Gráficas y funciones

Para encontrar intersecciones

intersecciones y: Sea $x = 0$ en la ecuación y resolvemos para y
intersecciones x: Sea $y = 0$ en la ecuación y resolvemos para x

Funciones de polinomios

$$f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0,$$

donde n es un entero no negativo.

Función lineal

$$f(x) = ax + b, a \neq 0$$

La gráfica de una función lineal es una recta.

Formas de ecuaciones de rectas:

Punto pendiente: $y - x_0 = m(x - x_0)$,
Pendiente ordenada al origen: $y = mx + b$,

donde m es la pendiente.

Función cuadrática

$$f(x) = ax^2 + bx + c, a \neq 0$$

La gráfica de una función cuadrática es una parábola.

Vértice (h, k) de una parábola

Complete el cuadrado en x para $f(x) = ax^2 + bx + c$ para obtener $f(x) = a(x - h)^2 + k$. De manera alterna, calcule las coordenadas

$$\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right).$$

Funciones par e impar

Par: $f(-x) = f(x)$; simetría de la gráfica: el eje y
Impar: $f(-x) = -f(x)$; simetría de la gráfica: el origen

Transformaciones rígidas

La gráfica de $y = f(x)$ para $c > 0$:

$y = f(x) + c$, desplazada hacia arriba c unidades
$y = f(x) - c$, desplazada hacia abajo c unidades
$y = f(x + c)$, desplazada hacia la izquierda c unidades
$y = f(x - c)$, desplazada hacia la derecha c unidades
$y = f(-x)$, reflexión sobre el eje y
$y = -f(x)$, reflexión sobre el eje x

Función racional

$$f(x) = \frac{p(x)}{q(x)} = \frac{a_n x^n + \cdots + a_1 x + a_0}{b_m x^m + \cdots + b_1 x + b_0},$$

donde $p(x)$ y $q(x)$ son funciones polinomiales.

Asíntotas

Si las funciones polinomiales $p(x)$ y $q(x)$ no tienen ningún factor en común, entonces la gráfica de la función racional

$$f(x) = \frac{p(x)}{q(x)} = \frac{a_n x^n + \cdots + a_1 x + a_0}{b_m x^m + \cdots + b_1 x + b_0}$$

tiene una

Asíntota vertical:

$x = a$ cuando $q(a) = 0$,

Asíntota horizontal:

$y = a_n/b_m$ cuando $n = m$ y $y = 0$ cuando $n < m$,

Asíntota oblicua:

$y = ax + b$ cuando $n = m + 1$.

La gráfica no tiene una asíntota horizontal cuando $n > m$. Una asíntota oblicua se encuentra mediante una división.

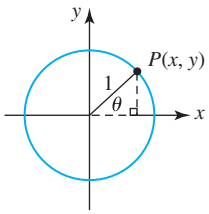
Función potencia

$$f(x) = x^n,$$

donde n es cualquier número real.

Revisión de trigonometría

Definición de seno y coseno de acuerdo con el círculo unitario



$$y = \operatorname{sen}\theta$$
$$x = \cos\theta$$

Otras funciones trigonométricas

$$\tan\theta = \frac{y}{x} = \frac{\operatorname{sen}\theta}{\cos\theta}, \quad \cot\theta = \frac{x}{y} = \frac{\cos\theta}{\operatorname{sen}\theta}$$

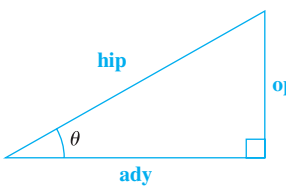
$$\sec\theta = \frac{1}{x} = \frac{1}{\cos\theta}, \quad \csc\theta = \frac{1}{y} = \frac{1}{\operatorname{sen}\theta}$$

Fórmulas de conversión

$$1 \text{ grado} = \frac{\pi}{180} \text{ radianes}$$

$$1 \text{ radián} = \frac{180}{\pi} \text{ grados}$$

Definición de seno y coseno de acuerdo con el triángulo recto



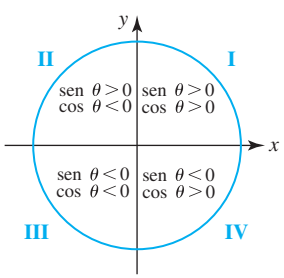
$$\operatorname{sen}\theta = \frac{\text{opu}}{\text{hip}}$$
$$\cos\theta = \frac{\text{ady}}{\text{hip}}$$

Otras funciones trigonométricas

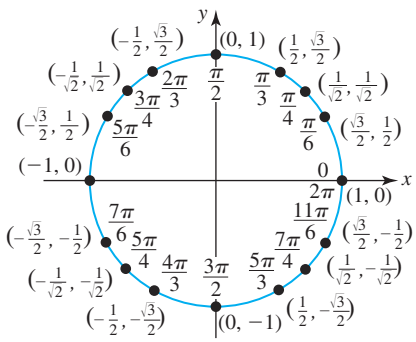
$$\tan\theta = \frac{\text{opu}}{\text{ady}}, \quad \cot\theta = \frac{\text{ady}}{\text{opu}}$$

$$\sec\theta = \frac{\text{hip}}{\text{ady}}, \quad \csc\theta = \frac{\text{hip}}{\text{opu}}$$

Signos de seno y coseno



Valores de seno y coseno para ángulos especiales



Límites para las funciones seno y coseno

$$-1 \le \operatorname{sen} x \le 1 \quad \text{y} \quad -1 \le \cos x \le 1$$

Periodicidad de las funciones trigonométricas

$$\operatorname{sen}(x + 2\pi) = \operatorname{sen} x, \quad \cos(x + 2\pi) = \cos x$$
$$\sec(x + 2\pi) = \sec x, \quad \csc(x + 2\pi) = \csc x$$
$$\tan(x + \pi) = \tan x, \quad \cot(x + \pi) = \cot x$$

Identidades de cofunción

$$\operatorname{sen}\left(\frac{\pi}{2} - x\right) = \cos x$$
$$\cos\left(\frac{\pi}{2} - x\right) = \operatorname{sen} x$$
$$\tan\left(\frac{\pi}{2} - x\right) = \cot x$$

Identidades pitagóricas

$$\operatorname{sen}^2 x + \cos^2 x = 1$$
$$1 + \tan^2 x = \sec^2 x$$
$$1 + \cot^2 x = \csc^2 x$$

Identidades par/impar

Par	Impar
$\cos(-x) = \cos x$	$\operatorname{sen}(-x) = -\operatorname{sen} x$
$\sec(-x) = \sec x$	$\csc(-x) = -\csc x$
	$\tan(-x) = -\tan x$
	$\cot(-x) = -\cot x$

Fórmulas de suma

$$\operatorname{sen}(x_1 + x_2) = \operatorname{sen} x_1 \cos x_2 + \cos x_1 \operatorname{sen} x_2$$

$$\cos(x_1 + x_2) = \cos x_1 \cos x_2 - \operatorname{sen} x_1 \operatorname{sen} x_2$$

$$\tan(x_1 + x_2) = \frac{\tan x_1 + \tan x_2}{1 - \tan x_1 \tan x_2}$$

Fórmulas de diferencia

$$\operatorname{sen}(x_1 - x_2) = \operatorname{sen} x_1 \cos x_2 - \cos x_1 \operatorname{sen} x_2$$

$$\cos(x_1 - x_2) = \cos x_1 \cos x_2 + \operatorname{sen} x_1 \operatorname{sen} x_2$$

$$\tan(x_1 - x_2) = \frac{\tan x_1 - \tan x_2}{1 + \tan x_1 \tan x_2}$$

Fórmulas del ángulo doble

$$\operatorname{sen} 2x = 2 \operatorname{sen} x \cos x$$

$$\cos 2x = \cos^2 x - \operatorname{sen}^2 x$$

Fórmulas alternas del ángulo doble para coseno

$$\cos 2x = 1 - 2 \operatorname{sen}^2 x$$

$$\cos 2x = 2 \cos^2 x - 1$$

Fórmulas del medio ángulo como se usa en cálculo

$$\operatorname{sen}^2 x = \tfrac{1}{2}(1 - \cos 2x)$$

$$\cos^2 x = \tfrac{1}{2}(1 + \cos 2x)$$

Leyes de los senos

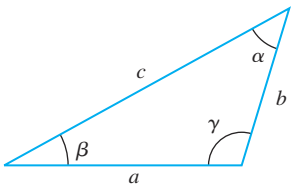
$$\frac{\operatorname{sen}\alpha}{a} = \frac{\operatorname{sen}\beta}{b} = \frac{\operatorname{sen}\gamma}{c}$$

Leyes de los cosenos

$$a^2 = b^2 + c^2 - 2bc \cos\alpha$$

$$b^2 = a^2 + c^2 - 2ac \cos\beta$$

$$c^2 = a^2 + b^2 - 2ab \cos\gamma$$



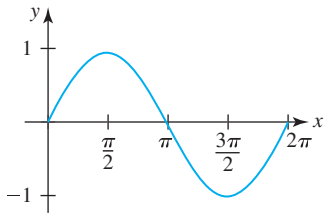
Funciones trigonométricas inversas

$y = \operatorname{sen}^{-1} x$ si y sólo si $x = \operatorname{sen} y$, $\quad -\pi/2 \le y \le \pi/2$

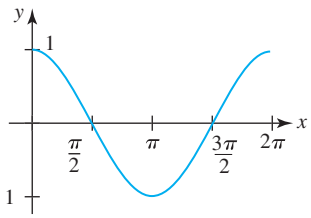
$y = \cos^{-1} x$ si y sólo si $x = \cos y$, $\quad 0 \le y \le \pi$

$y = \tan^{-1} x$ si y sólo si $x = \tan y$, $\quad -\pi/2 < y < \pi/2$

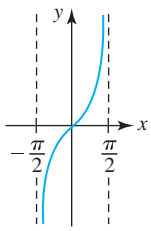
Ciclos para seno, coseno y tangente



seno



coseno



tangente

Funciones exponencial y logarítmica

El número *e*

$$e = 2.718281828459...$$

Definiciones del número *e*

$$e = \lim_{x\to\infty}\left(1 + \frac{1}{x}\right)^x$$

$$e = \lim_{h\to 0}(1 + h)^{1/h}$$

Función exponencial

$$f(x) = b^x,\ b > 0, b \neq 1$$

Función exponencial natural

$$f(x) = e^x$$

Función logarítmica

$$f(x) = \log_b x, \quad x > 0$$

donde $y = \log_b x$ es equivalente a $x = b^y$

Función logarítmica natural

$$f(x) = \log_e x = \ln x, \quad x > 0$$

donde $y = \ln x$ es equivalente a $x = e^y$

Leyes de logaritmos

$$\log_b MN = \log_b M + \log_b N$$

$$\log_b \frac{M}{N} = \log_b M - \log_b N$$

$$\log_b M^c = c\log_b M$$

Propiedades de logaritmos

$$\log_b b = 1, \qquad \log_b 1 = 0$$

$$\log_b b^x = x, \qquad b^{\log_b x} = x$$

Cambio de la base *b* a la base *e*

$$\log_b x = \frac{\ln x}{\ln b}$$

Funciones hiperbólicas

$$\operatorname{senh} x = \frac{e^x - e^{-x}}{2}, \quad \cosh x = \frac{e^x + e^{-x}}{2}$$

$$\tanh x = \frac{\operatorname{senh} x}{\cosh x}, \quad \coth x = \frac{\cosh x}{\operatorname{senh} x}$$

$$\operatorname{sech} x = \frac{1}{\cosh x}, \quad \operatorname{csch} x = \frac{1}{\operatorname{senh} x}$$

Funciones hiperbólicas inversas como logaritmos

$$\operatorname{senh}^{-1} x = \ln\left(x + \sqrt{x^2 + 1}\right)$$

$$\cosh^{-1} x = \ln\left(x + \sqrt{x^2 - 1}\right),\ x \geq 1$$

$$\tanh^{-1} x = \frac{1}{2}\ln\left(\frac{1 + x}{1 - x}\right),\ |x| < 1$$

$$\coth^{-1} x = \frac{1}{2}\ln\left(\frac{x + 1}{x - 1}\right),\ |x| > 1$$

$$\operatorname{sech}^{-1} x = \ln\left(\frac{1 + \sqrt{1 - x^2}}{x}\right),\ 0 < x \leq 1$$

$$\operatorname{csch}^{-1} x = \ln\left(\frac{1}{x} + \frac{\sqrt{1 + x^2}}{|x|}\right),\ x \neq 0$$

Identidades par/impar

Par	Impar
$\cosh(-x) = \cosh x$	$\operatorname{senh}(-x) = -\operatorname{senh} x$

Identidades adicionales

$$\cosh^2 x - \operatorname{senh}^2 x = 1$$

$$1 - \tanh^2 x = \operatorname{sech}^2 x$$

$$\coth^2 x - 1 = \operatorname{csch}^2 x$$

$$\operatorname{senh}(x_1 \pm x_2) = \operatorname{senh} x_1 \cosh x_2 \pm \cosh x_1 \operatorname{senh} x_2$$

$$\cosh(x_1 \pm x_2) = \cosh x_1 \cosh x_2 \pm \operatorname{senh} x_1 \operatorname{senh} x_2$$

$$\operatorname{senh} 2x = 2 \operatorname{senh} x \cosh x$$

$$\cosh 2x = \cosh^2 x + \operatorname{senh}^2 x$$

$$\operatorname{senh}^2 x = \tfrac{1}{2}(-1 + \cosh 2x)$$

$$\cosh^2 x = \tfrac{1}{2}(1 + \cosh 2x)$$

Diferenciación

Reglas

1. Constante: $\frac{d}{dx}c = 0$
2. Múltiplo constante: $\frac{d}{dx}cf(x) = cf'(x)$
3. Suma: $\frac{d}{dx}[f(x) \pm g(x)] = f'(x) \pm g'(x)$
4. Producto: $\frac{d}{dx}f(x)g(x) = f(x)g'(x) + g(x)f'(x)$
5. Cociente: $\frac{d}{dx}\frac{f(x)}{g(x)} = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$
6. Cadena: $\frac{d}{dx}f(g(x)) = f'(g(x))g'(x)$
7. Potencia: $\frac{d}{dx}x^n = nx^{n-1}$
8. Potencia: $\frac{d}{dx}[g(x)]^n = n[g(x)]^{n-1}g'(x)$

Funciones

Trigonométricas:

9. $\frac{d}{dx}\operatorname{sen} x = \cos x$
10. $\frac{d}{dx}\cos x = -\operatorname{sen} x$
11. $\frac{d}{dx}\tan x = \sec^2 x$
12. $\frac{d}{dx}\cot x = -\csc^2 x$
13. $\frac{d}{dx}\sec x = \sec x \tan x$
14. $\frac{d}{dx}\csc x = -\csc x \cot x$

Trigonométricas inversas:

15. $\frac{d}{dx}\operatorname{sen}^{-1} x = \frac{1}{\sqrt{1 - x^2}}$
16. $\frac{d}{dx}\cos^{-1} x = -\frac{1}{\sqrt{1 - x^2}}$
17. $\frac{d}{dx}\tan^{-1} x = \frac{1}{1 + x^2}$
18. $\frac{d}{dx}\cot^{-1} x = -\frac{1}{1 + x^2}$
19. $\frac{d}{dx}\sec^{-1} x = \frac{1}{|x|\sqrt{x^2 - 1}}$
20. $\frac{d}{dx}\csc^{-1} x = -\frac{1}{|x|\sqrt{x^2 - 1}}$

Hiperbólicas:

21. $\frac{d}{dx}\operatorname{senh} x = \cosh x$
22. $\frac{d}{dx}\cosh x = \operatorname{senh} x$
23. $\frac{d}{dx}\tanh x = \operatorname{sech}^2 x$
24. $\frac{d}{dx}\coth x = -\operatorname{csch}^2 x$
25. $\frac{d}{dx}\operatorname{sech} x = -\operatorname{sech} x \tanh x$
26. $\frac{d}{dx}\operatorname{csch} x = -\operatorname{csch} x \coth x$

Hiperbólicas inversas:

27. $\frac{d}{dx}\operatorname{senh}^{-1} x = \frac{1}{\sqrt{x^2 + 1}}$
28. $\frac{d}{dx}\cosh^{-1} x = \frac{1}{\sqrt{x^2 - 1}}$
29. $\frac{d}{dx}\tanh^{-1} x = \frac{1}{1 - x^2}$
30. $\frac{d}{dx}\coth^{-1} x = \frac{1}{1 - x^2}$
31. $\frac{d}{dx}\operatorname{sech}^{-1} x = -\frac{1}{x\sqrt{1 - x^2}}$
32. $\frac{d}{dx}\operatorname{csch}^{-1} x = -\frac{1}{|x|\sqrt{x^2 + 1}}$

Exponenciales:

33. $\frac{d}{dx}e^x = e^x$
34. $\frac{d}{dx}b^x = b^x(\ln b)$

Logarítmicas:

35. $\frac{d}{dx}\ln|x| = \frac{1}{x}$
36. $\frac{d}{dx}\log_b x = \frac{1}{x(\ln b)}$

Fórmulas de integración

Formas básicas

1. $\int u\,dv = uv - \int v\,du$
2. $\int u^n\,du = \frac{1}{n+1}u^{n+1} + C,\ n \neq -1$
3. $\int \frac{du}{u} = \ln|u| + C$
4. $\int e^u\,du = e^u + C$
5. $\int a^u\,du = \frac{1}{\ln a}a^u + C$
6. $\int \operatorname{sen} u\,du = -\cos u + C$
7. $\int \cos u\,du = \operatorname{sen} u + C$
8. $\int \sec^2 u\,du = \tan u + C$
9. $\int \csc^2 u\,du = -\cot u + C$
10. $\int \sec u \tan u\,du = \sec u + C$
11. $\int \csc u \cot u\,du = -\csc u + C$
12. $\int \tan u\,du = -\ln|\cos u| + C$
13. $\int \cot u\,du = \ln|\operatorname{sen} u| + C$
14. $\int \sec u\,du = \ln|\sec u + \tan u| + C$
15. $\int \csc u\,du = \ln|\csc u - \cot u| + C$
16. $\int \frac{du}{\sqrt{a^2-u^2}} = \operatorname{sen}^{-1}\frac{u}{a} + C$
17. $\int \frac{du}{a^2+u^2} = \frac{1}{a}\tan^{-1}\frac{u}{a} + C$
18. $\int \frac{du}{u\sqrt{u^2-a^2}} = \frac{1}{a}\sec^{-1}\left|\frac{u}{a}\right| + C$
19. $\int \frac{du}{a^2-u^2} = \frac{1}{2a}\ln\left|\frac{u+a}{u-a}\right| + C$
20. $\int \frac{du}{u^2-a^2} = \frac{1}{2a}\ln\left|\frac{u-a}{u+a}\right| + C$

Formas que implican $\sqrt{a^2+u^2}$

21. $\int \sqrt{a^2+u^2}\,du = \frac{u}{2}\sqrt{a^2+u^2} + \frac{a^2}{2}\ln|u+\sqrt{a^2+u^2}| + C$
22. $\int u^2\sqrt{a^2+u^2}\,du = \frac{u}{8}(a^2+2u^2)\sqrt{a^2+u^2} - \frac{a^4}{8}\ln|u+\sqrt{a^2+u^2}| + C$
23. $\int \frac{\sqrt{a^2+u^2}}{u}\,du = \sqrt{a^2+u^2} - a\ln\left|\frac{a+\sqrt{a^2+u^2}}{u}\right| + C$
24. $\int \frac{\sqrt{a^2+u^2}}{u^2}\,du = -\frac{\sqrt{a^2+u^2}}{u} + \ln|u+\sqrt{a^2+u^2}| + C$
25. $\int \frac{du}{\sqrt{a^2+u^2}} = \ln|u+\sqrt{a^2+u^2}| + C$
26. $\int \frac{u^2\,du}{\sqrt{a^2+u^2}} = \frac{u}{2}\sqrt{a^2+u^2} - \frac{a^2}{2}\ln|u+\sqrt{a^2+u^2}| + C$
27. $\int \frac{du}{u\sqrt{a^2+u^2}} = -\frac{1}{a}\ln\left|\frac{\sqrt{a^2+u^2}+a}{u}\right| + C$
28. $\int \frac{du}{u^2\sqrt{a^2+u^2}} = -\frac{\sqrt{a^2+u^2}}{a^2u} + C$
29. $\int \frac{du}{(a^2+u^2)^{3/2}} = \frac{u}{a^2\sqrt{a^2+u^2}} + C$

Formas que implican $\sqrt{a^2-u^2}$

30. $\int \sqrt{a^2-u^2}\,du = \frac{u}{2}\sqrt{a^2-u^2} + \frac{a^2}{2}\operatorname{sen}^{-1}\frac{u}{a} + C$
31. $\int u^2\sqrt{a^2-u^2}\,du = \frac{u}{8}(2u^2-a^2)\sqrt{a^2-u^2} + \frac{a^4}{8}\operatorname{sen}^{-1}\frac{u}{a} + C$
32. $\int \frac{\sqrt{a^2-u^2}}{u}\,du = \sqrt{a^2-u^2} - a\ln\left|\frac{a+\sqrt{a^2-u^2}}{u}\right| + C$
33. $\int \frac{\sqrt{a^2-u^2}}{u^2}\,du = -\frac{1}{u}\sqrt{a^2-u^2} - \operatorname{sen}^{-1}\frac{u}{a} + C$
34. $\int \frac{u^2\,du}{\sqrt{a^2-u^2}} = -\frac{u}{2}\sqrt{a^2-u^2} + \frac{a^2}{2}\operatorname{sen}^{-1}\frac{u}{a} + C$
35. $\int \frac{du}{u\sqrt{a^2-u^2}} = -\frac{1}{a}\ln\left|\frac{a+\sqrt{a^2-u^2}}{u}\right| + C$

36. $$\int \frac{du}{u^2\sqrt{a^2-u^2}} = -\frac{1}{a^2u}\sqrt{a^2-u^2} + C$$

37. $$\int (a^2-u^2)^{3/2}\,du = -\frac{u}{8}(2u^2-5a^2)\sqrt{a^2-u^2} + \frac{3a^4}{8}\operatorname{sen}^{-1}\frac{u}{a} + C$$

38. $$\int \frac{du}{(a^2-u^2)^{3/2}} = \frac{u}{a^2\sqrt{a^2-u^2}} + C$$

Formas que implican $\sqrt{u^2-a^2}$

39. $$\int \sqrt{u^2-a^2}\,du = \frac{u}{2}\sqrt{u^2-a^2} - \frac{a^2}{2}\ln|u+\sqrt{u^2-a^2}| + C$$

40. $$\int u^2\sqrt{u^2-a^2}\,du = \frac{u}{8}(2u^2-a^2)\sqrt{u^2-a^2} - \frac{a^4}{8}\ln|u+\sqrt{u^2-a^2}| + C$$

41. $$\int \frac{\sqrt{u^2-a^2}}{u}\,du = \sqrt{u^2-a^2} - a\cos^{-1}\frac{a}{u} + C$$

42. $$\int \frac{\sqrt{u^2-a^2}}{u^2}\,du = -\frac{\sqrt{u^2-a^2}}{u} + \ln|u+\sqrt{u^2-a^2}| + C$$

43. $$\int \frac{du}{\sqrt{u^2-a^2}} = \ln|u+\sqrt{u^2-a^2}| + C$$

44. $$\int \frac{u^2\,du}{\sqrt{u^2-a^2}} = \frac{u}{2}\sqrt{u^2-a^2} + \frac{a^2}{2}\ln|u+\sqrt{u^2-a^2}| + C$$

45. $$\int \frac{du}{u^2\sqrt{u^2-a^2}} = \frac{\sqrt{u^2-a^2}}{a^2u} + C$$

46. $$\int \frac{du}{(u^2-a^2)^{3/2}} = -\frac{u}{a^2\sqrt{u^2-a^2}} + C$$

Formas que implican $a+bu$

47. $$\int \frac{u\,du}{a+bu} = \frac{1}{b^2}(a+bu-a\ln|a+bu|) + C$$

48. $$\int \frac{u^2\,du}{a+bu} = \frac{1}{2b^3}[(a+bu)^2 - 4a(a+bu) + 2a^2\ln|a+bu|] + C$$

49. $$\int \frac{du}{u(a+bu)} = \frac{1}{a}\ln\left|\frac{u}{a+bu}\right| + C$$

50. $$\int \frac{du}{u^2(a+bu)} = -\frac{1}{au} + \frac{b}{a^2}\ln\left|\frac{a+bu}{u}\right| + C$$

51. $$\int \frac{u\,du}{(a+bu)^2} = \frac{a}{b^2(a+bu)} + \frac{1}{b^2}\ln|a+bu| + C$$

52. $$\int \frac{du}{u(a+bu)^2} = \frac{1}{a(a+bu)} - \frac{1}{a^2}\ln\left|\frac{a+bu}{u}\right| + C$$

53. $$\int \frac{u^2\,du}{(a+bu)^2} = \frac{1}{b^3}\left(a+bu-\frac{a^2}{a+bu}-2a\ln|a+bu|\right) + C$$

54. $$\int u\sqrt{a+bu}\,du = \frac{2}{15b^2}(3bu-2a)(a+bu)^{3/2} + C$$

55. $$\int \frac{u\,du}{\sqrt{a+bu}} = \frac{2}{3b^2}(bu-2a)\sqrt{a+bu} + C$$

56. $$\int \frac{u^2\,du}{\sqrt{a+bu}} = \frac{2}{15b^3}(8a^2+3b^2u^2-4abu)\sqrt{a+bu} + C$$

57. $$\int \frac{du}{u\sqrt{a+bu}} = \frac{1}{\sqrt{a}}\ln\left|\frac{\sqrt{a+bu}-\sqrt{a}}{\sqrt{a+bu}+\sqrt{a}}\right| + C, \text{ si } a>0$$
$$= \frac{2}{\sqrt{-a}}\tan^{-1}\sqrt{\frac{a+bu}{-a}} + C, \text{ si } a<0$$

58. $$\int \frac{\sqrt{a+bu}}{u}\,du = 2\sqrt{a+bu} + a\int \frac{du}{u\sqrt{a+bu}}$$

59. $$\int \frac{\sqrt{a+bu}}{u^2}\,du = -\frac{\sqrt{a+bu}}{u} + \frac{b}{2}\int \frac{du}{u\sqrt{a+bu}}$$

60. $$\int u^2\sqrt{a+bu}\,du = \frac{2u^n(a+bu)^{3/2}}{b(2n+3)} - \frac{2na}{b(2n+3)}\int u^{n-1}\sqrt{a+bu}\,du$$

61. $$\int \frac{u^n\,du}{\sqrt{a+bu}} = \frac{2u^n\sqrt{a+bu}}{b(2n+1)} - \frac{2na}{b(2n+1)}\int \frac{u^{n-1}\,du}{\sqrt{a+bu}}$$

62. $$\int \frac{du}{u^n\sqrt{a+bu}} = -\frac{\sqrt{a+bu}}{a(n-1)u^{n-1}} - \frac{b(2n-3)}{2a(n-1)}\int \frac{du}{u^{n-1}\sqrt{a+bu}}$$

Formas trigonométricas

63. $$\int \operatorname{sen}^2 u\,du = \frac{1}{2}u - \frac{1}{4}\operatorname{sen} 2u + C$$

64. $$\int \cos^2 u\,du = \frac{1}{2}u + \frac{1}{4}\operatorname{sen} 2u + C$$

65. $$\int \tan^2 u\,du = \tan u - u + C$$

66. $$\int \cot^2 u\,du = -\cot u - u + C$$

67. $$\int \operatorname{sen}^3 u\,du = -\frac{1}{3}(2+\operatorname{sen}^2 u)\cos u + C$$

68. $$\int \cos^3 u\,du = \frac{1}{3}(2+\cos^2 u)\operatorname{sen} u + C$$

69. $$\int \tan^3 u\,du = \frac{1}{2}\tan^2 u + \ln|\cos u| + C$$

70. $$\int \cot^2 u\,du = -\frac{1}{2}\cot^2 u - \ln|\operatorname{sen} u| + C$$

71. $$\int \sec^3 u\,du = \frac{1}{2}\sec u\tan u + \frac{1}{2}\ln|\sec u+\tan u| + C$$

72. $\int \csc^3 u\,du = -\frac{1}{2}\csc u \cot u + \frac{1}{2}\ln|\csc u - \cot u| + C$

73. $\int \operatorname{sen}^n u\,du = -\frac{1}{n}\operatorname{sen}^{n-1} u \cos u + \frac{n-1}{n}\int \operatorname{sen}^{n-2} u\,du$

74. $\int \cos^n u\,du = \frac{1}{n}\cos^{n-1} u \operatorname{sen} u + \frac{n-1}{n}\int \cos^{n-2} u\,du$

75. $\int \tan^n u\,du = \frac{1}{n-1}\tan^{n-1} u - \int \tan^{n-2} u\,du$

76. $\int \cot^n u\,du = \frac{-1}{n-1}\cot^{n-1} u - \int \cot^{n-2} u\,du$

77. $\int \sec^n u\,du = \frac{1}{n-1}\tan u \sec^{n-2} u + \frac{n-2}{n-1}\int \sec^{n-2} u\,du$

78. $\int \csc^n u\,du = \frac{-1}{n-1}\cot u \csc^{n-2} u + \frac{n-2}{n-1}\int \csc^{n-2} u\,du$

79. $\int \operatorname{sen} au \operatorname{sen} bu\,du = \frac{\operatorname{sen}(a-b)u}{2(a-b)} - \frac{\operatorname{sen}(a+b)u}{2(a+b)} + C$

80. $\int \cos au \cos bu\,du = \frac{\operatorname{sen}(a-b)u}{2(a-b)} + \frac{\operatorname{sen}(a+b)u}{2(a+b)} + C$

81. $\int \operatorname{sen} au \cos bu\,du = -\frac{\cos(a-b)u}{2(a-b)} - \frac{\cos(a+b)u}{2(a+b)} + C$

82. $\int u \operatorname{sen} u\,du = \operatorname{sen} u - u\cos u + C$

83. $\int u \cos u\,du = \cos u + u \operatorname{sen} u + C$

84. $\int u^n \operatorname{sen} u\,du = -u^n \cos u + n\int u^{n-1}\cos u\,du$

85. $\int u^n \cos u\,du = u^n \operatorname{sen} u - n\int u^{n-1}\operatorname{sen} u\,du$

86. $\int \operatorname{sen}^n u \cos^m u\,du = -\frac{\operatorname{sen}^{n-1} u \cos^{m+1} u}{n+m} + \frac{n-1}{n+m}\int \operatorname{sen}^{n-1} u \cos^m u\,du = \frac{\operatorname{sen}^{n+1} u \cos^{m-1} u}{n+m} + \frac{m-1}{n+m}\int \operatorname{sen}^n u \cos^{m-2} u\,du$

87. $\int \frac{du}{1-\operatorname{sen} au} = \frac{1}{a}\tan\left(\frac{\pi}{4} + \frac{au}{2}\right) + C$

88. $\int \frac{du}{1+\operatorname{sen} au} = -\frac{1}{a}\tan\left(\frac{\pi}{4} - \frac{au}{2}\right) + C$

89. $\int \frac{udu}{1-\operatorname{sen} au} = \frac{u}{a}\tan\left(\frac{\pi}{4} + \frac{au}{2}\right) + \frac{2}{a^2}\ln\left|\operatorname{sen}\left(\frac{\pi}{4} - \frac{au}{2}\right)\right| + C$

Formas trigonométricas inversas

90. $\int \operatorname{sen}^{-1} u\,du = u \operatorname{sen}^{-1} u + \sqrt{1-u^2} + C$

91. $\int \cos^{-1} u\,du = u\cos^{-1} u - \sqrt{1-u^2} + C$

92. $\int \tan^{-1} u\,du = u\tan^{-1} u - \frac{1}{2}\ln(1+u^2) + C$

93. $\int u \operatorname{sen}^{-1} u\,du = \frac{2u^2-1}{4}\operatorname{sen}^{-1} u + \frac{u\sqrt{1-u^2}}{4} + C$

94. $\int u\cos^{-1} u\,du = \frac{2u^2-1}{4}\cos^{-1} u - \frac{u\sqrt{1-u^2}}{4} + C$

95. $\int u\tan^{-1} u\,du = \frac{u^2+1}{2}\tan^{-1} u - \frac{u}{2} + C$

96. $\int u^n \operatorname{sen}^{-1} u\,du = \frac{1}{n+1}\left[u^{n+1}\operatorname{sen}^{-1} u - \int \frac{u^{n+1}\,du}{\sqrt{1-u^2}}\right], \quad n \neq -1$

97. $\int u^n \cos^{-1} u\,du = \frac{1}{n+1}\left[u^{n+1}\cos^{-1} u + \int \frac{u^{n+1}\,du}{\sqrt{1-u^2}}\right], \quad n \neq -1$

98. $\int u^n \tan^{-1} u\,du = \frac{1}{n+1}\left[u^{n+1}\tan^{-1} u - \int \frac{u^{n+1}\,du}{1+u^2}\right], \quad n \neq -1$

Formas exponenciales y logarítmicas

99. $\int ue^{au}\,du = \frac{1}{a^2}(au-1)e^{au} + C$

100. $\int u^n e^{au}\,du = \frac{1}{a}u^n e^{au} - \frac{n}{a}\int u^{n-1}e^{au}\,du$

101. $\int e^{au}\operatorname{sen} bu\,du = \frac{e^{au}}{a^2+b^2}(a \operatorname{sen} bu - b\cos bu) + C$

102. $\int e^{au}\cos bu\,du = \frac{e^{au}}{a^2+b^2}(a\cos bu + b \operatorname{sen} bu) + C$

103. $\int \ln u\,du = u\ln u - u + C$

104. $\int \frac{1}{u\ln u}\,du = \ln|\ln u| + C$

105. $\int u^n \ln u\,du = \frac{u^{n+1}}{(n+1)^2}[(n+1)\ln u - 1] + C$

106. $\int u^m \ln^n u\,du = \frac{u^{m+1}\ln^n u}{m+1} - \frac{n}{m+1}\int u^m \ln^{n-1} u\,du, \quad m \neq -1$

107. $\int \ln(u^2 + a^2)\,du = u\ln(u^2 + a^2) - 2u + 2a\tan^{-1}\frac{u}{a} + C$

108. $\int \ln|u^2 - a^2|\,du = u\ln|u^2 - a^2| - 2u + a\ln\left|\frac{u+a}{u-a}\right| + C$

109. $\int \frac{du}{a + be^u} = \frac{u}{a} - \frac{1}{a}\ln|a + be^u| + C$

Formas hiperbólicas

110. $\int \operatorname{senh} u\,du = \cosh u + C$

111. $\int \cosh u\,du = \operatorname{senh} u + C$

112. $\int \tanh u\,du = \ln(\cosh u) + C$

113. $\int \coth u\,du = \ln|\operatorname{senh} u| + C$

114. $\int \operatorname{sech} u\,du = \tan^{-1}(\operatorname{senh} u) + C$

115. $\int \operatorname{csch} u\,du = \ln|\tanh\tfrac{1}{2}u| + C$

116. $\int \operatorname{sech}^2 u\,du = \tanh u + C$

117. $\int \operatorname{csch}^2 u\,du = -\coth u + C$

118. $\int \operatorname{sech} u \tanh u\,du = -\operatorname{sech} u + C$

119. $\int \operatorname{csch} u \coth u\,du = -\operatorname{csch} u + C$

Formas que implican $\sqrt{2au - u^2}$

120. $\int \sqrt{2au - u^2}\,du = \frac{u - a}{2}\sqrt{2au - u^2} + \frac{a^2}{2}\cos^{-1}\left(\frac{a - u}{a}\right) + C$

121. $\int u\sqrt{2au - u^2}\,du = \frac{2u^2 - au - 3a^2}{6}\sqrt{2au - u^2} + \frac{a^3}{2}\cos^{-1}\left(\frac{a - u}{a}\right) + C$

122. $\int \frac{\sqrt{2au - u^2}}{u}\,du = \sqrt{2au - u^2} + a\cos^{-1}\left(\frac{a - u}{a}\right) + C$

123. $\int \frac{\sqrt{2au - u^2}}{u^2}\,du = -\frac{2\sqrt{2au - u^2}}{u} - \cos^{-1}\left(\frac{a - u}{a}\right) + C$

124. $\int \frac{du}{\sqrt{2au - u^2}} = \cos^{-1}\left(\frac{a - u}{a}\right) + C$

125. $\int \frac{u\,du}{\sqrt{2au - u^2}} = -\sqrt{2au - u^2} + a\cos^{-1}\left(\frac{a - u}{a}\right) + C$

126. $\int \frac{u^2\,du}{\sqrt{2au - u^2}} = -\frac{(u + 3a)}{2}\sqrt{2au - u^2} + \frac{3a^2}{2}\cos^{-1}\left(\frac{a - u}{a}\right) + C$

127. $\int \frac{du}{u\sqrt{2ua - u^2}} = -\frac{\sqrt{2au - u^2}}{au} + C$

Algunas integrales definidas

128. $\int_0^{\pi/2} \operatorname{sen}^{2n} x\,dx = \int_0^{\pi/2} \cos^{2n} x\,dx = \frac{\pi}{2}\frac{1 \cdot 3 \cdot 5 \cdots (2n-1)}{2 \cdot 4 \cdot 6 \cdots 2n},\ n = 1, 2, 3, \ldots$

129. $\int_0^{\pi/2} \operatorname{sen}^{2n+1} x\,dx = \int_0^{\pi/2} \cos^{2n+1} x\,dx = \frac{2 \cdot 4 \cdot 6 \cdots 2n}{1 \cdot 3 \cdot 5 \cdots (2n+1)},\ n = 1, 2, 3, \ldots$