Selecting Transmit Powers and Carrier Sense Thresholds in CSMA Protocols for Wireless Ad Hoc Networks*

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ABSTRACT

Carrier sense multiple access (CSMA) protocols, such as the IEEE 802.11 protocol, are widely used in wireless ad hoc networks. In this paper, we consider how the parameters of such protocols should be selected. Our primary goal is an increase in spatial reuse. By analyzing conditions for collision prevention, we argue that the product of the transmit power and the carrier sense threshold should remain constant. We then discuss the practical implementation of this principle within the framework of the IEEE 802.11 protocol. Finally, we provide simulation results that compare our scheme to other methods of selecting the protocol parameters. These results indicate that an increase in spatial reuse can indeed be achieved.

Categories and Subject Descriptors

C.2.2 [**Computer-Communication Networks**]: Network Protocols

General Terms

Algorithms, Design

Keywords

MAC protocols, CSMA, 802.11

1. INTRODUCTION

Power control is an important aspect of wireless network design. There are two primary goals of power control. The first goal is energy efficiency, which is the minimization of energy consumption in the network. The second goal is spatial reuse, which is the improvement of network throughput by allowing for multiple simultaneous transmissions. While energy efficiency and spatial reuse sometimes go hand-in-hand, this is not always the case.

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In this paper, we are interested in performing power control at the MAC layer. We will not develop an entirely new protocol. Instead, we will specify how to select the parameters of the existing IEEE 802.11 MAC protocol [1]. The 802.11 protocol has become the *de facto* standard for wireless networks and is a carrier sense multiple access (CSMA) protocol. There are two carrier sense mechanisms in the 802.11 protocol. The first is virtual carrier sensing. In virtual carrier sensing, the channel is reserved by including a network allocation vector (NAV) in the packet format. The use of short request-to-send (RTS) and clear-to-send (CTS) packets makes this mechanism particularly effective since the channel is reserved before a data packet is transmitted. The second carrier sensing mechanism is physical carrier sensing. In physical carrier sensing, each node uses a power threshold called the carrier sense (CS) threshold. A sender wishing to transmit first checks to see whether the power from other transmissions currently being received is above the CS threshold. If the threshold is exceeded, the transmission is deferred until a time when this is no longer the case.

We will focus on the physical carrier sensing mechanism in this paper with the primary goal of increasing spatial reuse. The rest of this paper is organized as follows: In Section 2, we give a brief outline of previous research in the area of power control for MAC protocols. In Section 3 we perform an analysis of collision prevention conditions for a CSMA protocol. The most important result of this analysis is that the product of the transmit power and carrier sense threshold parameters should remain constant. We then go on to discuss how this principle can be applied to the IEEE 802.11 protocol. In Section 4, we present simulation results that compare our method with other methods for selecting the transmit powers and CS thresholds. The results indicate that our scheme can indeed increase spatial reuse. In Section 5, we draw conclusions and discuss areas for future research.

2. PREVIOUS RESEARCH

There has been some previous research on the use of power control in MAC protocols. In [2], the PCMA protocol is defined. In this protocol, busy tones are sent by receivers on an out-of-band channel to communicate current interference margins. This informs potential transmitters of the maximum transmit power they can use without disturbing any ongoing transmissions. Within this maximum power constraint, the transmit power is chosen to be 4 dB above the minimum power currently required for successful transmission. This transmit power selection strategy is left unjustified. In this work, we will address collision prevention and transmit power selection simultaneously.

The PCDC and POWMAC protocols, described in [3] and [4] respectively, use similar collision prevention mechanisms. Both protocols use control frames sent at relatively high powers so that neighboring nodes are aware of ongoing transmissions. The selection of transmit powers in these schemes seeks to maximize the margin for future interference at each receiver. It is not clear that this selection strategy is best for maximizing spatial reuse. Thus it is possible that the scheme developed in this work may provide better results.

Our work is most similar to that in [5] and [6] since the authors of these papers also choose to work within the 802.11 framework. In these two papers, the goal is to find an optimal CS threshold to be used throughout the network. The selection of transmit power is not addressed. In contrast, our work will allow different links to use different CS thresholds and will also address the selection of transmit power. Thus, our approach is a much more holistic approach to parameter selection in CSMA protocols.

3. PROTOCOL DESIGN

3.1 Analytical Results

We first give our assumptions and definitions. We define a link as a transmitter-receiver pair. At the physical layer, we assume that a transmission on a link is successful if and only if the signal to interference plus noise ratio (SINR) at the receiver is at or above the threshold γ for the duration of the transmission. The thermal noise at each node is η. At the MAC layer, we will consider a CSMA protocol that uses only physical carrier sensing. A node that wishes to send a packet selects two parameters: a transmit power, denoted p_t, and a CS threshold, denoted p_{cs}. The sender is allowed to begin transmitting only if the total received power from ongoing transmissions (not including thermal noise) is less than or equal to the CS threshold.

By analyzing collision prevention conditions for our CSMA protocol, it is possible to derive relations that the parameters p_t and p_{cs} should satisfy. We give the key conclusions of this analysis in this section but provide the bulk of the derivations in the appendix. As seen in the appendix, our analysis relies in part on an approximation we refer to as the *collocation approximation*. A similar approximation is made in [5]. While this approximation does introduce some error, we will be able to account for this error in the design of our protocols.

The primary conclusion of our analysis is that the product of the transmit power and the CS threshold function should be a fixed constant for each transmitter throughout the network. We call this constant β and so we can therefore write

$$p_t \, p_{cs} = \beta \tag{1}$$

Using this constant product rule effectively bounds the amount of interference that a single interferer can pose to any link. One way to see this is to note that while transmitters using large transmit powers can cause large amounts of interference, these transmitters must also use small CS thresholds that cause them to me more cautious when transmitting. Because of this bounded interference, we can refer to a "worst-case interferer," an interferer that is ideally located to provide the maximum possible amount of interference to a given link.

Under the constant product rule, we need one more equation to uniquely specify p_t and p_{cs}. In the appendix, we derive the equation

$$p_{cs} = \frac{1}{k}\left(\frac{p_t \, g}{\gamma} - \eta\right) \tag{2}$$

where g is the channel gain from the transmitter to the receiver on the link and $k > 0$ is the number of worst-case interferers assumed. Note that the CS threshold is simply the total amount of interference that can be tolerated at the receiver divided by the factor k. Since p_{cs} decreases with increasing k, we conclude that p_t must increase with increasing k. An appropriate value of k for a given link is not known *a priori*, but collisions will be prevented if k is chosen sufficiently large. We can think of k as a factor of safety that accounts for two sources of interference. The first source of interference is the local topology surrounding the link, i.e., what potential interferers are present. The second source of interference is any approximation error that is introduced by the collocation approximation. We will address the selection of k when we discuss the details of our protocol.

As is discussed in the appendix, the value of β can be selected to trade-off energy efficiency with spatial reuse. The amount of spatial reuse is maximized by letting $\beta \to \infty$. However, this same spatial reuse can be achieved for every β by setting $\eta = 0$. Normally, η is not a parameter that can be changed. However, in our simulation results, we will artificially set $\eta = 0$ to explore the upper limit in spatial reuse.

3.2 Principles of Operation

We propose that on each link, the value of k for that link should be dynamically adjusted to find the minimum value of k that prevents collisions on that link. Note that because p_t is an increasing function of k, this is equivalent to finding the minimum value of p_t that prevents collisions. Thus, we could choose to adjust the transmit power directly without varying k. Nevertheless, we choose to vary k primarily because there are some values of k that seem reasonable. Recalling the use of hexagonal cells in cellular systems seems to indicate that perhaps k need not be larger than approximately six. However, since k is also used to compensate for the effects of the collocation approximation, larger values of k may be needed.

An advantage of our scheme should be emphasized. Recall that the constant product rule in (1) effectively bounds the amount of interference a single node can pose to a nearby link. As a result, the prevention of collisions on a given link is primarily a function of the transmit power chosen for that link. It is therefore reasonable to assume that a collision on a given link is due to insufficient transmit power being used on that link. Each link is then responsible for its own collision prevention. This is clearly desirable since otherwise links would have to somehow communicate their collision information to the nodes that caused the collisions.

A potential problem with our scheme is that fairness may be adversely affected. Of course, this depends on the definition of fairness being used. The source of the problem is that different senders in the same region may be using widely different CS thresholds. As a result, those with low CS thresholds are at a certain disadvantage since they need to wait for quieter conditions before transmitting. In the meantime, senders with high CS thresholds may access the channel, further postponing the transmissions of the disadvantaged transmitters. Note that short links will generally have an advantage over long links. The unfairness problem arises because we are aggressively insisting on many simultaneous transmissions.

3.3 Practical Issues

We have assumed that on every link, the sender knows the gain to the receiver. In practice, this gain will need to be estimated. One way this can be accomplished is by including the transmit power in RTS and/or data frames sent to the receiver. The receiver can then estimate the gain and relay it back to the sender in CTS and/or ACK frames. The receiver's transmit power and CS threshold when replying should be identical to that used originally by the sender. If the sender and receiver use different parameters, it becomes unclear to each who was at fault when a transmission is unsuccessful.

Although we have stated that we would like each link to use the minimum value of k that prevents collisions on that link. Unfortunately, this goal is not realistic because not all packet losses at the MAC layer are due to insufficient transmit power. Collisions can also occur when two nodes decide to transmit at the same time. The probability of this type of collision increases with the transmit power used by neighboring nodes. Thus, although increasing transmit power decreases collisions due to interference, collisions due to contention can actually increase. At this point, we do not have the necessary insight to design a control loop for k and we therefore postpone this discussion until Section 4.3.

4. SIMULATION RESULTS

4.1 Simulation Setup

The ns-2 network simulator was used for our simulations. This simulator provides an implementation of the 802.11 MAC protocol and a model of the wireless physical layer. To produce the desired results, some significant changes had to be made to these portions of the simulator. In the default implementation, the CS threshold parameter is used for two somewhat different functions. First, any single packet received with power above the threshold prohibits the transmitter from transmitting. Second, any single packet received above the threshold causes the receiver to begin receiving that packet. Because we wanted to vary the CS threshold without changing the behavior of the receiver, this second function was undesired. Moreover, we needed to keep track of the *cumulative* interference at the receiver (not just look at a single packet) to compute an accurate SINR.

In our modified code, a list of received powers for all active packets is kept at each node. A thermal noise parameter is also present. Thus, an SINR can be computed. Whether the receiver begins receiving a packet is then based on whether the SINR is at or above the threshold γ. If the SINR drops below γ at any time while a packet is being received, the packet is discarded and the receiver returns to idle. The CS threshold is used only by the transmitter when deciding whether to access the channel.

Table 1: Simulation parameters.

Parameter	Value
SINR Threshold (γ)	10 dB
Thermal Noise (η)	0 W
Link Rate	1 Mbps
Packet Size	512 Bytes
Slot Time	20 μs
Fixed CW Size	31 slots
RTS/CTS	Disabled
Effective Link Rate	767 Kbps
Transport Protocol	UDP
Offered Load Per Link	1 Mbps

As discussed in Section 3.3, replies from the receiver were transmitted with the same transmit power and CS threshold originally used by the sender. It was assumed that each sender and receiver had perfect knowledge of the gain on the link connecting them. This is not a problem because, as discussed in Section 3.3, this gain can be measured with little increase in overhead. The gain between any two nodes in the network was computed as a function of distance using the default two-ray ground reflection model in ns-2.

Many of the simulation parameters were the defaults in ns-2. A table of simulation parameters can be found in Table 1. The thermal noise was set to zero to maximize spatial reuse and to make the choice of β irrelevant. Also, there was no minimum or maximum power. The link rate was set to 1 Mbps. For simplicity, the contention window size was fixed (i.e. there was no exponential backoff). The use of RTS/CTS frames was disabled so that virtual carrier sensing did not pay a major role in the network. Indeed, with RTS/CTS exchanges disabled, virtual carrier sensing was used only for ACK protection after sending a data packet. The effective link rate listed in Table 1 is the throughput of a network containing a single link and is somewhat less than the link rate due to overhead. Note that the offered load on each link was equal to the link rate. Since this load could not be supported, the queues for each link quickly became backlogged. Thus, our simulations give results for heavily loaded networks. Unless otherwise specified, the duration of each simulation run was 30 s.

To generate a topology for simulation, 30 links were placed within a circle of radius 250 m. The link lengths were Rayleigh distributed. This distribution was selected because in a two-dimensional Poisson point process (which generates a network of uniform density), the distance to a nearest neighbor is a Rayleigh random variable. Although such a topology is not fully realistic for an ad hoc network, it is an irregular topology containing links of varying lengths and should be sufficient for testing MAC protocols. We simulated six different topologies: three with mean link length 50 m and three with mean link length 100 m. We will often

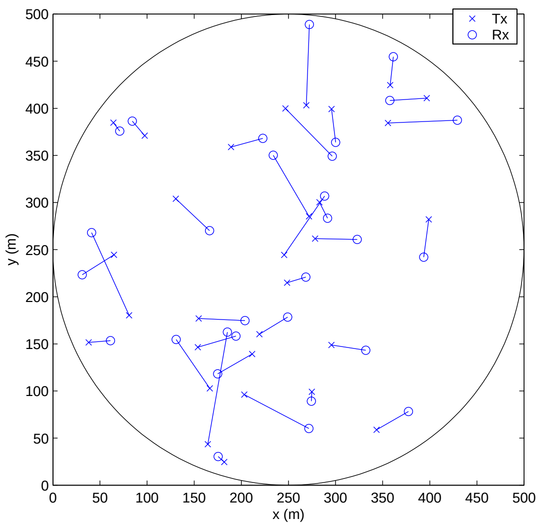

Figure 1: A random topology with mean link length 50 m.

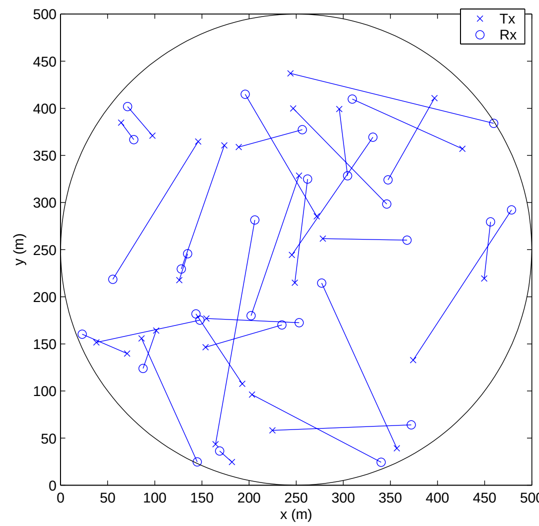

Figure 2: A random topology with mean link length 100 m.

refer to topologies with mean link lengths 50 m and 100 m as 50-m and 100-m topologies respectively. A sample 50-m topology is shown in Figure 1 while a sample 100-m topology is shown in Figure 2.

4.2 Schemes with Static Parameters

For each of the six topologies simulated, we considered the following three schemes:

1. *Fixed Receive (Rx) Power:* The transmit power on each link was chosen so that the receive power at the receiver on that link would be 3.652×10^{-10} W. The CS threshold was then varied uniformly across the network.

2. *Fixed Transmit (Tx) Power:* The transmit power was kept fixed at 0.282 W while the CS threshold was varied uniformly across the network.

3. *Static k:* Our scheme was used, varying k uniformly across the network. The value of β used was 1 W^2 but this value had no impact on the throughputs since there was no thermal noise.

We will give some of the results for the Static k scheme in this section since this will give us the necessary insight to design the Dynamic k scheme defined in the next section. However, we postpone comparing the various schemes until Sections 4.4 and 4.5.

Figure 3 shows the network throughput for the 50-m topologies as a function of k for the Static k scheme. In this figure, the network throughput is first plotted in millions of bits per second (Mbps) for each of the random topologies considered. The throughput is then plotted in millions of bit-meters per second (Mb-m/s) by weighting the throughput on each link by the length of that link. The interest in this second measure of throughput comes from the paper on the capacity of wireless ad hoc networks by Gupta and Kumar [7]. Note that by measuring throughput in Mb-m/s, we consider not only how much data is sent but also how far it travels. Note that the optimal point occurs at $k \approx 2$. This value may seem low for a random topology. The reason for this value is probably the sparseness of the topology when the mean link length is 50 m (see Figure 1). Although not shown here, the optimal point for the 100-m topologies was at $k \approx 3$ due to the higher density of links.

We can also examine the number of collisions that occurred as a function of k for the Static k scheme. This may help us in designing a scheme for adjusting k dynamically. We therefore define the failure rate, which is the fraction of packet transmission attempts that do not succeed. A transmission attempt occurs only when a node accesses the channel, i.e., when it has a packet to send and its CS threshold is not exceeded. Note that low failure rate does not necessarily imply higher throughput. For example, if a node is overly cautious in accessing the channel, its failure rate will be low but its throughput will also be low due to fewer transmission attempts.

For the Static k simulations using 50-m topologies, the failure rate is plotted as a function of k in Figure 4. The ranges $0 \leq k \leq 10$ and $10 \leq k \leq 100$ are plotted separately to better see the behavior. If we compare these data with the throughput curves in Figure 3, we see that the throughputs are maximized at approximately the same points where the failure rate curves first becomes flat. We will use this fact in Section 4.3. Figure 4 also shows that for large k there is a slow rise in failure rate. As mentioned in Section 3.3, this is because of an increase in contention collisions due to higher transmit powers. The failure rate curves for the 100-m topologies had the same characteristics as those for the

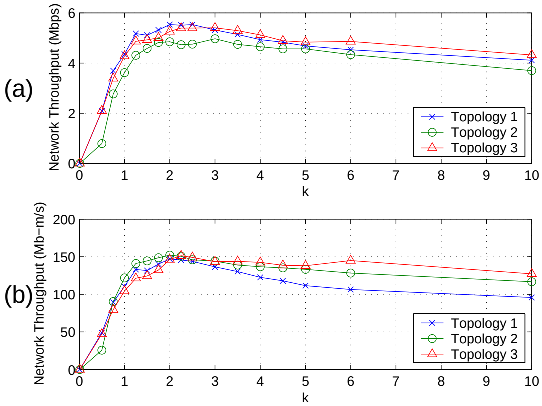

Figure 3: Throughputs for the Static k scheme and mean link length 50 m.

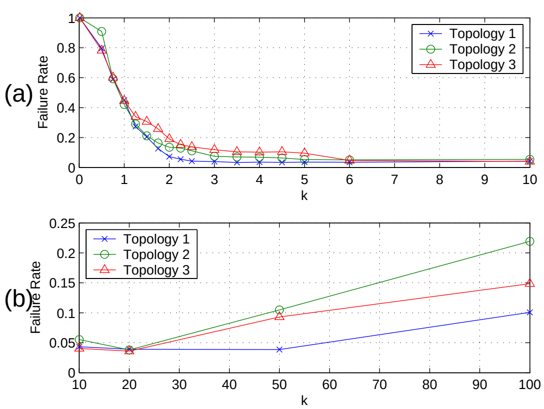

Figure 4: Failure rate versus k for the Static k scheme and mean link length 50 m.

50-m topologies.

4.3 The Dynamic k Scheme

We have seen that the optimal value of k depends on the topology being considered. In fact, even within a topology, different values of k may be needed since each link will see different amounts of interference. We would therefore like to run an algorithm at each link to adjust k dynamically. Our goal in designing this algorithm is to meet or exceed the maximum throughputs for the Static k scheme of previous section. Our previous results have indicated that throughput is maximized approximately when the failure rate first becomes flat. Thus, a reasonable control algorithm might try to reach a flat part of the failure rate curve while being biased toward low values of k.

We now describe such a control algorithm. This algorithm is the result of trial and error and should not be considered optimal in any sense. The algorithm has two phases. In the first phase, the link has an initial k of 0.1. On a transmission failure, k is increased by 0.1. On a transmission success, k is decreased by 0.1 unless this would cause k to become zero. This continues until at least five transmission attempts have been made and the failure rate averaged over all attempts is less than 0.75. This should happen eventually since the failure rate should converge in the mean to 0.5. The purpose of this phase is to reach a state where both successes and failures are occurring so that the next phase will be able to determine how to minimize the failure rate.

The second phase is based on a gradient-descent algorithm. Suppose k_i is the current value of k. First, 20 packet attempts are performed using $k = k_i$. The number of failures that occur in these 20 attempts is n_1. Next, 20 packet attempts are performed using $k = k_i + 0.5$. The number of failures is n_2. If $f(k)$ is the failure rate as a function of k, we can estimate the gradient at k_i as

$$\nabla f(k_i) = \frac{n_2 - n_1}{(0.5)(20)} \tag{3}$$

The next value of k to use, k_{i+1}, is determined through the update relation

$$k_{i+1} = k_i - 0.1(\nabla f(k_i) + 0.1) \tag{4}$$

Note that instead of trying to achieve $\nabla f = 0$, the algorithm tries to achieve $\nabla f = -0.1$. Since our previous results have shown that $f(k)$ decreases and then increases with increasing k, we see that this implies the desired bias toward low values of k.

We will refer to the use of the above control algorithm as the Dynamic k scheme. Simulations were run for the Dynamic k scheme for each of the six random topologies considered in Section 4.2. The length of each simulation was 120 s. For the most part, the value of k on each link converged to an equilibrium point within a few seconds. However, a small number of links in the 100-m topologies did not converge and the value of k continued to increase over the duration of the simulation. Changes to the control algorithm are needed to prevent this problem. For the 50-m topologies, it was found that the mean final value of k was 1.915. For the 100-m topologies, the mean final value of k was 3.201. These numbers are similar to the optimal values of k seen in the Static k simulations. We have left discussion of the throughput and fairness of the Dynamic k scheme to later sections so that comparisons can be made among all the various schemes.

4.4 Throughput Comparisons

In this section, we compare the network throughputs of the four schemes we have considered. Recall that the three schemes of Section 4.2 all had throughputs that depended on a free parameter (either the CS threshold or the value of k). For each scheme and topology, we use the maximum throughput obtained over all values of this free parameter for the purposes of comparison. Note that the maximum Mbps throughput and the maximum Mb-m/s throughput may not have occurred for the same value of the free parameter. For the Dynamic k scheme of Section 4.3, there are no free parameters and so this issue does not arise. Figure 5 compares the network throughputs for the 50-m topologies while Figure 6 compares the network throughputs for the 100-m topologies. In each figure, the throughputs are given first in Mbps and then in Mb-m/s.

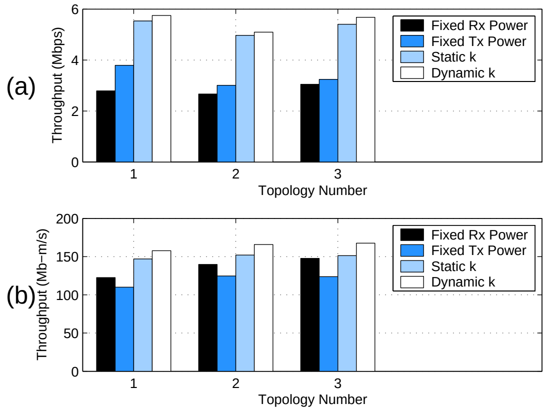

Figure 5: Throughput comparisons for mean link length 50 m.

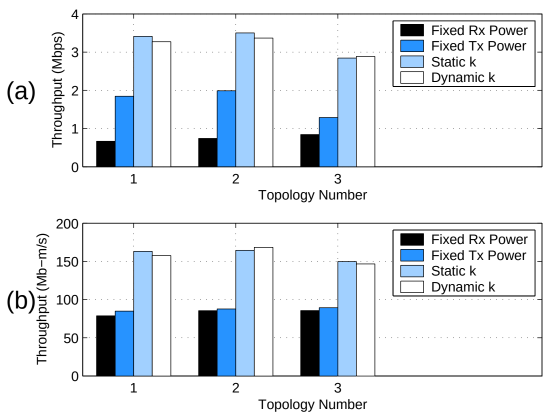

Figure 6: Throughput comparisons for mean link length 100 m.

From these data, we can see that the Static k and Dynamic k schemes consistently outperform the other two alternatives with regard to both types of throughput. For Mbps throughput, the Fixed Tx Power scheme is better than the Fixed Rx Power scheme. For Mb-m/s throughput, however, the two are roughly equal. The reason for this that the two are not equally fair, as will be seen in the next subsection.

For the 50-m topologies, we see that the Dynamic k scheme yields slightly better throughputs than the Static k scheme. The advantage of the Dynamic k scheme is probably that it allows different links to use different values of k. However, note that we do not see the same advantage of the Dynamic k scheme over the Static k scheme for the 100-m topologies. This is probably because, as seen earlier, the control algorithm for the Dynamic k scheme did not always converge properly. If a better algorithm were used, it is expected that the Dynamic k scheme would once again exhibit a slight improvement over the Static k scheme.

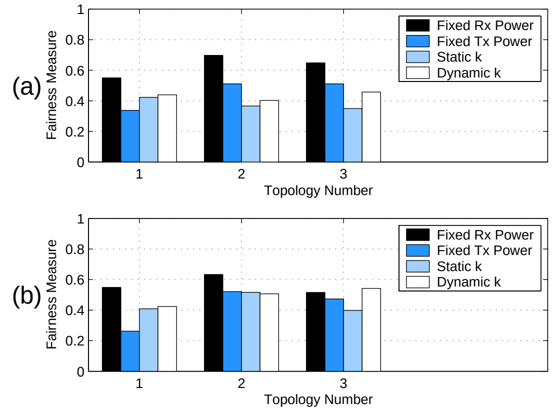

Figure 7: Fairness measures for mean link length 50 m.

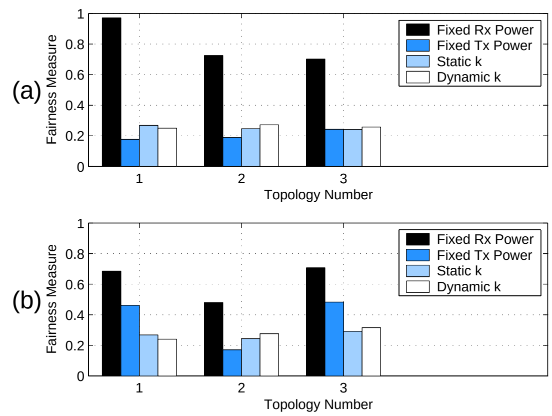

Figure 8: Fairness measures for mean link length 100 m.

4.5 Fairness Comparisons

There are many ways to quantify fairness. In this section, we will use the fairness measure proposed by Jain in [8]. If there are n flows and x_i is the throughput of the ith flow, then this fairness measure is given as

$$\text{Fairness Measure} = \frac{\left(\sum_{i=1}^{n} x_i\right)^2}{n \sum_{i=1}^{n} x_i^2} \tag{5}$$

Note that this is a number between zero and one with zero being the most unfair and one being the most fair. Note also that for our purposes x_i may be measured in either Mbps or Mb-m/s.

For each throughput plotted in Section 4.4, we can produce a corresponding fairness measure. Figure 7 depicts the fairness measures for the 50-m topologies while Figure 8 depicts the fairness measures for the 100-m topologies. From these data it is clear that the Fixed Rx Power scheme is the most fair. The other three schemes are approximately equally fair on average. However, the source of unfairness is not the same in all three cases, as we now explain.

As discussed in Section 3.2, the source of unfairness in our novel schemes (Static k and Dynamic k) is the use of different CS thresholds on different links. The result is that long links receive less throughput than short links. In the Fixed Tx Power scheme, a common CS threshold is used throughout the network. However, long links are still penalized. This is because the sender on a long link attempts to transmit to the receiver during times when there is too much interference at the receiver. We have seen that shorter links receive higher throughput even in the Mb-m/s case, when the throughputs on long links are given larger weight.

5. CONCLUSIONS

By analyzing conditions for collision prevention we have argued that in CSMA protocols, the product of the transmit power and the CS threshold should remain constant. We have discussed the practical implementation of this principle and seen that it can take place almost entirely within the framework of the existing 802.11 protocol. We have also provided some results that indicate that our scheme can lead to increased network throughput which indicates increased spatial reuse. The results also indicate that our scheme may lead to unfairness. However, this unfairness does not appear to be significantly worse than a scheme that uses a fixed transmit power and a fixed CS threshold throughout the network.

There are several areas for future research. Our scheme could be improved by designing a better control algorithm for adjusting the transmit power on each link. Our scheme could also be improved by developing techniques to mitigate unfairness. In addition, more realistic simulations should be performed. These simulations would incorporate some of the features of the 802.11 protocol that were disabled in this paper and would simulate more realistic network topologies and traffic models. The impact of our scheme on other network metrics and on higher-layer protocols should also be understood. It is possible that when this future work is performed, schemes which do not keep the product of the transmit power and CS threshold constant may have advantages. However, we expect that it will remain true that the CS threshold should decrease as the transmit power increases.

Our scheme should also be compared with other proposed MAC protocols that are not based on the 802.11 protocol. Examples of such protocols were discussed in Section 2. These comparisons may lead to new MAC protocol designs for wireless ad hoc networks.

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APPENDIX

Consider the arrangement of nodes shown in Figure 9. There are two links in this setup, indicated by solid arrows. We will consider the transmission from node A to node B to be the desired transmission with SIR threshold γ. The transmission from node C to node D is therefore potential interference. The gains on each link, as well as two other gains that will be important in our analysis, are shown in the figure. We will assume that these gains are bidirectional, although this assumption is not strictly required.

In this section, we denote the transmit power and CS threshold used on the link from A to B as p_{t1} and p_{cs1} respectively. Similarly, we denote the transmit power and CS threshold on the link from C to D as p_{t1} and p_{cs2} respectively. We assume that C has not yet begun transmitting when A's transmission begins and that the value of g_{cd} is not known to node A. We wish to establish conditions on p_{t1} so that A's transmission will be successful.

The fact that C is indeed allowed to interfere implies that its CS threshold has not been exceeded. Mathematically, we

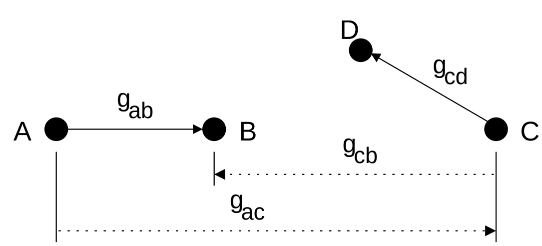

Figure 9: A wireless network with four nodes.

have

$$p_{t1}\, g_{ac} \leq p_{cs2} \tag{6}$$

Rearranging terms in this inequality yields a bound on g_{ac} as

$$g_{ac} \leq \frac{p_{cs2}}{p_{t1}} \tag{7}$$

We can also express the SINR requirement on node A's transmission as

$$\frac{p_{t1}\, g_{ab}}{k p_{t2}\, g_{cb} + \eta} \geq \gamma \tag{8}$$

Since there may be multiple interferers, we have introduced a nonnegative factor k into this equation which is the number of interferers assumed. Rearranging the terms in this inequality yields

$$p_{t1}\, g_{ab} \geq k\gamma p_{t2}\, g_{cb} + \gamma\eta \tag{9}$$

We would like to combine (7) and (9) in some meaningful way to obtain an inequality that is easily solved for a condition on p_{t1}. To do this, we choose to make an approximation in which we replace g_{cb} with g_{ac} in (9). By doing this, we have chosen to assume that nodes A and B see the same gain to node C. In effect, we are assuming that nodes A and B are essentially collocated while the gain between them remains at g_{ab}. We therefore refer to our approximation as the *collocation approximation*. It is quite possible for the collocation approximation to cause our analysis to become less conservative than before. This is a potential problem. However, we assume that nodes will be able to compensate for this effect by choosing an appropriately high value of k when selecting their transmit power.

Making our approximation, (9) becomes

$$p_{t1}\, g_{ab} \geq k\gamma p_{t2}\, g_{ac} + \gamma\eta \tag{10}$$

By substituting the maximum value of g_{ac} from (7), thus making node C a worst-case interferer to our desired transmission, this inequality becomes

$$p_{t1}\, g_{ab} \geq k\gamma p_{t2} \frac{p_{cs2}}{p_{t1}} + \gamma\eta \tag{11}$$

Because p_{t2} and p_{cs2} are unknown to node A, we need this inequality to hold for all possible values of these quantities. We therefore take the supremum of the right-hand side of (11) over p_{t2} and p_{cs2} to get the condition

$$p_{t1}\, g_{ab} \geq k\gamma \frac{\sup_{p_{t2}, p_{cs2}} \{p_{t2}\, p_{cs2}\}}{p_{t1}} + \gamma\eta \tag{12}$$

We can see from (12) that for p_{t1} to be finite, we need the supremum that appears in this equation to also be finite. We therefore impose the new constraint

$$p_t\, p_{cs} \leq \beta \tag{13}$$

where β is a finite constant. If we substitute β for the supremum in (12), we obtain a new and possibly stronger inequality. This inequality is quadratic in p_{t1} and has solution

$$p_{t1} \geq \frac{\gamma\eta + \sqrt{\gamma^2\eta^2 + 4k\gamma\beta g_{ab}}}{2 g_{ab}} \tag{14}$$

The value of β has not been specified and is left as a design parameter; we will discuss the selection of β shortly.

For maximum spatial reuse, it seems best to use low transmit powers and high CS thresholds, provided that the constraints of (13) and (14) are met. Using these guidelines, we arrive at the following formulas for the transmit power and CS threshold functions:

$$p_t = \frac{\gamma\eta + \sqrt{\gamma^2\eta^2 + 4k\gamma\beta g}}{2g} \tag{15}$$

$$p_{cs} = \frac{\beta}{p_t} \tag{16}$$

where g is the gain from the transmitter to the receiver on the link. Note that (16) can be rewritten as

$$p_t\, p_{cs} = \beta \tag{17}$$

which is the same as (1).

It is worthwhile to rearrange (15) to give a formula for the power seen at the receiver, as

$$p_t\, g = \frac{\gamma\eta + \sqrt{\gamma^2\eta^2 + 4k\gamma\beta g}}{2} \tag{18}$$

From (18), we can see that if $\beta = 0$, there is zero total interference margin at the receiver. We also know from (16) that if $\beta = 0$, the CS threshold is always 0. Thus, if $\beta = 0$ energy consumption is minimized but there is no spatial reuse. If β is increased, spatial reuse and energy consumption both increase. If we make the noise inconsequential by letting $\beta \to \infty$, the amount of spatial reuse approaches some (unquantified) limit. However, energy consumption continues to increase as $O(\sqrt{\beta})$. Thus we see that our design parameter β can be used to make a trade-off between energy consumption and spatial reuse. Note that we can also make the noise inconsequential by artificially setting $\eta = 0$, thus achieving the the same spatial reuse limit as when $\beta \to \infty$.

Through algebraic manipulation, we can find an interesting alternate interpretation of (15) and (16). Start with (16) and substitute (15) as

$$p_{cs} = \frac{\beta}{p_t} = \frac{2\beta g}{\gamma\eta + \sqrt{\gamma^2\eta^2 + 4k\gamma\beta g}} \tag{19}$$

Multiplying the numerator and denominator of this expression by a specific quantity yields (after simplification)

$$\begin{aligned} p_{cs} &= p_{cs} \frac{\sqrt{\gamma^2\eta^2 + 4k\gamma\beta g} - \gamma\eta}{\sqrt{\gamma^2\eta^2 + 4k\gamma\beta g} - \gamma\eta} \\ &= \frac{\sqrt{\gamma^2\eta^2 + 4k\gamma\beta g} - \gamma\eta}{2k\gamma} \end{aligned} \tag{20}$$

From (18) we see that we can rewrite this as

$$p_{cs} = \frac{p_t g - \gamma\eta}{k\gamma} = \frac{1}{k}\left(\frac{p_t g}{\gamma} - \eta\right) \tag{21}$$

which is the same as (2). Thus, we see that the CS threshold being used is the total interference margin at the receiver divided by k. The close relationship to the total interference margin is not surprising since under the collocation approximation the sender can perfectly estimate the current interference at the receiver to determine whether a potential transmission will be successful. The division by k simply accounts for the possibility of multiple interferers.