TP 7 - Inducción Matemática

1) $n=1$: $4^2-1=15$ ✓

$n=k$: Supongo Verdadero para algún $k \in \mathbb{Z}^+, k>1$

$$4^{2k}-1=15q_1 \quad q_1 \in \mathbb{Z}^+$$

$n=k+1$: ¿ $4^{2(k+1)}-1=15q$?

$$4^{2(k+1)}-1=4^{2k+2}-1=16.4^{2k}-1=(15+1).4^{2k}-1=$$

$$=\underbrace{1.4^{2k}-1}_{H.I.}+\underbrace{15.4^{2k}}_{\dot{15}}=15q \checkmark \quad q \in \mathbb{Z}^+$$

2) $$\sum_{i=1}^{n} i.5^i=\frac{5+(4n-1).5^{n+1}}{16}, \quad n \in \mathbb{N}$$

$n=1$: $1.5^1=5$; $\frac{5+(4.1-1).5^{1+1}}{16}=\frac{5+3.5^2}{16}=5$ ✓

$n=k$: Sup. Verdadero para algún $k \in \mathbb{Z}^+, k>1$

$$\sum_{i=1}^{k} i.5^i=\frac{5+(4k-1).5^{k+1}}{16}$$

$n=k+1$: ¿ $\sum_{i=1}^{k+1} i.5^i=\frac{5+[4(k+1)-1].5^{k+2}}{16}$?

¿ $\sum_{i=1}^{k+1} i.5^i=\frac{5+[4k+3].5^{k+2}}{16}$?

$$\sum_{i=1}^{k+1} i.5^i=\underbrace{\sum_{i=1}^{k} i.5^i}_{H.I}+(k+1).5^{k+1}=\frac{5+(4k-1).5^{k+1}}{16}+(k+1).5^{k+1}=$$

$$=\frac{5+(4k-1).5^{k+1}+16(k+1).5^{k+1}}{16}=\frac{5+(20k+15).5^{k+1}}{16}=$$

$$=\frac{5+5^1.(4k+3).5^{k+1}}{16}=\frac{5+5^{k+2}(4k+3)}{16} \checkmark$$

3) $\sum_{i=1}^{n} 1/\sqrt{i}>\sqrt{n} \quad \forall n \in \mathbb{N}, n \geq 2$

$n=2$) $1+1/\sqrt{2} \simeq 1{,}70>\sqrt{2}$

$n=k$) Supongo Verdadero para algún $k>2$, $k \in \mathbb{N}$

$$\sum_{i=1}^{k} 1/\sqrt{i}>\sqrt{k}$$

$n=k+1)$ $¿\sum_{i=1}^{k+1} 1/\sqrt{i} > \sqrt{k+1}\,?$

$$\sum_{i=1}^{k+1} 1/\sqrt{i} = \underbrace{\sum_{i=1}^{k} 1/\sqrt{i}}_{H.I} + 1/\sqrt{k+1} > \sqrt{k} + \frac{1}{\sqrt{k+1}} = \frac{\sqrt{k}.\sqrt{k+1}+1}{\sqrt{k+1}} >$$

$$> \frac{\sqrt{k}.\sqrt{k}+1}{\sqrt{k+1}} = \frac{\sqrt{k}^2+1}{\sqrt{k+1}} = \frac{(k+1)^1}{(k+1)^{1/2}} = (k+1)^{1/2} = \sqrt{k+1}$$

$$\therefore \sum_{i=1}^{k+1} 1/\sqrt{i} > \sqrt{k+1} \checkmark$$

4) $\sum_{i=1}^{n} i.i! = (n+1)! - 1 \quad \forall n \in \mathbb{N}$

$n=1)$ $1.1! = 1$; $(1+1)! - 1 = 1$ ✓

$n=k)$ Supongo Verdadero para algún $k>1$, $k \in \mathbb{N}$

$$\sum_{i=1}^{k} i.i! = (k+1)! - 1$$

$n=k+1)$ $¿\sum_{i=1}^{k+1} i.i! = (k+2)! - 1\,?$

$$\sum_{i=1}^{k+1} i.i! = \underbrace{\sum_{i=1}^{k} i.i!}_{H.I} + (k+1).(k+1)! = \underbrace{(k+1)!} - 1 + \underbrace{(k+1)(k+1)!} =$$ Factor Común $(k+1)!$

$$= (k+1)!\,[1+(k+1)] - 1 =$$

$$= (k+1)!\,(k+2) - 1 = (k+2)! - 1 \checkmark$$

5) $7^{n+2} + 8^{2n+1} = 57q$, $q \in \mathbb{N}$, $n \in \mathbb{N}$

$n=1)$ $7^3 + 8^3 = 855 = 57.15$ ✓

$n=k)$ Supongo Verdadero para algún $k>1$, $k \in \mathbb{N}$: $7^{k+2} + 8^{2k+1} = 57q_1$; $q_1 \in \mathbb{N}$

$n=k+1)$ $¿\,7^{k+3} + 8^{2(k+1)+1} = 7^{k+3} + 8^{2k+3} = 57q$, $q \in \mathbb{N}\,?$

$$7^{k+3} + 8^{2k+3} = 7.7^{k+2} + \underbrace{8^2}_{57+7}.8^{2k+1} = 7.7^{k+2} + 7.8^{2k+1} + 57.8^{2k+1} =$$

$$= 7.(\underbrace{7^{k+2} + 8^{2k+1}}_{H.I}) + 57.8^{2k+1} = 7.57q_1 + 57.8^{2k+1} =$$

$$= 57.q \checkmark$$

6) $\sum_{i=2}^{n} (i-1).i = \frac{n(n^2-1)}{3}$ $\quad n \in \mathbb{N}, n \geq 2$

$n=2)$ $(2-1).2 = 2$; $\frac{2(4-1)}{3} = 2$ ✓

$n=k)$ Sup Verdadero para algún $k>2, k\in\mathbb{N}$: $\sum_{i=2}^{k} (i-1).i = \frac{k(k^2-1)}{3}$

$n=k+1)$ ¿ $\sum_{i=2}^{k+1} (i-1).i = \frac{(k+1)[(k+1)^2-1]}{3}$?

¿ $\sum_{i=2}^{k+1} (i-1)i = \frac{(k+1)(k^2+2k)}{3} = \frac{(k+1)k(k+2)}{3}$?

$\sum_{i=2}^{k+1} (i-1)i = \underbrace{\sum_{i=2}^{k} (i-1).i}_{H.I.} + (k+1-1)(k+1) = \frac{k(k^2-1)}{3} + (k+1).k = \frac{k(k+1)(k-1)+3k(k+1)}{3} =$

$= \frac{k(k+1)}{3}(k-1+3) = \frac{k(k+1)(k+2)}{3}$ ✓

7) $n^2-1 = 8q$, $q\in\mathbb{N}$ n impar , $n = (2t+1)^2-1 = 4t^2+4t+1-1$

no lo por inducción

$= 4t^2+4t =$

$= 4\underbrace{t(t+1)}_{par} = 8q$

8) $n^2-7n+12 \geq 0$, $n>3$

$n=4)$ $4^2-7.4+12 = 0$ ✓

$n=k)$ Sup Verdadero para algún $k>4$, $k\in\mathbb{N}$: $k^2-7k+12\geq 0$

$n=k+1)$ ¿$(k+1)^2-7(k+1)+12\geq 0$?

$k^2+2k+1-7k-7+12 = \underbrace{k^2-7k+12}_{H.I} + (2k+1-7) =$

$= \underbrace{k^2-7k+12}_{\geq 0} + (2k-6)$

Como $k>4$, $2k-6>0$ por lo que $(k+1)^2-7(k+1)+12\geq 0$ ✓

9) $4^n \geq 16n^2$, $n\geq 4$, $n\in\mathbb{N}$

$n=4)$ $4^4 = 256$; $16.4^2 = 256$ ✓

$n=k)$ Sup Verdadero para algún $k>4$, $k\in\mathbb{N}$: $4^k\geq 16k^2$

$n=k+1)$ ¿ $4^{k+1}\geq 16(k+1)^2$?

H.I $\rightarrow$ $4^k \geq 16.k^2 > 16k$

$4^k \geq 16.k$

$4^k \geq 16.k > 16.1$

$4^k \geq 16.1$

$4.4^k \geq 16(k^2+2k+1)$ $\Rightarrow$ $4^{k+1}\geq 16(k+1)^2$ ✓

10) $\sum_{k=1}^{n} \frac{k}{(k+1)!} = 1 - \frac{1}{(n+1)!} \quad \forall n \in \mathbb{N}$.

$n=1)$ $\frac{1}{2!} = \frac{1}{2}$; $1 - \frac{1}{2!} = \frac{1}{2}$ ✓

$n=t)$ Sup Verdadero para algún $t>1$, $k \in \mathbb{N}$: $\sum_{k=1}^{t} \frac{k}{(k+1)!} = 1 - \frac{1}{(t+1)!}$

$n=t+1)$ ¿ $\sum_{k=1}^{t+1} \frac{k}{(k+1)!} = 1 - \frac{1}{(t+2)!}$?

$$\sum_{k=1}^{t+1} \frac{k}{(k+1)!} = \underbrace{\sum_{k=1}^{t} \frac{k}{(k+1)!}}_{H.I.} + \frac{t+1}{(t+2)!} = 1 - \frac{1}{(t+1)!} + \frac{t+1}{(t+2)!} = 1 + \frac{t+1}{(t+2)!} - \frac{(t+2)}{(t+2)(t+1)!} =$$

$$= 1 + \frac{t+1-t-2}{(t+2)!} = 1 - \frac{1}{(t+2)!} \checkmark$$

11) $\sum_{i=1}^{n} (2i-1) \cdot 3^{i} = (n-1) \cdot 3^{n+1} + 3 \quad \forall n \in \mathbb{N}$

$n=1)$ $(2 \cdot 1 - 1) \cdot 3^{1} = 3$; $(1-1) \cdot 3^{2} + 3 = 3$ ✓

$n=k)$ Sup. Verdadero para algún $k>1$, $k \in \mathbb{N}$: $\sum_{i=1}^{k} (2i-1) \cdot 3^{i} = (k-1) \cdot 3^{k+1} + 3$

$n=k+1)$ ¿ $\sum_{i=1}^{k+1} (2i-1) \cdot 3^{i} = k \cdot 3^{k+2} + 3$?

$$\sum_{i=1}^{k+1} (2i-1) \cdot 3^{i} = \underbrace{\sum_{i=1}^{k} (2i-1) \cdot 3^{i}}_{H.I} + [2(k+1)-1] \cdot 3^{k+1} = (k-1) \cdot 3^{k+1} + 3 + (2k+1) \cdot 3^{k+1} =$$

$$= 3^{k+1} \cdot (k-1+2k+1) + 3 =$$

$$= k \cdot 3^{k+1} + 3 \checkmark$$

TP8 – Recursión

1) $S_n = \frac{n^2+n+2}{2} \quad n \geq 1$

$S_1 = \frac{1^2+1+2}{2} = 2 \qquad S_3 = \frac{9+3+2}{2} = 7 \qquad S_5 = \frac{25+5+2}{2} = 16$

$S_2 = \frac{4+2+2}{2} = 4 \qquad S_4 = \frac{16+4+2}{2} = 11$

$$S_{n-1} = \frac{(n-1)^2+(n-1)+2}{2} = \frac{n^2-2n+\cancel{1}+n-\cancel{1}+2}{2} = \frac{n^2+n+2}{2} - \frac{2n}{2}$$

$$S_{n-1} = S_n - n \Rightarrow \boxed{S_n = S_{n-1} + n \quad n \geq 1}$$

2) a) $\begin{cases} a_1 = 2 \\ a_n = a_{n-1}+2 \end{cases} \quad n \geq 2$ b) $\begin{cases} a_1 = 0 \\ a_n = a_{n-1}+2 \end{cases} \quad n \geq 2$

3) $\sum_{i=0}^{2n} (-1)^{i+1} \cdot F_i = 1 - F_{2n-1} \quad n \in \mathbb{Z}^+$

$n=1$) $\sum_{i=0}^{2} (-1)^{i+1} \cdot F_i = -F_0 + F_1 - F_2 = 0+1-1 = 0$; $1 - F_1 = 1-1 = 0$ ✓

$n=k$) supongo verdadero para algún $k>1$, $k \in \mathbb{Z}$: $\sum_{i=0}^{2k} (-1)^{i+1} \cdot F_i = 1 - F_{2k-1}$

$n=k+1$) ¿ $\sum_{i=0}^{2(k+1)} (-1)^{i+1} \cdot F_i = 1 - F_{2(k+1)-1}$? ¿ $\sum_{i=0}^{2k+2} (-1)^{i+1} \cdot F_i = 1 - F_{2k+1}$?

$$\sum_{i=0}^{2k+2} (-1)^{i+1} \cdot F_i = \underbrace{\sum_{i=0}^{2k} (-1)^{i+1} F_i}_{HI} + (-1)^{2k+3} \cdot F_{2k+2} + (-1)^{2k+2} \cdot F_{2k+1} = 1 - F_{2k-1} - F_{2k+2} + F_{2k+1} =$$

$$= 1 - F_{2k-1} - (\cancel{F_{2k+1}} + F_{2k}) + \cancel{F_{2k+1}} = 1 - F_{2k-1} - F_{2k} = 1 - F_{2k+1}$$ ✓

4) $\sum_{i=1}^{n} H_i = (n+1) H_n - n, \quad n \in \mathbb{Z}^+$

$n=1$) $H_1 = 1$; $(1+1) \cdot H_1 - 1 = 2 - 1 = 1$ ✓

$n=k$) Supongo verdadero para algún $k>1$, $k \in \mathbb{Z}^+$

$$\sum_{i=1}^{k} H_i = (k+1) H_k - k$$

$n=k+1$) ¿ $\sum_{i=1}^{k+1} H_i = (k+2) H_{k+1} - (k+1)$?

$$\sum_{i=1}^{k+1} H_i = \underbrace{\sum_{i=1}^{k} H_i}_{H.I.} + H_{k+1} = (k+1) H_k - k + H_{k+1} = (k+1)\left(H_k + \frac{1}{k+1} - \frac{1}{k+1}\right) - k + H_{k+1}$$

$$= (k+1)\left(H_k + \frac{1}{k+1}\right) + (k+1)\left(-\frac{1}{k+1}\right) - k + H_{k+1} = (k+1) H_{k+1} - 1 - k + H_{k+1} =$$

$$= (k+1+1) H_{k+1} - (k+1) = (k+2) H_{k+1} - (k+1)$$ ✓

TP9 – DIVISIBILIDAD –

1) $\begin{array}{r|l} -789 & 37 \\ +814 & -22 \\ \hline 25 & \end{array}$ $\quad -789 = (-22).37 + 25$

2) a) $\begin{array}{r|l} 187 & 17 \\ & 11 \end{array}$ Rta: 11 enteros positivos menores o iguales a 187 son múltiplos de 17

b) $187 - 11 = 176$ no divisibles por 17

c) $\underset{(+)}{11} + \underset{(-)}{11} + \underset{\text{el cero}}{1} = 23$

3) a) $x|m \to m = k_1 x \; , k_1 \in \mathbb{Z}$
$x|n \to n = k_2 x \; , k_2 \in \mathbb{Z}$
$x|p \to p = k_3 x \; , k_3 \in \mathbb{Z}$

$am + bn + cp = ak_1 x + bk_2 x + ck_3 x =$
$= x(ak_1 + bk_2 + ck_3) \Rightarrow x | am + bn + cp$

b) si $a | b + a^2 \Rightarrow a|b$

$a|b+a^2 \to ak = b + a^2 \; ; \; ak - a^2 = b \; ; \; a(k-a) = b \Rightarrow a|b$

4) $k = (a^2)^2 \; ; \; 18|k$ ¿menor k?

$18|k \to k = 18.q \; , q \in \mathbb{Z}^+ \; , \; k = a^4$
$k = 2^1.3^2 q \to q = 2^3.3^2 \wedge k = 1296$

5) AF1 base hexadecimal
$10.16^2 + 15.16 + 1 = 2801$ base 10
101011110001 base 2
2 2 3 3 0 1 base 4
5 3 6 1 base 8

6) 51966 ⇒ CAFE ✓
60318 ⇒ EB9E ✗
64202 ⇒ FACA ✓

TP10 - MÁXIMO COMÚN DIVISOR

1) ¿$mcd(a,b)=1 \wedge a|bc \Rightarrow a|c$?

Si $mcd(a,b)=1 \Rightarrow a \nmid b \wedge a|bc \Rightarrow a|c$

2) $mcd(6,50)=2 \wedge 2 \nmid 17 \Rightarrow 6x+50y=17$ no tiene solución en $\mathbb{Z}$

3) $25x+2y=7$ $\quad mcd(25,2)=1 \wedge 1|7$ tiene solución

$25a+2b=1$ $\quad \begin{array}{l|l} 25 & \underline{2} \\ 1 & 12 \end{array}$ $\quad 1=25-2.12$

$$\left.\begin{array}{l} 25.1+2.(-12)=1 \\ 25.7+2.(-84)=7 \end{array}\right) \times 7$$

$$7=25.7+2.(-84) \pm 25.2.k = 25.(7-2k)+2.(-84+25k)$$

Sol: $\{(x,y) / x=7-2k, \ y=-84+25k, \ k \in \mathbb{Z}\}$

4) ¿$mcd(a,m)=1 \wedge mcd(b,m)=1 \Rightarrow mcd(ab,m)=1$?

Supongo $mcd(ab,m)=d \neq 1$

$d|ab$ $\quad$ a) Si $d|a$ Absurdo por Hip $\Rightarrow d=1$

$d|m$ $\quad$ b) Si $d|b$ Absurdo por Hip $\Rightarrow d=1$

c) Si $d|ab$ pero $d \nmid a \wedge d \nmid b$

$mcd(a,d)=a' \neq 1$

Pero $a'|d \wedge d|m \Rightarrow a'|m$ Absurdo porque $mcd(a,m)=1$ $\Rightarrow d=1$

5) $59677x+57353y=d$

$\begin{array}{l|l} 59677 & \underline{57353} \\ 2324 & 1 \end{array}$ $\quad 2324=59677-57353$

$\begin{array}{l|l} 57353 & \underline{2324} \\ 1577 & 24 \end{array}$ $\quad 1577=57353-24.2324$

$\begin{array}{l|l} 2324 & \underline{1577} \\ 747 & 1 \end{array}$ $\quad 747=2324-1577$

$\begin{array}{l|l} 1577 & \underline{747} \\ 83 & 2 \end{array}$ $\quad 83=1577-2.747$

$\begin{array}{l|l} 747 & \underline{83} \\ 0 & 9 \end{array}$ $\quad \boxed{d=83}$

$83=1577-2.747=(57353-24.2324)-$
$-2.(2324-1577)=$
$=57353-26.2324+2.1577=$
$=57353-26.(59677-57353)+2.(57353-$
$-24.2324)=$
$=29.57353-26.59677-48.2324=$
$=29.57353-26.59677-48.(59677-57353)=$
$=77.57353-74.59677=83$

$x=-74 \qquad y=77=7.11$

$7 \nmid 74 \wedge 11 \nmid 74 \Rightarrow mcd(-74,77)=1$

6) $mcd(1001,1331)=d \wedge 1001x-1331y=55$

$\begin{array}{l|l} 1331 & \underline{1001} \\ 330 & 1 \end{array}$ $\quad 330=1331-1001.1$

$\begin{array}{l|l} 1001 & \underline{330} \\ 11 & 3 \end{array}$ $\quad 11=1001-3.330$

$\begin{array}{l|l} 330 & \underline{11} \\ 0 & 30 \end{array}$ $\quad d=11$

$11=1001-3.330=1001-3.(1331-1001)=$

$$\left.11=4.1001-3.1331\right) \times 5$$

$55=1001.20-1331.15$

$55=20020-19965$ ✓

7) $d = \operatorname{mcd}(a,b) \Rightarrow \operatorname{mcd}\left(\frac{a}{d}, \frac{b}{d}\right) = 1$

$\operatorname{mcd}(a,b) = 1 \Rightarrow d = ax + by$ para algun $x, y \in \mathbb{Z}$. Como $\operatorname{mcd}(a,b) = d \Rightarrow$

$\Rightarrow a/d,\ b/d \in \mathbb{Z}$, $1 = (a/d)x + (b/d)x \Rightarrow \operatorname{mcd}(a/d, b/d) = 1$.

8) $\operatorname{mcd}(1820, 1369) = d$

$\begin{array}{l|l} 1820 & 1369 \\ \hline 451 & 1 \end{array}$ $\quad 451 = 1820 - 1369$

$\begin{array}{l|l} 1369 & 451 \\ \hline 16 & 3 \end{array}$ $\quad 16 = 1369 - 3 \cdot 451$

$\begin{array}{l|l} 451 & 16 \\ \hline 3 & 28 \end{array}$ $\quad 3 = 451 - 16 \cdot 28$

$\begin{array}{l|l} 16 & 3 \\ \hline 1 & 5 \end{array}$ $\quad 1 = 16 - 3 \cdot 5$

$1 = 16 - 3 \cdot 5 = (1369 - 3 \cdot 451) - 5 \cdot (451 - 16 \cdot 28) =$

$= 1369 - 8 \cdot 451 + 140 \cdot 16 =$

$= 1369 - 8 \cdot (1820 - 1369) + 140 \cdot (1369 - 3 \cdot 451) =$

$= 149 \cdot 1369 - 8 \cdot 1820 - 420 \cdot 451 =$

$= 149 \cdot 1369 - 8 \cdot 1820 - 420 \cdot (1820 - 1369) =$

$= 569 \cdot 1369 - 428 \cdot 1820 = \underline{1}$

9) $\operatorname{mcd}(84, 990) = d$

$\begin{array}{l|l} 990 & 84 \\ \hline 66 & 11 \end{array}$ $\quad 66 = 990 - 84 \cdot 11$

$\begin{array}{l|l} 84 & 66 \\ \hline 18 & 1 \end{array}$ $\quad 18 = 84 - 66 \cdot 1$

$\begin{array}{l|l} 66 & 18 \\ \hline 12 & 3 \end{array}$ $\quad 12 = 66 - 18 \cdot 3$

$\begin{array}{l|l} 18 & 12 \\ \hline 6 & 1 \end{array}$ $\quad 6 = 18 - 12$

$\begin{array}{l|l} 12 & 6 \\ \hline 0 & 2 \end{array}$ $\quad \boxed{d = 6}$

$6 = 18 - 12 = (84 - 66) - (66 - 18 \cdot 3) = 84 - 2 \cdot 66 + 3 \cdot 18 =$

$= 84 - 2 \cdot (990 - 84 \cdot 11) + 3 \cdot (84 - 66) =$

$= 26 \cdot 84 - 2 \cdot 990 - 3 \cdot 66 =$

$= 26 \cdot 84 - 2 \cdot 990 - 3 \cdot (990 - 84 \cdot 11) =$

$= 59 \cdot 84 - 5 \cdot 990 = 6$ $\times 2$

$84 \cdot 118 + 990 \cdot (-10) = 12$

$c = 12$, $x = 118$, $y = -10$

10) $14x + 23y = 210$ $\quad \operatorname{mcd}(14, 23) = 1 \wedge 1 \mid 210$ tiene solución

$14a + 23b = 1$

$\begin{array}{l|l} 23 & 14 \\ \hline 9 & 1 \end{array}$ $\quad 9 = 23 - 14 \cdot 1$

$\begin{array}{l|l} 14 & 9 \\ \hline 5 & 1 \end{array}$ $\quad 5 = 14 - 9 \cdot 1$

$\begin{array}{l|l} 9 & 5 \\ \hline 4 & 1 \end{array}$ $\quad 4 = 9 - 5 \cdot 1$

$\begin{array}{l|l} 5 & 4 \\ \hline 1 & 1 \end{array}$ $\quad 1 = 5 - 4 \cdot 1$

$1 = 5 - 4 = (14 - 9) - (9 - 5) = 14 - 2 \cdot 9 + 5 =$

$= 14 - 2 \cdot (23 - 14) + (14 - 9) = 4 \cdot 14 - 2 \cdot 23 - 9 =$

$= 4 \cdot 14 - 2 \cdot 23 - (23 - 14) = 5 \cdot 14 - 3 \cdot 23 = \underline{1}$ $\quad \times 210$

$14 \cdot 1050 + 23 \cdot (-630) = 210$

$14 \cdot 1050 + 23 \cdot (-630) \pm 14 \cdot 23 \cdot k = 210$

$14(\underbrace{1050 - 23k}_{x > 0}) + 23(\underbrace{-630 + 14k}_{y > 0}) = 210$

$X = 1050 - 23k > 0 \longrightarrow k < 1050/23 \leq 45$

$Y = -630 + 14k > 0 \longrightarrow k > 630/14 = 45 \; ; k \geq 46$

La única posibilidad es que $k = 45$, pero sólo estaría comprando

$X = 1050 - 23.45 = 15$ libros de \$14

$Y = -630 + 14.45 = 0$ ninguno de \$23

11) $c + p = 12$

$X = Y + 0{,}3$

c: cantidad de clavos

p: cantidad de pequeños cables

X : precio de cada clavo

Y : precio de cada pequeño cable

$Xc + Yp = 99$

$(Y+0{,}3)c + Yp = 99$

$Yc + 0{,}3c + Yp = 99$

$Y(c+p) + 0{,}3c = 99$

$12Y + 0{,}3c = 99$

$120Y + 3c = 990$

$\boxed{40Y + c = 330}$ $\quad mcd(40,1) = 1 \wedge 1 \mid 330$ ✓

$40 \cdot a + 1b = 1$

Sol particular: $a = 1$, $b = -39$

$Y_0 = 330$, $C_0 = -12870$ $\quad)330$

$40.330 - 12870 = 330$

$40.330 - 12870 \pm k.40.1 = 330$

$40.\underbrace{(330-k)}_{Y} + \underbrace{(-12870+40k)}_{C} = 330$

$330 - k > 0 \longrightarrow k < 330 \; ; k \leq 329$

$-12870 + 40k > 0 \longrightarrow k > 321{,}75 \; ; k \geq 322$

$-12870 + 40k < 12 \longrightarrow k < 322{,}05 \; ; k \leq 322$

k	c	p	x	Y
322	10	2	8,3	8

Única solución

10 clavos a 8,3 c/u

2 pequeños cables a 8 c/u

12) $1100 \leq$ pedido ≤ 1800 ; pedido $= 15.f + 33$

$1100 \leq C.54 \leq 1800$

$15f + 33 = 54C$

$\boxed{15f - 54C = -33}$ $\; \%(-3)$

$\boxed{18C - 5f = 11}$

18 | 5 → 3, resto 3 $\quad 3 = 18 - 5.3$

5 | 3 → 1, resto 2 $\quad 2 = 5 - 3.1$

3 | 2 → 1, resto 1

$1 = 3 - 2.1 = (18 - 5.3) - (5 - 3) = 18 - 4.5 + 3 = 18 - 4.5 + (18 - 5.3) =$

$1 = 2.18 - 7.5 \;) \times 11$

$11 = 18.22 - 5.77 \pm 18.5.k =$

$11 = 18.\overbrace{(22-5k)}^{c} - 5.\overbrace{(77-18k)}^{f}$

a) $\frac{1100}{54} \leq c \leq \frac{1800}{54}$

$21 \leq c \leq 33$

$21 \leq 22 - 5k \leq 33$

$-2{,}2 \leq k \leq -0{,}2$

$-1 \leq k \leq -1$

Unica solución $k = -1$

$C = 22 + 5 = 27$ contenedores

$f = 77 + 18 = 95$ furgonetas

pedido : $54.27 = 1458$

pedido : $15.95 + 33 = 1458$ ✓

TP11 - Teorema Fundamental de la Aritmética

1) $ab = 2^8 \cdot 3^9 \cdot 5^4 \cdot 7^{11}$ $\quad mcd(a,b) = 2^3 \cdot 5 \cdot 7^3$

$mcm(a,b) \cdot mcd(a,b) = a \cdot b \Rightarrow mcm(a,b) = ab/mcd(a,b)$

$mcm(a,b) = 2^8 \cdot 3^9 \cdot 5^4 \cdot 7^{11} \cdot 2^{-3} \cdot 5^{-1} \cdot 7^{-3} = 2^5 \cdot 3^9 \cdot 5^3 \cdot 7^8$

Quedan univocamente determinados porque

2) $mcd(3^7 \cdot 5^4 \cdot 7 \cdot 11^6 ; 3^4 \cdot d \cdot 11^3 \cdot 13^2) = 2835 = 5 \cdot 7 \cdot 3^4$

$d = 5 \cdot 7 \cdot 11^{-3}$

3) $15435 = 5^1 \cdot 7^3 \cdot 3^2$ tiene $(1+1) \cdot (3+1) \cdot (2+1) = 24$ divisores positivos

4) $a = 2 \cdot 3^3 \cdot 7^2 \cdot 13^2$ $\quad c = mcd(a,b) = 3^2 \cdot 7^1 \Rightarrow$ tiene $3 \cdot 2 = 6$ divisores positivos

$b = 3^2 \cdot 5 \cdot 7 \cdot 11$

$d = mcm(a,b) = 2^1 \cdot 3^3 \cdot 5^1 \cdot 7^2 \cdot 11^1 \cdot 13^2$

tiene $2 \cdot 4 \cdot 2 \cdot 3 \cdot 2 \cdot 3 = 288$ divisores positivos