

Opportunistic Spectrum Access Using Partially Overlapping Channels: Graphical Game and Uncoupled Learning

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Abstract—This article investigates the problem of distributed channel selection in opportunistic spectrum access (OSA) networks with partially overlapping channels (POC) using a game-theoretic learning algorithm. Compared with traditional non-overlapping channels (NOC), POC can increase the full-range spectrum utilization, mitigate interference and improve the network throughput. However, most existing POC approaches are centralized, which are not suitable for distributed OSA networks. We formulate the POC selection problem as an interference mitigation game. We prove that the game has at least one pure strategy NE point and the best pure strategy NE point minimizes the aggregate interference in the network. We characterize the achievable performance of the game by presenting an upper bound for aggregate interference of all NE points. In addition, we propose a simultaneous uncoupled learning algorithm with heterogeneous exploration rates to achieve the pure strategy NE points of the game. Simulation results show that the heterogeneous exploration rates lead to faster convergence speed and the throughput improvement gain of the proposed POC approach over traditional NOC approach is significant. Also, the proposed uncoupled learning algorithm achieves satisfactory performance when compared with existing coupled and uncoupled algorithms.

Index Terms—Opportunistic spectrum access, cognitive radio networks, distributed channel selection, partially overlapping channels, exact potential game, simultaneous uncoupled learning algorithm.

I. INTRODUCTION

OPPORTUNISTIC spectrum access (OSA) is an efficient solution to lessen the spectrum shortage problem facing wireless communications today [1]–[3]. Generally, there are multiple channels in OSA systems while each user can only access a small part at a time [4]. Thus, careful design of channel selection is key to improve the network throughput since it can eliminate mutual interference among the users [5]. Due to the nonideal shaping filter, the transmitted signal in a channel always causes spectrum leak in its adjacent

channels. Thus, two operational channels should keep enough separation to avoid adjacent channel interference. These channels are called non-overlapping channels, which are also called orthogonal channels. Existing work mainly focuses on assigning non-overlapping channels to interfering nodes, e.g., [6]–[10]. In these studies, two interfering nodes can simultaneously transmit only if they are allocated with non-overlapping channels. However, due to the limited number of non-overlapping channels, e.g., there are total eleven channels but only three non-overlapping channels in the IEEE 802.11b-based OSA networks, interference can not be completely eliminated, which results in severe throughput drop. Thus, it is important to develop new channel usage mechanisms to improve the network throughput with non-overlapping channels.

We resort to utilizing partially overlapping channels, in which the operational channels are not necessary to be orthogonal [11]. As a result, the number of operational channels increases significantly. It has been shown that using partially overlapping channels can increase the full-range spectrum utilization and improve the network throughput [12]–[19]. In most existing research, the partially overlapping channels are allocated in a centralized manner, using e.g., graph coloring [12], genetic algorithm [18], and other optimization technologies [15], [19]. These centralized approaches essentially need a central controller and information exchange among users. However, there is no central control in distributed OSA networks, and information exchange is costly and even not feasible in some scenarios. Therefore, the task of assigning partially overlapping channels in distributed OSA networks remains unsolved and is challenging.

We first analyze the relationship between the experienced interference and the achieved throughput under the framework of partially overlapping channels, and then formulate the partially overlapping channel selection problem as an interference mitigation game. The reasons for using such a game model are threefold: (i) the users make decisions distributively and autonomously, (ii) their decisions are interactive, and (iii) mitigating the interference leads to increased network throughput. The formulated game belongs to graphical games [8], in which the utility of a user is only affected by its nearby neighbors. We prove that the proposed game is an exact potential game which has at least one pure strategy Nash equilibrium (NE); more importantly, the optimal pure strategy Nash equilibria minimize the aggregate interference.

Although this result is promising, the task of achieving

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pure strategy Nash equilibria of the game is challenging. Most existing algorithms in game theory are coupled, e.g., the best response [17], fictitious play [20], spatial adaptive play [41] and regret learning [5], since they need information about other users. Notably, these coupled algorithms can not be applied to the considered distributed OSA networks. Users in OSA systems are characterized by being able to monitor the environment, learn from past experiences, and make intelligent decisions [21], [22]. Following this methodology, we propose an uncoupled learning algorithm to achieve the Nash equilibria of the formulated interference mitigation game. The main contributions of this article are summarized as follows:

- 1) We formulate the partially overlapping channel selection problem as an interference mitigation game, in which the utility function of each user is defined as the negative value of its experienced interference. We prove that the game has at least one pure strategy NE point and the best pure strategy NE point minimizes the aggregate interference in the network. In addition, we characterize the achievable performance of the game by presenting an upper bound for aggregate interference of all NE points.
- 2) We propose a simultaneous uncoupled learning algorithm to achieve the pure strategy NE points of the game. The proposed algorithm is only relying on the individual information of a user and does not need information about other players and all the players simultaneously perform learning. Also, heterogeneous exploration rates are used by exploiting the feature of spatially located players, which leads to faster convergence speed.
- 3) We present comprehensive simulation results to validate the superiority of the proposed approach. It has been shown that in both random and grid network topologies, the throughput improvement of partially overlapping channels over non-overlapping channels is significant. Also, the proposed uncoupled learning algorithm achieves satisfactory performance when compared with existing coupled and uncoupled algorithm.

The rest of the article is organized as follows. In Section II, we give a brief review of related work. In Section III, we present the interference model and formulate the network interference mitigation problem. In Section IV, we present the interference mitigation game and investigate the properties of its NE, and then propose an uncoupled learning algorithm for achieving its Nash equilibria. In Section V, simulation results are presented. Finally, we present discussion and draw conclusion in Section VI.

II. RELATED WORK

The problem of channel selection in OSA systems has been extensively investigated in the literature, using e.g., partially observable Markovian decision process [4], optimal stopping theory [23], and multi-armed bandit problem [24]. These work mainly focused on characterizing the behavior of single user in OSA systems. To capture the interactions among the multiple users, different kinds of game-theoretic channel selection approaches have been investigated [5]–[10]. The above references, however, only considered non-overlapping channels and did not exploit partially overlapping channels.

The pioneer idea of using partially overlapping channels can be found in a series of studies [11], [12]. It has been demonstrated that using partially overlapping channels can improve the full-range channel usage, reduce the number of interfering users, and enhance the network throughput. Since the traditional interference models for non-overlapping channel are not suitable for overlapping channels, new interference models are needed. In [11], [12], a simple interference factor is defined as the amount of the spectral overlap of two channels. This interference factor only considers the channel separation but does not take into account the physical distance. In a recent work [18], the channel separation and physical distance are jointly considered and a binary interference model is defined to indicate whether two users interfere with each other or not. Based on this, a continuous interference model was proposed in [19]. The interference model used in this article can be regarded as a general version of that in [18].

Although partially overlapping channels have draw great attention, most existing algorithms in the literature are centralized [12], [18], [19]. In addition, although a game-theoretic approach for utilizing partially overlapping channel selection in wireless mesh networks has also been reported in [17], it needs cooperation and information exchange among users. In this article, we propose a fully distributed learning algorithm, which is suitable for distributed OSA networks.

Generally, most algorithms for game models in the literature are coupled which need information about other users. Recently, developing uncoupled learning algorithms has been an active topic for both communication engineering community and game community. Specifically, an algorithm with local information exchange between neighboring users can be found in our recent work [8] and a Q-learning based channel selection algorithm for cognitive radio systems was reported in [25]. In parallel, some uncoupled learning algorithms were proposed in the pure research of game theory [26]–[28].

Note that there are some previous work which also used potential games to study the non-overlapping channel selection problem, e.g., [29]–[31]. The main differences in methodology are: i) we formulated the overlapping channel selection problem as a graphical game, and ii) the proposed learning algorithm in our work is simultaneous and uncoupled.

III. NETWORK AND INTERFERENCE MODELS AND PROBLEM FORMULATION

A. Network Model

We consider a wireless canonical network consisting of K nodes. In canonical networks, each node represents a collection of entities located in a relatively small region with intra-node communications [32]–[34]. Examples of canonical networks are an IEEE 802.11-based WLAN access point (AP) together with its clients [34] and a cluster head together with its members [33].

Due to the attractive features such as easy deployment and increasing popularity, the IEEE 802.11b-based networks are always utilized as the hardware platforms for OSA technologies [8]. In this article, we also take the IEEE 802.11b-based network as a research instance. However, it should be pointed out the investigations presented in this article are general and

thereby can be extended to many other scenarios. In this network, an AP together with its associated clients form a basic service set (BSS). All clients belonging to the same BSS operate on the same channel chosen by the AP, and share that channel using the well-known distributed coordination function. As a result, each AP in the network is a decision-making agent responsible for choosing the operational channel to eliminate mutual interference with other APs.

Suppose that there are M channels in the considered network ($M = 11$ in the IEEE 802.11-based WLAN), which are denoted as $\{c_1, \dots, c_M\}$. Each AP chooses one channel for its intra-node communications. The objective is to maximize the aggregate throughput of all APs. Clearly, the throughput of each AP is jointly determined by its own transmitting power and the interference caused by other APs. We assume that each AP has the same transmitting power. As a result, we will focus on the problem of channel selection in this article, which determines mutual interference among the APs.

B. Binary interference with partially overlapping channels

There are total eleven channels available and each channel is with 5MHz bandwidth. Recent reported measurement results reveal that spectrum leak in IEEE 802.11-based WLAN is quite severe [19]. Specifically, the transmit power mask of a channel with center frequency F_c is given by [18]:

$$P(f) = \begin{cases} 0\text{dB}, & |f - F_c| \leq 11\text{MHz} \\ -30\text{dB}, & 11\text{MHz} < |f - F_c| < 22\text{MHz} \\ -50\text{dB}, & |f - F_c| > 22\text{MHz} \end{cases} \quad (1)$$

It is noted that if the separation of the center frequencies of two channels is larger than 22MHz, there is nearly no mutual interference. Therefore, there are called non-overlapping channels. In other words, the separation of two orthogonal channels in terms of channel number, is at least 5. Thus, there are at most three non-overlapping channels in the IEEE 802.11-based networks.

Measurement results recently reported in [18] reveal that interference in IEEE 802.11-based WLAN are jointly determined by the following two factors: (i) the channel separation, and (ii) the physical distance. To describe clearly, we first analyze the interference model for a simple scenario involving only two nodes (APs) and then extend it to general scenarios involving multiple nodes. To study the effect of mutual interference on the throughput, the authors in [18] defined a throughput loss metric $\eta = \frac{s_1 + s_2}{s'_1 + s'_2}$, where s_1 and s_2 is the throughput of node 1 and node 2 respectively when the other node is inactive, and s'_1 and s'_2 is their throughput when both nodes are active. This metric reflects the throughput loss due to mutual interference. The relationship between η and their physical distance is measured. It was reported in [18] that it exhibits a binary property with respect to their physical distance. Specifically, η changes sharply from severe interference (around 0.5) to no interference (around 1) with a slight increase in their physical distance.

Suppose that two nodes choose channels a_1 and a_2 respectively, then the channel separation between them in terms of channel number is given by:

$$\delta = |a_1 - a_2|. \quad (2)$$

TABLE I
THE INTERFERENCE RANGE FOR DIFFERENT TRANSMITTING RATES.

Channel Separation (δ)	0	1	2	3	4	5
Interference Range (2Mb/s)	2R	1.125R	0.75R	0.375R	0.125R	0
Interference Range (5.5Mb/s)	2R	R	0.625R	0.375R	0.125R	0
Interference Range (11Mb/s)	2R	R	0.5R	0.345R	0.125R	0

For example, if channels 1 and 4 are chosen, we have $\delta = 3$. To capture the binary interference model, an interference range $R_I(\delta)$ in which η is around 0.5 can be defined for a specific channel separation distance δ . Specifically, as long as two nodes are located in the interference range of each other, their throughput loss ratio is almost 0.5 regardless of the change in physical distance; also, when they are located beyond the interference range, their throughput loss ratio is zero regardless of the change in physical distance.

Without loss of generality, denote the communication range as R . For a specific data rate, the relationship between the interference range $R_I(\delta)$ and the channel separation δ are summarized in Table I. Take the data rate 2Mb/s as an example: (i) when $\delta = 0$, i.e., the two nodes use the same channel, the interference range is two times the communication range, i.e., $R_I(0) = 2R$, (ii) when $\delta = 5$, i.e., they use orthogonal channels, the interference range is zero which means that there is no interference no matter what the distance is, and (iii) when the channel separation δ increases from 0 to 5, the interference range decreases monotonically. For other data rates, e.g., 5.5Mb/s and 11Mb/s, the same trend also holds with a slight difference in the values of interference ranges.

The binary feature of mutual interference between two nodes motivates us to propose a new metric to study the effect of mutual interference on the throughput. Specifically, the achieved normalized throughput of a node can be expressed as $T = \frac{1}{1+\alpha}$, where α is the following interference indicator:

$$\alpha = \begin{cases} 1, & d \leq R_I(\delta) \\ 0, & d > R_I(\delta) \end{cases} \quad (3)$$

where d is the distance between the two nodes. The above formulation can be explained as follows. On one hand, when the nodes are located in the interference range, they can hear each other even they are not using the same channel and hence only one user can transmit at a time. Thus, each of them achieves averagely one half throughput. On the other hand, when their physical distance is greater than the interference range, they can transmit simultaneously, which means that there is no throughput loss. It is noted that this formulation coincides well with the measured throughput [18].

We extend the binary interference model from two nodes to the scenarios of multiple nodes. Denote the physical distance between nodes k and n as d_{kn} , and their channel separation in terms of channel number as δ_{kn} . Then, the interference indicator α_{kn} between nodes k and n is given by:

$$\alpha_{kn} = \begin{cases} 1, & d_{kn} \leq R_I(\delta_{kn}) \\ 0, & d_{kn} > R_I(\delta_{kn}) \end{cases} \quad (4)$$

Based on this, the achieved normalized throughput of node k is determined by $T_k = \frac{1}{1+I_k}$, where $I_k = \sum_{n \neq k} \alpha_{nk}$ is the aggregate interference experienced by node k .

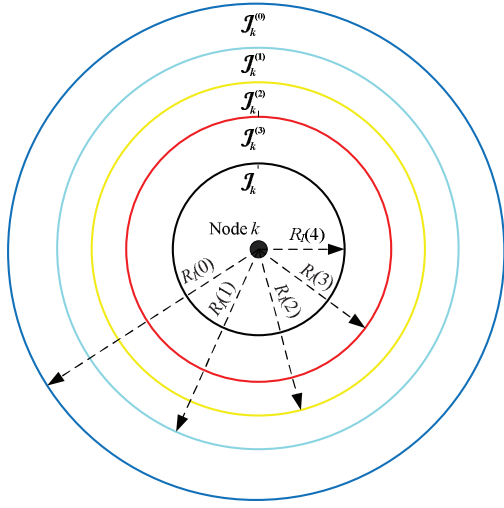


Fig. 1. An illustrative diagram of the defined five node sets.

The above analysis reveals the relationship between the aggregate interference and the achieved normalized throughput. Specifically, minimizing the aggregate interference can maximize the throughput. Thus, in the rest of this article, we will focus on the problem of channel selection for minimizing the aggregate interference in the network. In other words, the objective is to minimize the number of interfering nodes of each node. Note that some previous work also considered similar optimization objectives for orthogonal channel assignment/selection problems, e.g., [8], [10], [35]. In particular, it has been shown that the achievable rate is approximately linear with the number of interfering users in some conditions (see equation (4) in [35]).

C. Problem formulation for channel selection with partially overlapping channels

Denote the node set as \mathcal{K} , i.e., $\mathcal{K} = \{1, 2, \dots, K\}$. For a specific node, say node k , the number of interfering nodes is determined by the channel selection profiles of its nearby nodes. For easy analysis, we define the node set of node k in the interference range of channel separation $\delta = 4$ as $\mathcal{J}_k^{(4)}$, i.e., $\mathcal{J}_k^{(4)} = \{j \in \mathcal{K} : d_{jk} \leq R_I(4)\}$, where $R_I(4)$ is given in Table I. Furthermore, we define the node set of node k in the range of $R_I(3)$ but beyond the range of $R_I(4)$ as $\mathcal{J}_k^{(3)} = \{j \in \mathcal{K} : R_I(4) < d_{jk} \leq R_I(3)\}$. Similarly, $\mathcal{J}_k^{(2)} = \{j \in \mathcal{K} : R_I(3) < d_{jk} \leq R_I(2)\}$, $\mathcal{J}_k^{(1)} = \{j \in \mathcal{K} : R_I(2) < d_{jk} \leq R_I(1)\}$ and $\mathcal{J}_k^{(0)} = \{j \in \mathcal{K} : R_I(1) < d_{jk} \leq R_I(0)\}$. It is noted that $\mathcal{J}_k = \mathcal{J}_k^{(4)} \cup \mathcal{J}_k^{(3)} \cup \mathcal{J}_k^{(2)} \cup \mathcal{J}_k^{(1)} \cup \mathcal{J}_k^{(0)}$ represents the node set that potentially interferes with node k . The classification of the five node sets is determined by the network topology and is irrespective of the chosen channels of the nodes. An illustrative diagram of the five node sets is shown in Fig. 1.

Denote the channel set as \mathcal{M} , i.e., $\mathcal{M} = \{1, 2, \dots, M\}$. Assume that node k chooses channel $a_k \in \mathcal{M}$ for its intra-communication. Denote the channel separation in terms of channel number between nodes k and j as $\delta(a_k, a_j)$. Denote $s_k^{(i)}$ as the number of interfering nodes in the set $\mathcal{J}_k^{(i)}$,

$\forall i = 0, 1, \dots, 4$, which is calculated as:

$$s_k^{(i)} = \sum_{j \in \mathcal{J}_k^{(i)}} \sigma_{kj}^{(i)}, \quad (5)$$

where $\sigma_{kj}^{(i)}$ is defined as:

$$\sigma_{kj}^{(i)} = \begin{cases} 1, & \delta(a_k, a_j) \leq i \\ 0, & \text{otherwise} \end{cases} \quad (6)$$

That is, if the channel separation between nodes k and j is not greater than i , $\sigma_{kj}^{(i)}$ takes one, and zero otherwise. This is obtained by the binary interference model. Then, under a channel selection profile (a_1, \dots, a_K) , the achieved normalized throughput of node k is given by:

$$T_k(a_1, \dots, a_K) = \frac{1}{1 + s_k}, \quad (7)$$

where s_k is the total number of interfering nodes in \mathcal{J}_k , i.e.,

$$s_k = \sum_{i=0}^4 \sum_{j \in \mathcal{J}_k^{(i)}} \sigma_{kj}^{(i)} = \sum_{i=0}^4 s_k^{(i)}. \quad (8)$$

Motivated by previous studies addressing the minimization of the number of interfering nodes in wireless networks with orthogonal channels, e.g., [8], [10], [35], we formulated a partially overlapping channel selection problem for aggregate interference minimization. Specifically, the objective is to achieve the optimal channel selection profile such that the aggregate interference in the network is minimized, i.e.,

$$\mathbf{P1:} \quad \min \sum_{k \in \mathcal{K}} s_k \quad (9)$$

Remark 1: The used optimization objective, as given in (9), can be regarded as the aggregate interference level in the network. There are two reasons for using such an optimization objective. From the user side, minimizing the number of interfering nodes of node k , i.e., s_k , is equivalent to maximizing its achieved normalized throughput, as specified by (7). Also, from the network side, minimizing the aggregate interference would maximize the network throughput. However, we note that a rigorous connection between the aggregate interference and the network throughput can not be obtained since it is topology-dependent.

IV. GRAPHICAL GAME MODEL AND UNCOUPLED LEARNING ALGORITHM

A. Graphical game model

Since the players choose the channels distributively and autonomously, we can formulate the channel selection problem as a non-cooperative game. Specifically, the game is denoted as $\mathcal{G} = \{\mathcal{K}, \mathcal{A}_k, \mathcal{J}_k, u_k\}$, where \mathcal{K} is the player (node) set, \mathcal{A}_k is the action space of player k , \mathcal{J}_k characterizes the topology with regard to the potential interfering players of player k , and u_k is the utility function of player k . The action space of all the players is exactly the channel set, i.e., $\mathcal{A}_k \equiv \mathcal{M}$, $\forall k \in \mathcal{K}$. Generally, the utility function in a game is denoted as $u_k(a_k, a_{-k})$, where a_k is the action of player k and a_{-k} is the chosen action profile of all the players excluding player k . It is noted that in the considered network, the utility function of

any given player k is only affected by its action and the action profile of the nodes in \mathcal{J}_k . Thus, the utility function of player k in the game can be expressed as $u_k(a_k, a_{\mathcal{J}_k})$, where $a_{\mathcal{J}_k}$ is the action profile of the players in \mathcal{J}_k . This kind of game models is called local interactive game [8] or graphical game [36], which has began to draw attention in wireless communications recently. In the proposed channel selection game, we define the utility function as:

$$u_k(a_k, a_{\mathcal{J}_k}) = -s_k, \quad (10)$$

where s_k is the number of interfering nodes of node k as defined in (8). Then, the proposed channel selection game with partially overlapping channels is expressed as:

$$\mathcal{G} : \quad \max u_k(a_k, a_{\mathcal{J}_k}), \forall k \in \mathcal{K} \quad (11)$$

B. Analysis of Nash equilibrium (NE) of the game

In this subsection, we present the concept of Nash equilibrium (NE), which is the most well-known stable solution in game models, and analyze its properties in terms of existence and performance. A channel selection profile $a_{NE} = (a_1^*, \dots, a_N^*)$ is a pure strategy NE if and only if no player can improve its utility function by deviating unilaterally [37], i.e.,

$$u_k(a_k^*, a_{\mathcal{J}_k}^*) \geq u_k(a_k, a_{\mathcal{J}_k}^*), \forall k \in \mathcal{K}, \forall a_k \in \mathcal{A}_k \quad (12)$$

It is seen from (4) that the interference between two nodes is symmetrical. Such a channel model lends a potential game formulation for a globally interactive game, as shown in [38]. However, we again emphasize that the formulated game is a graphical game, which eventually requires some new technologies to analyze its properties.

Theorem 1. *The channel selection game \mathcal{G} is an exact potential game which has at least one pure strategy NE point.*

Proof: To prove this statement, we need to prove that there exists a potential function such that the change in the individual utility function caused by any player's unilateral deviation is the same as that in the potential function [39]. Specifically, the potential function is defined as:

$$\Phi(a_1, \dots, a_K) = -\frac{1}{2} \sum_{k \in \mathcal{K}} s_k, \quad (13)$$

which is exactly the negative value of the half aggregate interference, as shown in (9).

For easy analysis, we define the set of players that cause interference to player k in $\mathcal{J}_k^{(i)}$ as follows:

$$\mathcal{I}_k^{(i)}(a_k, a_{\mathcal{J}_k}) = \{j \in \mathcal{J}_k^{(i)} : \sigma_{kj}^{(i)} = 1\}, i = 0, 1, \dots, 4, \quad (16)$$

which immediately implies the following equation:

$$|\mathcal{I}_k^{(i)}(a_k, a_{\mathcal{J}_k})| = s_k^{(i)}, i = 0, 1, \dots, 4. \quad (17)$$

That is, the total number of players in $\mathcal{I}_k^{(i)}(a_k, a_{\mathcal{J}_k})$ is exactly the interference experienced by player k in $\mathcal{J}_k^{(i)}$.

Now, suppose that player k unilaterally changes its action from a_k to a'_k while all other players keep their actions unchanged. Also, the set of player that cause interference to player k after its unilateral action change is denoted as

$\mathcal{I}_k^{(i)}(a'_k, a_{\mathcal{J}_k})$. Denote \bar{A} as the complementary set of A . For every $i = 0, 1, \dots, 4$, we define $\mathcal{J}_k^{(i)}$ as the full set and divide the players in $\mathcal{J}_k^{(i)}$ into the following four sets: (i) $\mathcal{I}_k^{(i)}(a_k, a_{\mathcal{J}_k}) \cap \bar{\mathcal{I}}_k^{(i)}(a'_k, a_{\mathcal{J}_k})$, (ii) $\bar{\mathcal{I}}_k^{(i)}(a_k, a_{\mathcal{J}_k}) \cap \mathcal{I}_k^{(i)}(a'_k, a_{\mathcal{J}_k})$, (iii) $\mathcal{I}_k^{(i)}(a_k, a_{\mathcal{J}_k}) \cap \mathcal{I}_k^{(i)}(a'_k, a_{\mathcal{J}_k})$, and (iv) $\bar{\mathcal{I}}_k^{(i)}(a_k, a_{\mathcal{J}_k}) \cap \bar{\mathcal{I}}_k^{(i)}(a'_k, a_{\mathcal{J}_k})$. On explanation, the players in the first set only interfere with player k before it unilaterally changes action. The players in the second set only interfere with player k after the unilateral action change. The players in the third set interfere with player k both before and after the unilateral action change, while the players in the fourth set interfere with player k neither before nor after the unilateral action change. It is noted that the four divided player sets are exclusive and complementary.

Based on the above classification and analysis, the change in the utility function of player k can be expressed by (14) and the change in the potential function caused by the unilateral action changing of player k is given by (15), which are presented in the top of next page. In deducing (15), we use the following results:

$$X_j = \begin{cases} 1, & \forall j \in \mathcal{I}_k^{(i)}(a_k, a_{\mathcal{J}_k}) \cap \bar{\mathcal{I}}_k^{(i)}(a'_k, a_{\mathcal{J}_k}) \\ -1, & \forall j \in \bar{\mathcal{I}}_k^{(i)}(a_k, a_{\mathcal{J}_k}) \cap \mathcal{I}_k^{(i)}(a'_k, a_{\mathcal{J}_k}) \\ 0, & \forall j \in \mathcal{I}_k^{(i)}(a_k, a_{\mathcal{J}_k}) \cap \mathcal{I}_k^{(i)}(a'_k, a_{\mathcal{J}_k}) \\ 0, & \forall j \in \bar{\mathcal{I}}_k^{(i)}(a_k, a_{\mathcal{J}_k}) \cap \bar{\mathcal{I}}_k^{(i)}(a'_k, a_{\mathcal{J}_k}) \end{cases} \quad (18)$$

which is obtained from the definitions of the four player sets.

Equations (14) and (15) show that the change in the utility function of player k is the same as that in the potential function. Thus, according to the definition given in [39], it is known that the proposed channel selection game is an exact potential game with Φ serving as the potential function. Exact potential game is a special kind of game models since it admits several promising features, among which the most important feature is that every exact potential game has at least one pure strategy NE point. Thus, Theorem 1 is proved. ■

To characterize the achievable performance of the proposed interference minimization game, the aggregate interference of a pure strategy NE point $a_{NE} = \{a_1^*, \dots, a_N^*\}$ is given by:

$$U(a_{NE}) = \sum_{n \in \mathcal{K}} s_k(a_n^*, a_{\mathcal{J}_k}^*), \quad (19)$$

where s_k is calculated by (8).

The players in the game selfishly maximize their individual utility functions, as specified by (11). This may lead to inefficiency and dilemma, which is known as *tragedy of commons* [40]. Although Theorem 1 states that this game has at least one pure NE point, the total number of NE points of the game is generally hard to obtain [37]. Furthermore, the task of analyzing the achievable performance of NE points of a general exact potential game is interesting but challenging. By exploiting the inherent structure of the proposed game, we characterize the achievable performance bounds of NE points of the proposed game in the following.

Theorem 2. *The global minimum of the aggregate interference of the network constitutes a pure strategy NE point of \mathcal{G} .*

$$\Delta u_k = u_k(a_k, a_{\mathcal{J}_k}) - u_k(a'_k, a_{\mathcal{J}_k}) = \sum_{i=0}^4 (|\mathcal{I}_k^{(i)}(a_k, a_{\mathcal{J}_k}) \cap \bar{\mathcal{I}}_k^{(i)}(a'_k, a_{\mathcal{J}_k})| - |\bar{\mathcal{I}}_k^{(i)}(a_k, a_{\mathcal{J}_k}) \cap \mathcal{I}_k^{(i)}(a'_k, a_{\mathcal{J}_k})|) \quad (14)$$

$$\begin{aligned} \Delta \Phi &= \Phi(a_k, a_{-k}) - \Phi(\bar{a}_k, a_{-k}) \\ &= \frac{1}{2} \sum_{i=0}^4 (|\mathcal{I}_k^{(i)}(a_k, a_{\mathcal{J}_k}) \cap \bar{\mathcal{I}}_k^{(i)}(a'_k, a_{\mathcal{J}_k})| - |\bar{\mathcal{I}}_k^{(i)}(a_k, a_{\mathcal{J}_k}) \cap \mathcal{I}_k^{(i)}(a'_k, a_{\mathcal{J}_k})|) \\ &\quad + \frac{1}{2} \sum_{i=0}^4 \left\{ \sum_{j \in \mathcal{I}_k^{(i)}(a_k, a_{\mathcal{J}_k}) \cap \bar{\mathcal{I}}_k^{(i)}(a'_k, a_{\mathcal{J}_k})} \underbrace{\left(\sum_{n=0}^4 |\mathcal{I}_j^{(n)}(a_j, a_k, a_{\mathcal{J}_j \setminus k})| - \sum_{n=0}^4 |\mathcal{I}_j^{(n)}(a_j, a'_k, a_{\mathcal{J}_j \setminus k})| \right)}_{X_j} \right\} \\ &\quad + \sum_{j \in \bar{\mathcal{I}}_k^{(i)}(a_k, a_{\mathcal{J}_k}) \cap \mathcal{I}_k^{(i)}(a'_k, a_{\mathcal{J}_k})} X_j + \sum_{j \in \mathcal{I}_k^{(i)}(a_k, a_{\mathcal{J}_k}) \cap \bar{\mathcal{I}}_k^{(i)}(a'_k, a_{\mathcal{J}_k})} X_j + \sum_{j \in \bar{\mathcal{I}}_k^{(i)}(a_k, a_{\mathcal{J}_k}) \cap \bar{\mathcal{I}}_k^{(i)}(a'_k, a_{\mathcal{J}_k})} X_j \Big\} \\ &= \sum_{i=0}^4 (|\mathcal{I}_k^{(i)}(a_k, a_{\mathcal{J}_k}) \cap \bar{\mathcal{I}}_k^{(i)}(a'_k, a_{\mathcal{J}_k})| - |\bar{\mathcal{I}}_k^{(i)}(a_k, a_{\mathcal{J}_k}) \cap \mathcal{I}_k^{(i)}(a'_k, a_{\mathcal{J}_k})|) \end{aligned} \quad (15)$$

Proof: Denote a_{opt} as an optimal channel selection profile that maximizes the potential function, i.e.,

$$a_{opt} \in \arg \max_a \Phi(a_1, \dots, a_K). \quad (20)$$

According to the relationship between the potential function and the system optimization objective, as specified by (13), it is known that a_{opt} also minimizes the aggregate interference in the network, i.e.,

$$a_{opt} \in \arg \min_a \sum_{k \in \mathcal{K}} s_k \quad (21)$$

For any exact potential game, a maximizer of the potential function constitutes a pure strategy NE point [39]. Therefore, a_{opt} serves as the optimal pure strategy NE of the game. ■

The above theorem characterizes the best performance of the proposed graphical game. However, as it may have some pure strategy NE points that are not optimal, it is also important to characterize the worst performance of the game. Specifically, the following theorem characterizes the upper bound of aggregate interference at any NE solution.

Theorem 3. *For any network topology, the achieved aggregate interference is upper bounded by $U(a_{NE}) < \sum_{k \in \mathcal{K}} ((\sum_{i=0}^4 (2i+1)|\mathcal{J}_k^{(i)}|)/M)$, where $\mathcal{J}_k^{(i)}$ is the defined interfering user set with channel separation i and M is the number of available channels.*

Proof: For a pure strategy NE $a_{NE} = \{a_1^*, \dots, a_K^*\}$, the following equation holds:

$$s_k(a_k^*, a_{\mathcal{J}_k}^*) \leq s_k(a_k, a_{\mathcal{J}_k}^*), \forall a_k \in \mathcal{A}_k, \forall k \in \mathcal{K}, \quad (22)$$

which can be directly obtained by the definition of NE as specified by (12). By summarizing the two-sides of (22) over \mathcal{A}_k , we have:

$$|\mathcal{A}_k| s_k(a_k^*, a_{\mathcal{J}_k}^*) \leq \sum_{a_k \in \mathcal{A}_k} s_k(a_k, a_{\mathcal{J}_k}^*), \quad (23)$$

The right-hand side of this equation can be regarded as the virtual aggregate interference experienced by player k if it would transmit on all the channels simultaneously while

all other players keep their channel selections unchanged. To calculate its value, we re-write it as follows:

$$\begin{aligned} \sum_{a_k \in \mathcal{A}_k} s_k(a_k, a_{\mathcal{J}_k}^*) &= \sum_{a_k \in \mathcal{A}_k} \sum_{i=0}^4 s_k^{(i)}(a_k, a_{\mathcal{J}_k}^*) \\ &= \sum_{i=0}^4 \sum_{a_k \in \mathcal{A}_k} s_k^{(i)}(a_k, a_{\mathcal{J}_k}^*), \end{aligned} \quad (24)$$

which can be calculated according to the following cases:

- All players in $\mathcal{J}_k^{(0)}$ cause co-channel interference to player k , which implies that

$$\sum_{a_k \in \mathcal{A}_k} s_k^{(0)}(a_k, a_{\mathcal{J}_k}^*) = |\mathcal{J}_k^{(0)}| \quad (25)$$

- In $\mathcal{J}_k^{(i)}$, $i = 1, 2, 3, 4$, players surely cause co-channel interference to player k ; furthermore, they also cause partially overlapping channel interference to player k . According to the interference model defined in (6), we have:

$$\sum_{a_k \in \mathcal{A}_k} s_k^{(i)}(a_k, a_{\mathcal{J}_k}^*) < (2i+1)|\mathcal{J}_k^{(i)}|, i = 1, 2, 3, 4 \quad (26)$$

Therefore, the following inequality holds:

$$\sum_{a_k \in \mathcal{A}_k} s_k(a_k, a_{\mathcal{J}_k}^*) < \sum_{i=0}^4 (2i+1)|\mathcal{J}_k^{(i)}|. \quad (27)$$

Accordingly, it follows that:

$$U(a_{NE}) = \sum_{n \in \mathcal{K}} s_n(a_n^*, a_{\mathcal{J}_n}^*) < \sum_{k \in \mathcal{K}} \left(\frac{\sum_{i=0}^4 (2i+1)|\mathcal{J}_k^{(i)}|}{M} \right), \quad (28)$$

which proves Theorem 3. ■

Intuitively, Theorem 3 shows that increasing the total number of channels, i.e., M , can lower the aggregate interference in the network, which enhances the network throughput. This is exactly the inherent idea of using partially overlapping channels. In addition, the upper bound presented in this theorem is relatively loose since the network topology is general. In fact, as shown later, the performance of the game is quite good.

C. Simultaneous log-linear learning algorithm with heterogeneous rates

There are a large number of learning algorithms for potential games in the literature, e.g., the best response [17], fictitious play [20], spatial adaptive play [8] and regret learning [5]. However, these algorithms are not practical for distributed wireless networks since they need to know the actions of other players in past plays of the game. In other words, these algorithms are coupled. To overcome the constraint of requiring information about other players, some uncoupled algorithms that are only relying on the individual information of a user, e.g., Q-learning [25], stochastic learning automata [21], [44] and MAX-logit algorithm [30], have begun to draw attention recently in wireless communication community. However, these algorithms may only converge to a suboptimal solution or only one player is allowed to update its action at a time. Thus, a simultaneous and uncoupled algorithm that can achieve the optimal solution is desirable.

To coincide with the distributed nature of the considered wireless network, we propose a simultaneous log-learning algorithm in which the players simultaneously update their actions based on the individual received payoffs after each play. The proposed algorithm is described in Algorithm 1. Specifically, an iteration of the algorithm consists of multiple slots with equal length. In every slot, a perfect carrier sense multiple access (CSMA) mechanism is applied at each player. The dynamics of the proposed learning algorithm can be explained as follows. The players explore new actions with probability δ_k , as given in (29). After the exploration, they update their selections according to the log-linear learning strategy over the selections of the last two iterations, as shown in (30). The proposed algorithm does not need information about other players and is only relying on the individual information of each player. In other words, this algorithm is simultaneous, fully distributed and uncoupled.

1) *Estimate the utility function in each iteration:* In the update rule specified by (30), the player needs to know its received utility functions in the last two iterations. Suppose that each iteration of the proposed algorithm consists of N slots, and denote $N_k(i)$ as the number of slots in which player k successfully captures the channel in the i th iteration. Then, the throughput of player k can be expressed as $T_k = \frac{N_k(i)}{N} = \frac{1}{1+s_k(i)}$, which implies that the aggregate interference experienced by player k can be estimated as $\hat{s}_k(i) = \frac{N}{N_k(i)} - 1$. Thus, the received utility in each iteration in the algorithm can be estimated by:

$$\hat{u}_k(i) = 1 - \frac{N}{N_k(i)}, \quad (31)$$

The proposed simultaneous log-linear learning algorithm is motivated by the spatial adaptive play (SAP) [8], [42], [43], which is a coupled learning algorithm for game models. In addition, there is an improved algorithm called binary SAP [41] (also called B-logit in [30]), in which no information exchange is needed. However, only one player is allowed to perform learning in this algorithm, which means that some coordination mechanisms are needed to schedule the learning. In fact, the proposed algorithm can be regarded as a simultaneous version of the binary SAP. It is shown that

Algorithm 1: simultaneous log-linear learning algorithm

Initialization: Set the iteration index $i = 0$, let each player k , $\forall k \in \mathcal{K}$, randomly select a channel $a_k(0) \in \mathcal{A}_k$, and set the binary flag $x_k(0) = 0$, $\forall k \in \mathcal{K}$.

All players simultaneously execute the following procedure:

Loop for $i = 1, \dots$,

Exploration:

If $x_k(i-1) = 0$, player k updates its selection according to the following rule:

$$\Pr[a_k(i) = a] = \begin{cases} \frac{\delta_k}{|\mathcal{A}_k|-1}, & a \in \{\mathcal{A}_k \setminus a_k(i-1)\} \\ 1 - \delta_k, & a = a_k(i-1), \end{cases} \quad (29)$$

where $|\mathcal{A}_k|$ represents the cardinality of the set \mathcal{A}_k , i.e., the number of available channels in the network, and δ_k can be regarded as the exploration rate of player k . Furthermore, set $x_k(i) = 1$ if $a_k(i) \neq a_k(i-1)$, and $x_k(i) = 0$ otherwise.

End if

Update:

If $x_k(i-1) = 1$, player k updates the selection according to the following rule:

$$\begin{aligned} \Pr[a_k(i) = a_k(i-1)] &= \frac{\exp\{\hat{u}_k(i-1)\beta\}}{X} \\ \Pr[a_k(i) = a_k(i-2)] &= \frac{\exp\{\hat{u}_k(i-2)\beta\}}{X}, \end{aligned} \quad (30)$$

where β is the learning parameter, $\hat{u}_k(i-1)$ and $\hat{u}_k(i-2)$ represent the received utility function of the player k in iterations $i-1$ and $i-2$ respectively, as specified by (31), and $X = \exp\{\hat{u}_k(i-1)\beta\} + \exp\{\hat{u}_k(i-2)\beta\}$. Furthermore, set $x_k(i) = 0$.

End if

End loop

the (binary) SAP converges to the set of the global maximizer of the potential function with arbitrarily high probability [41]. Also, the proposed algorithm can be regarded as a variant of the payoff-based learning algorithm given in [28]. The key difference is that all players choose the same exploration rate in [28] while heterogeneous exploration rates are used in the proposed algorithm. The reason for setting heterogeneous rates is to accelerate the learning speed by exploiting the feature that the players are spatially located. The following theorem characterizes the convergence and optimality of the proposed simultaneous log-linear learning algorithm.

Theorem 4. *If the exploration parameters are chosen as $\delta_k = \exp(-\beta m_k)$, $\forall k \in \mathcal{K}$, the proposed algorithm asymptotically converges to the optimal channel selection profile that minimizes the aggregate interference for sufficiently large m_k .*

Proof: Refer to Appendix A. ■

The proposed learning algorithm can be intuitively explained as follows. At the beginning phase, the players occa-

sionally explore new selections with the expectation of finding a better selection. After the exploration, a log-linear strategy is applied to update its selection. The log-linear strategy is also refereed as Boltzmann exploration strategy [45], where players choose actions of higher utilities with greater probabilities than those of lower utilities. Most importantly, it is known that Boltzmann exploration strategy is an efficient way to escape from local optimal points and finally achieves the global optimum. To guarantee convergence, the probability of exploration should decrease as the algorithm iterates. Eventually, when all players stop exploration, the system evolves into a stable state. Thus, the value of the learning parameter m_k is advisable to be set to small values at the beginning phase which keeps increasing as the algorithm iterates. The simplest form of the linear strategy is $m(i) = m_0 + i\Delta m$, where m_0 is the initial value, Δm is the step size, and i is the iteration number.

2) *Discussion on the heterogeneous learning rates:* It is noted that the payoff-based learning algorithm given in [28] is designed for globally interactive games where an action of a player affects all other players. However, we found that exploiting the feature of the spatially locations of the players would accelerate the learning speed, which motivates us to set the heterogeneous exploration rates for the users. The number of potentially interfering users of player k is given by $D_k = |\mathcal{J}_k|$, where \mathcal{J}_k is defined in Section III.C. Specifically, the players with less value of D_k are advisable to have larger exploration rates while those with large value of D_k are advisable to have smaller rates. The reason is as follows: the actions of the players with large value of D_k have more impact on the system, and their action changes will lead to more perturbations which finally results in lower learning speed. Thus, we set the heterogeneous exploration rates for player k as $m_k(i) = \frac{\max\{D_1, \dots, D_k\}}{D_k} m(i)$. It will be shown later that the convergence speed of the proposed algorithm with heterogeneous exploration rates is much faster than that of the original algorithm with homogeneous rates.

V. SIMULATION RESULTS AND DISCUSSION

A. Scenario setup

In all simulations, the IEEE 802.11b with 2Mb/s data rate is applied. All the nodes are located in a 1000m×1000m square area, and the interference range of co-channel communications is set to 200m [13], i.e., $2R = 200$ m. Thus, the interference ranges of different channel separations are given by $R_I(1) = 112.5$ m, $R_I(2) = 75$ m, $R_I(3) = 37.5$ m and $R_I(4) = 12.5$ m, which are determined by the ratios shown in Table I. In the simulation study, we first present the convergence behaviors of the proposed simultaneous log-linear learning algorithm, and then investigate the throughput performance.

To make it more general, we consider two kinds of networks. The first is the random topology, in which the nodes are randomly located in the square. The second is the grid topology, in which the nodes are located in the junction points of a grid. We investigate the impact of the node density on the performance of the proposed approach. To achieve this, we can increase the number of nodes arbitrarily in the random topology. For the grid topology, we increase the number of nodes in a square rule, i.e., the number of nodes is determined

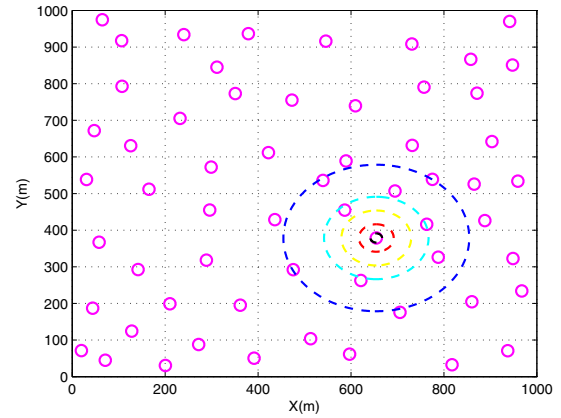


Fig. 2. An example of simulated random topology with 60 nodes. The small solid circles represent the nodes while the large dashed circles represent the interference regions of different channel separations.

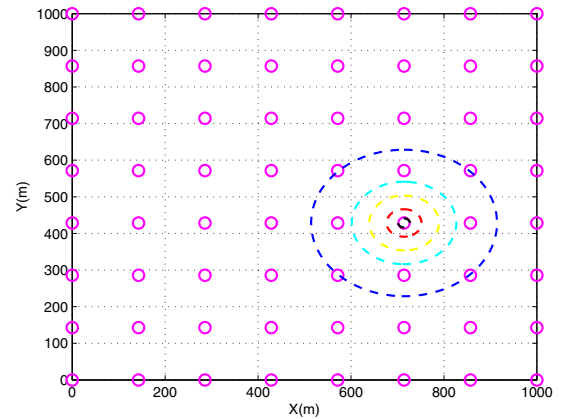


Fig. 3. An example of simulated grid topology with 64 nodes. The small solid circles represent the nodes while the large dashed circles represent the interference regions of different channel separations.

by $K = l^2$, where l is a natural number. Examples of the simulated random and grid topologies are given by Fig. 2 and Fig. 3 respectively.

To evaluate the throughput improvement of partially overlapping channels (POC) over non-overlapping channels (NOC), we compare the throughput performance of the proposed POC approach with that of an existing optimal NOC approach which employed the spatial adaptive play (SAP) [8]. In that approach, the channel selection problem can also be formulated as a potential game with the aggregate interference serving as the potential function. In fact, the NOC selection problem can be regarded as a simplified version of the POC selection problem and potential-game theoretic analysis for NOC can be found in the referred work.

In addition, since the formulated POC selection problem is an exact potential game with its optimal NE points minimizing the aggregate interference, as characterized by Theorems 1 and 2, some other coupled and uncoupled algorithms that converge to the pure strategy NE points can also be used as referred algorithms. For comparison, we also evaluate the performance of the SAP algorithm [8], the MAX-logit algorithm [30], the B-logit algorithm [41] and the multi-agent Q-learning algo-

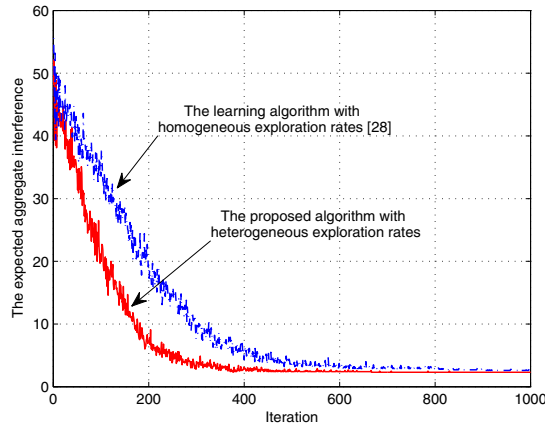


Fig. 4. The comparison results of the convergence speed of the proposed learning algorithm with heterogeneous exploration rates and that with homogeneous rates.

rithm¹ [25]. In classification, SAP is coupled and sequential (only exactly one player performs learning at a time), both MAX-logit and B-logit are uncoupled and sequential, and both the proposed and Q-learning are uncoupled and simultaneous (multiple players perform learning at a time).

B. Convergence behavior

In this subsection, we present the convergence behaviors of the proposed simultaneous log-linear learning algorithm. The parameters in the learning algorithm are set to $\beta = -8$ and $m = 0.1 + 0.0095i$, where i is the iteration number. These parameters have been optimized by experiments.

The convergence behaviors for two learning algorithms with homogeneous and heterogeneous exploration rates are shown in Fig. 4. These results are obtained by taking the expected value of 20 independent trials. It is noted that the proposed learning algorithm converges in about 400 iterations while the algorithm with homogeneous rates converges in about 600 iterations. Also, it is noted from the figure that, the aggregate interference of both algorithms decreases gradually as they iterate. These results validate the convergence of the proposed simultaneous learning algorithm and the fast convergence speed caused by the heterogeneous exploration rates.

C. Throughput performance

In this subsection, we evaluate the achieved performance of the proposed uncoupled learning algorithm using POC. In particular, we increase the node density by adding more nodes in the square. Then, we compare the achieved throughput of POC with that of NOC. Also, we compare the proposed uncoupled learning algorithm with other coupled and uncoupled algorithms.

1) *Random topology*: The comparison results of the expected network throughput of random topologies are shown in Fig. 5. The number of nodes increases from 40 to

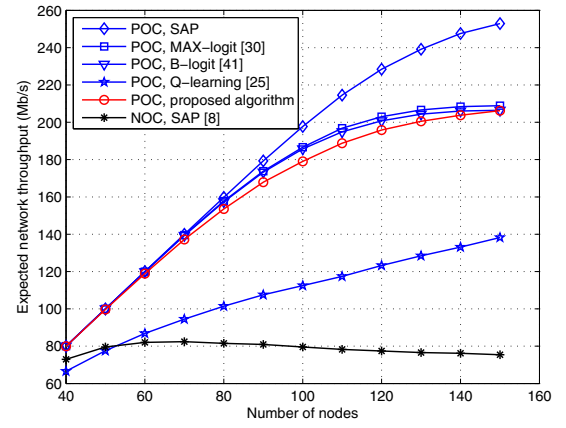


Fig. 5. The comparison results of the expected network throughput of random topologies.

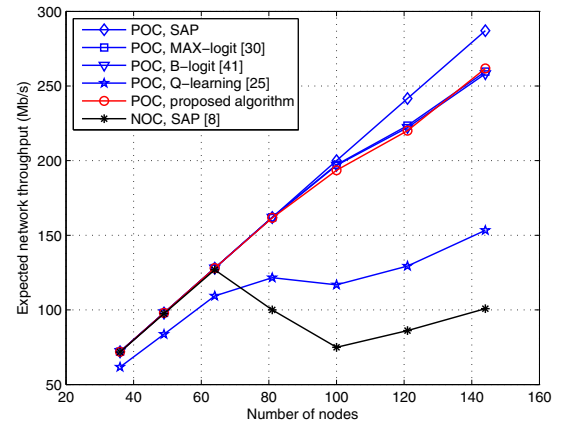


Fig. 6. The comparison results of the expected network throughput of grid topologies.

150. The results are obtained by independently simulating 500 topologies and then taking the expected values.

It is noted from Fig. 5 that the expected network throughput of POC is significantly larger than that of NOC. The expected network throughput of NOC is around 80 Mb/s with slight fluctuation. Specifically, it increases slightly when the number of nodes increases from 40 to 60, while it decreases slightly when the number of nodes increases from 60 to 160. On the contrary, the expected throughput of POC increases significantly and consistently as the number of nodes increase. The results validate that the throughput improvement of POC over NOC is significant.

Also, it is noted from the figure that the proposed algorithm achieves satisfactory performance when compared with other existing learning algorithms. In particular, for relatively sparse networks, e.g., $K \leq 80$, the proposed uncoupled algorithm achieves almost the same performance with that of SAP. As the node density becomes large, the performance of the proposed uncoupled algorithm is slightly worse than that of SAP. For existing uncoupled algorithms, it is noted that the throughput performance of the proposed algorithm is close to those of MAX-logit and B-logit, which are uncoupled with single learning player at a time. Furthermore, it is noted that the proposed algorithm achieves much higher throughput than that of Q-learning, which is also uncoupled and synchronous.

¹For the referred Q-learning algorithm, we found that it never converges when the original algorithm presented in [25] is directly applied. To achieve its convergence, we modify the original Q-learning algorithm slightly by setting the learning parameter as $\gamma = \frac{1}{5+0.045i}$, where i is the iteration number.

2) *Grid topology*: The comparison results of the expected aggregate interference and the expected throughput of grid topologies are shown in Fig. 6. The number of nodes increases from 36 to 169. The results are obtained by independently running 500 trials and then taking the expected values.

It is noted that the expected interference of POC is significantly less than that of NOC and the expected network throughput of POC is much higher than that of NOC, which exhibits the same trend as that in random topologies. Specifically, the expected network throughput of POC outperforms NOC significantly, especially when the number of nodes is large. For scenarios with small number of nodes, e.g., $K = 36, 49, 64, 81$, the throughput gap is trivial. The reason is that the interference in the grid topologies is light when the number of nodes is smaller due to the inherent structure. Thus, the gap of the expected network throughput of POC and NOC is small.

An interesting result in Fig. 6 is that the expected network of NOC exhibits a singular trend. Specifically, it increases when the number of nodes, i.e., K , increases from 36 to 64, decreases when K increases from 64 to 100, and again increases when K increases from 100 to 144. This might be caused by the grid network topologies. Since the nodes in the grid networks are fixed, some scales of the networks may lead to singular result. For small K , the interference in the network is light and the non-overlapping channels is enough; thus, the expected network throughput increases with K . However, in some scales of networks, e.g., $K = 100$, the topologies lead to some singular results and the exact reasons are hard to analyze.

It is also noted that the proposed learning algorithm achieves satisfactory performance when compared with existing coupled or uncoupled algorithms. which exhibits the same trend as that in random topologies

VI. CONCLUSION

We investigated the problem of distributed channel selection in opportunistic spectrum access (OSA) networks with partially overlapping channels (POC) using a game-theoretic uncoupled learning algorithm. The motivation is that compared with traditional non-overlapping channels (NOC), POC can increase the full-range spectrum utilization, alleviate interference and improve the network throughput. However, most existing POC approaches are centralized, which are not suitable for distributed OSA networks. We formulated the POC selection problem as an interference mitigation game, in which the utility function of each user is defined as the negative value of its experienced interference. We proved that the game has at least one pure strategy NE point and the best pure strategy NE point minimizes the aggregate interference in the network. We proposed a simultaneous and uncoupled learning algorithm to achieve the pure strategy NE points of the game. Simulation results show that the throughput improvement gain of the proposed POC approach over traditional NOC approach is significant. Also, the proposed uncoupled algorithm achieves satisfactory performance when compared with existing coupled and uncoupled algorithms.

However, there is an important issue that is still open and should be further investigated in future. Specifically, although

the binary interference model provides with a simple and well approximated approach to study the mutual interference, it is noted that only physical distance is considered. Thus, it would be more precise and interesting to take into account the effect of random channel fading on the interference model. This may be solved by extending the binary model to a real-valued one.

APPENDIX A PROOF OF THEOREM 4

Denote the system state at the i th iteration as $z(i) = [a(i-1), a(i), x(i)]$, where $a(i)$ is the channel selection profile of all the users at the i th iteration, i.e., $a(i) = \{a_1(i), \dots, a_K(i)\}$, and $x(i)$ is the vector of the flag variables used in the algorithm, i.e., $x(i) = \{x_1(i), \dots, x_K(i)\}$. According to the procedure of the proposed algorithm, the sequence $z(i)$ is a Markovian process. Since the precise stationary distribution of $z(i)$ is hard to obtain, we study its stochastic stable state using the theory of resistance trees [28], [46].

A. Brief summary of resistance trees

In this subsection, we present a brief summary of the key theoretic results of resistance trees². Denote P^0 as the probability transition matrix of a finite state Markovian chain, which is referred to as the “unperturbed” process. Consider a “perturbed” process in which the size of perturbations can be characterized by a scalar $\varepsilon > 0$, and denote the associated transition probability matrix as P^ε . We call the process P^ε a *regular perturbed Markovian process* if i) P^ε is ergodic for all sufficiently small value of ε and ii) P^ε approaches P^0 at an exponential rate. Mathematically, the above conditions for two arbitrary states z_1 and z_2 can be expressed as:

$$\lim_{\varepsilon \rightarrow 0^+} P^\varepsilon_{z_1 \rightarrow z_2} = P^0_{z_1 \rightarrow z_2}, \quad (\text{A.1})$$

and

$$0 < \lim_{\varepsilon \rightarrow 0^+} \frac{P^\varepsilon_{z_1 \rightarrow z_2}}{\varepsilon R(z_1 \rightarrow z_2)} < \infty, \quad (\text{A.2})$$

for some nonnegative real number $R(z_1 \rightarrow z_2)$, which is called the resistance of the transition from z_1 to z_2 [46].

Suppose that the state space of the perturbed Markovian chain is Z . We can construct a complete directed graph with $|Z|$ vertices with each state being a vertex, where $|A|$ represents the cardinality of set A . Furthermore, the weight on the directed edge from i to j is characterized by $\rho_{ij} = R(z_i \rightarrow z_j)$. For an arbitrary state z_j , a rooted tree can be constructed such that there is a unique directed path from every other state to z_j . The resistance of a rooted tree is the sum the resistances of every connected vertices, and the minimum resistance over all trees rooted at state z_j is defined as its *stochastic potential*. The stochastically stable state of a regular perturbed Markovian process is determined by the following theorem (See Lemma 1 in [46]).

Theorem 5. *For each $\varepsilon > 0$, let μ^ε be the unique stationary distribution of a regular perturbed Markovian process P^ε . The $\lim_{\varepsilon \rightarrow 0} \mu^\varepsilon$ exists and the limiting distribution μ^0 is a stationary distribution of the associated unperturbed Markovian process P^0 . Furthermore, the stochastically stable states are precisely those states with minimum stochastic potential.*

²For detailed analysis and investigation on resistance trees, refer to [46].

B. The asymptotical optimality of the proposed simultaneous log-linear learning algorithm

We first characterize the resistance of the proposed simultaneous log-linear learning algorithm and then show that a minimum resistance tree must be rooted at an action profile that maximizes the potential function of the proposed channel selection game. The following proof follows the lines of proof given in [28].

Theorem 6. *For the proposed simultaneous log-linear learning algorithm, the resistance of a transition from state $z_1 = [a(i-1), a(i), x(i)]$ to $z_2 = [a(i), a(i+1), x(i+1)]$ is given by:*

$$R(z_1 \rightarrow z_2) = \sum_k x_k(i+1)m_k + \sum_{k: x_k(i)=1, a_k(i+1)=a_k(i-1)} (V_k(i-1, i) - u_k(i-1)) + \sum_{k: x_k(i)=1, a_k(i+1)=a_k(i)} (V_k(i-1, i) - u_k(i)), \quad (\text{A.3})$$

where $V_k(i-1, i) = \max\{u_k(i-1), u_k(i)\}$.

Proof: According to the action update of the proposed learning algorithm, the state transitions for an arbitrary player k are determined by:

$$x_k(i) = 0 \Rightarrow \begin{cases} x_k(i+1) = 0, a_k(i+1) = a_k(i) \\ x_k(i+1) = 1, a_k(i+1) \in \{\mathcal{A}_k \setminus a_k(i)\}, \end{cases} \quad (\text{A.4})$$

Note that the above two cases correspond to the *Exploration* and *Update* process of the proposed algorithm respectively.

Denote $\varepsilon = \exp(-\beta)$, we then have $\delta_k = \varepsilon^{m_k}$. The transition probability from z_1 to z_2 is given by:

$$P_{z_1 \rightarrow z_2}^\varepsilon = \left(\prod_{k: x_k(i)=0, x_k(i+1)=0} (1 - \varepsilon^{m_k}) \right) \times \left(\prod_{k: x_k(i)=0, x_k(i+1)=1} \frac{\varepsilon^{m_k}}{|\mathcal{A}_k| - 1} \right) \times \left(\prod_{k: x_k(i)=1, a_k(i+1)=a_k(i-1)} \frac{\varepsilon^{-u_k(i-1)}}{\varepsilon^{-u_k(i-1)} + \varepsilon^{-u_k(i)}} \right) \times \left(\prod_{k: x_k(i)=1, a_k(i+1)=a_k(i)} \frac{\varepsilon^{-u_k(i)}}{\varepsilon^{-u_k(i-1)} + \varepsilon^{-u_k(i)}} \right), \quad (\text{A.5})$$

where the first item is the aggregate probability of players that do not explore, the second is that for players that explore, the third is that for players that update with the old selection, and the fourth is that for players that update with the new selection.

Denote $V_k(i-1, i) = \max\{u_k(i-1), u_k(i)\}$. Multiplying the numerator and denominator of the third and the fourth items in (A.5) of $\varepsilon^{V_k(i-1, i)}$, we obtain:

$$P_{z_1 \rightarrow z_2}^\varepsilon = \left(\prod_{k: x_k(i)=0, x_k(i+1)=0} (1 - \varepsilon^{m_k}) \right) \times \left(\prod_{k: x_k(i)=0, x_k(i+1)=1} \frac{\varepsilon^{m_k}}{|\mathcal{A}_k| - 1} \right) \times \left(\prod_{k: x_k(i)=1, a_k(i+1)=a_k(i-1)} \frac{\varepsilon^{V_k(i-1, i) - u_k(i-1)}}{\varepsilon^{V_k(i-1, i) - u_k(i-1)} + \varepsilon^{V_k(i-1, i) - u_k(i)}} \right) \times \left(\prod_{k: x_k(i)=1, a_k(i+1)=a_k(i)} \frac{\varepsilon^{V_k(i-1, i) - u_k(i)}}{\varepsilon^{V_k(i-1, i) - u_k(i-1)} + \varepsilon^{V_k(i-1, i) - u_k(i)}} \right) \quad (\text{A.6})$$

Accordingly, if we choose $R(z_1 \rightarrow z_2)$ as that shown in (A.3), we have

$$\lim_{\varepsilon \rightarrow 0^+} \frac{P_{z_1 \rightarrow z_2}^\varepsilon}{\varepsilon^{R(z_1 \rightarrow z_2)}} = (|\mathcal{A}_k| - 1)^{-\sum_k x_k(i+1)} = (M - 1)^{-\sum_k x_k(i+1)}, \quad (\text{A.7})$$

where M is the number of channels. Clearly, the above equation satisfies the following condition:

$$0 < \lim_{\varepsilon \rightarrow 0^+} \frac{P_{z_1 \rightarrow z_2}^\varepsilon}{\varepsilon^{R(z_1 \rightarrow z_2)}} < \infty, \quad (\text{A.8})$$

which means that the resistance of transition from z_1 to z_2 is exactly shown in (A.3). ■

With the resistance of arbitrary two states is now obtained, we are ready to study the minimum resistance tree of the perturbed Markovian process of the proposed learning algorithm. An interesting result is that the above resistance is very similar to that obtained in [28], where the assumption of $m_k = m, \forall k \in \mathcal{K}$ is used. Thus, following the similar lines given in [28], the asymptotical optimality of the proposed simultaneous log-linear learning algorithm can be obtained accordingly. Specifically, it was pointed out in [28] that the a minimum resistance tree must be rooted at an action profile that maximizes the potential function (See Claim 6.4 therein), which equally means that the all channel selection profiles that maximize the potential function are stochastically stable.

Denote the set of potential maximizer as $\mathcal{A}_{opt} = \{a \in \mathcal{A} : \Phi(a) = \arg \max \Phi(a)\}$. We explain the stochastically stable states with regard to the empirical distribution of the proposed algorithm. It is noted that the channel selection profile also evolves as a discrete Markovian process $\{a(0), a(1), \dots, a(i)\}$, where $a(i) \in \mathcal{A}, \forall i = \{0, 1, 2, \dots\}$. Let us define the empirical distribution of an action profile $a \in \mathcal{A}$ as follows:

$$z(a; Y) = \frac{1}{Y} \# \{i \leq Y : a(i) = a\}, \quad (\text{A.9})$$

where $\#\{\cdot\}$ denotes the number of times the event inside the bracket occurs and $a(i)$ is the action profile at iteration i . Notably, the stationary distribution of the joint action profiles $\mu(a) \in \Delta(\mathcal{A})$ is related to the empirical distribution by:

$$\lim_{Y \rightarrow \infty} z(a; Y) = \mu(a). \quad (\text{A.10})$$

Mathematically, the statement that the stochastically stable states are the set of potential maximizers implies the following equalization [8]:

$$\lim_{Y \rightarrow \infty} \sum_{a \in \mathcal{A}_{opt}} \mu(a) = 1. \quad (\text{A.11})$$

Now, using again the relationship between the potential function and the aggregate interference as specified by (9) and (13), Theorem 4 follows.

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