Windowing High-Resolution ADC Data – Part 1

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Abstract
Analyzing data from ADCs requires the use of windowing functions for spectral estimation and analysis but different windows suit different purposes. National Semiconductor's new WaveVision 5 software provides a family of mathematically simple windowing functions that spans a fundamental tradeoff and provides the flexibility to meet a wide range of user applications. Part 1 of this article is a discussion on spectral analysis that presents the need for data windowing as well as the effect of windowing functions in the time and frequency domain.

Introduction

In applications ranging from sensor networks to communication systems, analyzing sampled data from an analog-to-digital converter (ADC) in the frequency domain crucially depends on understanding the spectrum power at various frequencies. When gauging performance metrics such as the signal-to-noise ratio (SNR) or total harmonic distortion (THD), one must make critical assumptions that the energy at certain frequencies is in fact the signal, noise, or distortion.

Using a finite time span of data makes the spectrum analysis difficult because truncating a signal has the effect of smearing power across the frequency spectrum and corrupting the distinction between noise and signal power, making metric calculations inaccurate. Many methods exist to minimize smearing, or "spectral leakage" such as collecting very large data sets or requiring frequency coherency, but these are often impractical. The best option is to create a reasonable approximation of the infinite-time signal spectrum using windowing functions.

Not uncommon in the signal processing world, windowing functions provide the means of containing the spectral leakage across the spectrum. With the leakage successfully contained and with knowledge of the consequences of using windowing functions the analysis of the spectrum is straightforward. Unfortunately, there is not a single windowing function that is suitable for all situations. Tradeoffs must be understood to choose the window that is most appropriate for the application.

Part 1 of this article on data windowing presents an introduction to the spectral analysis of time-finite data segments and establishes the need for data windowing. Fundamental tradeoffs and specific qualities of windows are described as well as the characteristics of a few common windows.

Part 2 moves the context to windowing functions used in National Semiconductor's new ADC evaluation software platform called WaveVision 5. Using WaveVision 5, a user can take advantage of a flexible family of Cosine-Sum windowing functions to analyze the performance of ADCs that have a wide span of speeds and resolutions. These windows can be applied to data captured during single-tone test and is useful for evaluating data captured from a variety of different applications.

Time-Finite Data Segments

A continuous sinusoidal segment can be represented as the multiplication of a time-infinite sinusoid and a finite length pulse. From signal processing theory and using the Fourier Transform it can be shown that the continuous frequency spectrum, X(ω), of the sinusoid segment can be expressed as the convolution of dirac delta functions associated with the sinusoid and a *sinc* function associated with the pulse as given in (**1**), see [1].

In this equation, ω_0 is the radial frequency of the sinusoid and T_{pulse} is the duration of the pulse. The resulting spectrum has a large main lobe near the frequency location of the sinusoid and surrounding side lobes.

Figure 1 shows the transformation of the continuous sinusoidal segment from the time domain to the frequency domain and illustrates the important point that a sinusoidal segment has energy across the entire spectrum and is not concentrated into one frequency. All the power of a sinusoid is contained at a single frequency *only* if the signal is infinite in duration.

Data from an ADC is collected in samples and is not a continuous waveform although the same power spreading phenomenon is observed with the sampled data. **Figure 2** shows the continuous frequency spectrum of the sampled signal that is acquired using another spectrum analysis tool called the Discrete-Time Fourier Transform (DTFT). One can observe the symmetry that is characteristic of sampled signal spectra and, once again, the power of the sinusoidal segment spans the whole spectrum.

$$X(\omega) = \pi\left[\delta(\omega + \omega_0) + \delta(\omega - \omega_0)\right] * T_{pulse}\,\mathrm{sinc}\left(\omega\,\frac{T_{pulse}}{2\pi}\right) \qquad (1)$$

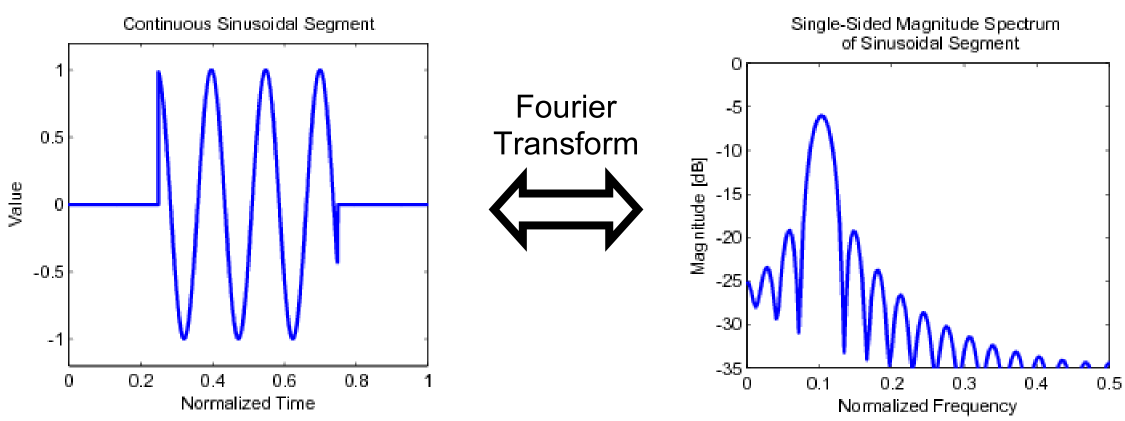


Figure 1: Spectral leakage of a continuous, finite-time sinusoidal segment

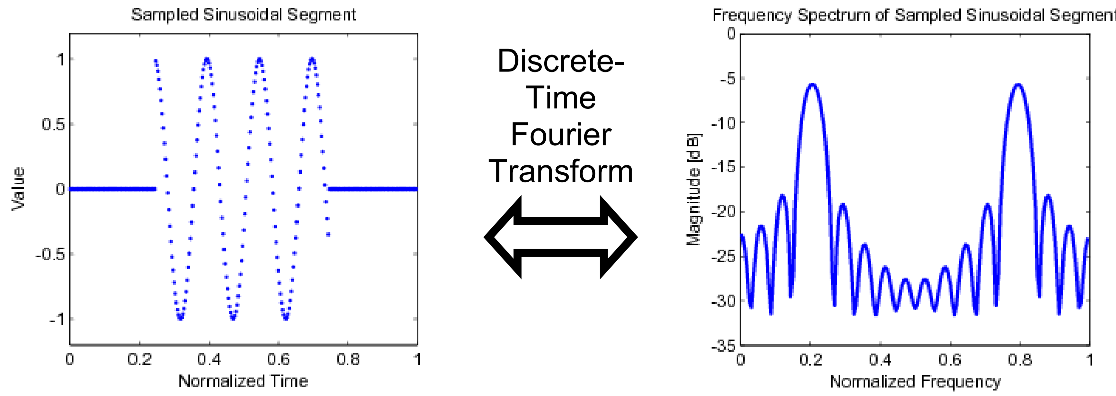


Figure 2: Spectral leakage of a sampled, finite-time sinusoidal segment

In most cases other than the simplest mathematical exercises a Discrete Fourier Transform (DFT) must be used for spectrum analyses where both the time-domain data and frequency-domain spectrum are sampled in discrete steps. The DFT, as opposed to the DTFT, results in a sampled representation of the continuous frequency spectrum and loses the spectrum information that occurs between frequency "bins."

Figure 3 illustrates this loss of information for sinusoidal segments with two different frequencies. Figure 3(a) shows the continuous and sampled spectrum of a sinusoidal signal that contains a non-integer number of sinusoidal periods and is therefore said to be *non-coherent* with the sampling frequency. For illustrative purposes, the continuous spectrum is scaled in frequency and magnitude to coincide with the sampled values of the 16-point DFT. Notice how the red, sampled points of the DFT fall on the leakage side lobes of the *sinc* shaping, creating an appearance of a skirt around the largest tone. Note also that the sample on the main lobe falls short of the lobe maximum.

Figure 3(b) shows the case where the sinusoidal segment has an integer number of periods so the frequency is *coherent* with the sampling frequency. In this case the points of the DFT fall on the zeros of the *sinc* function and all information in the side lobes is lost.

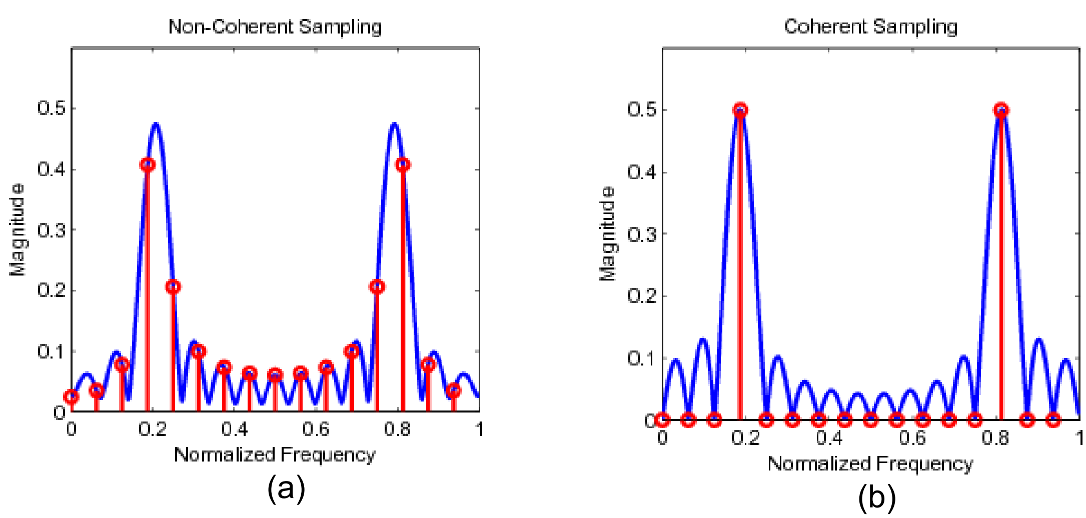


Figure 3: Loss of side lobe information when using the DFT on (a) non-coherent and (b) coherent signal frequencies

This example has two important points. First, as mentioned before, any finite-time data segment *always* exhibits spectral leakage as side lobes in the continuous spectrum. Second, the loss of side lobe information when using a DFT for coherent frequencies results in a spectrum that more closely resembles the spectrum of an infininte-time segment. In the coherent case the user can easily identify the power of the well-contained fundamental signal in a single frequency bin.

The DFT spectrum of non-coherent sinusoidal segment exhibits a leakage skirt, but the level of the skirt is dependent on the relative closeness of the sinusoid frequency to the nearest coherent frequency which occurs at the center of a DFT frequency bin. **Figure 4** compares the leakage skirts of sinusoids that are offset from the bin center by the given amount. Obviously, larger frequency offsets from the bin center result in more pronounced leakage skirts.

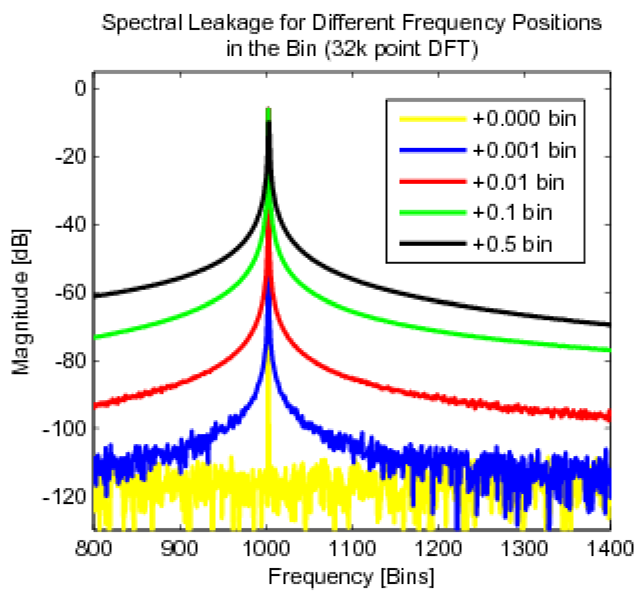
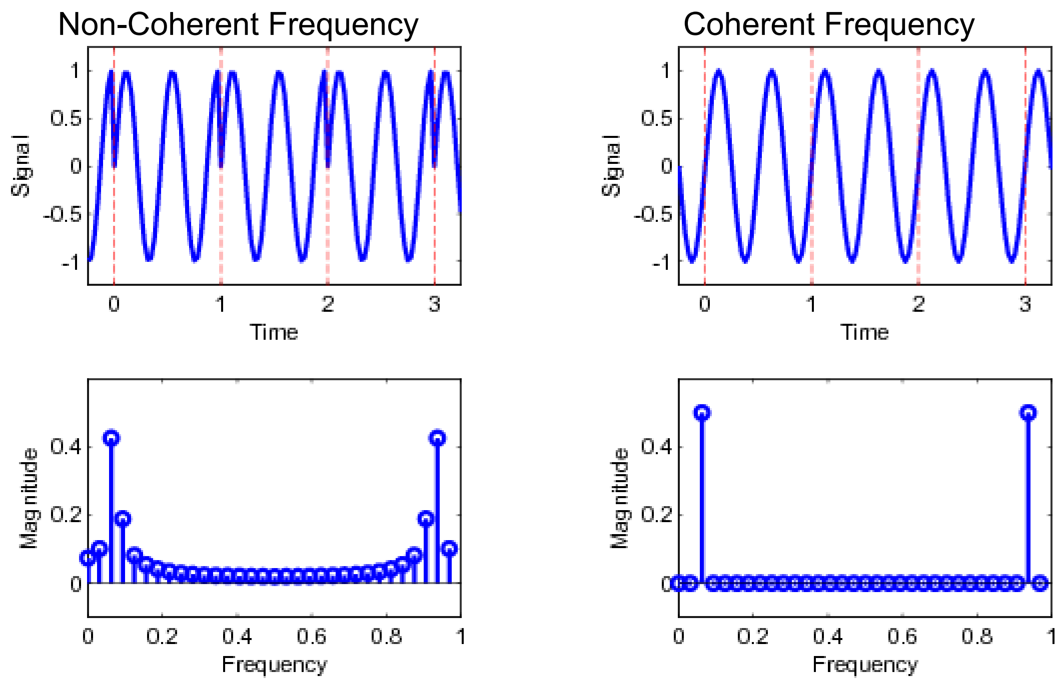


Figure 4: Leakage skirt as a function of the sinusoidal frequency relative to the frequency bin center

The convenient loss of the side lobe information for coherent frequencies gives the illusion that the data extends infinitely in time. If one were to cascade the sinusoidal segment in time as in **Figure 5** (a) and (b) for the sake of observation, non-coherent frequencies would have a discontinuity at the boundary, giving rise to additional frequency content. Coherent frequencies would have a smooth transition and display only one sinusoid frequency component.



(a) (b)

Figure 5: Spectral leakage as a result of end-point discontinuities when cascading the data segment in time

Testing ADCs with coherent frequencies is preferred so that the DFT output spectrum can be easily analyzed. With a coherent ADC input frequency, all harmonic tones at the output will also be coherent and the output spectrum will contain clear tones with no spectral leakage and a clean, flat noise power density.

Unfortunately, requiring coherent frequencies is impractical in large systems and broadband applications. Therefore the data must be manipulated to limit the spectral leakage to a small bandwidth. Windowing functions are necessary for this purpose.

Windowing Functions

In the previous section, it is shown that the appearance of spectral leakage in the DFT spectrum is attributed to discontinuity between the end points of the transient data record. Data acquired from an ADC is also referred to as “Rectangular” windowed data because the data record is a limited time block of the signal and the amplitude of the data is not shaped. Therefore, the ADC data is prone to endpoint discontinuities. The purpose of applying a windowing function is to reduce the discontinuity by attenuating the values at the beginning and end of the data record. With the discontinuity attenuated, most of the spectral leakage is confined to a smaller frequency range.

Applying a windowing function to an N-point sampled signal is as simple as multiplying the signal by the N-point windowing function as shown in (**2**). Multiplying a non-coherent sinusoidal segment by the common Hann window of (**3**), see [2], results in the transient waveform and zoomed spectrum of **Figure 6**. In this case the fundamental lobe spans only a few of bins but the side lobe suppression is merely -31.5 dB. This window does not have sufficient side lobe suppression for an accurate performance analysis of medium to high resolution ADCs because the side lobes distort the spectrum. The profile of the side lobes must fall below the noise power density of the ADC for accurate analysis results.

$$x_w[n] = x[n] \cdot w[n] \tag{2}$$

$$w[n] = 0.5 - 0.5\cos\left(2\pi \frac{n}{N}\right) \quad \text{for } 0 \leq n \leq N\text{-}1 \tag{3}$$

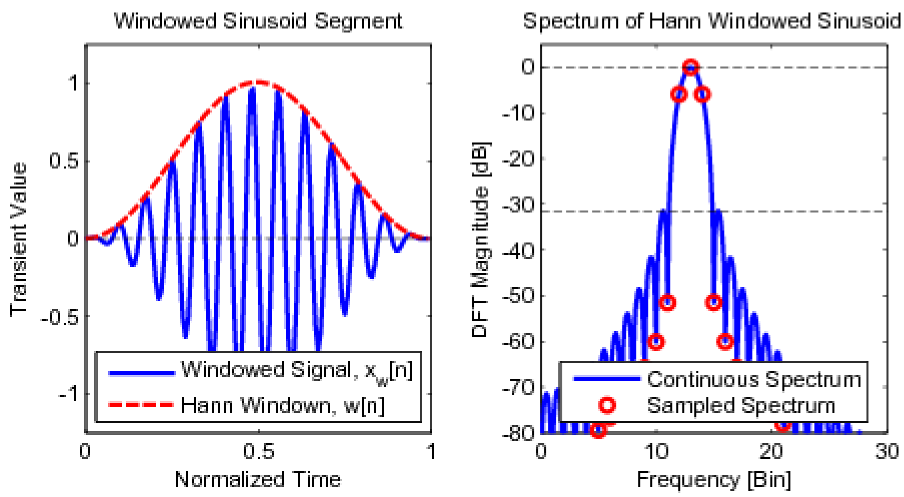


Figure 6: The transient waveform and zoomed spectrum of a sinusoid data segment windowed with the Hann window.

The shape and distribution of the contained leakage depends on the form of the windowing function itself. Multiplying two functions in the time domain is equivalent to convolution in the frequency domain so the spectral shape of a sinusoid takes on the inherent shape and frequency distribution of the window's frequency domain representation.

The power of a windowed sinusoid is typically lumped into a large main lobe and distributed into surrounding side lobes that are linked together in a fundamental tradeoff balancing the bandwidth of the main lobe and the maximum side lobe level. Decreasing the side-lobes for higher *dynamic range* increases the main lobe bandwidth, causing lower *frequency resolution* because the lobe can mask signals in adjacent frequencies. In the spectra of **Figure 7**, two different windowing functions are applied to two-tone data to illustrate this tradeoff. Applying the Hamming window gives sufficient distinction between the two frequency tones but results in poor side lobe suppression whereas applying the Blackman-Harris window fuses the two-tone power into one block but gives very low side lobes levels.

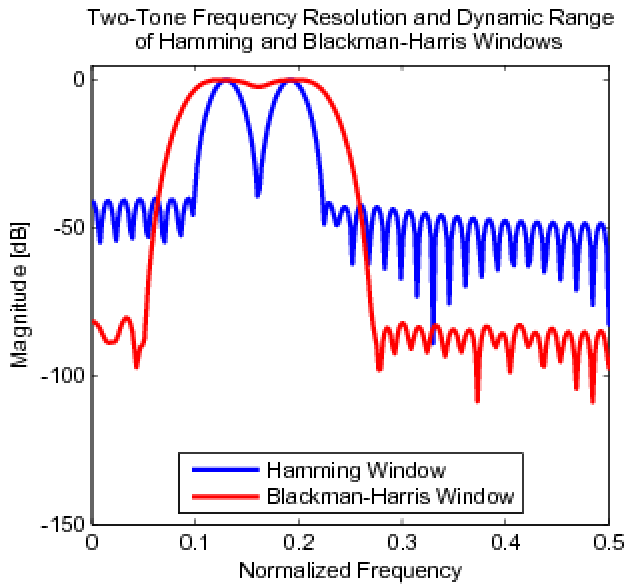


Figure 7: Dynamic range and frequency resolution comparison between the Hamming and Blackman-Harris windows

The harsh reality is that the analysis of non-coherent frequency data from a high dynamic range ADC requires a windowing function that keeps the window's side lobe signature below the noise power density of the ADC. This prevents the distinction between signals whose frequencies are very close.

The profiles of four common windows are shown in **Figure 8** demonstrating their varying main lobe widths and side lobe levels. Given the case of a high speed 16-bit ADC the SNR may be as good as 78 dBFS (dB relative to full scale). If a ~32k sample data set is the largest set that can be accommodated by data acquisition hardware, then the noise power density of the DFT spectrum will be near -120 dBFS/bin. The side lobe levels of the four presented windows are no better than -100 dB so these windows cannot be applied to the ADC data of this example while guaranteeing accurate results across all input frequencies.

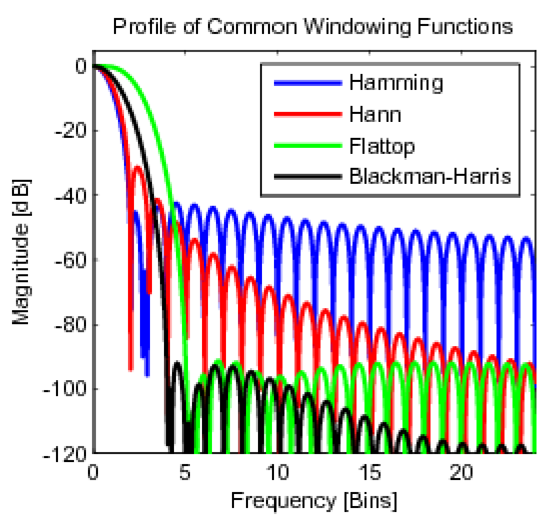
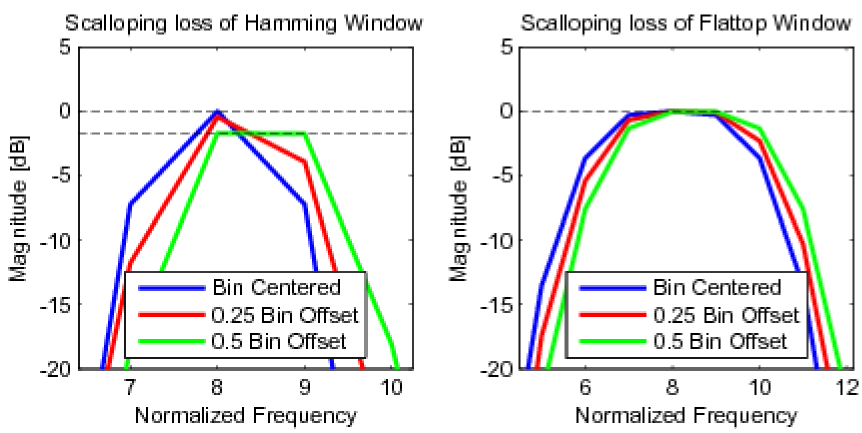


Figure 8: Profile comparison of four common windowing functions

Additional Properties of Windowing Functions

In addition to the dynamic range and frequency resolution of windowing functions, two additional properties are relevant to the analysis of spectrums that come from ADC data. These are the window's scalloping loss and gain factors.

Scalloping loss is the variation of the main lobe maximum value depending on the frequency location of the signal in a DFT frequency bin. As illustrated earlier in Figure 3, non-coherent frequencies in the spectrum do not get sampled at the peak of their main lobes. The same situation applies to windowed data. **Figure 9**(a) demonstrates the variation of the main lobe height for the Hamming window. When the frequency is on the edge of the bin (0.5 bin offset) the maximum value of the main lobe is 1.8 dB less than if the frequency is bin-centered. The Flattop window of Figure 9(b) has the unique property that the scalloping loss is very small and is popular when determining the power of a sinusoidal tone by simply observing the maximum value of the main lobe instead of considering all the power within the lobe.



(a) (b)

Figure 9: Scalloping loss of windowing functions

When applying a windowing function to a data set, care must be taken to properly scale the window so that the total power of the data is conserved. This will ensure that the correct fundamental power and noise power is calculated. For this to occur, the *incoherent gain*, see [**2**], of the window is scaled to unity.

Another import gain factor is the *processing gain*. It can be easily visualized as the change in the peak amplitude of a coherent sinusoidal signal in the frequency domain when a windowing function is applied to the data. Without a windowing function, the coherent signal will appear as a single-frequency tone with amplitude A_{sig}. After the window is applied, the signal will appear as a main lobe with a maximum value of A_{sig}/PG_w where PG_w is the *processing gain*. This reduction of the main lobe maximum conserves power by compensating for the fact that the signal power spans a wider bandwidth. More details including the importance of these gain quantities will be covered in the second installment of this article.

Conclusion

Using windowing functions to analyze the performance of ADCs is essential in situations where input frequencies are non-coherent. In this article we've presented the arguments behind this necessity and covered the resulting effects of applying windowing functions. We demonstrated that there is not a "one size fits all" window and argued that different windows are good for different applications based on the required dynamic range or frequency resolution of the window.

In the next installment of this article we present a family of windows used in the new National Semiconductor WaveVision 5 ADC evaluation software. The windows are specifically intended to span the dynamic range / frequency resolution tradeoff, allowing the user to perform analyses of data from most any application.

References

[1] E. W. Kamen and B. S. Heck, "Fundamentals of Signal and Systems," Prentice Hall, Upper Saddle River, NJ, 2000.

[2] F. J. Harris, "On the Use of Windows for Harmonic Analysis with the Discrete Fourier Transform," Proceedings of the *IEEE*, vol. 66, pp. 51-83, Jan. 1978.

About the Author

Josh Carnes is an applications engineer with National Semiconductor's Strategic Signal Path Group, based in Ft. Collins, Colorado. He received his BSEE and MSEE degrees from Oregon State University in 2004 and 2007, respectively, with research focusing on low-voltage pipelined ADC design techniques. His interests include cellular base station subsystems, wireless communications, as well as automated testing and analysis of ADCs.