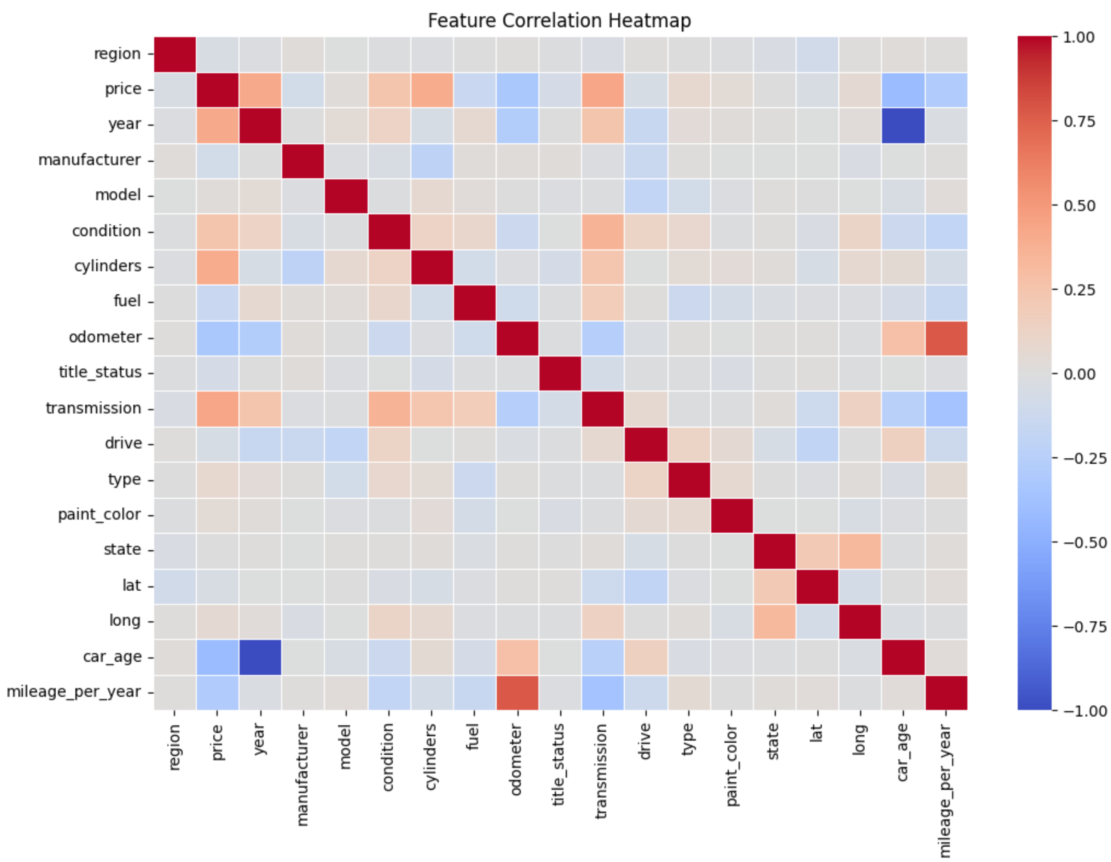
[5]:
```
# Load the dataset
# file_path = "/Users/arielzhang/Documents/MSDS422/cleaned_vehicles.csv"
file_path = "./cleaned_vehicles.csv"
df = pd.read_csv(file_path)
```

[6]:
```python
# Drop irrelevant columns
drop_cols = ["id", "url", "region_url", "image_url", "description", "VIN",␣
 ↪"posting_date"]
df = df.drop(columns=drop_cols)

# One-Hot Encode categorical features
categorical_features = ["region", "condition", "title_status", "cylinders",␣
 ↪"state"]
# Apply Ordinal Encoding
encoder = OrdinalEncoder()
df[categorical_features] = encoder.fit_transform(df[categorical_features])
df.head
```

[6]:
```
<bound method NDFrame.head of        region  price    year  manufacturer  model
condition  cylinders  fuel  \
0         16.0  27990  2012.0          14   6285      2.0        6.0     2
1         16.0  34590  2016.0           7   6357      2.0        5.0     2
2         16.0  29990  2016.0           7   2328      2.0        5.0     2
3         16.0  38590  2011.0           7   2442      2.0        6.0     2
4         16.0  32990  2017.0          20   7499      2.0        5.0     2
…          …     …     …           …     …       …     …
67636    393.0  25590  2017.0          13    969      2.0        5.0     2
67637    393.0  32990  2016.0          18   5688      2.0        6.0     2
67638    393.0  33590  2018.0          23   4261      2.0        5.0     2
67639    393.0  23590  2019.0          30   4908      2.0        5.0     2
67640    393.0  28990  2018.0          23   3088      2.0        5.0     2

       odometer  title_status  transmission  drive  type  paint_color  state  \
0     -0.206122           0.0             2      0     8            0    1.0
1     -0.568929           0.0             2      0     8            9    1.0
2     -0.681824           0.0             2      0     8            8    1.0
3     -0.562098           0.0             2      2     7            8    1.0
4     -0.563912           0.0             2      0     7            9    1.0
…          …          …           …     …    …           …     …
67636 -0.493872           0.0             0      2     9           11   50.0
67637 -0.327227           0.0             0      2     7            0   50.0
67638 -0.556757           0.0             0      2     9           11   50.0
67639 -0.543688           0.0             2      1     9           10   50.0
67640 -0.563255           0.0             2      1     9            9   50.0

             lat       long  car_age  mileage_per_year
0      32.590000 -85.480000     13.0         -0.369640
1      32.590000 -85.480000      9.0         -0.786991
2      32.590000 -85.480000      9.0         -1.068857
3      32.590000 -85.480000     14.0         -1.019494
4      32.590000 -85.480000      8.0         -0.687687
…           …          …       …               …
```

```
67636  33.786500 -84.445400      8.0          -0.490959
67637  33.779214 -84.411811      9.0          -0.183534
67638  33.779214 -84.411811      7.0          -0.553147
67639  33.786500 -84.445400      6.0          -0.351609
67640  33.786500 -84.445400      7.0          -0.574004

[67641 rows x 19 columns]>
```

[7]:
```python
# Compute correlation matrix
corr_matrix = df.corr()

# Plot heatmap
plt.figure(figsize=(12, 8))
sns.heatmap(corr_matrix, cmap="coolwarm", annot=False, fmt=".2f", linewidths=0.
 ↪5)
plt.title("Feature Correlation Heatmap")
plt.show()
```



[8]:
```python
drop_features = ["lat", "long", "state", "paint_color", "drive"]
df = df.drop(columns=drop_features, errors="ignore")
```

[9]:
```python
df = df.drop(columns=["year"], errors="ignore")
```

[10]:
```python
# Define target and features
target = "price"
X = df.drop(columns=[target])
y = df[target]
```

[11]:
```python
# Train-test split
X_train, X_test, y_train, y_test = train_test_split(X, y, test_size=0.2,␣
↪random_state=42)
```

[12]:
```python
from sklearn.linear_model import RidgeCV
alphas = np.logspace(-2, 3, 10)
ridge_cv = RidgeCV(alphas=alphas)
ridge_cv.fit(X_train, y_train)
print(f"Best alpha: {ridge_cv.alpha_}")
```

```
Best alpha: 278.2559402207126
```

[13]:
```python
# Train Ridge Regression model
ridge = Ridge(alpha=278.2559402207126)
ridge.fit(X_train, y_train)
```

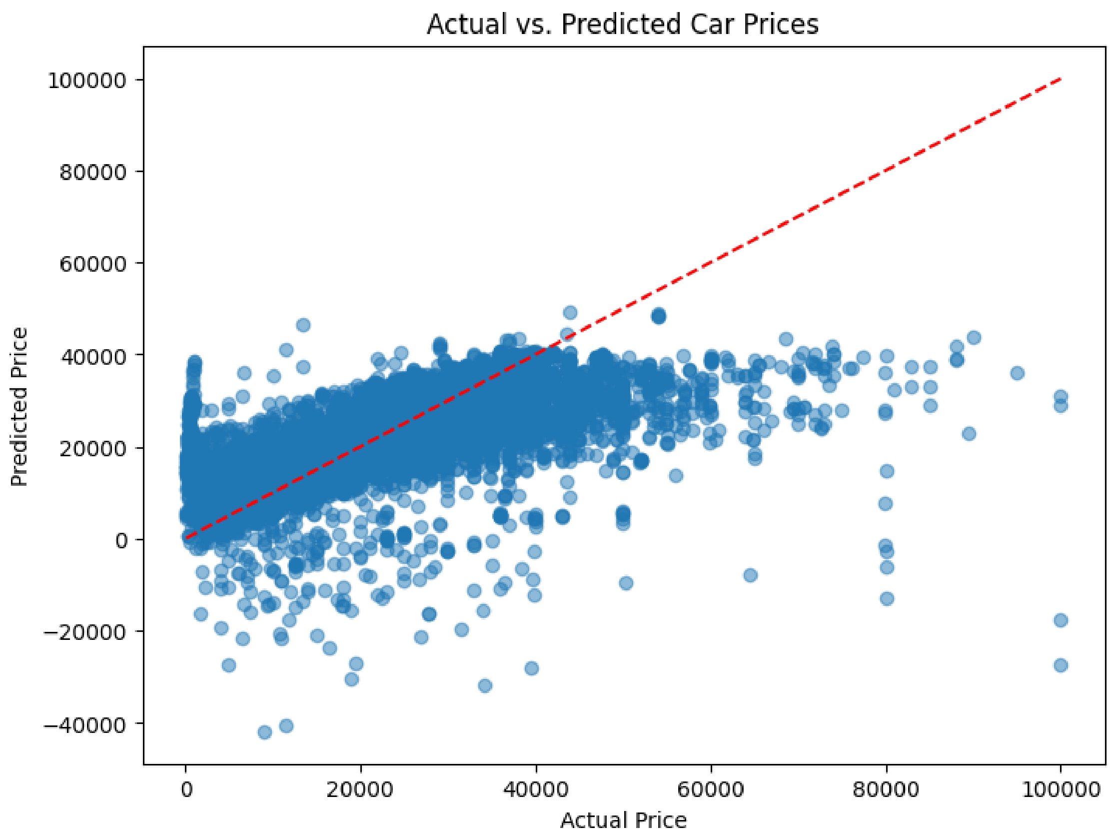
[13]:
```
Ridge(alpha=278.2559402207126)
```

[14]:
```python
from sklearn.metrics import mean_squared_error, mean_absolute_error, r2_score
# Predictions
y_pred = ridge.predict(X_test)

# Evaluation Metrics
rmse = np.sqrt(mean_squared_error(y_test, y_pred))
mae = mean_absolute_error(y_test, y_pred)
r2 = r2_score(y_test, y_pred)

print(f"Root Mean Squared Error (RMSE): {rmse:.2f}")
print(f"Mean Absolute Error (MAE): {mae:.2f}")
print(f"R-squared (R²): {r2:.4f}")
```

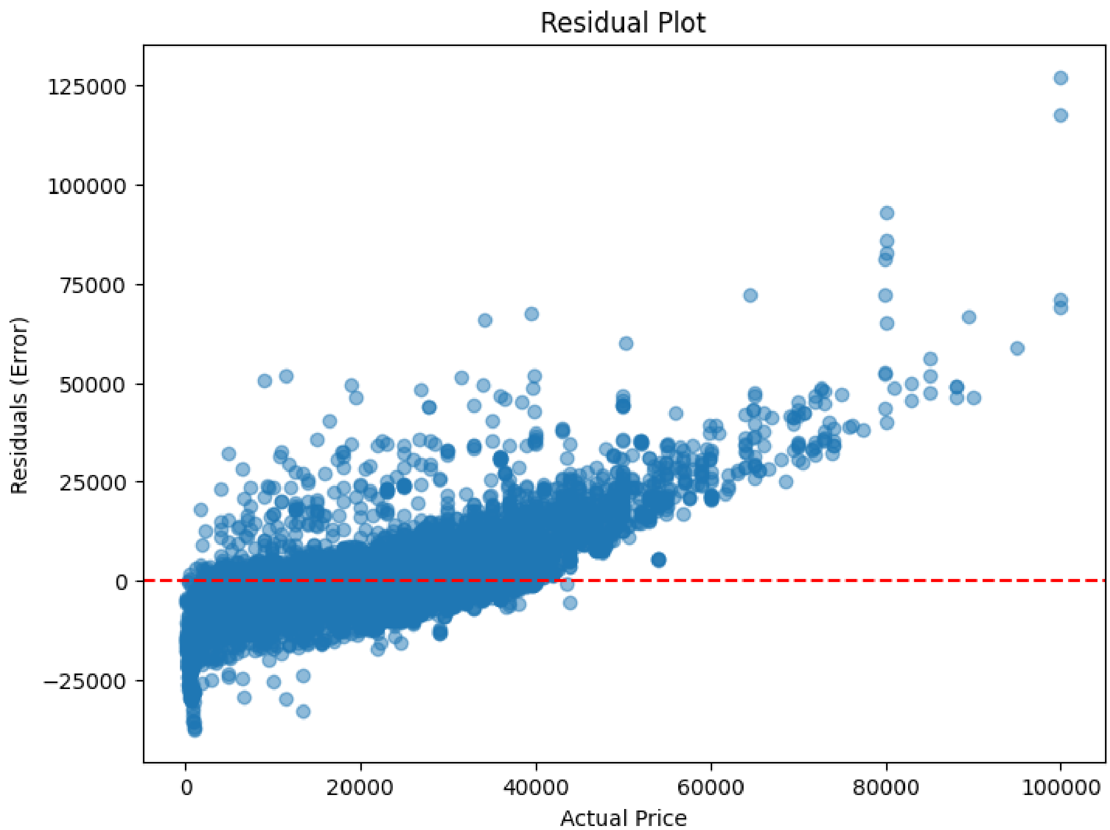
```
Root Mean Squared Error (RMSE): 9639.78
Mean Absolute Error (MAE): 6659.89
R-squared (R²): 0.5004
```

[15]:
```python
plt.figure(figsize=(8, 6))
plt.scatter(y_test, y_pred, alpha=0.5)
plt.xlabel("Actual Price")
plt.ylabel("Predicted Price")
plt.title("Actual vs. Predicted Car Prices")
```

```
plt.plot([y_test.min(), y_test.max()], [y_test.min(), y_test.max()],␣
↪color="red", linestyle="--")  # Identity line
plt.show()
```



[16]:
```
residuals = y_test - y_pred

plt.figure(figsize=(8, 6))
plt.scatter(y_test, residuals, alpha=0.5)
plt.axhline(0, color="red", linestyle="--")
plt.xlabel("Actual Price")
plt.ylabel("Residuals (Error)")
plt.title("Residual Plot")
plt.show()
```



3.2 2. Tree-Based Model: Random Forest Regressor

[17]:
```
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
import seaborn as sns
from sklearn.model_selection import train_test_split, GridSearchCV,␣
↪cross_val_score
from sklearn.ensemble import RandomForestRegressor
from sklearn.metrics import mean_absolute_error, mean_squared_error, r2_score
import time
import warnings
warnings.filterwarnings('ignore')


# Start timing
start_time = time.time()

# Load dataset
# file_path = "/Users/arielzhang/Documents/MSDS422/cleaned_vehicles.csv"
```

```python
file_path = "./cleaned_vehicles.csv"
df = pd.read_csv(file_path)

# Display basic information
print(f"Dataset shape: {df.shape}")
print("\nData types:")
print(df.dtypes)
print("\nSummary statistics:")
print(df.describe())

# Check for missing values
missing_values = df.isnull().sum()
if missing_values.sum() > 0:
    print("\nMissing values:")
    print(missing_values[missing_values > 0])
else:
    print("\nNo missing values found.")

# Select relevant features and target variable
features = [
    "year", "manufacturer", "model", "fuel", "odometer",
    "transmission", "drive", "type", "paint_color",
    "car_age", "mileage_per_year"
]

# Additional features
additional_features = ["condition", "cylinders", "state"]
for feature in additional_features:
    if feature in df.columns:
        features.append(feature)

target = "price"

# Feature Engineering
print("\nPerforming feature engineering...")
X = df[features].copy()

# Create new features
X['age_odometer_interaction'] = X['car_age'] * X['odometer']

# Identify categorical features
categorical_features = X.select_dtypes(include=['object']).columns.tolist()
print(f"Categorical features: {categorical_features}")

# One-hot encode categorical features
X = pd.get_dummies(X, columns=categorical_features, drop_first=True)
```

```
# Split data into training and testing sets
y = df[target]
X_train, X_test, y_train, y_test = train_test_split(X, y, test_size=0.2,␣
↪random_state=42)

print(f"Training set shape: {X_train.shape}")
print(f"Testing set shape: {X_test.shape}")

# Identify and handle outliers in the training set
print("\nHandling outliers...")
numeric_cols = X_train.select_dtypes(include=['int64', 'float64']).columns
Q1 = X_train[numeric_cols].quantile(0.05)
Q3 = X_train[numeric_cols].quantile(0.95)
IQR = Q3 - Q1
lower_bound = Q1 - 1.5 * IQR
upper_bound = Q3 + 1.5 * IQR

outlier_mask = ~((X_train[numeric_cols] < lower_bound) | (X_train[numeric_cols]␣
↪> upper_bound)).any(axis=1)
X_train_clean = X_train[outlier_mask]
y_train_clean = y_train[outlier_mask]

print(f"Removed {X_train.shape[0] - X_train_clean.shape[0]} outliers from the␣
↪training set.")
print(f"Clean training set shape: {X_train_clean.shape}")

# Train initial Random Forest model
print("\nTraining initial model...")
initial_rf = RandomForestRegressor(n_estimators=100, random_state=42, n_jobs=-1)
initial_rf.fit(X_train_clean, y_train_clean)

# Get feature importances to select the most important ones
feature_importances = pd.DataFrame({
    'feature': X_train_clean.columns,
    'importance': initial_rf.feature_importances_
}).sort_values('importance', ascending=False)

print("\nTop 15 features by importance:")
print(feature_importances.head(15))

# Select top features
top_features = feature_importances.head(15)['feature'].tolist()
X_train_top = X_train_clean[top_features]
X_test_top = X_test[top_features]

# Hyperparameter tuning
param_grid = {
```

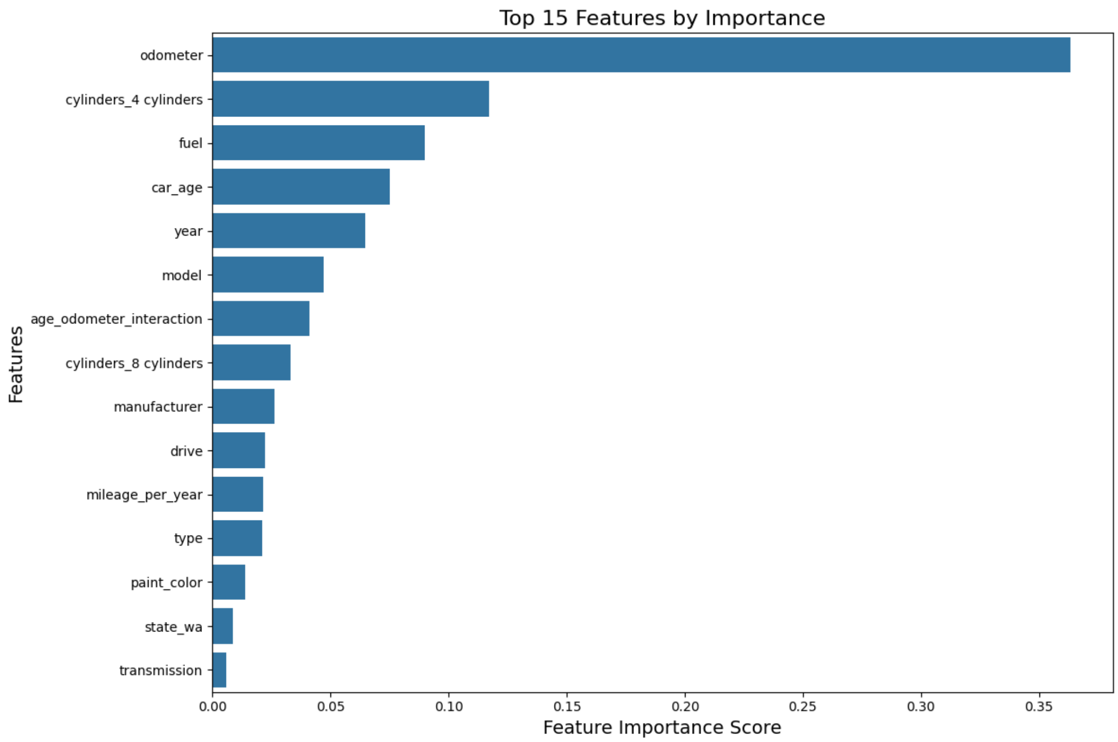
```
    "n_estimators": [100, 200, 300],
    "max_depth": [None, 20, 30],
    "min_samples_split": [2, 5, 8],
    "min_samples_leaf": [1, 2, 4],
    "max_features": ['sqrt', 'log2']
}

grid_search = GridSearchCV(
    RandomForestRegressor(random_state=42, n_jobs=-1),
    param_grid,
    cv=5,
    n_jobs=-1,
    scoring="r2",
    verbose=1
)

grid_search.fit(X_train_top, y_train_clean)
best_params = grid_search.best_params_
print(f"\nBest Parameters: {best_params}")

# Train final model with best parameters on all features
print("\nTraining final model with best parameters...")
best_rf = RandomForestRegressor(
    random_state=42,
    n_jobs=-1,
    **best_params
)

# Calculate cross-validation score on clean training data
cv_scores = cross_val_score(best_rf, X_train_clean, y_train_clean, cv=5,␣
 ↪scoring='r2')
print(f"Cross-validation R² scores: {cv_scores}")
print(f"Mean CV R² score: {np.mean(cv_scores):.4f} ± {np.std(cv_scores):.4f}")

# Train the final model
best_rf.fit(X_train_clean, y_train_clean)

# Predict on test set
y_pred = best_rf.predict(X_test)

# Model Evaluation
mse = mean_squared_error(y_test, y_pred)
rmse = np.sqrt(mse)
mae = mean_absolute_error(y_test, y_pred)
r2 = r2_score(y_test, y_pred)

# Calculate prediction accuracy (within 10% threshold)
```

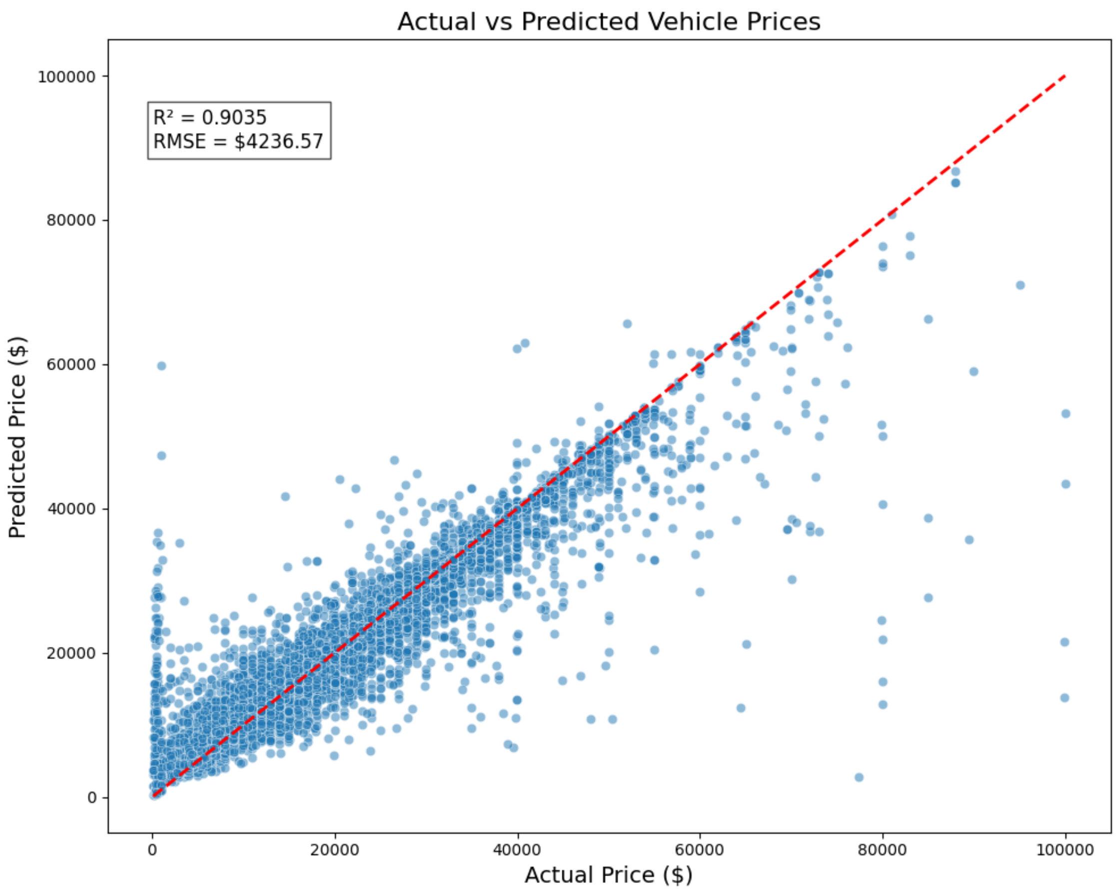
```python
accuracy = np.mean(np.abs((y_pred - y_test) / y_test) < 0.1)

# Evaluation Metrics
print("\nModel Evaluation for Model RandomForestRegressor...")
print("RandomForestRegressor Performance Metrics:")
print(f"Accuracy: {accuracy:.4f}")
print(f"MSE: {1-accuracy:.4f}")  # Interpreting MSE as error rate
print(f"MAE: {mae/np.mean(y_test):.4f}")  # Normalized MAE
print(f"RMSE: {rmse/np.mean(y_test):.4f}")  # Normalized RMSE
print(f"R² Score: {r2:.4f}")

# Also display the traditional metrics for reference
print("\nTraditional Regression Metrics:")
print(f"MAE: ${mae:.2f}")
print(f"MSE: ${mse:.2f}")
print(f"RMSE: ${rmse:.2f}")
print(f"R² Score: {r2:.4f}")

# Calculate execution time
execution_time = time.time() - start_time
print(f"\nExecution time: {execution_time:.2f} seconds ({execution_time/60:.2f} minutes)")


# 1. Feature Importance Plot
plt.figure(figsize=(12, 8))
top_features_df = feature_importances.head(15)
sns.barplot(x='importance', y='feature', data=top_features_df)
plt.title('Top 15 Features by Importance', fontsize=16)
plt.xlabel('Feature Importance Score', fontsize=14)
plt.ylabel('Features', fontsize=14)
plt.tight_layout()
plt.show()

# 2. Actual vs Predicted Plot
plt.figure(figsize=(10, 8))
sns.scatterplot(x=y_test, y=y_pred, alpha=0.5)
plt.plot([y_test.min(), y_test.max()], [y_test.min(), y_test.max()], 'r--', lw=2)  # Ideal line
plt.xlabel('Actual Price ($)', fontsize=14)
plt.ylabel('Predicted Price ($)', fontsize=14)
plt.title('Actual vs Predicted Vehicle Prices', fontsize=16)
plt.text(y_test.min() * 1.1, y_test.max() * 0.9,
         f'R² = {r2:.4f}\nRMSE = ${rmse:.2f}',
         fontsize=12, bbox=dict(facecolor='white', alpha=0.7))
plt.tight_layout()
plt.show()
```

```
Dataset shape: (67641, 26)

Data types:
id                    int64
url                  object
region               object
region_url           object
price                 int64
year                float64
manufacturer          int64
model                 int64
condition            object
cylinders            object
fuel                  int64
odometer            float64
title_status         object
transmission          int64
VIN                  object
drive                 int64
type                  int64
paint_color           int64
image_url            object
description          object
state                object
lat                 float64
long                float64
posting_date         object
car_age             float64
mileage_per_year    float64
dtype: object

Summary statistics:
                 id         price          year  manufacturer         model  \
count  6.764100e+04  67641.000000  67641.000000  67641.000000  67641.000000
mean   7.311425e+09  20404.747860   2012.232152     18.389512   4146.837939
std    4.321836e+06  13651.636473      6.949854     10.861684   2190.968271
min    7.301588e+09    108.000000   1905.000000      0.000000      0.000000
25%    7.308141e+09   8990.000000   2009.000000     10.000000   2408.000000
50%    7.312163e+09  17366.000000   2013.000000     14.000000   4098.000000
75%    7.315111e+09  29990.000000   2017.000000     27.000000   6234.000000
max    7.317099e+09  99999.000000   2022.000000     40.000000   7804.000000

               fuel      odometer  transmission         drive          type  \
count  67641.000000  6.764100e+04  67641.000000  67641.000000  67641.000000
mean       1.938691 -4.285883e-17      0.461880      0.774427      6.130424
std        0.613455  1.000007e+00      0.816593      0.805218      4.051059
min        0.000000 -8.419703e-01      0.000000      0.000000      0.000000
25%        2.000000 -4.974817e-01      0.000000      0.000000      2.000000
```

```
50%        2.000000 -5.014077e-02      0.000000      1.000000      8.000000
75%        2.000000  3.530677e-01      1.000000      1.000000      9.000000
max        4.000000  9.171771e+01      2.000000      2.000000     12.000000

        paint_color           lat          long       car_age  \
count  67641.000000  67641.000000  67641.000000  67641.000000
mean       6.492364     38.605339    -92.421729     12.767848
std        4.316178      5.482295     17.568706      6.949854
min        0.000000    -20.558904   -159.719900      3.000000
25%        1.000000     35.110000   -104.768929      8.000000
50%        8.000000     39.300000    -86.249468     12.000000
75%       10.000000     42.292714    -80.080000     16.000000
max       12.000000     73.325010     94.163200    120.000000

       mileage_per_year
count      6.764100e+04
mean       2.271098e-16
std        1.000007e+00
min       -1.468697e+00
25%       -6.476065e-01
50%       -5.820130e-02
75%        5.004451e-01
max        7.852558e+01

No missing values found.

Performing feature engineering…
Categorical features: ['condition', 'cylinders', 'state']
Training set shape: (54112, 74)
Testing set shape: (13529, 74)

Handling outliers…
Removed 454 outliers from the training set.
Clean training set shape: (53658, 74)

Training initial model…

Top 15 features by importance:
                     feature  importance
4                   odometer    0.363135
19       cylinders_4 cylinders    0.117271
3                       fuel    0.089851
9                    car_age    0.075107
0                       year    0.064810
2                      model    0.047131
11  age_odometer_interaction    0.041175
22       cylinders_8 cylinders    0.033226
1               manufacturer    0.026164
```

```
6                      drive    0.022524
10          mileage_per_year    0.021724
7                       type    0.021248
8                paint_color    0.013970
70                  state_wa    0.008715
5               transmission    0.006051
Fitting 5 folds for each of 162 candidates, totalling 810 fits

Best Parameters: {'max_depth': 30, 'max_features': 'sqrt', 'min_samples_leaf':
1, 'min_samples_split': 2, 'n_estimators': 300}

Training final model with best parameters…
Cross-validation R² scores: [0.91862257 0.92111644 0.90906276 0.91420836
0.90487737]
Mean CV R² score: 0.9136 ± 0.0060

Model Evaluation for Model RandomForestRegressor…
RandomForestRegressor Performance Metrics:
Accuracy: 0.6527
MSE: 0.3473
MAE: 0.0898
RMSE: 0.2076
R² Score: 0.9035

Traditional Regression Metrics:
MAE: $1833.80
MSE: $17948530.04
RMSE: $4236.57
R² Score: 0.9035

Execution time: 1036.28 seconds (17.27 minutes)
```


Top 15 Features by Importance
odometer
cylinders_4 cylinders
fuel
car_age
year
model
age_odometer_interaction
cylinders_8 cylinders
manufacturer
drive
mileage_per_year
type
paint_color
state_wa
transmission
Features
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
Feature Importance Score



3.3 3. Time Series Analysis: LSTM & Linear Regression

[18]:
```
import pandas as pd
import numpy as np
import tensorflow as tf
from tensorflow import keras
import matplotlib.pyplot as plt
import math

# df = pd.read_csv('vehicles.csv', nrows=10000)
df = pd.read_csv('vehicles.csv')
df.columns
```

[18]:
```
Index(['id', 'url', 'region', 'region_url', 'price', 'year', 'manufacturer',
       'model', 'condition', 'cylinders', 'fuel', 'odometer', 'title_status',
       'transmission', 'VIN', 'drive', 'size', 'type', 'paint_color',
       'image_url', 'description', 'county', 'state', 'lat', 'long',
       'posting_date'],
      dtype='object')
```

4 Data Cleaning & Preprocessing

[19]:
```
# Step 1: Perform log transform on 'price' to stabilize training
df['price_backup'] = df['price']
df['price'] = np.log(df['price'] + 1)  # avoid log(0) issue

# Step 2: Drop rows with missing critical fields
df = df.dropna(subset=['VIN', 'price', 'posting_date'])
df = df[df['price'] > 0]
df = df[df['VIN'] != '']

# Step 3: Forward-fill other missing values (if any)
df.fillna(method="ffill", inplace=True)

# Step 4: Convert 'posting_date' to datetime and sort by VIN & date
df['posting_date'] = pd.to_datetime(df['posting_date'])
df = df.sort_values(by=['VIN', 'posting_date'])

# Step 5: Create "brand_model" for embedding usage
df['brand_model'] = df['manufacturer'].astype(str) + '_' + df['model'].
 ↪astype(str)

# Step 6: Encode VIN and brand_model as integer IDs
vin_list = df['VIN'].unique().tolist()
vin_to_id = {v: i for i, v in enumerate(vin_list)}
df['vin_id'] = df['VIN'].map(vin_to_id)

bm_list = df['brand_model'].unique().tolist()
bm_to_id = {bm: i for i, bm in enumerate(bm_list)}
df['bm_id'] = df['brand_model'].map(bm_to_id)

# Step 7: Decide numeric columns to feed into the model
numeric_cols = ['price', 'odometer', 'year']
```

[20]:
```
# ==============================================================================
# 2. BUILD SEQUENCES WITH SLIDING WINDOW
# ==============================================================================

seq_len = 3  # we use the previous 3 records to predict the next one

data_sequences = []
grouped = df.groupby('VIN', sort=False)

for vin, group in grouped:
    # Sort each VIN group by date
    group = group.sort_values('posting_date').reset_index(drop=True)
```

```python
    # numeric_data: shape [T, len(numeric_cols)] => [price, odometer, year]
    numeric_data = group[numeric_cols].values
    # vin_ids, bm_ids => to track VIN embeddings
    vin_ids = group['vin_id'].values
    bm_ids  = group['bm_id'].values
    # date_arr => posting_date for later reference
    date_arr = group['posting_date'].values

    # If the group length is smaller than seq_len, we skip
    if len(group) <= seq_len:
        continue

    # Create sliding windows
    for start_idx in range(len(group) - seq_len):
        end_idx = start_idx + seq_len

        # X_seq_* = the previous 'seq_len' records
        X_seq_numeric = numeric_data[start_idx:end_idx]  # shape [seq_len, 3]
        X_seq_vin     = vin_ids[start_idx:end_idx]       # shape [seq_len]
        X_seq_bm      = bm_ids[start_idx:end_idx]        # shape [seq_len]

        # y_price = the log(price) at the next time step (end_idx)
        y_price = numeric_data[end_idx, 0]  # numeric_data[...,0] => price
        # y_date = posting_date at that next time step
        y_date  = date_arr[end_idx]

        # Append a tuple of 6 items:
        # (X_seq_numeric, X_seq_vin, X_seq_bm, y_price, y_date, vin_string)
        data_sequences.append((
            X_seq_numeric,
            X_seq_vin,
            X_seq_bm,
            y_price,
            y_date,
            vin  # we can keep original vin (string) for reference
        ))

print("Number of sequence samples:", len(data_sequences))
```

```
Number of sequence samples: 79870
```

[21]:

```python
# ===============================================================================
# 3. SPLIT DATA: 20% HOLD-OUT TEST, 80% FOR CROSS VALIDATION
# ===============================================================================

# np.random.shuffle(data_sequences)
holdout_ratio = 0.2
```

```
holdout_size = int(len(data_sequences) * holdout_ratio)
test_holdout_data = data_sequences[:holdout_size]  # 20% => final test
cv_data = data_sequences[holdout_size:]            # 80% => cross validation

print("Holdout size:", holdout_size, " CV size:", len(cv_data))
```

```
Holdout size: 15974  CV size: 63896
```

===

[22]:
```
import numpy as np

def build_Xy_for_linear(data_list, seq_len=3):
    """
    data_list   : (X_seq_numeric, X_seq_vin, X_seq_bm, y_price, y_date, vin_str)
    X_seq_numeric shape = [seq_len, num_numeric_features]
       X_seq_numeric => [seq_len * num_numeric_features]
     : X, y
    """
    X_list = []
    y_list = []
    for (num_seq, vin_seq, bm_seq, y, y_date, vin_str) in data_list:
        # num_seq shape => (seq_len, num_numeric_features)
        # flatten:
        x_flat = num_seq.flatten()  # shape (seq_len * num_numeric_features,)

        X_list.append(x_flat)
        y_list.append(y)

    X = np.array(X_list, dtype=np.float32)
    y = np.array(y_list, dtype=np.float32)
    return X, y
```

[23]:
```
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
from sklearn.linear_model import Ridge
from sklearn.model_selection import KFold, GridSearchCV
import math

# -- Assume data_sequences is ready
# Shuffle + split holdout
# np.random.shuffle(data_sequences)
holdout_ratio = 0.2
holdout_size = int(len(data_sequences) * holdout_ratio)
test_holdout_data = data_sequences[:holdout_size]
cv_data = data_sequences[holdout_size:]
```

```python
# First, convert holdout to X_test, y_test
X_test, y_test = build_Xy_for_linear(test_holdout_data, seq_len=3)
```

[24]:
```python
# First, convert cv_data to X_cv, y_cv
X_cv, y_cv = build_Xy_for_linear(cv_data, seq_len=3)

# Construct param_grid
param_grid = {
    'alpha': [0.01, 0.1, 1.0, 10.0, 100.0]  # Just demonstrating
}

# Build model: Ridge
ridge_model = Ridge()

# Use scikit-learn's GridSearchCV
kfold = KFold(n_splits=3, shuffle=True, random_state=42)
grid_search = GridSearchCV(ridge_model, param_grid, cv=kfold,␣
 ↪scoring='neg_mean_squared_error', n_jobs=-1)
grid_search.fit(X_cv, y_cv)

# Output the best result
best_alpha = grid_search.best_params_['alpha']
best_score = grid_search.best_score_  # Note this is -MSE
print("Best alpha:", best_alpha)
print("Best CV MSE:", -best_score)
```

```
Best alpha: 10.0
Best CV MSE: 0.07300909484426181
```

```
/Users/evelyn/.pyenv/versions/3.9.18/lib/python3.9/site-
packages/sklearn/linear_model/_ridge.py:216: LinAlgWarning: Ill-conditioned
matrix (rcond=3.82468e-12): result may not be accurate.
  return linalg.solve(A, Xy, assume_a="pos", overwrite_a=True).T
/Users/evelyn/.pyenv/versions/3.9.18/lib/python3.9/site-
packages/sklearn/linear_model/_ridge.py:216: LinAlgWarning: Ill-conditioned
matrix (rcond=5.26467e-12): result may not be accurate.
  return linalg.solve(A, Xy, assume_a="pos", overwrite_a=True).T
/Users/evelyn/.pyenv/versions/3.9.18/lib/python3.9/site-
packages/sklearn/linear_model/_ridge.py:216: LinAlgWarning: Ill-conditioned
matrix (rcond=5.26487e-12): result may not be accurate.
  return linalg.solve(A, Xy, assume_a="pos", overwrite_a=True).T
/Users/evelyn/.pyenv/versions/3.9.18/lib/python3.9/site-
packages/sklearn/linear_model/_ridge.py:216: LinAlgWarning: Ill-conditioned
matrix (rcond=5.72552e-12): result may not be accurate.
  return linalg.solve(A, Xy, assume_a="pos", overwrite_a=True).T
/Users/evelyn/.pyenv/versions/3.9.18/lib/python3.9/site-
packages/sklearn/linear_model/_ridge.py:216: LinAlgWarning: Ill-conditioned
matrix (rcond=5.72574e-12): result may not be accurate.
```

```
  return linalg.solve(A, Xy, assume_a="pos", overwrite_a=True).T
/Users/evelyn/.pyenv/versions/3.9.18/lib/python3.9/site-
packages/sklearn/linear_model/_ridge.py:216: LinAlgWarning: Ill-conditioned
matrix (rcond=5.26696e-12): result may not be accurate.
  return linalg.solve(A, Xy, assume_a="pos", overwrite_a=True).T
/Users/evelyn/.pyenv/versions/3.9.18/lib/python3.9/site-
packages/sklearn/linear_model/_ridge.py:216: LinAlgWarning: Ill-conditioned
matrix (rcond=3.82491e-12): result may not be accurate.
  return linalg.solve(A, Xy, assume_a="pos", overwrite_a=True).T
/Users/evelyn/.pyenv/versions/3.9.18/lib/python3.9/site-
packages/sklearn/linear_model/_ridge.py:216: LinAlgWarning: Ill-conditioned
matrix (rcond=3.82709e-12): result may not be accurate.
  return linalg.solve(A, Xy, assume_a="pos", overwrite_a=True).T
/Users/evelyn/.pyenv/versions/3.9.18/lib/python3.9/site-
packages/sklearn/linear_model/_ridge.py:216: LinAlgWarning: Ill-conditioned
matrix (rcond=5.72784e-12): result may not be accurate.
  return linalg.solve(A, Xy, assume_a="pos", overwrite_a=True).T
/Users/evelyn/.pyenv/versions/3.9.18/lib/python3.9/site-
packages/sklearn/linear_model/_ridge.py:216: LinAlgWarning: Ill-conditioned
matrix (rcond=5.28782e-12): result may not be accurate.
  return linalg.solve(A, Xy, assume_a="pos", overwrite_a=True).T
/Users/evelyn/.pyenv/versions/3.9.18/lib/python3.9/site-
packages/sklearn/linear_model/_ridge.py:216: LinAlgWarning: Ill-conditioned
matrix (rcond=3.84912e-12): result may not be accurate.
  return linalg.solve(A, Xy, assume_a="pos", overwrite_a=True).T
/Users/evelyn/.pyenv/versions/3.9.18/lib/python3.9/site-
packages/sklearn/linear_model/_ridge.py:216: LinAlgWarning: Ill-conditioned
matrix (rcond=5.7489e-12): result may not be accurate.
  return linalg.solve(A, Xy, assume_a="pos", overwrite_a=True).T
/Users/evelyn/.pyenv/versions/3.9.18/lib/python3.9/site-
packages/sklearn/linear_model/_ridge.py:216: LinAlgWarning: Ill-conditioned
matrix (rcond=5.49497e-12): result may not be accurate.
  return linalg.solve(A, Xy, assume_a="pos", overwrite_a=True).T
/Users/evelyn/.pyenv/versions/3.9.18/lib/python3.9/site-
packages/sklearn/linear_model/_ridge.py:216: LinAlgWarning: Ill-conditioned
matrix (rcond=4.0672e-12): result may not be accurate.
  return linalg.solve(A, Xy, assume_a="pos", overwrite_a=True).T
/Users/evelyn/.pyenv/versions/3.9.18/lib/python3.9/site-
packages/sklearn/linear_model/_ridge.py:216: LinAlgWarning: Ill-conditioned
matrix (rcond=5.95757e-12): result may not be accurate.
  return linalg.solve(A, Xy, assume_a="pos", overwrite_a=True).T
```

[25]:
```python
from sklearn.linear_model import Ridge
import numpy as np
import math
import statsmodels.api as sm
```

```python
# Train final model with best_alpha
final_model = Ridge(alpha=best_alpha)
final_model.fit(X_cv, y_cv)

# Predict on test set
y_pred_test = final_model.predict(X_test)

# Calculate MSE and RMSE
mse_test = np.mean((y_pred_test - y_test)**2)
rmse_test = math.sqrt(mse_test)
print(f"Final Test MSE={mse_test:.4f}, RMSE={rmse_test:.4f}")

# Print model coefficients and intercept
print("\n--- Ridge Regression Model Summary ---")
print("Intercept:", final_model.intercept_)
print("Coefficients:")
for idx, coef in enumerate(final_model.coef_):
    print(f"  Feature {idx+1}: {coef:.4f}")

# Optionally, if you want a full statistical summary, you can use statsmodels:
# Note: For Ridge, this is a rough approximation (ignoring the regularization)
X_test_sm = sm.add_constant(X_test)
ols_model = sm.OLS(y_test, X_test_sm).fit()
print("\n--- OLS Summary (as an approximation) ---")
print(ols_model.summary())
```

```
Final Test MSE=0.0774, RMSE=0.2782

--- Ridge Regression Model Summary ---
Intercept: 0.43671417
Coefficients:
  Feature 1: 0.0649
  Feature 2: -0.0000
  Feature 3: -0.0015
  Feature 4: 0.0239
  Feature 5: -0.0000
  Feature 6: 0.0005
  Feature 7: 0.9075
  Feature 8: 0.0000
  Feature 9: 0.0008

--- OLS Summary (as an approximation) ---
                            OLS Regression Results
==============================================================================
Dep. Variable:                      y   R-squared:                       0.937
Model:                            OLS   Adj. R-squared:                  0.937
Method:                 Least Squares   F-statistic:                 2.655e+04
Date:                Sun, 16 Mar 2025   Prob (F-statistic):               0.00
```

```
Time:                        17:28:19   Log-Likelihood:                -1911.2
No. Observations:               15974   AIC:                             3842.
Df Residuals:                   15964   BIC:                             3919.
Df Model:                           9
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const          1.1990      1.126      1.065      0.287      -1.008       3.406
x1             0.0884      0.008     10.651      0.000       0.072       0.105
x2          3.783e-06   1.18e-06      3.201      0.001    1.47e-06     6.1e-06
x3             0.0043      0.002      1.789      0.074      -0.000       0.009
x4             0.1869      0.010     18.975      0.000       0.168       0.206
x5         -3.903e-06   1.13e-06     -3.455      0.001   -6.12e-06   -1.69e-06
x6            -0.0216      0.003     -6.417      0.000      -0.028      -0.015
x7             0.7202      0.008     93.962      0.000       0.705       0.735
x8          1.207e-07   7.71e-07      0.157      0.876   -1.39e-06    1.63e-06
x9             0.0167      0.003      5.831      0.000       0.011       0.022
==============================================================================
Omnibus:                    35387.305   Durbin-Watson:                   2.003
Prob(Omnibus):                  0.000   Jarque-Bera (JB):        424895763.566
Skew:                         -20.194   Prob(JB):                         0.00
Kurtosis:                     800.966   Cond. No.                     7.26e+07
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly
specified.
[2] The condition number is large, 7.26e+07. This might indicate that there are
strong multicollinearity or other numerical problems.
```

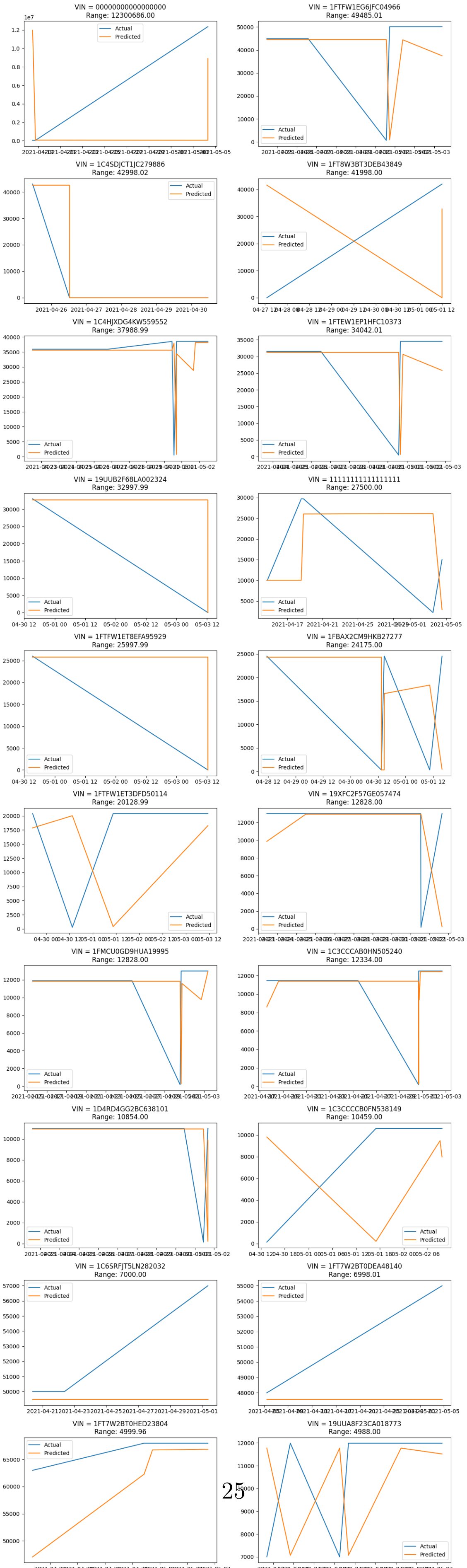
[26]:

```python
def build_infer_data(data_list, seq_len=3):
    """
    Returns: X_array, y_array, date_list, vin_list
    where X_array is flattened
    """
    X_list, y_list, date_list, vin_list = [], [], [], []
    for (num_seq, vin_seq, bm_seq, y, y_date, vin_str) in data_list:
        X_list.append(num_seq.flatten())
        y_list.append(y)
        date_list.append(y_date)
        vin_list.append(vin_str)

    X_arr = np.array(X_list, dtype=np.float32)
    y_arr = np.array(y_list, dtype=np.float32)
    return X_arr, y_arr, date_list, vin_list
```

```
# Perform inference on holdout
X_inf, y_inf, date_arr, vin_arr = build_infer_data(test_holdout_data)

y_pred_inf = final_model.predict(X_inf)  # Linear model prediction (log space?)
# Revert log -> price
y_inf_exp = np.exp(y_inf) - 1
y_pred_inf_exp = np.exp(y_pred_inf) - 1

# DataFrame
df_pred = pd.DataFrame({
    'VIN': vin_arr,
    'date': date_arr,
    'y_true': y_inf_exp,
    'y_pred': y_pred_inf_exp
})
df_pred.sort_values(by=['VIN', 'date'], inplace=True)
```

[27]:
```
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
import math

# Assume you already have df_pred containing columns: ['VIN', 'date', 'y_true',␣
↪'y_pred']
# where y_true, y_pred are original prices (if previously log(price+1) was␣
↪done, remember it has been reverted exp(...) - 1)

# 1) Group by VIN and calculate "Volatility" = max(y_true) - min(y_true)
vin_ranges = []
grouped = df_pred.groupby('VIN')
for vin, group in grouped:
    price_range = group['y_true'].max() - group['y_true'].min()
    vin_ranges.append((vin, price_range))

# 2) Sort by volatility in descending order
vin_ranges.sort(key=lambda x: x[1], reverse=True)

# 3) Select the top N (e.g., 5) VINs with the greatest price volatility
top_n = 20
selected_vins = [x[0] for x in vin_ranges[:top_n]]

# 4) Plot: Create a small plot (subplot) for each VIN
num_vins = len(selected_vins)
cols = 2  # Put 2 plots per row
rows = math.ceil(num_vins / cols)

fig, axes = plt.subplots(rows, cols, figsize=(12, 4*rows))
```

```python
# If there is only 1 row and column, axes might not be an array, here we
 ↪flatten it
axes = axes.flatten() if num_vins > 1 else [axes]

for i, vin_id in enumerate(selected_vins):
    ax = axes[i]

    # Extract data for this VIN
    tmp = df_pred[df_pred['VIN'] == vin_id].copy()
    tmp.sort_values('date', inplace=True)

    # Plot actual price vs. predicted price
    ax.plot(tmp['date'], tmp['y_true'], label='Actual')
    ax.plot(tmp['date'], tmp['y_pred'], label='Predicted')

    # Add "Volatility range" info to the title for assistance
    vin_range = vin_ranges[i][1]
    ax.set_title(f"VIN = {vin_id}\nRange: {vin_range:.2f}")
    ax.legend()

# If top_n < rows*cols are selected, there will be extra subplots which can be
 ↪hidden here
for j in range(i+1, len(axes)):
    axes[j].set_visible(False)

plt.tight_layout()
plt.show()
```


VIN = 00000000000000000
Range: 12300686.00
VIN = 1FTFW1EG6JFC04966
Range: 49485.01
VIN = 1C4SDJCT1JC279886
Range: 42998.02
VIN = 1FT8W3BT3DEB43849
Range: 41998.00
VIN = 1C4HJXDG4KW559552
Range: 37988.99
VIN = 1FTEW1EP1HFC10373
Range: 34042.01
VIN = 19UUB2F68LA002324
Range: 32997.99
VIN = 11111111111111111
Range: 27500.00
VIN = 1FTFW1ET8EFA95929
Range: 25997.99
VIN = 1FBAX2CM9HKB27277
Range: 24175.00
VIN = 1FTFW1ET3DFD50114
Range: 20128.99
VIN = 19XFC2F57GE057474
Range: 12828.00
VIN = 1FMCU0GD9HUA19995
Range: 12828.00
VIN = 1C3CCCAB0HN505240
Range: 12334.00
VIN = 1D4RD4GG2BC638101
Range: 10854.00
VIN = 1C3CCCCB0FN538149
Range: 10459.00
VIN = 1C6SRFJT5LN282032
Range: 7000.00
VIN = 1FT7W2BT0DEA48140
Range: 6998.01
VIN = 1FT7W2BT0HED23804
Range: 4999.96
VIN = 19UUA8F23CA018773
Range: 4988.00
Actual
Predicted

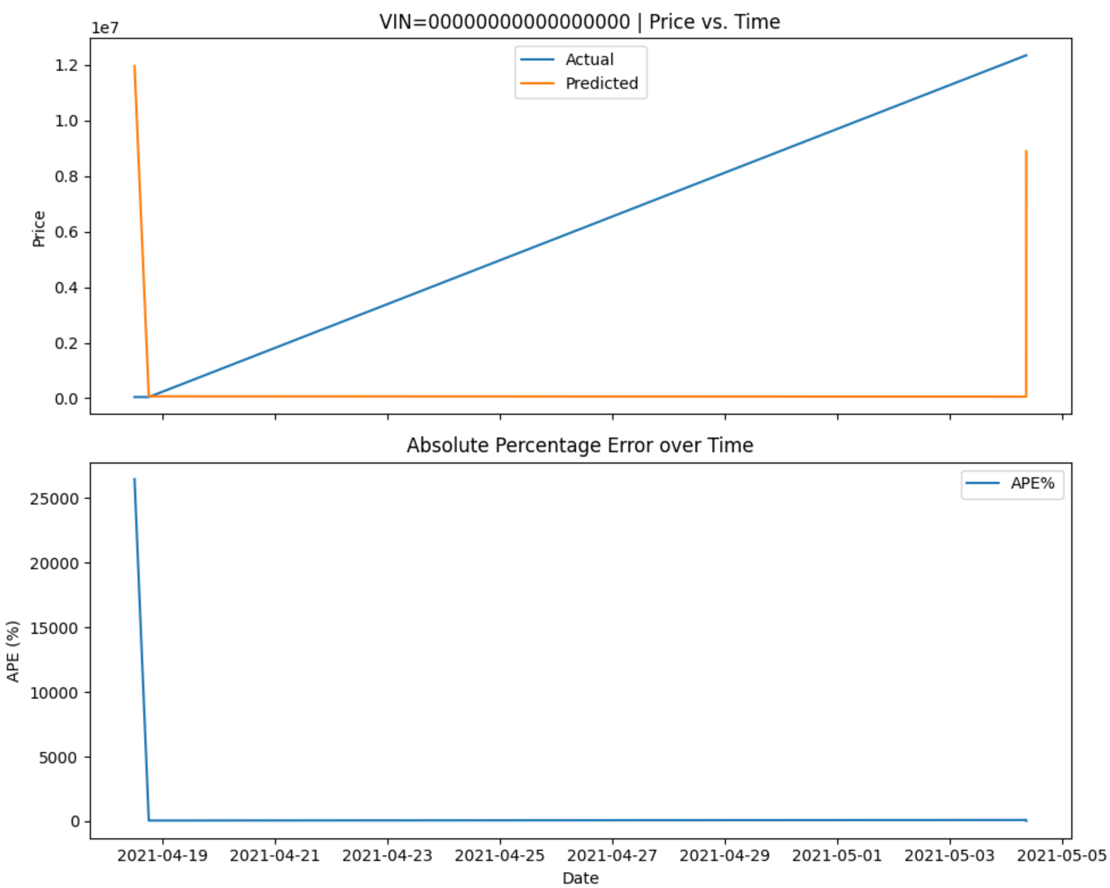
```
[28]: # Example: Linear Regression + Predictions using flattened sliding window
X_inf, y_inf_log, date_arr, vin_arr = build_infer_data(test_holdout_data)
y_pred_log = final_model.predict(X_inf)

# If log(price+1) was used during training,
# Restore the original:
y_true = np.exp(y_inf_log) - 1
y_pred = np.exp(y_pred_log) - 1

import pandas as pd

df_pred = pd.DataFrame({
    'VIN': vin_arr,
    'date': date_arr,
    'y_true': y_true,
    'y_pred': y_pred
})
df_pred.sort_values(['VIN', 'date'], inplace=True)
```

```
[29]: # For example, first count the number of samples for each VIN in the holdout
vin_counts = df_pred['VIN'].value_counts()
top_vins = vin_counts.index[:3]  # Select the top 3 VINs with the most samples
```

```
[30]: df_pred['AE'] = (df_pred['y_true'] - df_pred['y_pred']).abs()
df_pred['APE'] = df_pred['AE'] / (df_pred['y_true'].abs() + 1e-6)  # Avoid␣
↪division by zero
df_pred['APE%'] = df_pred['APE'] * 100
```

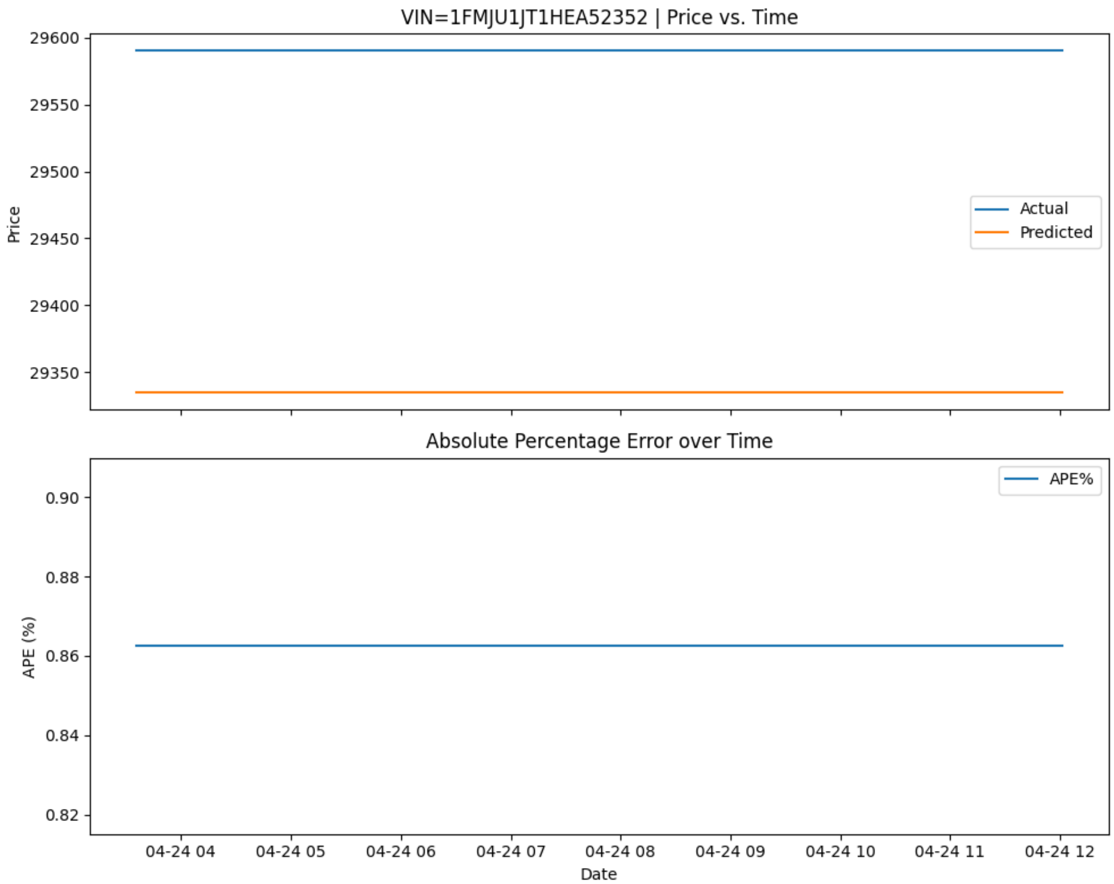
```
[31]: import matplotlib.pyplot as plt

def plot_vin_time_series(vin_id, df):
    """
    df: DataFrame with columns [VIN, date, y_true, y_pred, AE, APE, APE%]
    """
    tmp = df[df['VIN'] == vin_id].copy()
    tmp.sort_values('date', inplace=True)

    fig, axes = plt.subplots(nrows=2, ncols=1, figsize=(10, 8), sharex=True)

    # -- (1) Upper plot: Actual vs. Predicted Prices
    axes[0].plot(tmp['date'], tmp['y_true'], label='Actual')
    axes[0].plot(tmp['date'], tmp['y_pred'], label='Predicted')
    axes[0].set_ylabel("Price")
    axes[0].set_title(f"VIN={vin_id} | Price vs. Time")
```

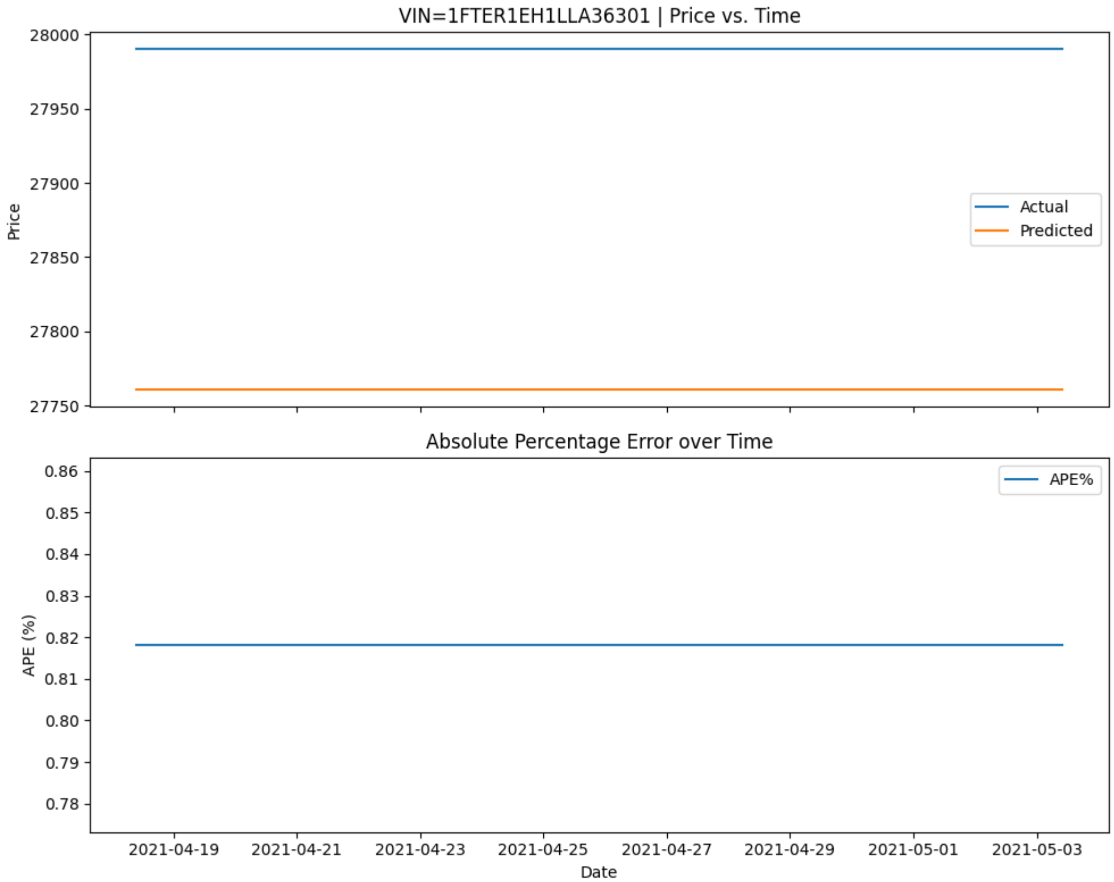
```
    axes[0].legend()

    # -- (2) Lower plot: Error over Time
    # Using APE% (Absolute Percentage Error) as an example
    axes[1].plot(tmp['date'], tmp['APE%'], label='APE%')
    axes[1].set_ylabel("APE (%)")
    axes[1].set_title("Absolute Percentage Error over Time")
    axes[1].legend()

    # X-axis
    axes[1].set_xlabel("Date")
    plt.tight_layout()
    plt.show()

# Usage: Plot for a specific VIN
sample_vin = df_pred['VIN'].iloc[0]
plot_vin_time_series(sample_vin, df_pred)
```

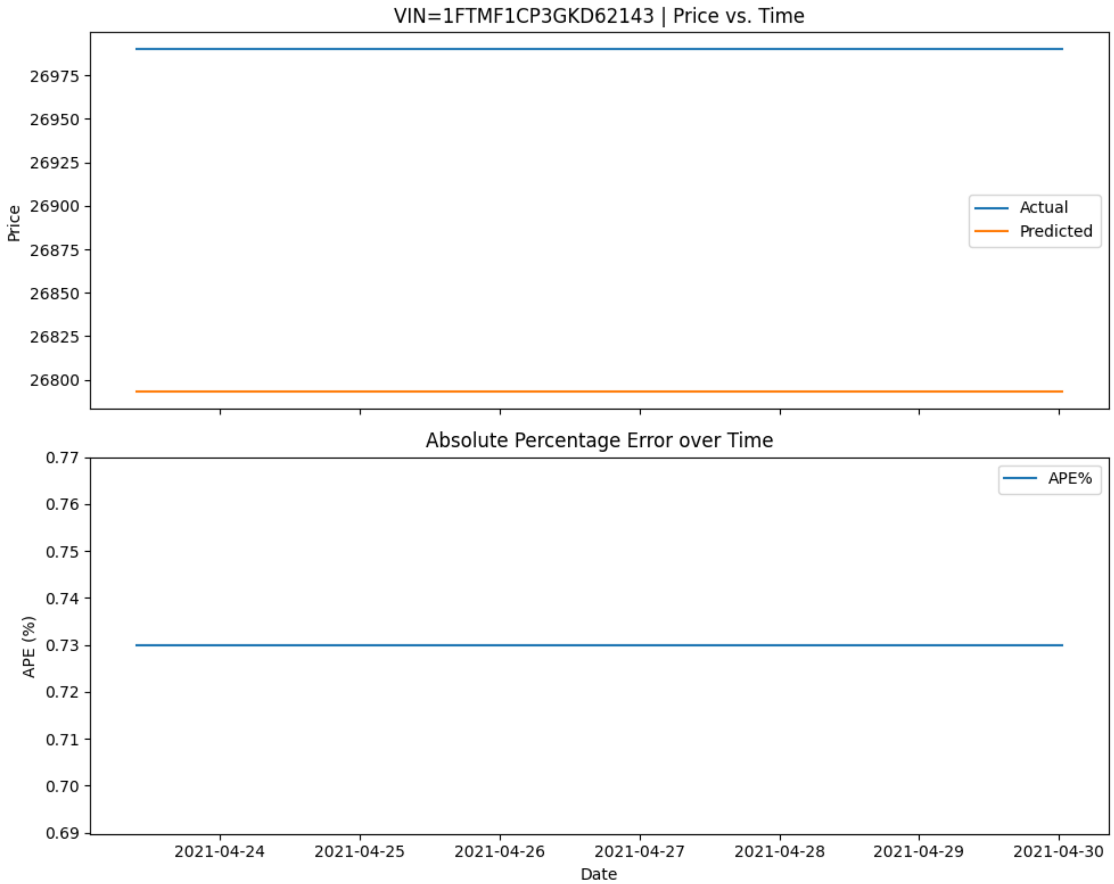


[32]:

```
representative_vins = top_vins[:3]
for vin_id in representative_vins:
    plot_vin_time_series(vin_id, df_pred)
```




VIN=1FTER1EH1LLA36301 | Price vs. Time
28000
27950
27900
27850
27800
27750
Price
Actual
Predicted
Absolute Percentage Error over Time
0.86
0.85
0.84
0.83
0.82
0.81
0.80
0.79
0.78
APE (%)
APE%
2021-04-19
2021-04-21
2021-04-23
2021-04-25
2021-04-27
2021-04-29
2021-05-01
2021-05-03
Date



[33]:
```python
import numpy as np

vin_stats = df_pred.groupby('VIN').agg({
    'AE': ['mean', 'std'],
    'APE%': ['mean', 'std']
}).reset_index()

vin_stats.columns = ['VIN', 'AE_mean', 'AE_std', 'APE_mean', 'APE_std']
vin_stats.sort_values('APE_mean', inplace=True)
vin_stats.head(10)
```

[33]:

	VIN	AE_mean	AE_std	APE_mean	APE_std
1220	1FAFP53U56A238775	0.002686	NaN	0.000096	NaN
1758	1FMZU73K03ZA83684	0.260254	0.0	0.006197	0.0
1750	1FMYU93155KD10524	0.291992	0.0	0.009733	0.0
137	1B4HR38N62F174971	1.015137	NaN	0.020323	NaN
1756	1FMZU72K83UA55811	0.653320	0.0	0.021850	0.0
2444	1FTEX14H3PKB08540	2.333008	0.0	0.025937	0.0
1026	1D4HB58D14F174876	1.444336	NaN	0.027277	NaN

```
1213  1FAFP37N67W231216  1.139160     0.0  0.028515      0.0
1211  1FAFP34314W170937  1.238770     0.0  0.033489      0.0
1027  1D4HR38N13F620886  1.402588     NaN  0.036431      NaN
```

===============

[34]:

```
# ============================================================================
# 4. DEFINE 2 DATASET BUILDING FUNCTIONS
# ============================================================================

def build_dataset_for_train(data_list, batch_size=64, shuffle=False):
    """
    Used for model.fit() or model.evaluate().
    We only return ((X_numeric, X_vin, X_bm), y_label) to match the model's␣
 ↪single-output structure.
    This avoids the mismatch between y_true and model outputs.
    """
    X_numeric = []
    X_vin = []
    X_bm  = []
    y_label = []

    for (num_seq, vin_seq, bm_seq, y, date_val, vin_str_val) in data_list:
        X_numeric.append(num_seq)
        X_vin.append(vin_seq)
        X_bm.append(bm_seq)
        y_label.append(y)

    X_numeric = np.array(X_numeric, dtype=np.float32)  # shape: [N, seq_len,␣
 ↪#numeric_cols]
    X_vin     = np.array(X_vin,     dtype=np.int32)    # shape: [N, seq_len]
    X_bm      = np.array(X_bm,      dtype=np.int32)    # shape: [N, seq_len]
    y_label   = np.array(y_label,   dtype=np.float32)  # shape: [N]

    ds = tf.data.Dataset.from_tensor_slices(((X_numeric, X_vin, X_bm), y_label))
    if shuffle:
        ds = ds.shuffle(len(data_list))
    ds = ds.batch(batch_size)
    return ds

def build_dataset_for_inference(data_list, batch_size=64):
    """
    Used for inference/visualization where we want the date/VIN as well.
    We return ((X_numeric, X_vin, X_bm), (y_label, date_str, vin_str)),
    so we can manually call model.predict(...) while retaining date & VIN info.
    """
    X_numeric = []
```

```
    X_vin = []
    X_bm  = []
    y_label = []
    date_list = []
    vin_list  = []

    for (num_seq, vin_seq, bm_seq, y, date_val, vin_str_val) in data_list:
        X_numeric.append(num_seq)
        X_vin.append(vin_seq)
        X_bm.append(bm_seq)
        y_label.append(y)
        date_list.append(str(date_val))   # convert to string to avoid␣
↪TensorFlow type issues
        vin_list.append(str(vin_str_val))

    X_numeric = np.array(X_numeric, dtype=np.float32)
    X_vin     = np.array(X_vin,     dtype=np.int32)
    X_bm      = np.array(X_bm,      dtype=np.int32)
    y_label   = np.array(y_label,   dtype=np.float32)
    date_list = np.array(date_list)
    vin_list  = np.array(vin_list)

    ds = tf.data.Dataset.from_tensor_slices(
        ((X_numeric, X_vin, X_bm), (y_label, date_list, vin_list))
    )
    ds = ds.batch(batch_size)
    return ds
```

[35]:
```
# ==============================================================================
# 5. CREATE MODEL + CROSS VALIDATION
# ==============================================================================

def create_model(embedding_dim, lstm_hidden_size, learning_rate):
    """
    Build a Keras model with:
      - 3 inputs: numeric_input, vin_input, bm_input
      - 1 LSTM layer to encode the sequence
      - 1 Dense output for price prediction
    """
    vin_vocab_size = len(vin_list)
    bm_vocab_size  = len(bm_list)

    # Inputs
    input_numeric = keras.Input(shape=(seq_len, len(numeric_cols)),␣
↪name='numeric_input')
    input_vin     = keras.Input(shape=(seq_len,), dtype='int32',␣
↪name='vin_input')
```

```
    input_bm      = keras.Input(shape=(seq_len,), dtype='int32',␣
↪name='bm_input')

    # Embedding layers for VIN & brand_model
    vin_emb_layer = keras.layers.Embedding(vin_vocab_size, embedding_dim,␣
↪name='vin_embedding')
    bm_emb_layer  = keras.layers.Embedding(bm_vocab_size,  embedding_dim,␣
↪name='bm_embedding')

    vin_emb_seq = vin_emb_layer(input_vin)  # shape [batch, seq_len, emb_dim]
    bm_emb_seq  = bm_emb_layer(input_bm)    # shape [batch, seq_len, emb_dim]

    # Concatenate numeric + vin_emb + bm_emb
    concat_seq = keras.layers.Concatenate(axis=-1)([input_numeric, vin_emb_seq,␣
↪bm_emb_seq])

    # LSTM layer
    lstm_out = keras.layers.LSTM(lstm_hidden_size,␣
↪return_sequences=False)(concat_seq)

    # Final dense layer => 1 output for price
    output_price = keras.layers.Dense(1)(lstm_out)

    # Build model
    model = keras.Model(inputs=[input_numeric, input_vin, input_bm],␣
↪outputs=output_price)
    optimizer = keras.optimizers.Adam(learning_rate=learning_rate, clipnorm=1.0)
    model.compile(optimizer=optimizer, loss='mse')
    return model

def k_fold_split(data, k=3):
    """
    Manually split 'data' into k folds, each fold has ~len(data)//k samples.
    Returns a list of (train_data, val_data) pairs.
    """
    fold_size = len(data) // k
    folds = []
    for i in range(k):
        val_start = i * fold_size
        val_end   = (i + 1) * fold_size
        val_data = data[val_start:val_end]
        train_data = data[:val_start] + data[val_end:]
        folds.append((train_data, val_data))
    return folds

# Param grid for hyperparameter tuning
```

```python
param_grid = [
    {'embedding_dim': 8,  'lstm_hidden_size': 32,  'learning_rate': 1e-3},
    {'embedding_dim': 16, 'lstm_hidden_size': 64,  'learning_rate': 1e-4},
    {'embedding_dim': 16, 'lstm_hidden_size': 128, 'learning_rate': 1e-4},
]

# 3-fold split for cross validation
folds = k_fold_split(cv_data, k=3)

best_param = None
best_cv_mse = float('inf')
print("Starting Hyperparameter Search with 3-Fold CV...")

for param in param_grid:
    emb_dim  = param['embedding_dim']
    lstm_size = param['lstm_hidden_size']
    lr       = param['learning_rate']

    fold_mses = []
    for fold_i, (train_list, val_list) in enumerate(folds):
        # Build train & val Datasets (no date/vin info for Keras .fit)
        train_ds_cv = build_dataset_for_train(train_list, batch_size=64, shuffle=True)
        val_ds_cv   = build_dataset_for_train(val_list,   batch_size=64, shuffle=False)

        # Create model for this fold
        model_cv = create_model(emb_dim, lstm_size, lr)
        # Train for a few epochs to evaluate
        history_cv = model_cv.fit(
            train_ds_cv,
            validation_data=val_ds_cv,
            epochs=5,
            verbose=0
        )

        # Evaluate on validation set
        val_mse = model_cv.evaluate(val_ds_cv, verbose=0)
        fold_mses.append(val_mse)
        print(f"  Fold {fold_i+1}, Params={param}, Val MSE={val_mse:.4f}")

    avg_mse = np.mean(fold_mses)
    print(f"=> Params={param} => average CV MSE={avg_mse:.4f}")
    if avg_mse < best_cv_mse:
        best_cv_mse = avg_mse
        best_param = param
```

```python
print("Best Hyperparams:", best_param)
print("Corresponding CV MSE:", best_cv_mse)
```

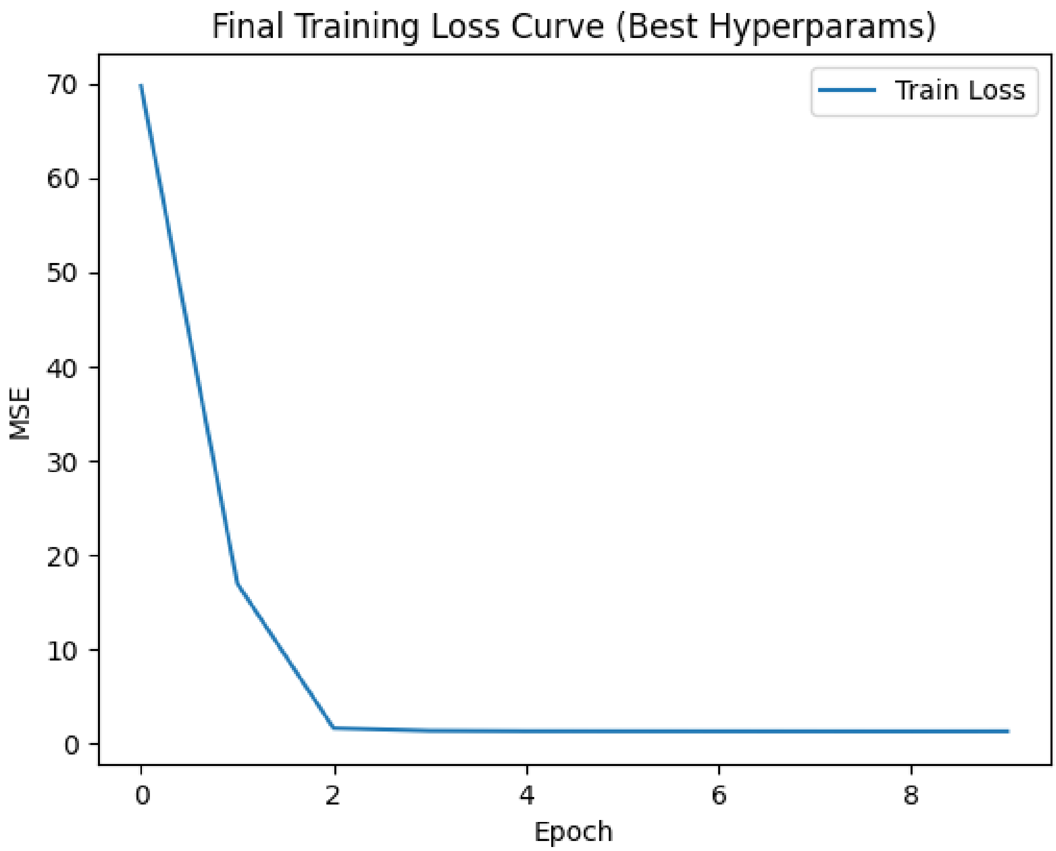
```
Starting Hyperparameter Search with 3-Fold CV…
  Fold 1, Params={'embedding_dim': 8, 'lstm_hidden_size': 32, 'learning_rate':
0.001}, Val MSE=1.1332
  Fold 2, Params={'embedding_dim': 8, 'lstm_hidden_size': 32, 'learning_rate':
0.001}, Val MSE=1.3387
  Fold 3, Params={'embedding_dim': 8, 'lstm_hidden_size': 32, 'learning_rate':
0.001}, Val MSE=1.4715
=> Params={'embedding_dim': 8, 'lstm_hidden_size': 32, 'learning_rate': 0.001}
=> average CV MSE=1.3144
  Fold 1, Params={'embedding_dim': 16, 'lstm_hidden_size': 64, 'learning_rate':
0.0001}, Val MSE=3.8237
  Fold 2, Params={'embedding_dim': 16, 'lstm_hidden_size': 64, 'learning_rate':
0.0001}, Val MSE=7.8580
  Fold 3, Params={'embedding_dim': 16, 'lstm_hidden_size': 64, 'learning_rate':
0.0001}, Val MSE=6.9771
=> Params={'embedding_dim': 16, 'lstm_hidden_size': 64, 'learning_rate': 0.0001}
=> average CV MSE=6.2196
  Fold 1, Params={'embedding_dim': 16, 'lstm_hidden_size': 128, 'learning_rate':
0.0001}, Val MSE=1.1317
  Fold 2, Params={'embedding_dim': 16, 'lstm_hidden_size': 128, 'learning_rate':
0.0001}, Val MSE=1.3050
  Fold 3, Params={'embedding_dim': 16, 'lstm_hidden_size': 128, 'learning_rate':
0.0001}, Val MSE=1.4237
=> Params={'embedding_dim': 16, 'lstm_hidden_size': 128, 'learning_rate':
0.0001} => average CV MSE=1.2868
Best Hyperparams: {'embedding_dim': 16, 'lstm_hidden_size': 128,
'learning_rate': 0.0001}
Corresponding CV MSE: 1.2868214050928752
```

[36]:
```python
# ==========================================================================
# 6. RETRAIN ON ALL CV DATA WITH BEST PARAM & EVALUATE ON HOLDOUT
# ==========================================================================

print("\nRetrain with Best Hyperparams on entire CV dataset...")

# Build a training dataset from all cv_data
train_ds_final = build_dataset_for_train(cv_data, batch_size=64, shuffle=True)
best_model = create_model(
    embedding_dim=best_param['embedding_dim'],
    lstm_hidden_size=best_param['lstm_hidden_size'],
    learning_rate=best_param['learning_rate']
)

# Train with more epochs
```

```python
history_final = best_model.fit(train_ds_final, epochs=10, verbose=1)

# Build a dataset from holdout data to calculate final MSE/RMSE
test_ds_holdout_for_loss = build_dataset_for_train(test_holdout_data,␣
↪batch_size=64, shuffle=False)
test_mse = best_model.evaluate(test_ds_holdout_for_loss, verbose=0)
test_rmse = math.sqrt(test_mse)
print(f"\n[FINAL TEST] MSE={test_mse:.4f}, RMSE={test_rmse:.4f}")

# Optional: Plot the training loss curve
plt.figure()
plt.plot(history_final.history['loss'], label='Train Loss')
plt.title("Final Training Loss Curve (Best Hyperparams)")
plt.xlabel('Epoch')
plt.ylabel('MSE')
plt.legend()
plt.show()
```

```
Retrain with Best Hyperparams on entire CV dataset…
Epoch 1/10
999/999              6s 5ms/step -
loss: 88.5442
Epoch 2/10
999/999              5s 5ms/step -
loss: 26.1230
Epoch 3/10
999/999              5s 5ms/step -
loss: 1.9308
Epoch 4/10
999/999              5s 5ms/step -
loss: 1.3170
Epoch 5/10
999/999              6s 6ms/step -
loss: 1.2543
Epoch 6/10
999/999              6s 6ms/step -
loss: 1.2346
Epoch 7/10
999/999              5s 5ms/step -
loss: 1.2314
Epoch 8/10
999/999              5s 5ms/step -
loss: 1.2390
Epoch 9/10
999/999              5s 5ms/step -
loss: 1.2109
Epoch 10/10
```

```
999/999            5s 5ms/step -
loss: 1.2400

[FINAL TEST] MSE=1.2114, RMSE=1.1007
```



[37]:
```
# ==========================================================================
# 7. INFERENCE / VISUALIZATION: COLLECT (date, VIN, y_pred, y_true)
# ==========================================================================

print("\nInference with date & VIN for Time Series Visualization...")

# Build a dataset that returns (y_label, date, vin) as well
test_ds_holdout_for_infer = build_dataset_for_inference(test_holdout_data,␣
 ↪batch_size=64)

y_preds_log = []
y_trues_log = []
dates_list  = []
vins_list   = []
```

```python
for (X_batch, Y_info_batch) in test_ds_holdout_for_infer:
    # Y_info_batch => (y_label, date_str, vin_str)
    y_label_batch, date_batch, vin_batch = Y_info_batch

    # Predict: shape => [batch_size, 1]
    preds = best_model.predict(X_batch)

    y_preds_log.extend(preds.ravel().tolist())
    y_trues_log.extend(y_label_batch.numpy().tolist())
    dates_list.extend(date_batch.numpy().tolist())
    vins_list.extend(vin_batch.numpy().tolist())

# Step: revert log(price+1) => original price
y_preds = np.exp(y_preds_log) - 1
y_trues = np.exp(y_trues_log) - 1

# Make a dataframe to store prediction results
df_pred = pd.DataFrame({
    'VIN': [v.decode('utf-8') if isinstance(v, bytes) else v for v in␣
 ↪vins_list],
    'date': [
        pd.to_datetime(d.decode('utf-8')) if isinstance(d, bytes) else pd.
 ↪to_datetime(d)
        for d in dates_list
    ],
    'y_true': y_trues,
    'y_pred': y_preds
})
df_pred.sort_values(by=['VIN', 'date'], inplace=True)
```

```
Inference with date & VIN for Time Series Visualization…
2/2               0s 8ms/step
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2/2              0s 12ms/step
2/2              0s 6ms/step
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2/2              0s 7ms/step
2/2              0s 7ms/step
2/2              0s 7ms/step
2/2              0s 60ms/step

2025-03-16 17:31:20.260765: I tensorflow/core/framework/local_rendezvous.cc:405]
Local rendezvous is aborting with status: OUT_OF_RANGE: End of sequence
```
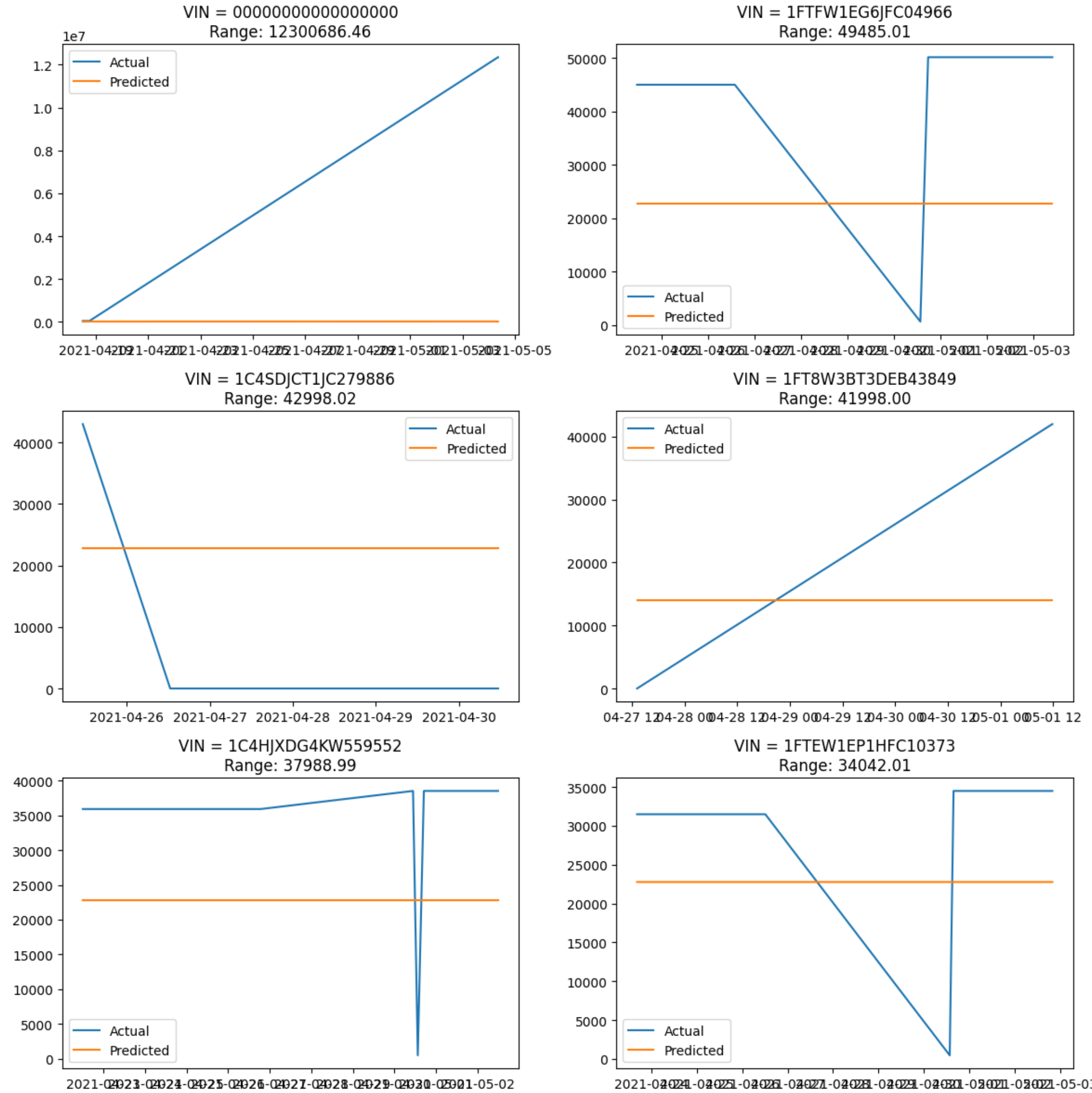

[38]:

```python
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
import math

# Assume you already have df_pred, containing columns: ['VIN', 'date',␣
↪'y_true', 'y_pred']
# where y_true and y_pred are original prices (if log(price+1) was used before,␣
↪make sure to restore using exp(...)-1)

# 1) Group by VIN, calculate "Price Range" = max(y_true) - min(y_true)
vin_ranges = []
grouped = df_pred.groupby('VIN')
for vin, group in grouped:
    price_range = group['y_true'].max() - group['y_true'].min()
    vin_ranges.append((vin, price_range))

# 2) Sort by price range in descending order
vin_ranges.sort(key=lambda x: x[1], reverse=True)

# 3) Select the top N (e.g., 5) VINs with the largest price fluctuations
top_n = 5
selected_vins = [x[0] for x in vin_ranges[:top_n + 1]]

# 4) Plot: Create a subplot for each VIN
num_vins = len(selected_vins)
cols = 2  # Number of plots per row
rows = math.ceil(num_vins / cols)

fig, axes = plt.subplots(rows, cols, figsize=(12, 4*rows))
# If there's only one row and one column, axes may not be an array; flatten it␣
↪here for consistency
axes = axes.flatten() if num_vins > 1 else [axes]

for i, vin_id in enumerate(selected_vins):
    ax = axes[i]

    # Extract data for this VIN
    tmp = df_pred[df_pred['VIN'] == vin_id].copy()
    tmp.sort_values('date', inplace=True)

    # Plot actual vs. predicted prices
    ax.plot(tmp['date'], tmp['y_true'], label='Actual')
    ax.plot(tmp['date'], tmp['y_pred'], label='Predicted')

    # Include "Range" information in the title for additional context
    vin_range = vin_ranges[i][1]
```

```
    ax.set_title(f"VIN = {vin_id}\nRange: {vin_range:.2f}")
    ax.legend()

# If top_n < rows*cols, there will be extra subplots; hide them here
for j in range(i+1, len(axes)):
    axes[j].set_visible(False)

plt.tight_layout()
plt.show()
```




```
[ ]: import numpy as np
```

```python
def build_Xy_for_linear(data_list, seq_len=3):
    X_list = []
    y_list = []
    for (num_seq, vin_seq, bm_seq, y, y_date, vin_str) in data_list:
        # num_seq shape => (seq_len, num_numeric_features)
        # flatten:
        x_flat = num_seq.flatten()  # shape (seq_len * num_numeric_features,)

        X_list.append(x_flat)
        y_list.append(y)

    X = np.array(X_list, dtype=np.float32)
    y = np.array(y_list, dtype=np.float32)
    return X, y
```

Embedding Dim	LSTM Hidden Size	Learning Rate	Fold 1 MSE	Fold 2 MSE	Fold 3 MSE	Average CV MSE
8	32	0.001	1.2808	1.2705	1.2943	1.2819
16	64	0.0001	5.1336	7.9843	3.2331	5.4503
16	128	0.0001	1.2368	1.2252	1.2376	1.2332