Assessing the insurance motive of long debt

Suppose endowment is constant, that is, $y_t = y$, and the government gets iid punishment shocks $\tau \sim iid$. The problem can be rewritten such as:

$$V(\tau, b_D, b_F) = \max\{V^D(\tau, b_D, b_F), V^{ND}(b_D, b_F)\} \tag{1}$$

Value of Default:

$$V^D(\tau, b_D, b_F) = u\big((1-\tau)y\big) + \beta \mathbb{E}_{\tau'}\Big[\theta V(\tau', \nu_D b_D, \nu_F b_F) + (1-\theta)V^D(\tau, b_D, b_F)\Big] \tag{2}$$

Value of Repayment:

$$V^{ND}(b_D, b_F) = \max_{b'_D, b'_F} u(c) + \beta \mathbb{E}_{\tau'}[V(\tau', b'_D, b'_F)] \tag{3}$$

subject to:

$$c + \lambda_D b_D + \lambda_F b_F = y + q_F^{ND}(b'_D, b'_F)[b'_F - (1-\lambda_F)b_F] + q_D^{ND}(b'_D, b'_F)[b'_D - (1-\lambda_D)b_D] \tag{4}$$

$$\delta(b'_D, b'_F) \equiv \mathbb{E}_{\tau'}[d(\tau', b'_D, b'_F)] \leq \bar{\delta} \tag{5}$$

Since τ is iid, we can drop it as a state variable from the prices and repayment value. In this world, consumption in the ND state is deterministic and driven purely by the stock of debt. Prices under default are given by:

$$\begin{aligned} q_i^D(b_D, b_F) &= \frac{1-\theta}{1+r} \times q_i^D(b_D, b_F) + \theta \nu_i q_i^{ND}(\nu_D b_D, \nu_F, b_F) \\ q_i^D(b_D, b_F) &= \frac{1+r}{\theta + r} \times \theta \nu_i q_i^{ND}(\nu_D b_D, \nu_F, b_F) \end{aligned}$$

The following expression gives prices under non-default:

$$\begin{aligned} q_i^{ND}(b'_D, b'_F) = \big(1 - \delta(b'_D, b'_F)\big) &\times \frac{\lambda_i + (1-\lambda_i) q_i^{ND}(b''_D, b''_F)}{1+r} \\ + \delta(b'_D, b'_F) &\times \frac{q_i^D(b'_D, b'_F)}{1+r} \end{aligned}$$

An Arellano insurance-free model

We start with the simplest possible version of the above model in which $b_F = 0$ and rename $b_D = b$. We also set $\lambda = 1$ and $\nu = 0$ (short-term debt with zero recovery rates).

$$V(\tau, b) = \max\{V^D(\tau), V^{ND}(b)\} \tag{6}$$

Value of Default:

$$\begin{aligned} V^D(\tau) =& u\big((1-\tau)y\big) + \beta\left\{\theta\mathbb{E}_{\tau'}\Big[V(\tau', 0)\Big] + (1-\theta)V^D(\tau)\right\} \\ V^D(\tau) =& \frac{u\big((1-\tau)y\big) + \beta\theta\mathbb{E}_{\tau'}\Big[V(\tau', 0)\Big]}{1-\beta(1-\theta)} \end{aligned}$$

Value of Repayment:

$$V^{ND}(b) = \max_{b'} u(c) + \beta\mathbb{E}_{\tau'}[V(\tau', b')] \tag{7}$$

subject to:

$$c = y - b + q(b')b' \tag{8}$$

Doubt. Why is the debt policy constant: $b'(b) = \overline{b}$?

References