# The Binary World

Binary values and number systems
Chapter 2

# Dichotomy

- Logic:
A statement is either True or False
Often represented by a T or F.

- Electrical Circuits:
A lamp will be On or Off
A circuit can have current flow or no flow (switch on or off)

- Polarity:
Positive or negative.

- Binary – composed of, relating to, or involving 2.

# Alternative Number Systems

How many ones make up 642???

600 + 40 + 2 ?

i.e. 6 hundreds, 4 tens, and 2 ones

$6x10^2 + 4x10^1 + 2x10^0$

What would it mean if we counted in eights?

$6x8^2 + 4x8^1 + 2x8^0$

Then it would mean 418 counting in tens.

# Positional Notation

642 in base 10 *positional notation* is:

$$
\begin{aligned}
& 6 \times 10^2 = 6 \times 100 = 600 \\
+\ & 4 \times 10^1 = 4 \times 10 = 40 \\
+\ & 2 \times 10^0 = 2 \times 1 = 2 \qquad = 642 \text{ in base } 10
\end{aligned}
$$

This number is in base 10

The power indicates the position of the number

- 6 hundreds and 4 tens and 2 ones is the representation of this number in BASE 10.
- The base of a number determines the number of digits and the value of digit positions.
- What is special about 10?

# Positional Notation

R is the base of the number

As a formula:

$$d_n * R^{n-1} + d_{n-1} * R^{n-2} + \dots + d_2 * R + d_1$$

n is the number of digits in the number

d is the digit in the $i^{th}$ position in the number

$642$ is $6_3 * 10^2 + 4_2 * 10 + 2_1$

# Thought problem…

- Can you show that this representation must always be unique?

# Binary Numbers

- Binary numbers are numbers where
  the **BASE IS 2**
- This means there can only be two digits **0** and **1** (Why?)
- Counting in Binary:
  0, 1, 10, 11, 100, 101, 110, 111, 1000, 1001 and so on…
- Why does Base 2 interest us so much?

# Octal and Hexadecimal

- Octal is where we use BASE 8
- The digits are then 0, 1, 2, 3, 4, 5, 6, 7

- Hexadecimal is where we use BASE 16
- The digits are then 0, 1, 2,3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F

# Counting

| Binary | Octal | Decimal |
|---|---|---|
| 0 | 0 | 0 |
| 1 | 1 | 1 |
| 10 | 2 | 2 |
| 11 | 3 | 3 |
| 100 | 4 | 4 |
| 101 | 5 | 5 |
| 110 | 6 | 6 |
| 111 | 7 | 7 |
| 1000 | 10 | 8 |
| 1001 | 11 | 9 |
| 1010 | 12 | 10 |

# Example

- What is **1010** (as a base 10 number) if the base is 2, 8, 10 and 16?
- **10**
- **520**
- **1010**
- **4112**

# REVIEW – The Binary World

1. Computers represent data in binary code:

   off/on, false/true, 0/1;
2. Thus we would like to represent <u>numbers</u> in such a format;
3. We can represent numbers in <u>any base</u>. We are used to base 10 but this is habit; it is not driven by mathematics;

   E.g. 642 is $6_3 * 10^2 + 4_2 * 10 + 2_1$
4. In base R the number $d_n\, d_{n-1} \ldots d_2\, d_1$ means:

   $d_n * R^{n-1} + d_{n-1} * R^{n-2} + \ldots + d_2 * R + d_1$
5. Arithmetic works in any base exactly as in base 10.

# Positional Notation

As a formula:

$$d_n * R^{n-1} + d_{n-1} * R^{n-2} + \dots + d_2 * R + d_1$$

R is the base of the number

n is the number of digits in the number

d is the digit in the $i^{th}$ position in the number

$642_{10}$ is

$$1202 = 1_4 * 8^3 + 2_3 * 8^2 + 0_2 * 8^1 + 2_1$$

$$1_{10}0_9 1_8 0_7 0_6 0_5 0_4 0_3 1_2 0_1$$

# Arithmetic

Addition in Base 2:
We want to add the two numbers

1010111 and
1001011

Remember that there are only 2 digits in binary, 0 and 1

1 + 1 is 0 with a carry

Carry Values

```
 1 0 1 1 1 1 1
   1 0 1 0 1 1 1
 +1 0 0 1 0 1 1
 1 0 1 0 0 0 1 0
```

# Subtraction in binary

- Let's take 111011 away from 1010111.

# Subtraction in binary

*Remember borrowing?* Apply that concept here:

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| | | 1 | 2 | | | | |
| | | 2 | 0 | 2 | | | |
| | 1 | 0 | 1 | 0 | 1 | 1 | 1 |
| − | | 1 | 1 | 1 | 0 | 1 | 1 |
| | 0 | 0 | 1 | 1 | 1 | 0 | 0 |

# Converting Binary to Octal

Mark groups of *three* (from right)

This is because three binary digits exactly represent the octal digits 0, 1, 2, 3, 4, 5, 6, 7

i.e. 000, 001, 010, 011, 100, 101, 110, 111

Example: Convert 10101011 to octal

| | | |
|---|---|---|
| 010 | 101 | 011 |
| 2 | 5 | 3 |

10101011 in base 2 is 253 in base 8

# Converting Binary to Hexadecimal

Mark groups of *four* (from right)

This is because each group of four binary digits represents one hexadecimal digit

Example: Convert 10101011(binary) to hex.

$\underline{1010}$ $\underline{1011}$

A B

10101011 is AB in base 16

# Converting Decimal to Other Bases

Algorithm for converting number in base 10 to other bases:

*While (the quotient is not zero)*

- *Divide the decimal number by the new base*
- *Make the remainder the next digit to the left in the answer*
- *Replace the original decimal number with the quotient*

# Converting Decimal to Other Bases

- We write out the number in decimal, but expand it in terms of the new base R:

$N = d_n * R^{n-1} + d_{n-1} * R^{n-2} + ... + d_2 * R + d_1$

- Divide by the base and take the remainder:

$N/R = d_n * R^{n-2} + d_{n-1} * R^{n-3} + ... + d_3R + d_2$

Remainder is $d_1$.

Repeating:

$N/R^2 = d_n * R^{n-3} + d_{n-1} * R^{n-4} + ... + d_4R + d_3$

Remainder is $d_2$.

AND SO ON…

# Converting Decimal to Octal

*What is 1988 (base 10) in base 8?*

# Converting Decimal to Octal

| | | | |
|---|---|---|---|
| 248 | 31 | 3 | 0 |
| 8 ⟌ 1988 | 8 ⟌ 248 | 8 ⟌ 31 | 8 ⟌ 3 |
| 16 | 24 | 24 | 0 |
| 38 | 08 | **7** | **3** |
| 32 | 8 | | |
| 68 | **0** | | |
| 64 | | | |
| **4** | | | |

**Answer is : 3 7 0 4**

# Converting Decimal to Hexadecimal

What is 3567 (base 10) in base 16?

# Converting Decimal to Hexadecimal

| | | |
|---|---|---|
| 222 | 13 | 0 |
| 16 )3567 | 16 )222 | 16 )13 |
| 32 | 16 | 0 |
| 36 | 62 | 13 |
| 32 | 48 | |
| 47 | 14 | |
| 32 | | |
| 15 | | |

**So result is DEF**

# To the Right of the Positional "point"

- We can extend the idea of working to different bases to the fractional part of a number.

- What does 645.456 mean in Base 10?

- Why not do this in other bases???

# Representation in Base 2

$d_n * 2^{n-1} + d_{n-1} * 2^{n-2} + \ldots + d_2 * 2^1 + d_1 * 2^0$ (radix point) $+ d_{-1} * 2^{-1} + d_{-2} * 2^{-2} + d_{-3} * 2^{-3} + \ldots$

Thus (binary) 101.011means:

$2^2 + 2^0 + 1/2^2 + 1/2^3$

$= 5\ {}^3/_8$

What would hexadecimal A5.FF be in decimal?

$165\ {}^{255}/_{256}$

# Converting Fractions

- Suppose the fraction is in Base 10 and we want to convert to another base.
  - *Multiply the decimal part of the number by the new base*
  - *The integer part is the next digit to the right in the answer*
  - *Replace the original decimal number with the fractional part*
  - *Continue until…*

# Converting Fractions

$d_n * 2^{n-1} + d_{n-1} * 2^{n-2} + \ldots + d_2 * 2^1 + d_1 * 2^0$ (radix point) $+ d_{-1} * 2^{-1} + d_{-2} * 2^{-2} + d_{-3} * 2^{-3} + \ldots$

- Keep only the radix part:
- (radix point) $+ d_{-1} * 2^{-1} + d_{-2} * 2^{-2} + d_{-3} * 2^{-3} + \ldots$

- Multiply by 2:
- $d_{-1}$ (radix point) $+ d_{-2} * 2^{-1} + d_{-3} * 2^{-2} + \ldots$

# Example

- Convert 0.1to binary, hexadecimal, and octal
- $.0\overline{0011}$,
- $.1\overline{9}$
- $.0\overline{6314}$

# Example

- Subtraction of Binary fractions
- 10110.000
- − 1010.011
- ANS 1011.101

# Binary and Computers

Bit (Binary digit)

Byte 8 bits

The number of bytes in a word determines the word length of the computer, but it is usually a multiple of 8

32-bit machines
64-bit machines

# What you should understand/be able to do

- Understand the importance of Binary digits to data representation in computers;
- Understand the positional representation of numbers in different bases;
- Understand the relationship between base 2, and base 8 and base 16 numbers;
- Convert from base R to base 10;
- Convert from base 10 to base R;
- Do arithmetic in different bases.