Univerzitet u Tuzli

Fakultet elektrotehnike

Automatika I robotika

ZADAĆA 2

Predmet: Stohastički sistemi i estimacije

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Asistent: Azra Grudić Ribić

Mjesto i datum: Tuzla,25.12.2022. Student: Belma Nurkić

Zadatak 1 (3 boda)

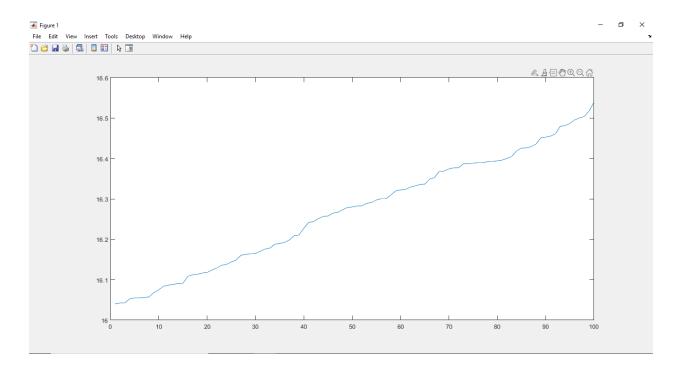
Izvršiti estimaciju parametara vremenske serije koristeći nekoliko različitih AR, MA, ARMA i ARIMA

modela (mijenjati parametre AR, MA i I) za dati Excel fajl, dati grafičku interpretaciju, kao i matematsku

formu i napraviti usporedbu, komentar.

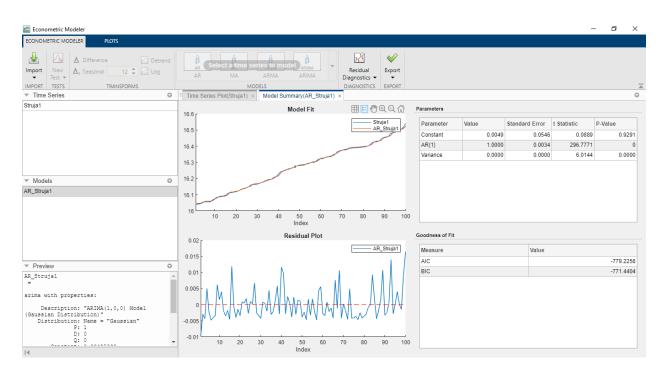
Matlab kod za učitavanje potrebnih podataka:

```
Struja=[xlsread('Podaci3', 'Podaci', 'B3001:B3100');];
Napon=[xlsread('Podaci3', 'Podaci', 'C3001:C3100');];
Snaga=[xlsread('Podaci3', 'Podaci', 'D3001:D3100');];
Otpor=[xlsread('Podaci3', 'Podaci', 'E3001:E3100');];
```



Slika1: Signal ulazne veličine, struje

Slika2: Matematicki model AR prvog reda za ulaznu veličinu struju



Slika3: Grafički prikaz AR modela prvog reda za ulaznu veličinu struju

```
AR =

arima with properties:

Description: "ARIMA(4,0,0) Model (Gaussian Distribution)"
Distribution: Name = "Gaussian"

P: 4

D: 0

Q: 0

Constant: 0.00459393

AR: {0.994212 0.0162527 0.047201 -0.0576658} at lags [1 2 3 4]

SAR: {}

MA: {}

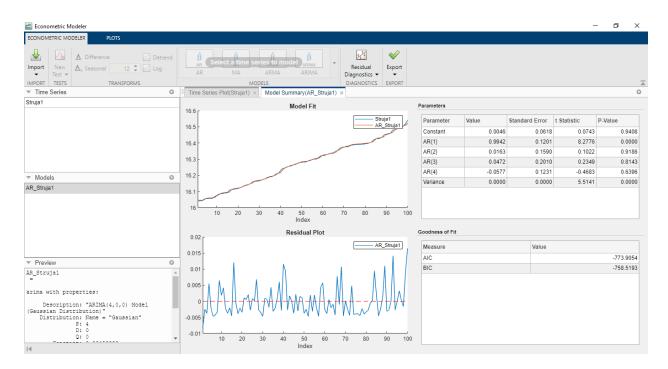
SMA: {}

Seasonality: 0

Beta: [1×0]

Variance: 2.3025e-05
```

Slika4: Matematički prikaz AR modela 4. Reda za struju



Slika5: Grafički prikaz AR modela 4. reda za struju

```
Command Window
          variance: Nan
  MA =
    arima with properties:
       Description: "ARIMA(0,0,1) Model (Gaussian Distribution)"
      Distribution: Name = "Gaussian"
                  P: 0
                  D: 0
                  Q: 1
          Constant: 16.262
                 AR: {}
               SAR: {}
                MA: {0.893979} at lag [1]
               SMA: {}
       Seasonality: 0
              Beta: [1×0]
          Variance: 0.00612903
fx >>
```

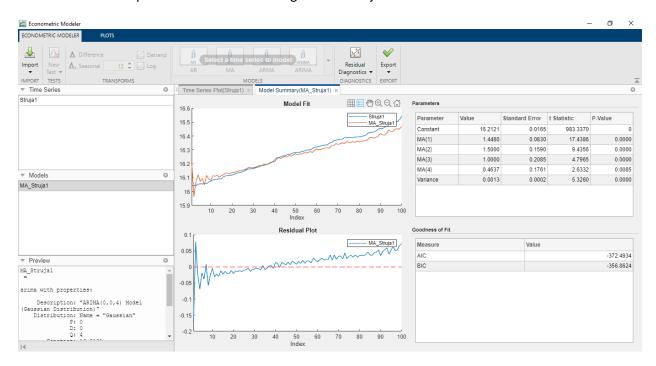
Slika6: Matematički prikaz MA modela prvog reda za struju



Slika7: Grafički prikaz MA modela prvog reda za struju

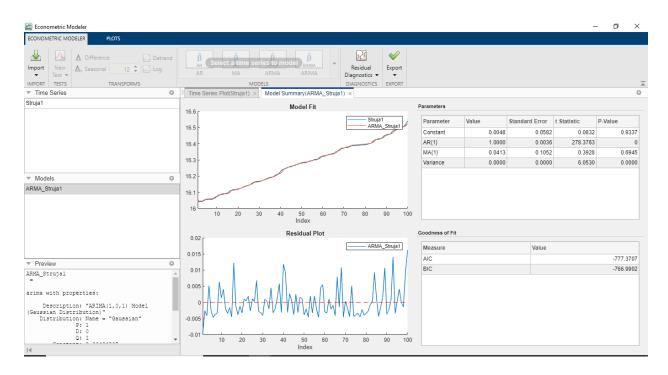
```
Command Window
  MA =
     arima with properties:
        Description: "ARIMA(0,0,4) Model (Gaussian Distribution)"
       Distribution: Name = "Gaussian"
                   P: 0
                   D: 0
                   0: 4
           Constant: 16.2121
                  AR: {}
                 SAR: {}
                  MA: {1.44799 1.5 1 0.463687} at lags [1 2 3 4]
        Seasonality: 0
               Beta: [1×0]
           Variance: 0.00125225
f_{\underline{x}} >>
```

Slika8: Matematički prikaz MA modela četvrtog reda za struju



Slika9: Grafički prikaz MA modela četvrtog reda za struju

Slika10: Matematički prikaz ARMA modela prvog reda za struju



Slika11: Grafički prikaz ARMA modela prvog reda za struju

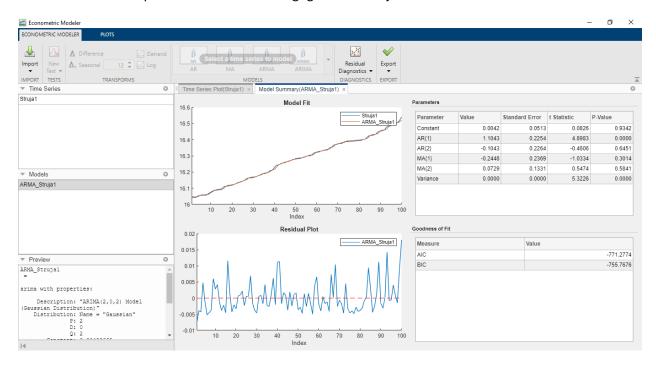
```
Description: "ARIMA(2,0,2) Model (Gaussian Distribution)"
Distribution: Name = "Gaussian"
P: 2
D: 0
Q: 2
Constant: 0.00423665
AR: {1.10428 -0.104285} at lags [1 2]
```

MA: {-0.244843 0.0728523} at lags [1 2]

SMA: {}
Seasonality: 0
Beta: [1×0]
Variance: 2.3594e-05

arima with properties:

Slika12: Matematički prikaz ARMA modela drugog reda za struju

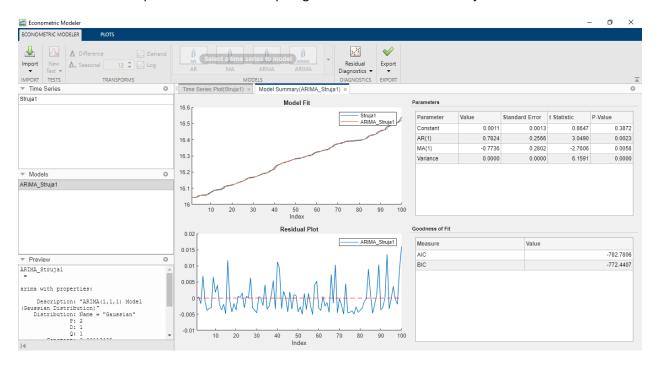


Salika12: Grafički prikaz ARMA modela drugog reda za struju

```
arima with properties:
```

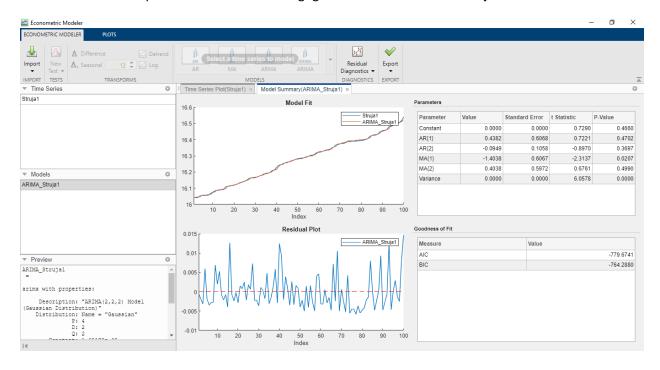
Beta: [1×0] Variance: 2.15333e-05

Slika13: Matematički prikaz ARIMA modela prvog reda za ulaznu veličinu struju

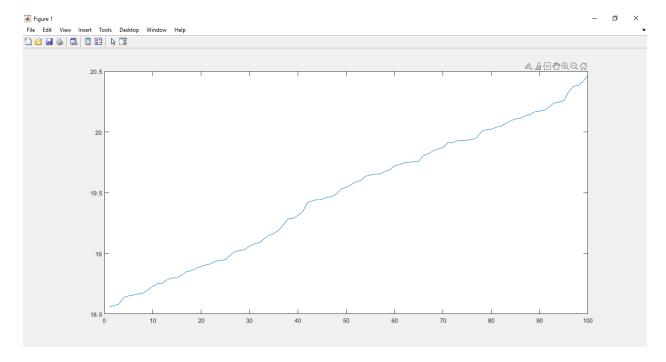


Slika14: Grafički prikaz ARIMA modela prvog reda za ulaznu veličinu struju

Slika15: Matematički prikaz ARIMA modela drugog reda za ulaznu veličinu struju

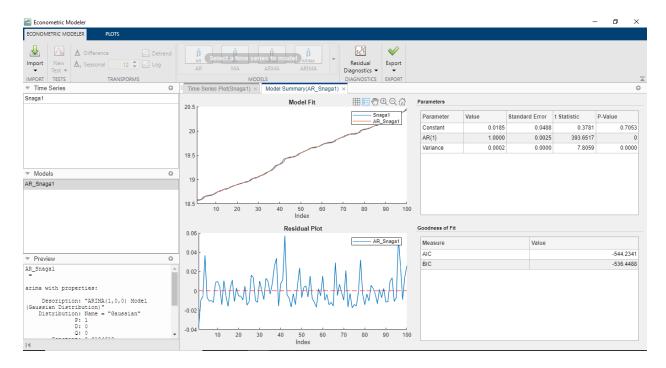


Slika16: Grafički prikaz ARIMA modela drugog reda za ulaznu veličinu struju



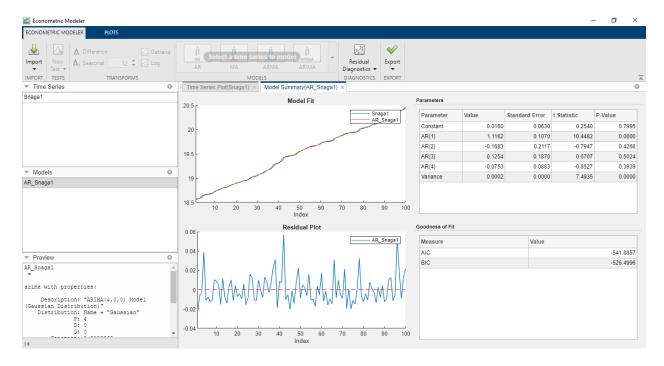
Slika17: Izgled izlaznog signala snage

Slika18: Matematički prikaz AR modela prvog reda za snagu



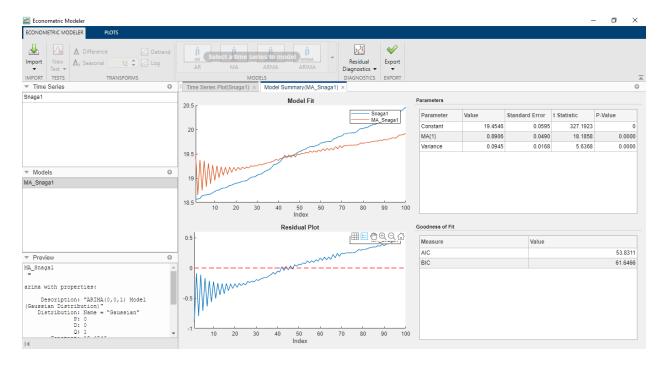
Slika19: Grafički prikaz AR modela prvog reda za snagu

Slika20: Matematički prikaz AR modela četvrtog reda za snagu



Slika21: Grafički prikaz AR modela četvrtog reda za snagu

Slika22: Matematički prikaz MA modela prvog reda za snagu



Slika23: Grafički prikaz MA modela prvog reda za snagu

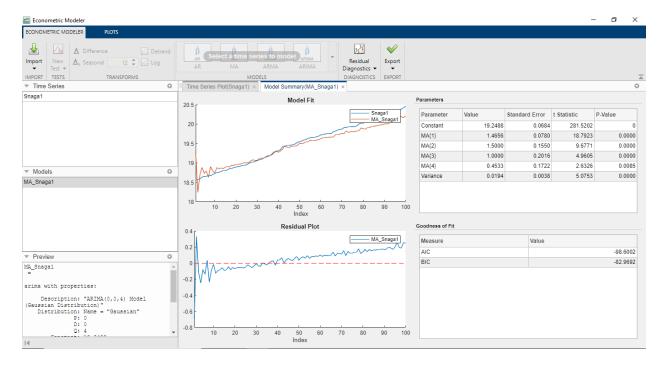
```
MA =

arima with properties:

Description: "ARIMA(0,0,4) Model (Gaussian Distribution)"
Distribution: Name = "Gaussian"

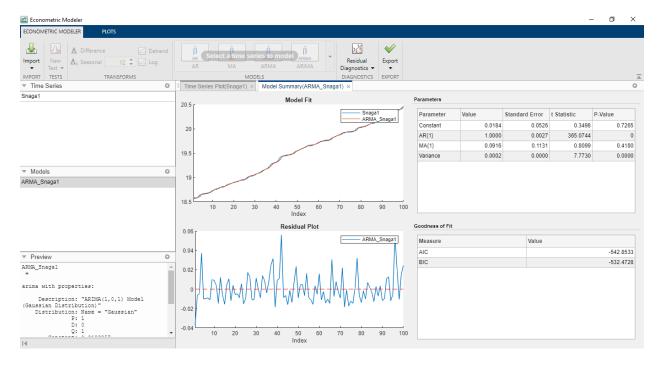
P: 0
D: 0
Q: 4
Constant: 19.2488
AR: {}
SAR: {}
MA: {1.46565 1.5 1 0.453258} at lags [1 2 3 4]
SMA: {}
Seasonality: 0
Beta: [1×0]
Variance: 0.0193729
```

Slika24: Matematički prikaz AR modela črtvrtog reda za snagu



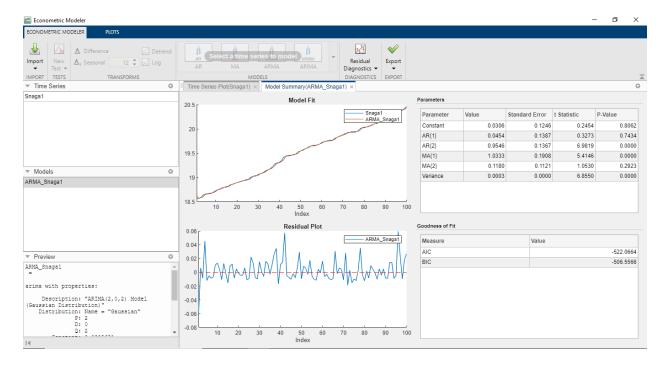
Slika25: Grafički prikaz MA modela četvrtog reda za snagu

Slika26: Matematički prikaz ARMA modela prvog reda za snagu



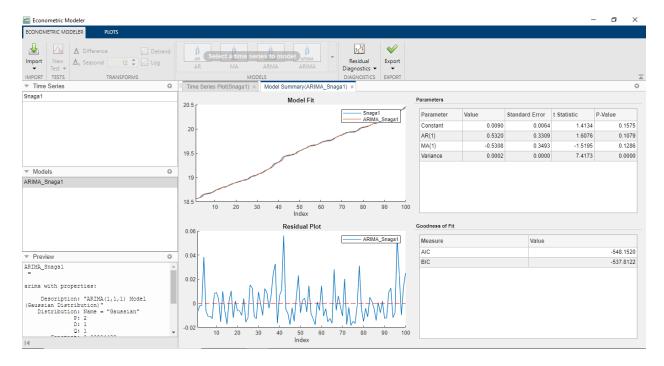
Slika27: Grafički prikaz ARMA modela prvog reda za snagu

Slika28: Matematički prikaz ARMA modela drugog reda za snagu



Slika29: Grafički prikaz ARMA modela za snagu

Slika30: Matematički prikaz ARIMA modela prvog reda za snagu



Slika31: Grafički prikaz ARIMA modela prvog za snagu

```
ARIMA =

arima with properties:

Description: "ARIMA(2,2,2) Model (Gaussian Distribution)"
Distribution: Name = "Gaussian"

P: 4

D: 2

Q: 2

Constant: 0.00010413

AR: {0.199998 -0.218762} at lags [1 2]

SAR: {}

MA: {-1.10618 0.214879} at lags [1 2]

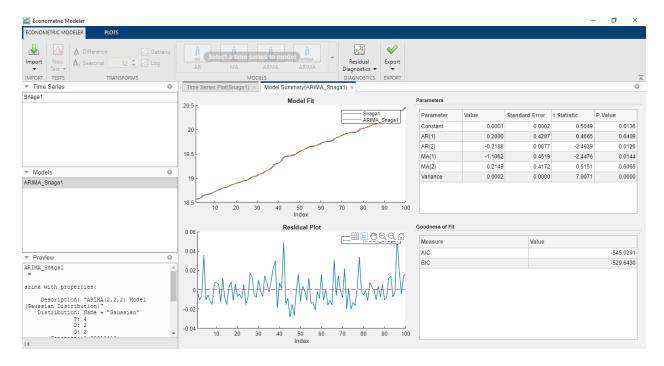
SMA: {}

Seasonality: 0

Beta: [1×0]

Variance: 0.000223031
```

Slika32: Matematički prikaz ARIMA modela drugog reda za snagu



Slika33: Grafički prikaz ARIMA modela drugog reda za snagu

Komentar:

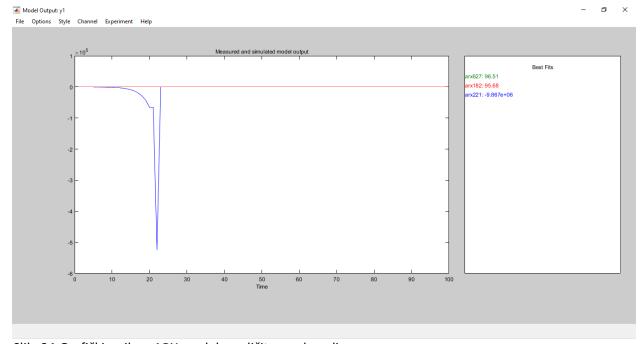
U zadatku vršimo estimacije struje I snage koristeći nekoliko različitih AR, MA,ARMA I ARIM modela pomoću EM appa. Matematicki prikaz modela dobili smo koristeći naredme atima I estimate, mijenjajući parameter za odgovarajuće modele. Prvo su prikazani modeli struje zazim snage. Prvo je prikazan AR model struje gdje primjetimo da povećanjem reda ne utičemo puno na promjenu estmacije. Istu situaciju primjetimo I kod snage. Kod MA modela već primjetimo promjenu kod oba signal. Ovaj model je bolji jer kod njega imamo manju grešku. Slično kod ARMA modela se dobija veoma mala greška. Svaki od predhodni modela možemo dobiti kao specijalan slucaj ARIMA modela, takodjer primjetimo da povećanjem reda ARIMA model daje manju grešku.

Zadatak 2 (3 boda)

Izvršiti estimaciju parametara modelske strukture u Matlabu, za ulazno-izlazne podatke (za dati Excel fajl), napraviti grafičku usporedbu nekoliko linearnih (ARX, ARMAX) modela (mijenjati parametre na, nb i nk) i nelinearnih (NLARX) modela (mijenjati kombinacije regresora) uz prikaz najboljeg polinoma na osnovu RMSE greške i dati komentar.

Matlab kod za učittavanje podataka:

```
Struja=[xlsread('Podaci3', 'Podaci', 'B3001:B3100');];
Napon=[xlsread('Podaci3', 'Podaci', 'C3001:C3100');];
Snaga=[xlsread('Podaci3', 'Podaci', 'D3001:D3100');];
Otpor=[xlsread('Podaci3', 'Podaci', 'E3001:E3100');];
sys=iddata([Snaga,Otpor], [Struja,Napon],1);
```



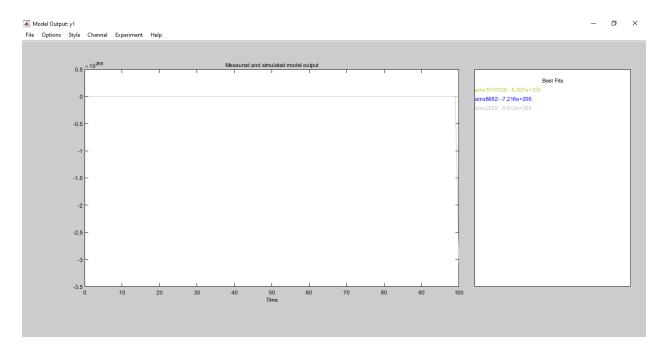
Slika34:Grafički prikaz ARX modela različitog reda polinoma

```
amx8882 =
  Discrete-time ARMAX model:
       Model for output "yl": A(z)y_1(t) = -A_i(z)y_i(t) + B(z)u(t) + C(z)e_1(t)
                   \texttt{A(z)} = 1 - 117.4 \ \texttt{z}^{-1} + 115.3 \ \texttt{z}^{-2} + 0.1788 \ \texttt{z}^{-3} + 0.2042 \ \texttt{z}^{-4} - 0.1354 \ \texttt{z}^{-5} - 0.3358 \ \texttt{z}^{-6} + 0.09207 \ \texttt{z}^{-7} 
                    \texttt{A\_2}\left(\texttt{z}\right) \ = \ 14.26 \ \texttt{z}^{\mathsf{-}1} \ - \ 15.99 \ \texttt{z}^{\mathsf{-}2} \ - \ 0.3924 \ \texttt{z}^{\mathsf{-}3} \ + \ 0.8421 \ \texttt{z}^{\mathsf{-}4} \ + \ 0.05188 \ \texttt{z}^{\mathsf{-}5} \ - \ 0.3043 \ \texttt{z}^{\mathsf{-}6} \ + \ 0.02074 \ \texttt{z}^{\mathsf{-}7} \ + \ 0.00188 \ \texttt{z}^{\mathsf{-}8} \ + \
                 B1(z) = -128.3 z^{-1} + 125.9 z^{-2}
                   B2(z) = -2063 z^{-1} + 2064 z^{-2}
                  \texttt{C(z)} = 1 + 0.1917 \ \texttt{z}^{-1} - 0.1135 \ \texttt{z}^{-2} + 0.5129 \ \texttt{z}^{-3} + 0.5528 \ \texttt{z}^{-4} + 0.005924 \ \texttt{z}^{-5} - 0.2444 \ \texttt{z}^{-6} + 0.03052 \ \texttt{z}^{-7} + 0.03052 \ 
            \texttt{A(z)} = \texttt{1} + \texttt{11.97} \ \texttt{z^{-1}} - \texttt{14.06} \ \texttt{z^{-2}} - \texttt{0.2247} \ \texttt{z^{-3}} + \texttt{0.3232} \ \texttt{z^{-4}} + \texttt{0.001434} \ \texttt{z^{-5}} - \texttt{0.5521} \ \texttt{z^{-6}} + \texttt{0.1009} \ \texttt{z^{-7}} 
                                                                                                                                                                                                                                                                                                                                                                                                                                      + 0.3377 z^-8
                    \texttt{A} \ \texttt{1(z)} \ = \ -12.68 \ \texttt{z}^{-1} \ + \ 12.02 \ \texttt{z}^{-2} \ + \ 0.2897 \ \texttt{z}^{-3} \ - \ 0.2382 \ \texttt{z}^{-4} \ + \ 0.08567 \ \texttt{z}^{-5} \ - \ 0.3862 \ \texttt{z}^{-6} \ + \ 0.1971 \ \texttt{z}^{-7} 
                                                                                                                                                                                                                                                                                                                                                                                                                                       + 0.1461 z^-8
 amx2222 =
  Discrete-time ARMAX model:
             Model for output "yl": A(z)y_1(t) = -A_i(z)y_i(t) + B(z)u(t) + C(z)e_1(t)
                          A(z) = 1 - 487.2 z^{-1} + 490.3 z^{-2}
                          A 2(z) = 195.6 z^{-1} - 190.3 z^{-2}
                          B1(z) = -421.3 z^{-1} + 430.2 z^{-2}
                          B2(z) = -1.013e04 z^{-1} + 1.013e04 z^{-2}
                          C(z) = 1 - 0.1143 z^{-1} - 0.2304 z^{-2}
             Model for output "y2": A(z)y = -Ai(z)y i(t) + B(z)u(t) + C(z)e = 2(t)
                          A(z) = 1 - 4.848 z^{-1} + 3.292 z^{-2}
                          A 1(z) = 26.35 z^{-1} - 26.6 z^{-2}
                           B1(z) = 27.55 z^{-1} - 28.65 z^{-2}
                           B2(z) = 481.8 z^{-1} - 477.2 z^{-2}
                           C(z) = 1 + 0.74 z^{-1} - 0.05469 z^{-2}
```

Name: amx2222

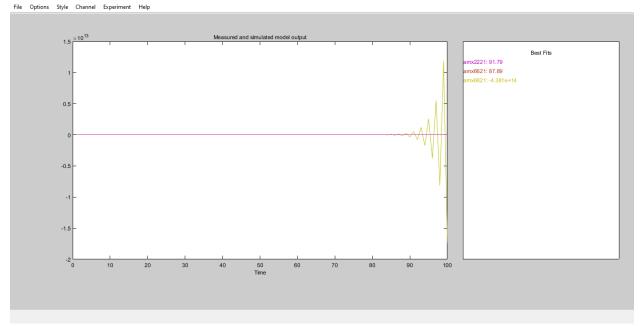
Sample time: 1 seconds

Na predhodne 3 slike imamo prikaz eksportovanih ARX modela u workspaceu gdje pozivom odgovarajućeg modela vidimo koliko je uspješna estimacija odnosno koliko iznosi MSE a samim time I RMSE greška. Za prvi model greska je MSE: 0.0002359, drugi MSE: 0.0002364 I treći MSE: 0.0003459.



Slika35: Grafički prikaz ARMAX modela različite složenosti





Slika35: Grafički prikaz ARMAX modela različite složenosti

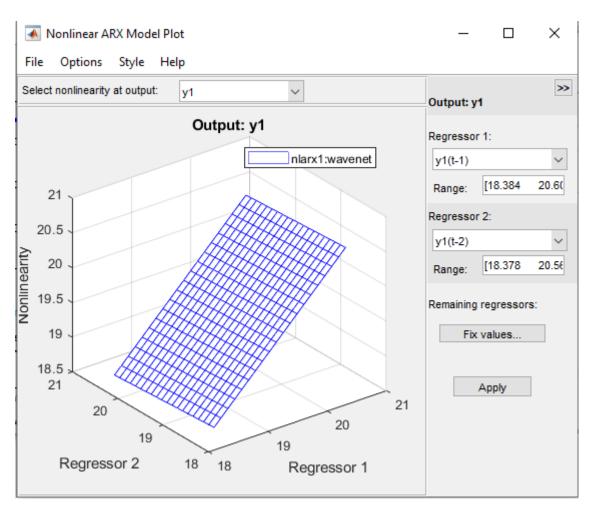
```
Name: amx8882
Sample time: 1 seconds

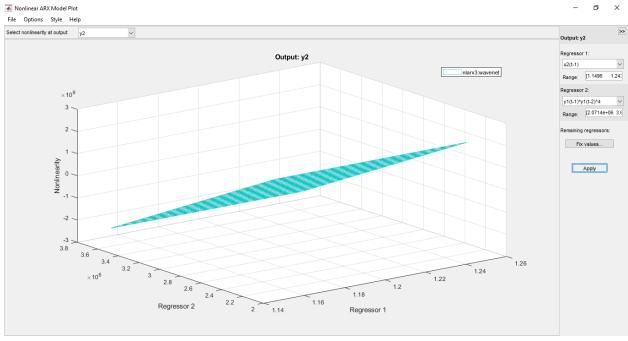
Parameterization:
   Polynomial orders: na=[8 8;8 8] nb=[2 2;2 2] nc=[7;7] nk=[1 1;1 1]
   Number of free coefficients: 54
   Use "polydata", "getpvec", "getcov" for parameters and their uncertainties.

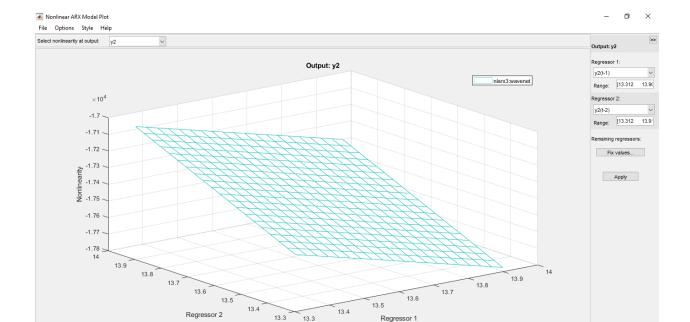
Status:
Estimated using PEM on time domain data "sys".
Fit to estimation data: [96.81;95.4]% (prediction focus)
FPE: 7.93e-08, MSE: 0.0003459
>> amx2222
```

```
amx8882 =
Discrete-time ARMAX model:
      \label{eq:model_for_output} \mbox{Model for output "yl": } \mbox{$\mathbb{A}(z)$\,y$_l(t) = - $\mathbb{A}_l(z)$\,y$_l(t) + $\mathbb{B}(z)$\,u(t) + $\mathbb{C}(z)$\,e$_l(t) } 
              \texttt{A\_2}(\texttt{z}) \ = \ 14.26 \ \texttt{z}^-\texttt{1} \ - \ 15.99 \ \texttt{z}^-\texttt{2} \ - \ 0.3924 \ \texttt{z}^-\texttt{3} \ + \ 0.8421 \ \texttt{z}^-\texttt{4} \ + \ 0.05188 \ \texttt{z}^-\texttt{5} \ - \ 0.3043 \ \texttt{z}^-\texttt{6} \ + \ 0.02074 \ \texttt{z}^-\texttt{7} \ ) 
              B1(z) = -128.3 z^{-1} + 125.9 z^{-2}
             B2(z) = -2063 z^{-1} + 2064 z^{-2}
             \texttt{C(z)} = \texttt{1} + \texttt{0.1917} \ \texttt{z} - \texttt{1} - \texttt{0.1135} \ \texttt{z} - \texttt{2} + \texttt{0.5129} \ \texttt{z} - \texttt{3} + \texttt{0.5528} \ \texttt{z} - \texttt{4} + \texttt{0.005924} \ \texttt{z} - \texttt{5} - \texttt{0.2444} \ \texttt{z} - \texttt{6} + \texttt{0.03052} \ \texttt{z} - \texttt{7} - \texttt{0.2444} \ \texttt{2} - \texttt{6} + \texttt{0.03052} \ \texttt{2} - \texttt{3} + \texttt{0.5528} \ \texttt{3} - \texttt{3} - \texttt{3} + \texttt{0.5528} \ \texttt{3} - \texttt{3} -
       Model for output "y2": A(z)y_2(t) = -A_i(z)y_i(t) + B(z)u(t) + C(z)e_2(t)
              A(z) = 1 + 11.97 \ z^{-1} - 14.06 \ z^{-2} - 0.2247 \ z^{-3} + 0.3232 \ z^{-4} + 0.001434 \ z^{-5} - 0.5521 \ z^{-6} + 0.1009 \ z^{-7} 
  amx2222 =
 Discrete-time ARMAX model:
          Model for output "yl": A(z)y_1(t) = -A_i(z)y_i(t) + B(z)u(t) + C(z)e_1(t)
                    A(z) = 1 - 487.2 z^{-1} + 490.3 z^{-2}
                    A 2(z) = 195.6 z^{-1} - 190.3 z^{-2}
                    B1(z) = -421.3 z^{-1} + 430.2 z^{-2}
                    B2(z) = -1.013e04 z^{-1} + 1.013e04 z^{-2}
                    C(z) = 1 - 0.1143 z^{-1} - 0.2304 z^{-2}
           Model for output "y2": A(z)y 2(t) = -Ai(z)yi(t) + B(z)u(t) + C(z)e 2(t)
                    A(z) = 1 - 4.848 z^{-1} + 3.292 z^{-2}
                    A 1(z) = 26.35 z^{-1} - 26.6 z^{-2}
                    B1(z) = 27.55 z^{-1} - 28.65 z^{-2}
                    B2(z) = 481.8 z^{-1} - 477.2 z^{-2}
                     C(z) = 1 + 0.74 z^{-1} - 0.05469 z^{-2}
```

Na predhodne 3 slike imamo prikaz eksportovanih ARX modela u workspaceu gdje pozivom odgovarajućeg modela vidimo koliko je uspješna estimacija odnosno koliko iznosi MSE a samim time I RMSE greška. Za prvi model greska je MSE: 0.0002359, drugi MSE: 0.0002364 I treći MSE: 0.004081







Na predhodne tri slike prikazan nelinearan ARX model različite složenost

```
nlarx1 =
 Nonlinear ARX model with 2 outputs and 2 inputs
  Inputs: ul, u2
   Outputs: yl, y2
 Regressors:
   Linear regressors in variables yl, y2, u1, u2
   List of all regressors
 Output functions:
   Output 1: Wavelet Network with 32 units
   Output 2: Wavelet Network with 35 units
 Name: nlarx1
 Sample time: 1 seconds
 Status:
 Estimated using NLARX on time domain data "sys".
 Fit to estimation data: [99.05;98.02]% (prediction focus)
FPE: 2.442e-09, MSE: 3.525e-05
```

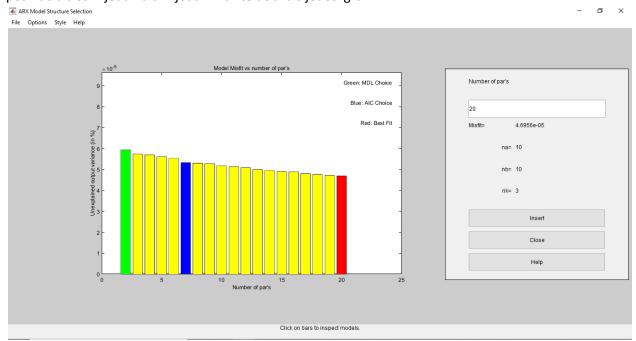
```
nlarx2 =
Nonlinear ARX model with 2 outputs and 2 inputs
  Inputs: ul, u2
  Outputs: yl, y2
Regressors:
  1. Linear regressors in variables yl, y2, ul, u2
  2. Order 2 regressors in variables yl, y2, ul, u2
  List of all regressors
Output functions:
  Output 1: Wavelet Network with 78 units
  Output 2: Wavelet Network with 79 units
Name: nlarx2
Sample time: 1 seconds
Status:
Estimated using NLARX on time domain data "sys".
Fit to estimation data: [100;100]% (prediction focus)
MSE: 1.121e-16
nlarx3 =
Nonlinear ARX model with 2 outputs and 2 inputs
  Inputs: ul, u2
  Outputs: yl, y2
Regressors:
  1. Linear regressors in variables y1, y2, u1, u2
  2. Order 5 regressors in variables y1, y2, u1, u2
 List of all regressors
Output functions:
  Output 1: Wavelet Network with 73 units
  Output 2: Wavelet Network with 73 units
Name: nlarx3
Sample time: 1 seconds
Status:
Estimated using NLARX on time domain data "sys".
Fit to estimation data: [99.99;74.08]% (prediction focus)
MSE: 0.001491
```

Na prehodne tri slike imamo prikaz estimacije I možemo zapayiti koliko je uspješna estimacija u odnosu na greske.

Komentar:

U drugom zadatku vršimo estimaciju parametara modelske strukture za ulazno-izlazne podatke pomoću system identification appa . Prethodno je potrebno podatke učitati u varijablu tipa iddata I tako ih spremiti za estimaciju. Za ARX model koristimo različite redove polinoma I upoređujući ih zaključujemo koji najviše odgovara. Sličan postupak provodimo i za ARMAX i NLARX modele. Ponovo mijenjamo parametre pa upoređujemo dobijene rezultate. Svaki od dobijenih modela eksportujemo u workspace te pokretanjem pojedinog modela dobijamo informacije o uspješnosti estimacije odnosno o MSE a samim tim i RMSE greški. (RMSE = VMSE). Računajući RMSE greške zaključujemo da je najbolji ARX627 model njegova RMSE greška je 0.01.

Dodatak, obzirom da mi opcija order selection nije bila ponuđena za sistem za dva ulaza I izlaza, posmatrala sam jedan ulaz I jedan izlaz te dobila sljedeci grafik.



Komentar:

Za ARX model odabiremo opciju order selection pa estimate i na taj način dobijamo grafički prikaz usporedbe složenosti modela i grešaka. Vidimo da najmanju grešku ima model označen crvenom bojom, skroz desno, ali je on i najsloženiji. Međutim njegova greška je zanemarivo manja u odnosu na neke modele koji su jednostavniji pa je bolje odabrati neki jednostavniji sa neznatno većom greškom. S druge strane, prvi model na lijevoj strani ima najveću grešku ali je i najjednostavniji. S toga je potrebno napraviti kompromis te izabrati model sa optimalnom složenosti i prihvatljivom greškom.