Midterm 2 is Friday May 16

- Midterm on May 16 2025
- 2:10pm-3:00pm
- multi-choice and text questions

- Covers material through (lectures 10-18)
  - **Stars** (chap. 15,17,18), **degeneracy pressure** S4.3-S4.4), Virial Theorem, Time scale
- Exam is not cumulative, but much of this material builds on what we learned in the first 1/3rd of the course

Special Relativity

Or, weird things happen when you move at extremely fast speeds.

Chapter S2

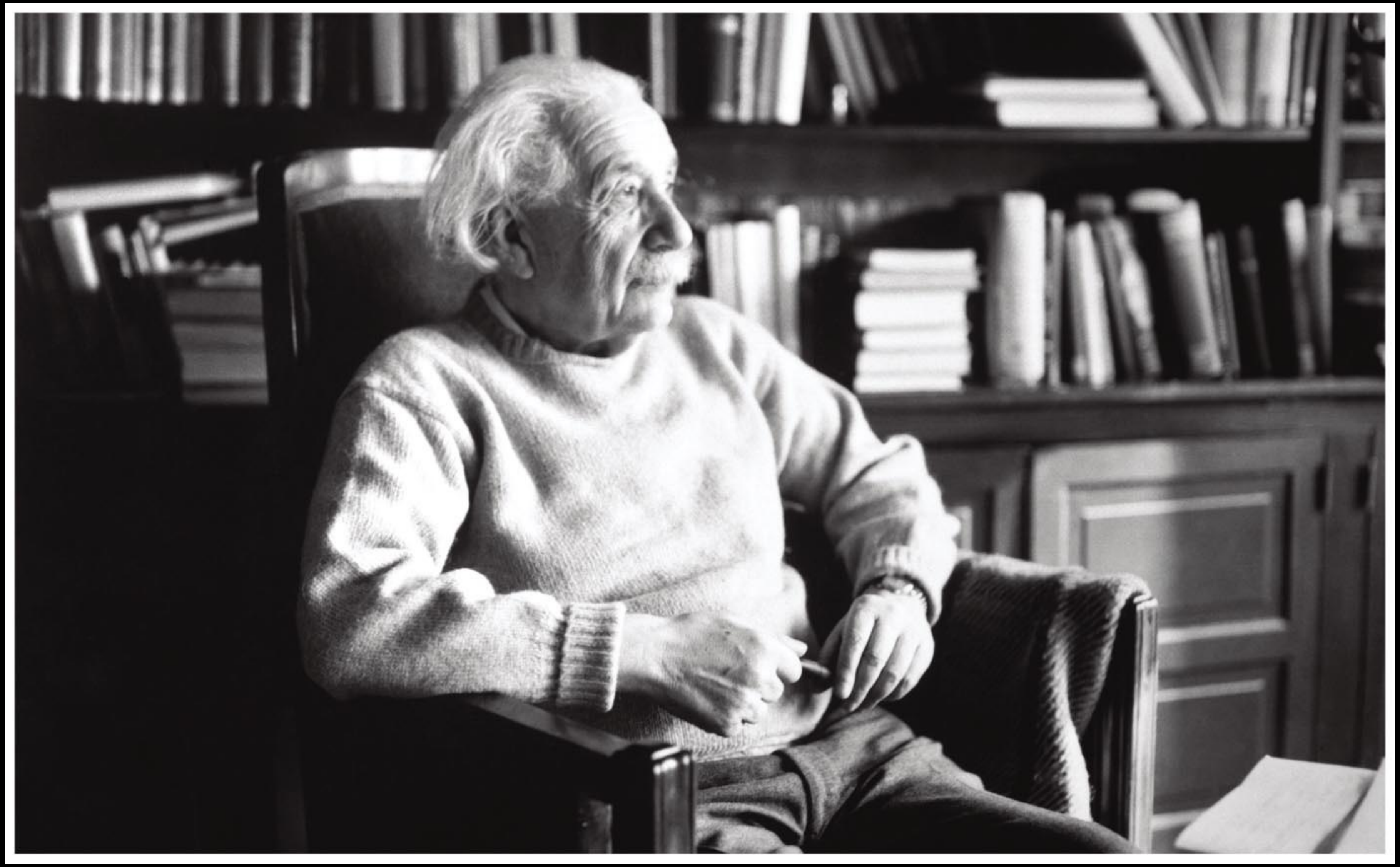
Gravity

- Newton's theory of gravity (1729)
  - Gravity is a force: $F_G = G\dfrac{M_1 M_2}{d^2}$
  - Predicted the existence and position of the planet Neptune! (based on motions of the planet Uranus, which implied an additional force)
  - But, cannot fully explain the orbit of Mercury (perihelion precession does not match prediction)

- Einstein's theory of gravity (general relativity, 1915)
  - Gravity is a curvature of space, not a force
  - Explains the precession of Mercury's orbit, deflection of light around the Sun, and much much more…

Special Relativity: Space and Time

Chapter S2



Einstein's theories of relativity
What is "special" about special relativity?

- **Special Theory of Relativity (1905)**
  - **Special** case which does not consider the effects of gravity (only works when there is no strong gravity)
  - Shows that usual notions of space and time must be revised for speeds approaching light speed (c)
  - Gives the equation $E = mc^2$
- **General Theory of Relativity (1915)**
  - **General** case which expands the ideas of special theory to include a surprising new view of gravity

Questions to be answered

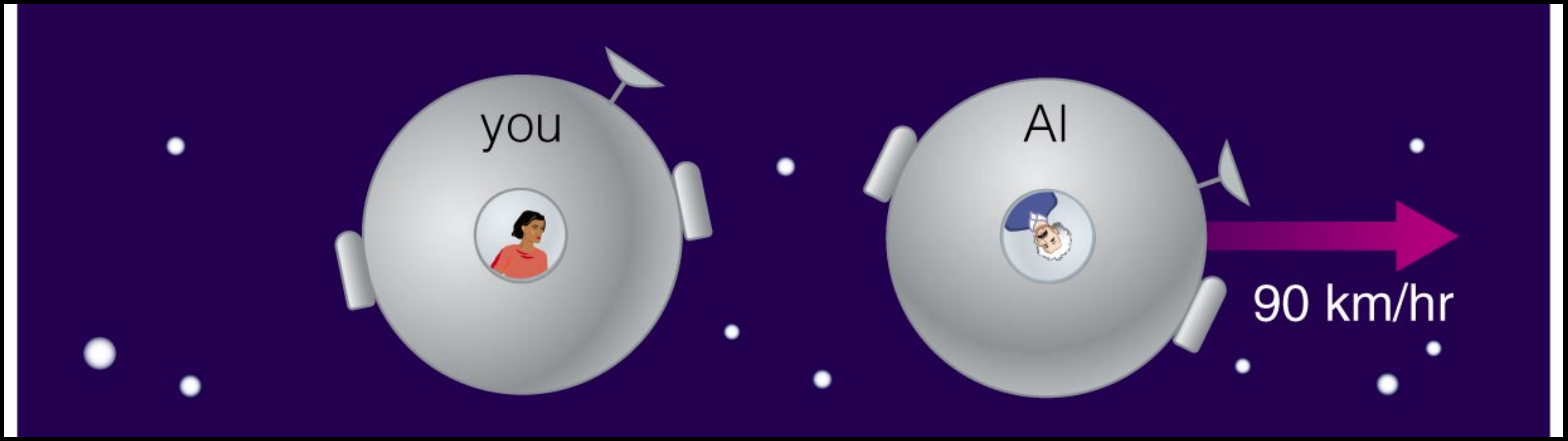
- What are the major ideas of special relativity?
- What is “relative” about relativity?
- What is absolute about relativity?

What is relative about relativity?

Key idea: “inertial reference frame”

Inertial = not accelerating

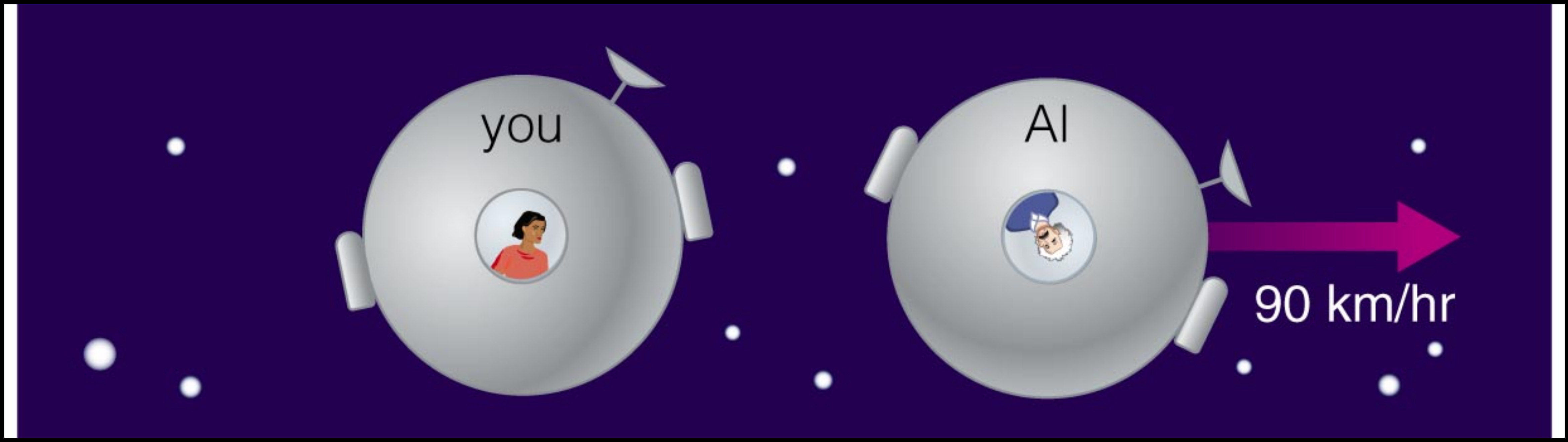
What is relative about relativity?



You and Al are traveling through space in your spaceships. Which of the following is true?

- Al is moving away from you at 90 km/hr

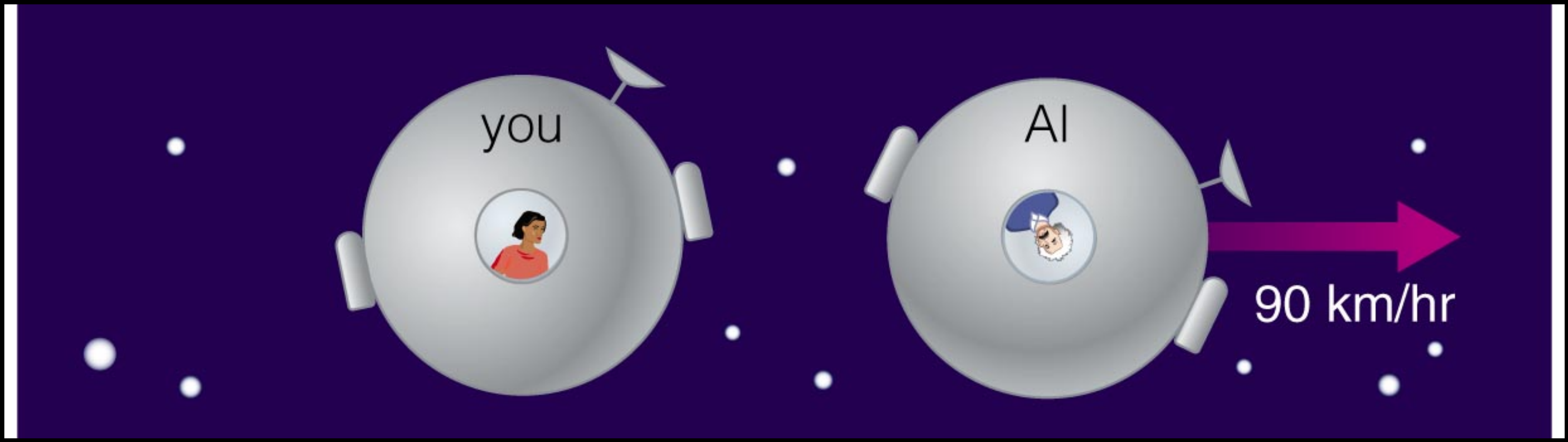
What is relative about relativity?



You and Al are traveling through space in your spaceships. Which of the following is true?

- Al is moving away from you at 90 km/hr
- You are moving away from Al at 90 km/hr

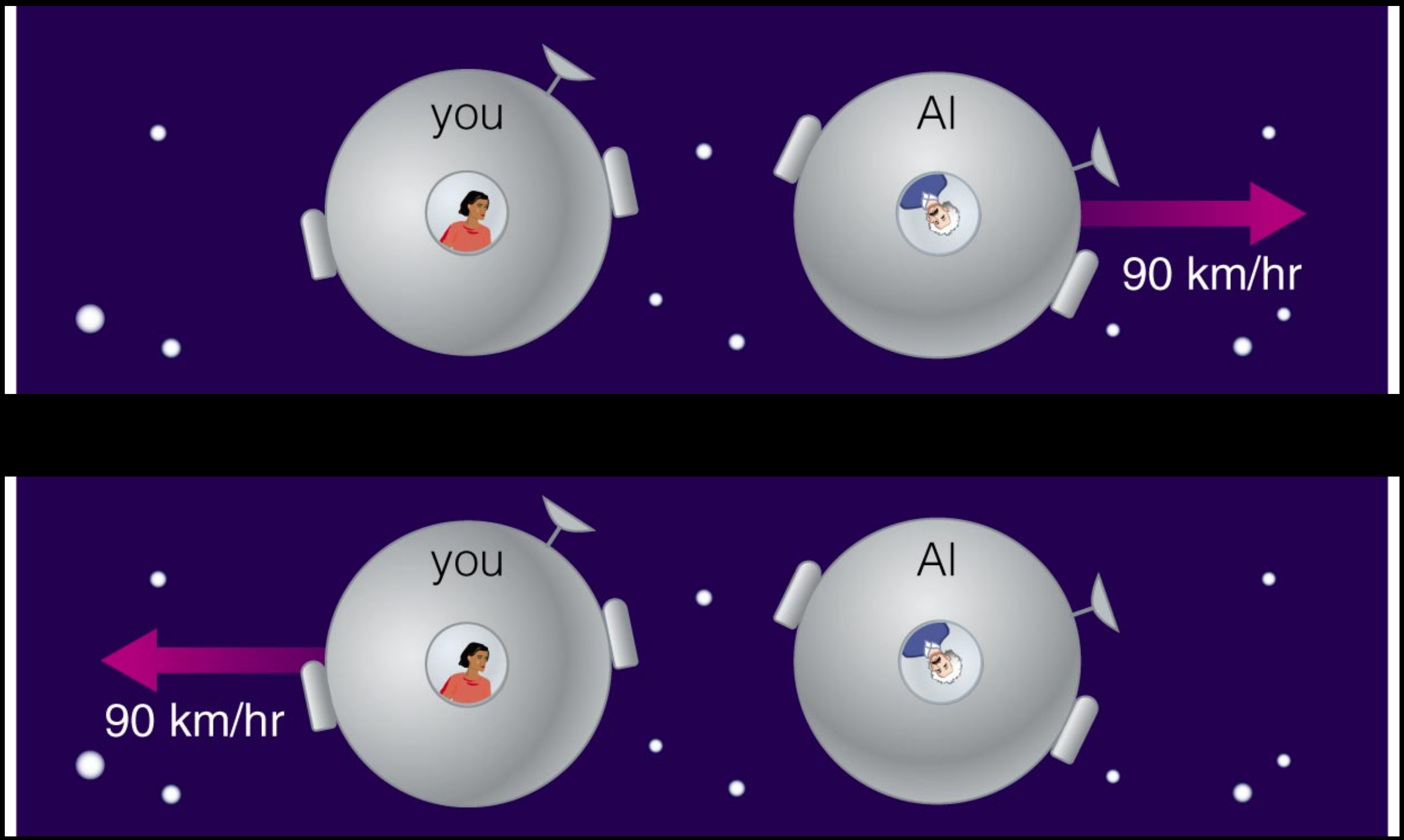
What is relative about relativity?



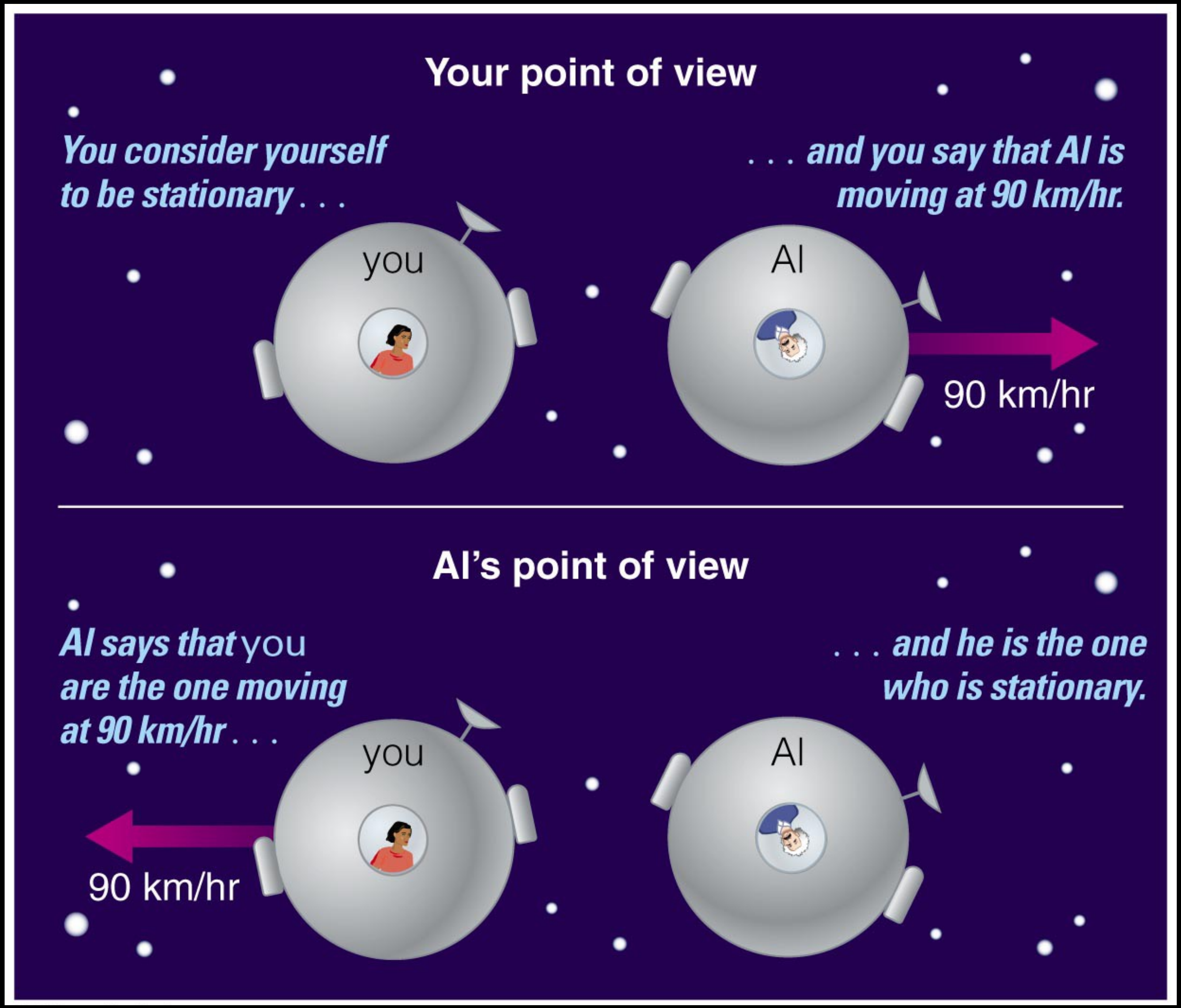
You and Al are traveling through space in your spaceships. Which of the following is true?

- Al is moving away from you at 90 km/hr
- You are moving away from Al at 90 km/hr
- You are both moving away from a central point at 45 km/hr

What is relative about relativity?

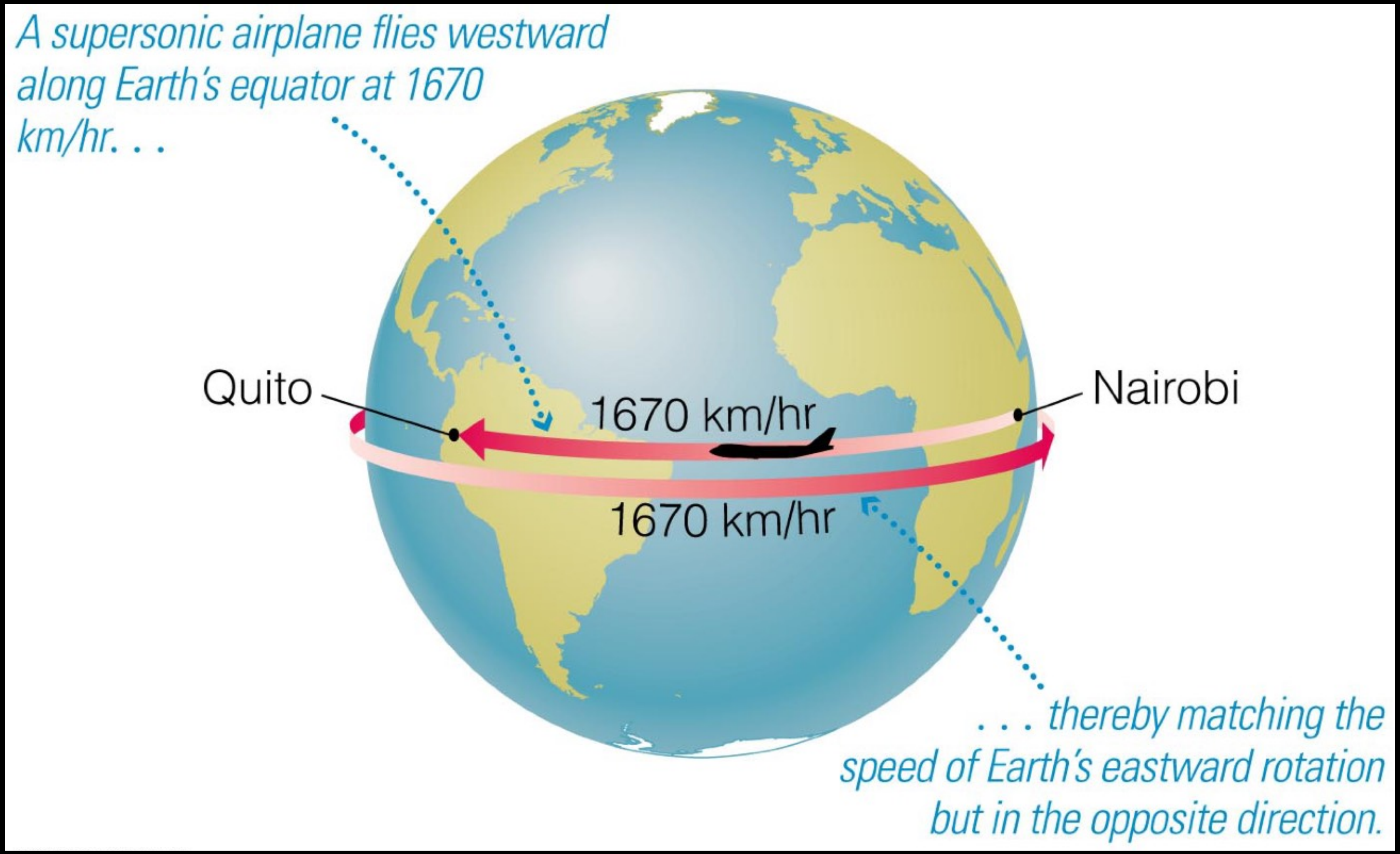


What is relative about relativity?



All motion is **relative** to our frame of reference. We must define motion using a **reference frame**.

Example: flying airplane



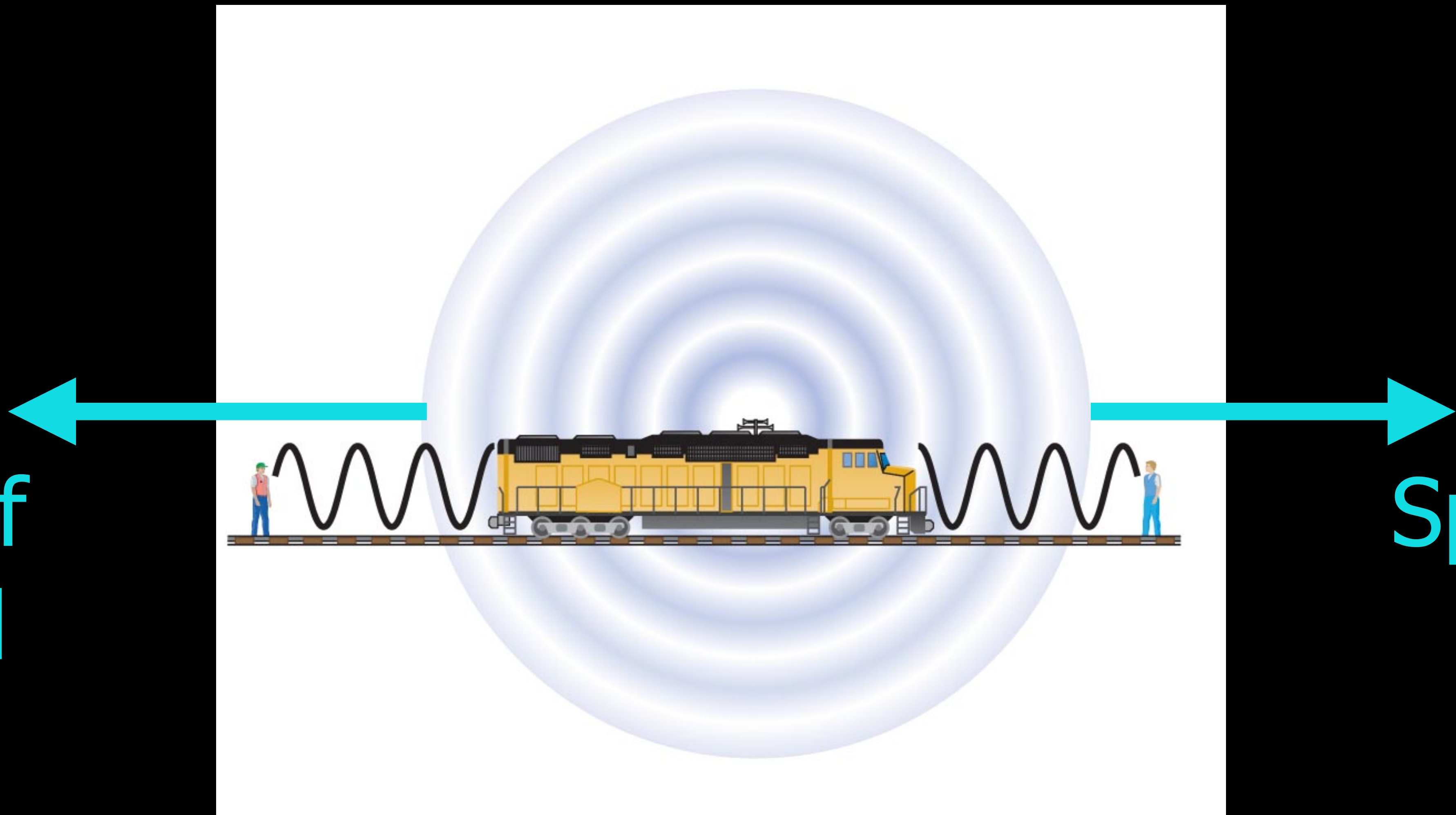
- According to observer on Earth, the plane is moving west at 1670 km/hr
- According to observer in space, the plane is standing still! (but Earth is rotating underneath the plane)

What is absolute about relativity?

Now it gets weird…

Recall: motion and the Doppler effect

Speed of sound

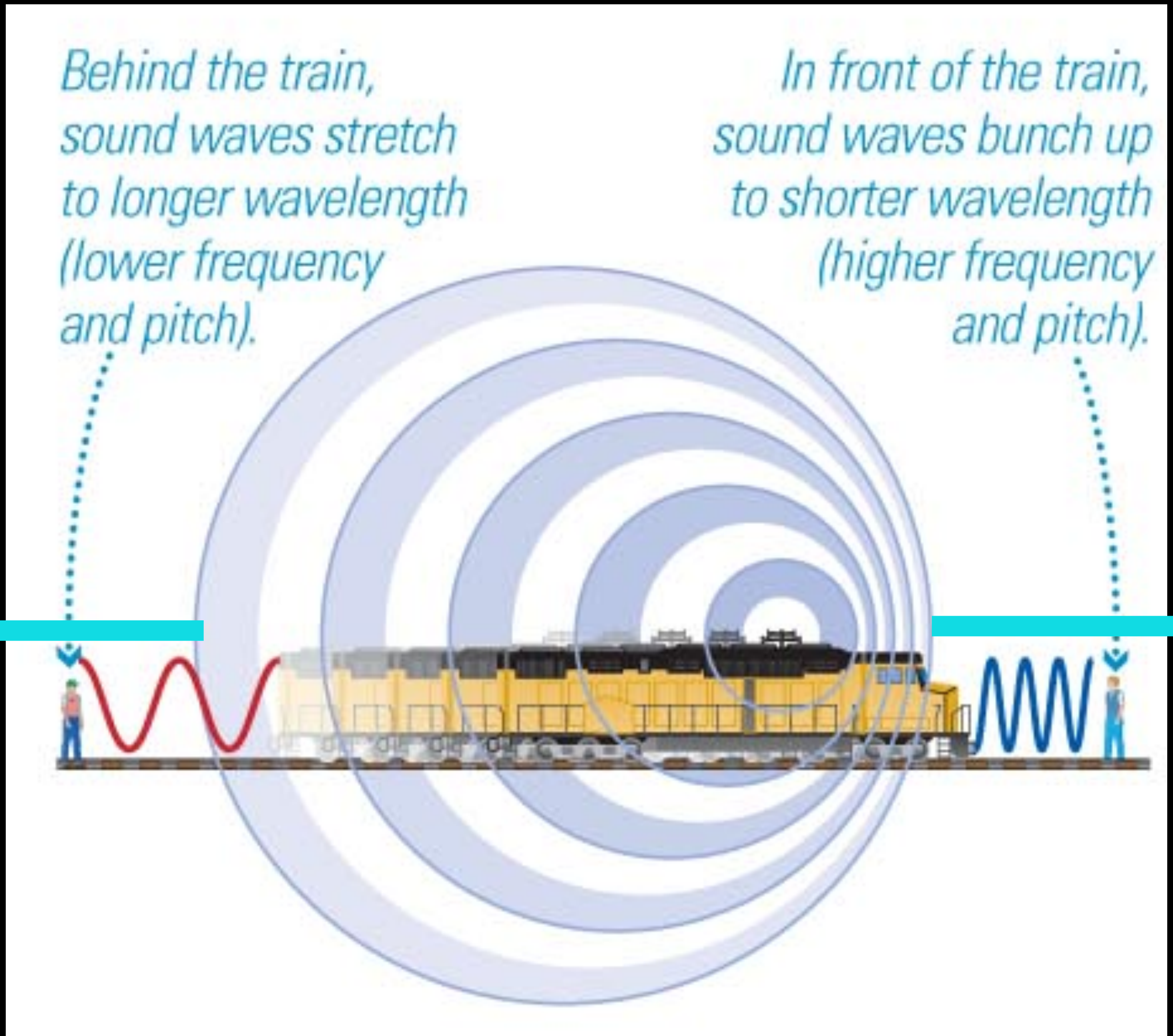


Speed of sound

a The whistle sounds the same no matter where you stand near a stationary train.

Recall: motion and the Doppler effect

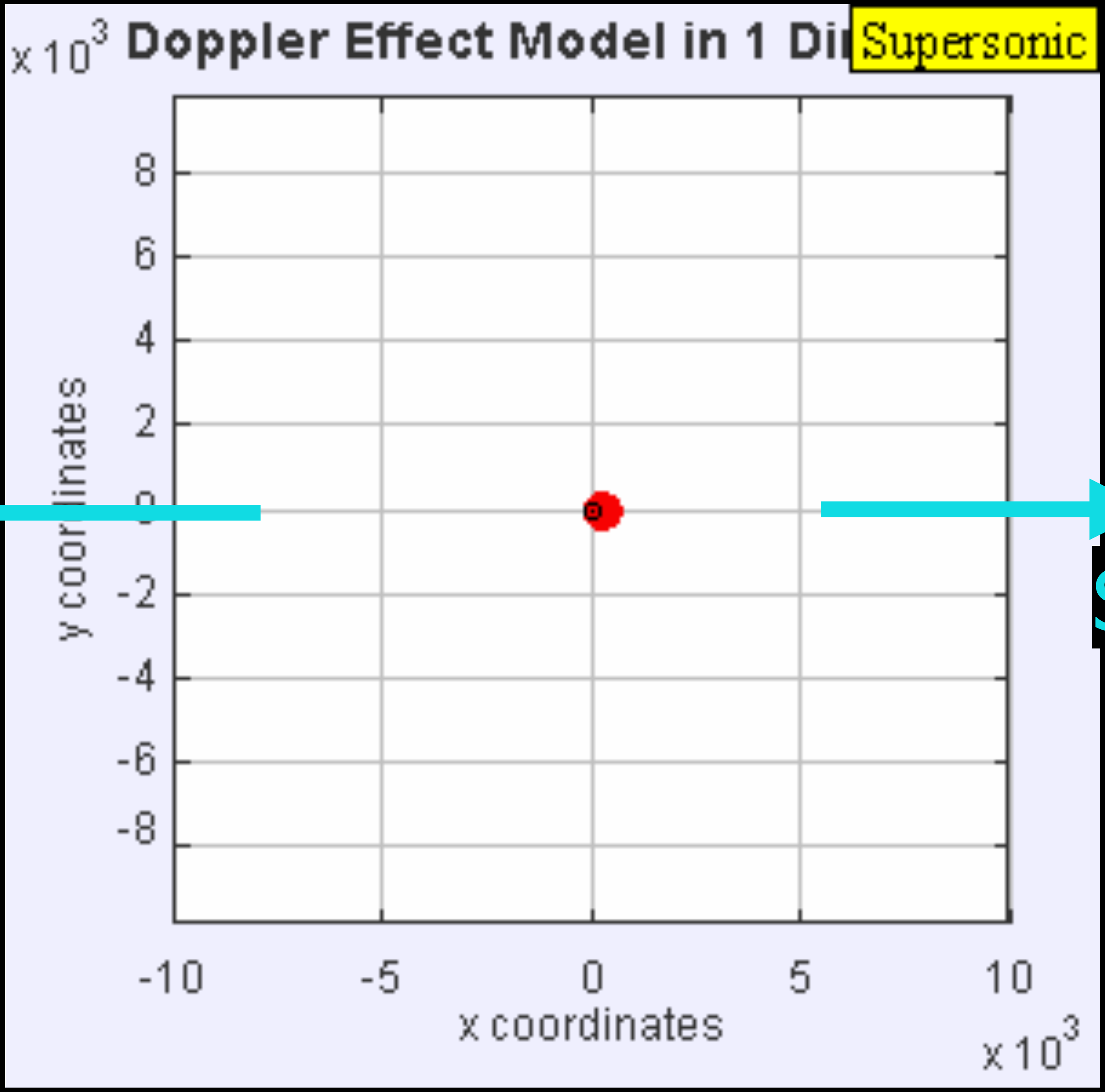
Speed of sound



b For a moving train, the sound you hear depends on whether the train is moving toward you or away from you.

Speed of sound

Recall: motion and the Doppler effect

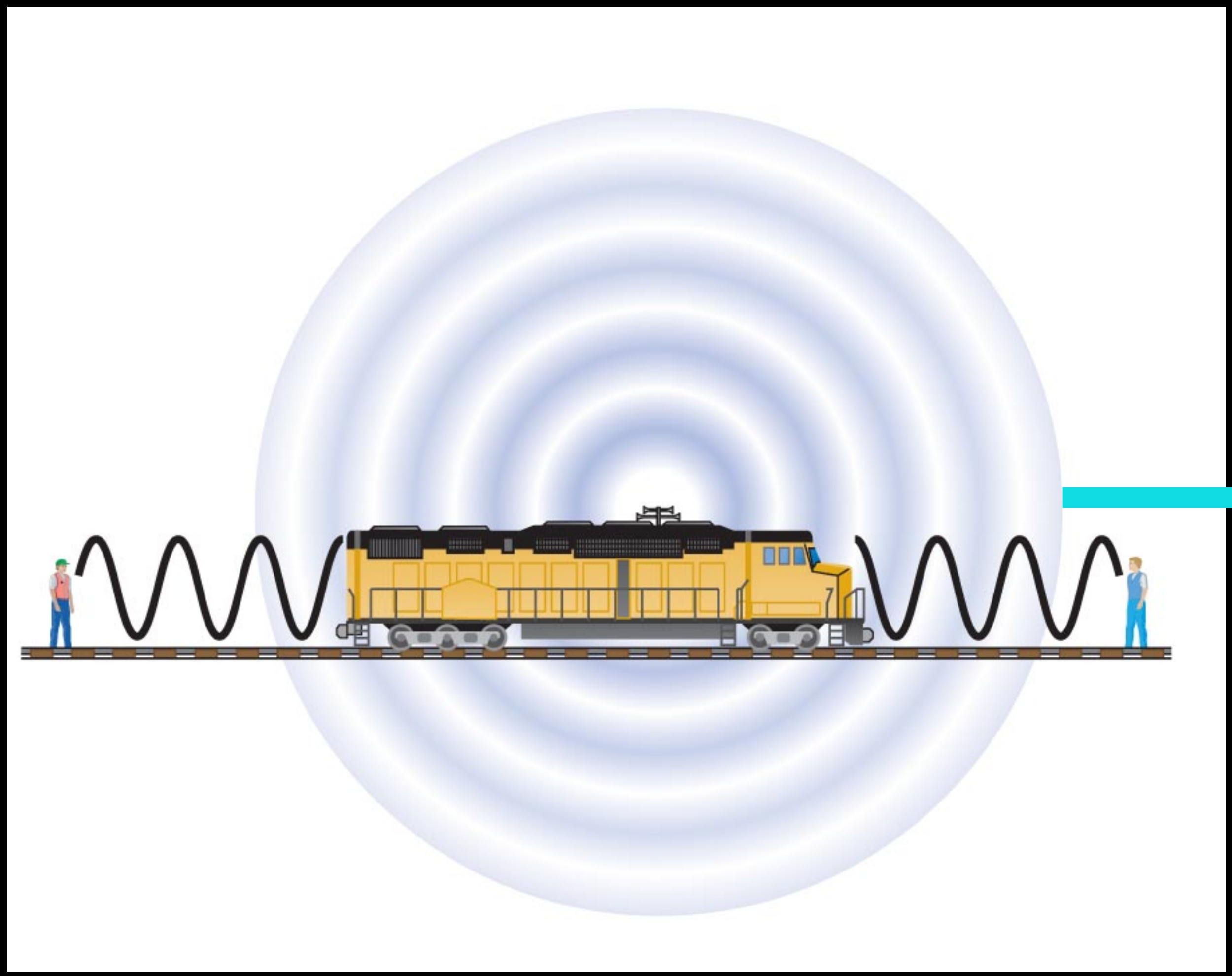


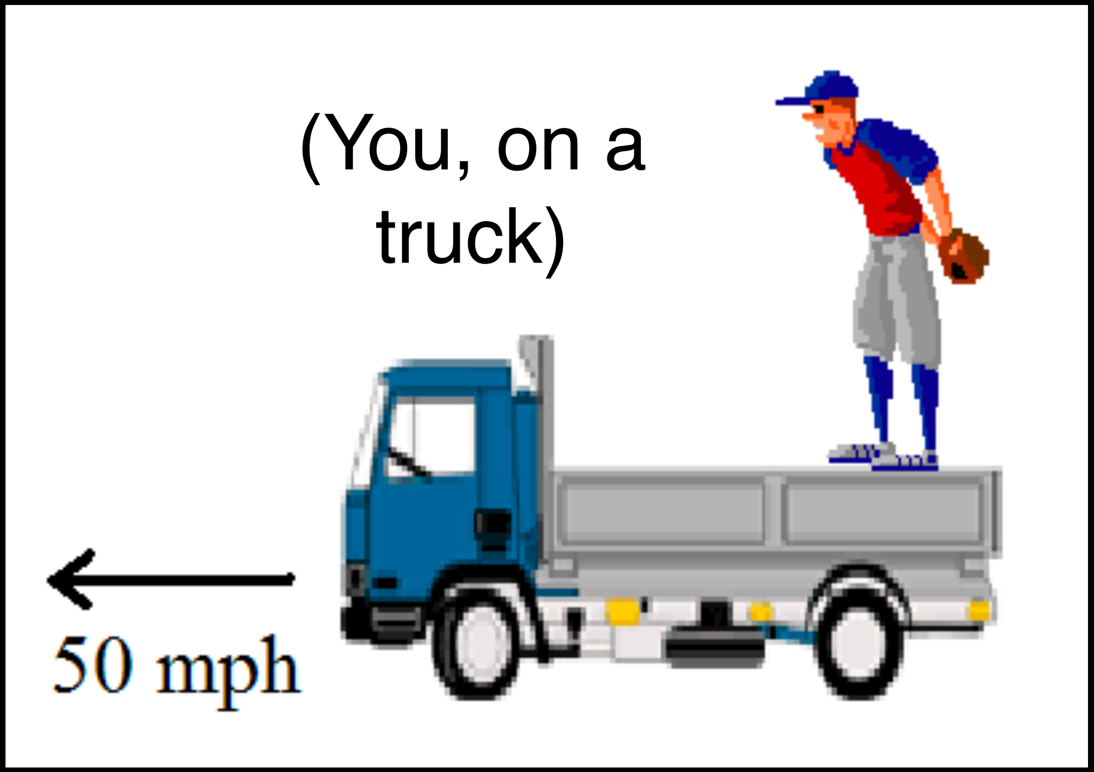
Speed of sound

Speed of sound

Sound waves from a supersonic jet

What about now?



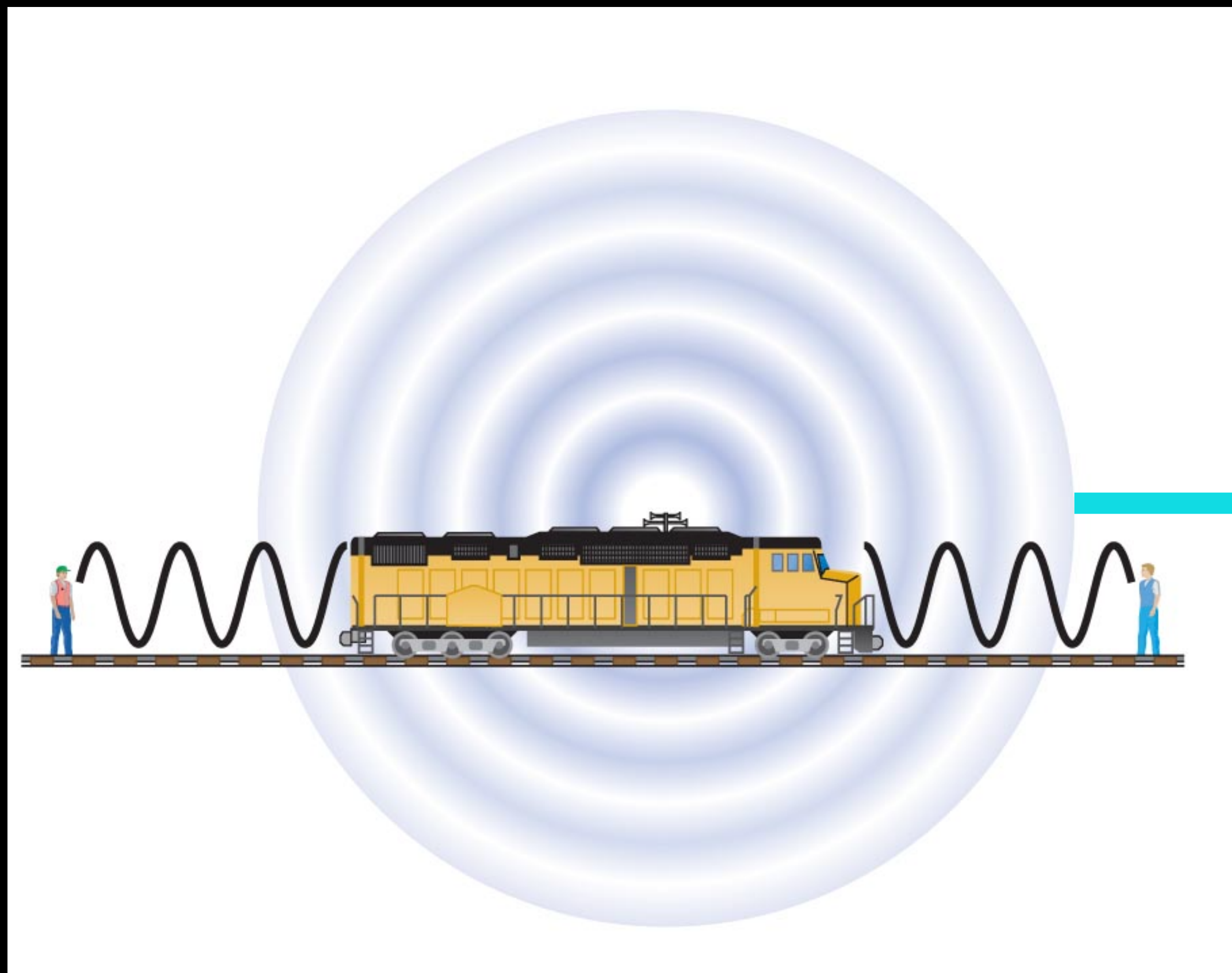


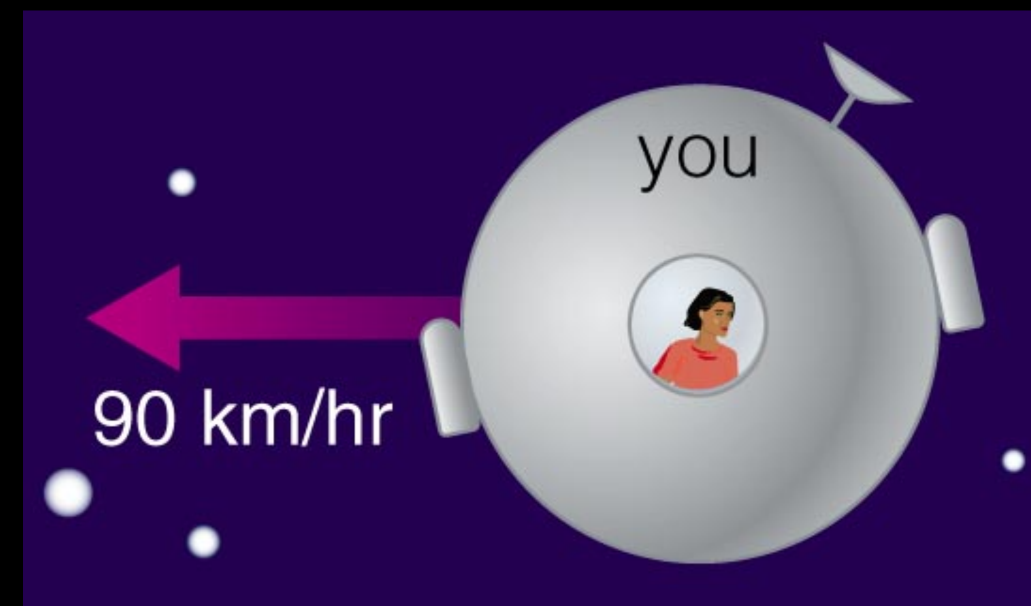
Speed of sound

What you perceive:

- Sound wave appears to be moving toward you at speed = (speed of sound + 50 mph)
- Sound wave will be Doppler shifted (to shorter wavelength, and higher frequency)

What about now?



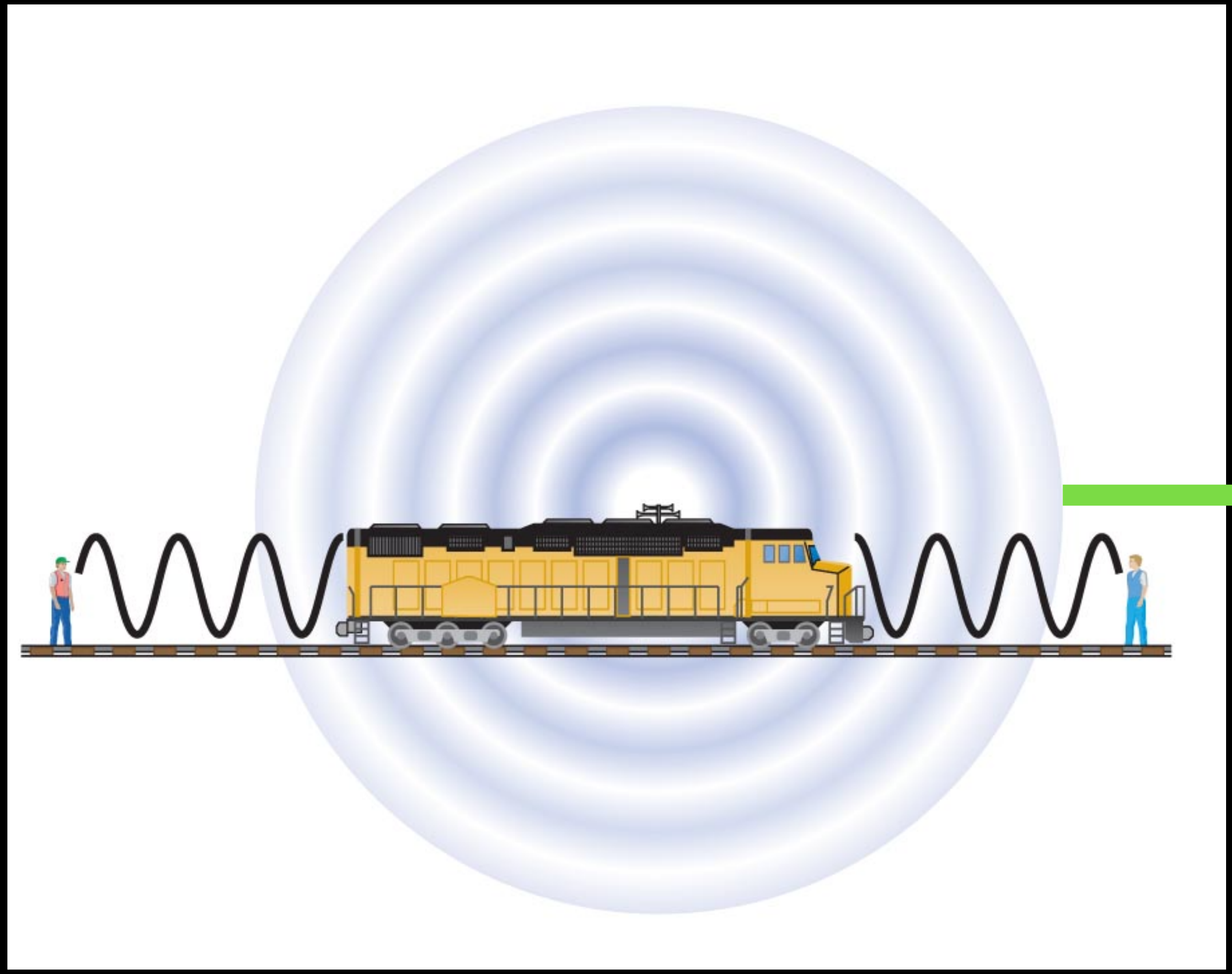


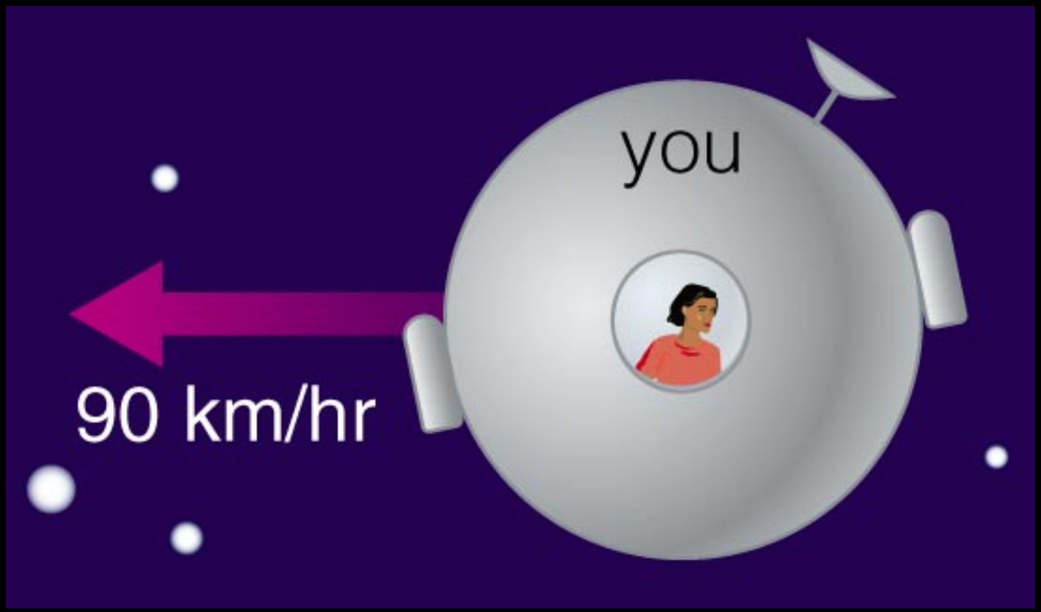
Speed of sound

What you perceive:

- Sound wave appears to be moving toward you at speed = (speed of sound + 90 km/hr)
- Sound wave will be Doppler shifted (to shorter wavelength, and higher frequency)

What about now?



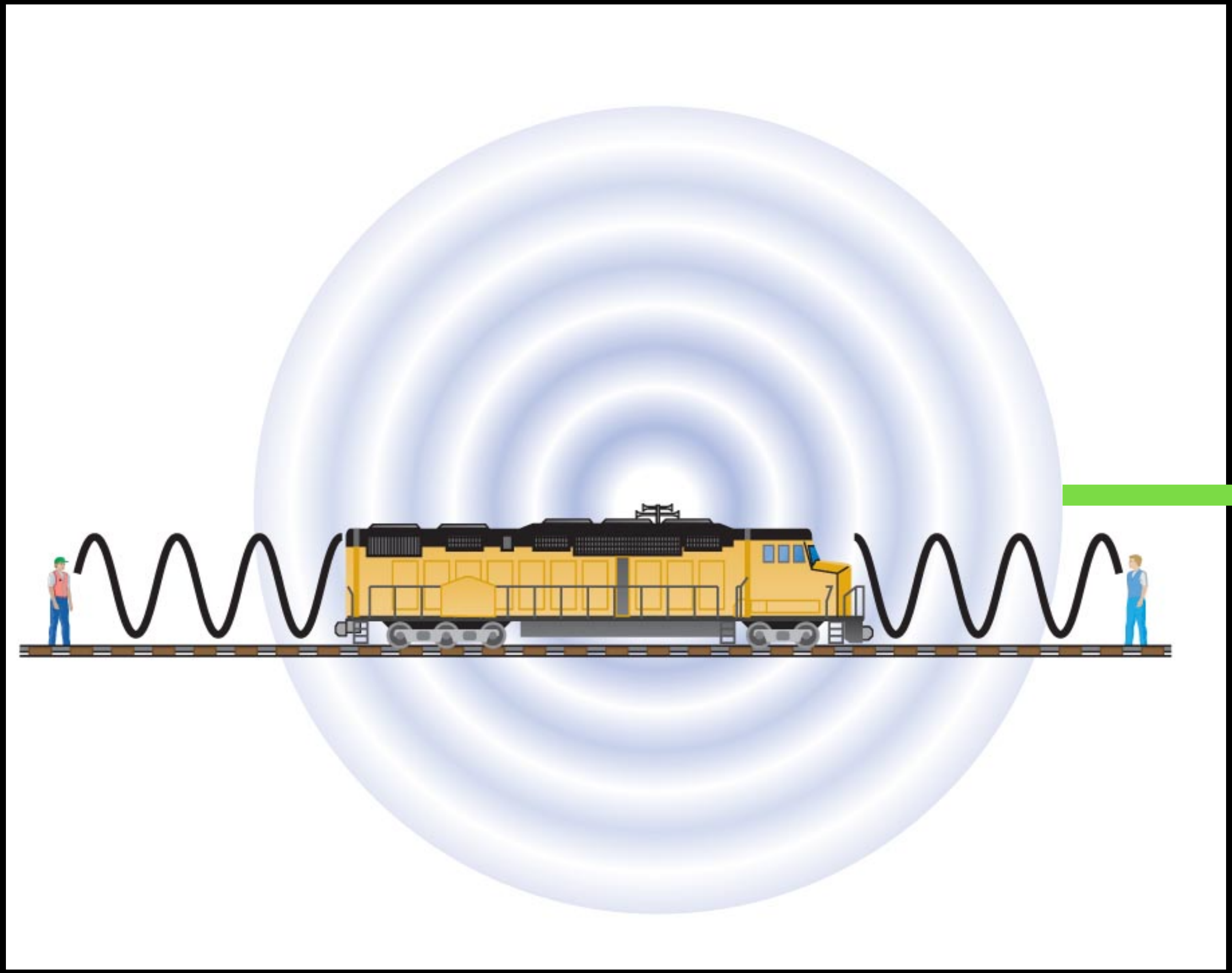


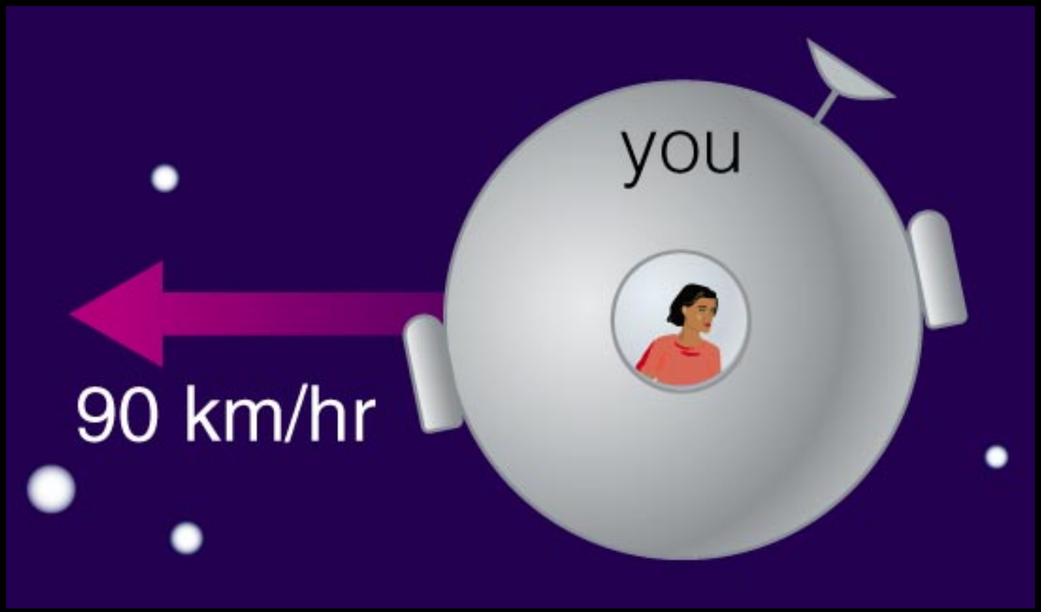
Speed of light (c)

What you perceive:

- Light appears to be moving toward you at speed = **???**
- Light will be Doppler shifted (**???**)

What about now?



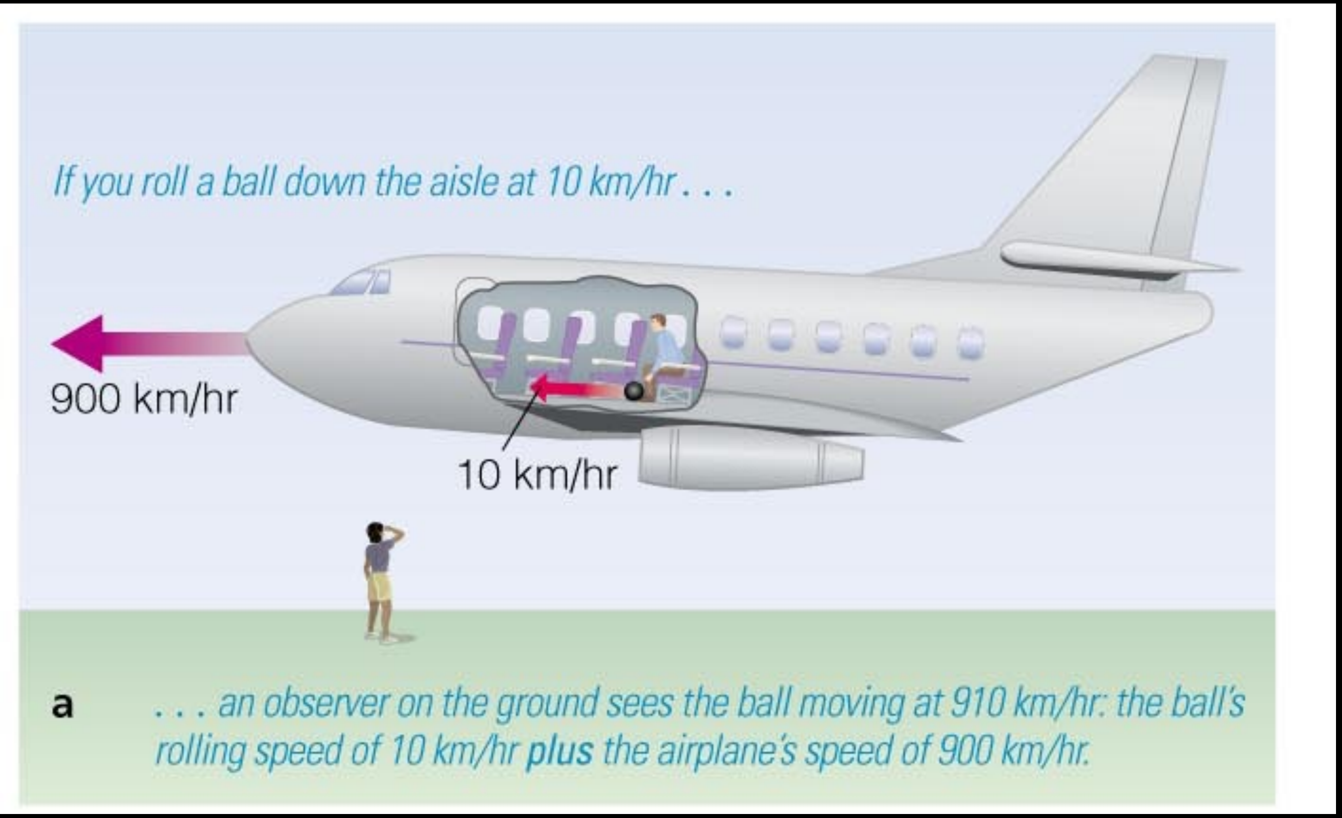


Speed of light (c)

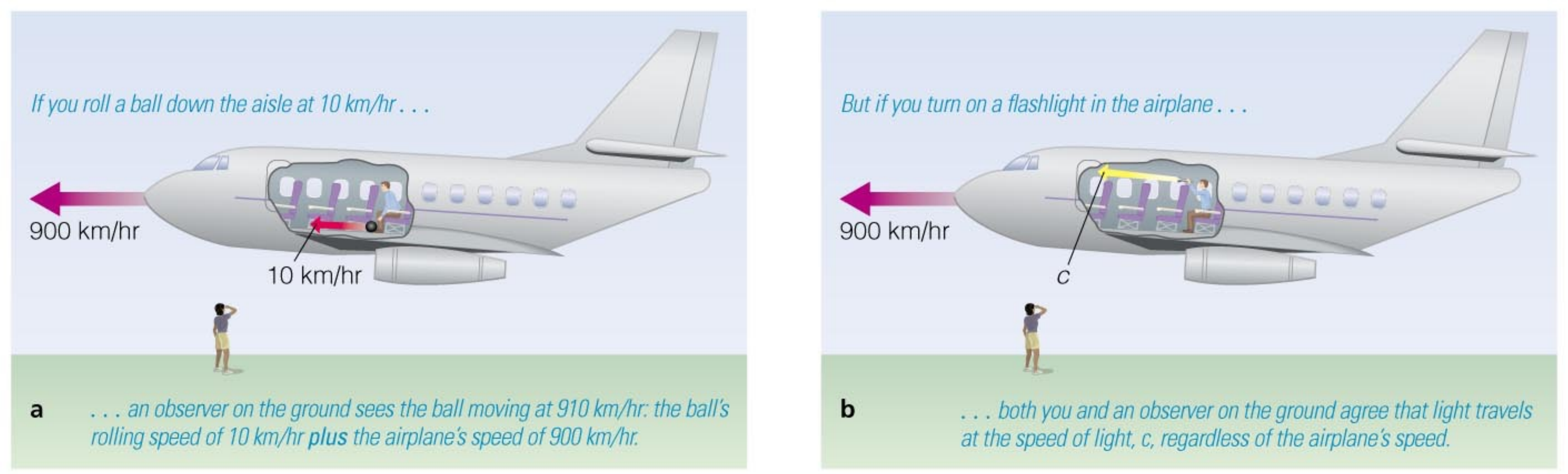
What you perceive:

- Light appears to be moving toward you at speed = c (**NOT** at c + 90 km/hr !!!)
- Light will be Doppler shifted (to shorter wavelength, and higher frequency)

We **always** perceive light to be moving at **exactly** the same speed



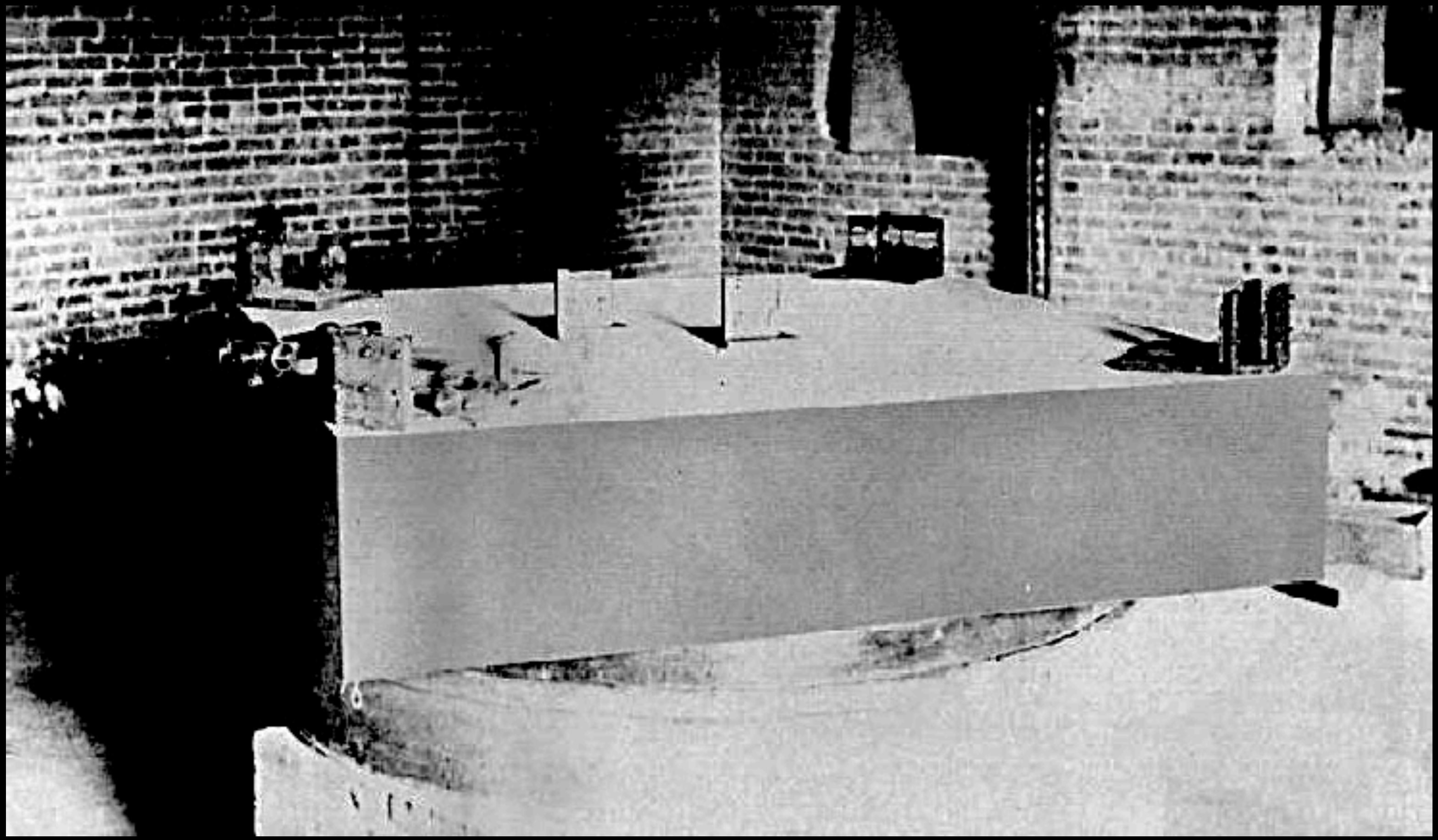
We **always** perceive light to be moving at **exactly** the same speed



What you perceive:

- Light **always** appears to be moving at the same speed c, regardless of your motion
- Light will be Doppler shifted depending on the relative motion between you and the source of light

Speed of light does not change?!?!



First determined experimentally by Michelson and Morley (1880s). Verified many times.

- Speed of light is the same in all directions, regardless of motion
- Huge surprise at the time!! It showed that we did not understand light at all!

What is absolute about relativity?

1. The speed of light!
2. The laws of physics!

All predictions of relativity follow from these two basic ideas!!!

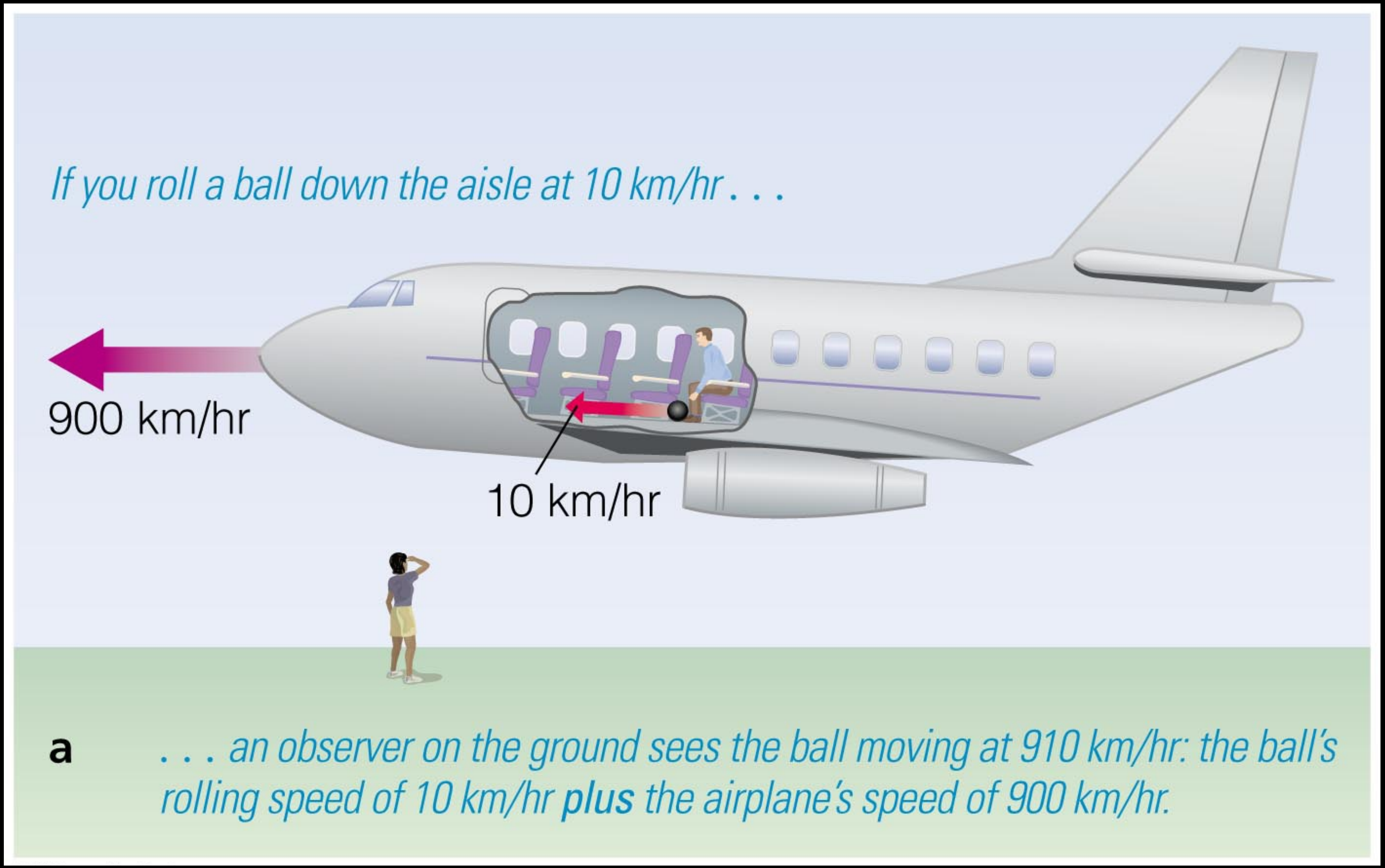
Thought experiments

Einstein explored the consequences of the absoluteness of light speed using "thought experiments”

We will attempt to understand the consequences with thought experiments, by visualizing spaceships in freely floating reference frames (no gravity and no acceleration)

One key result: **nothing can ever move faster than the speed of light.**

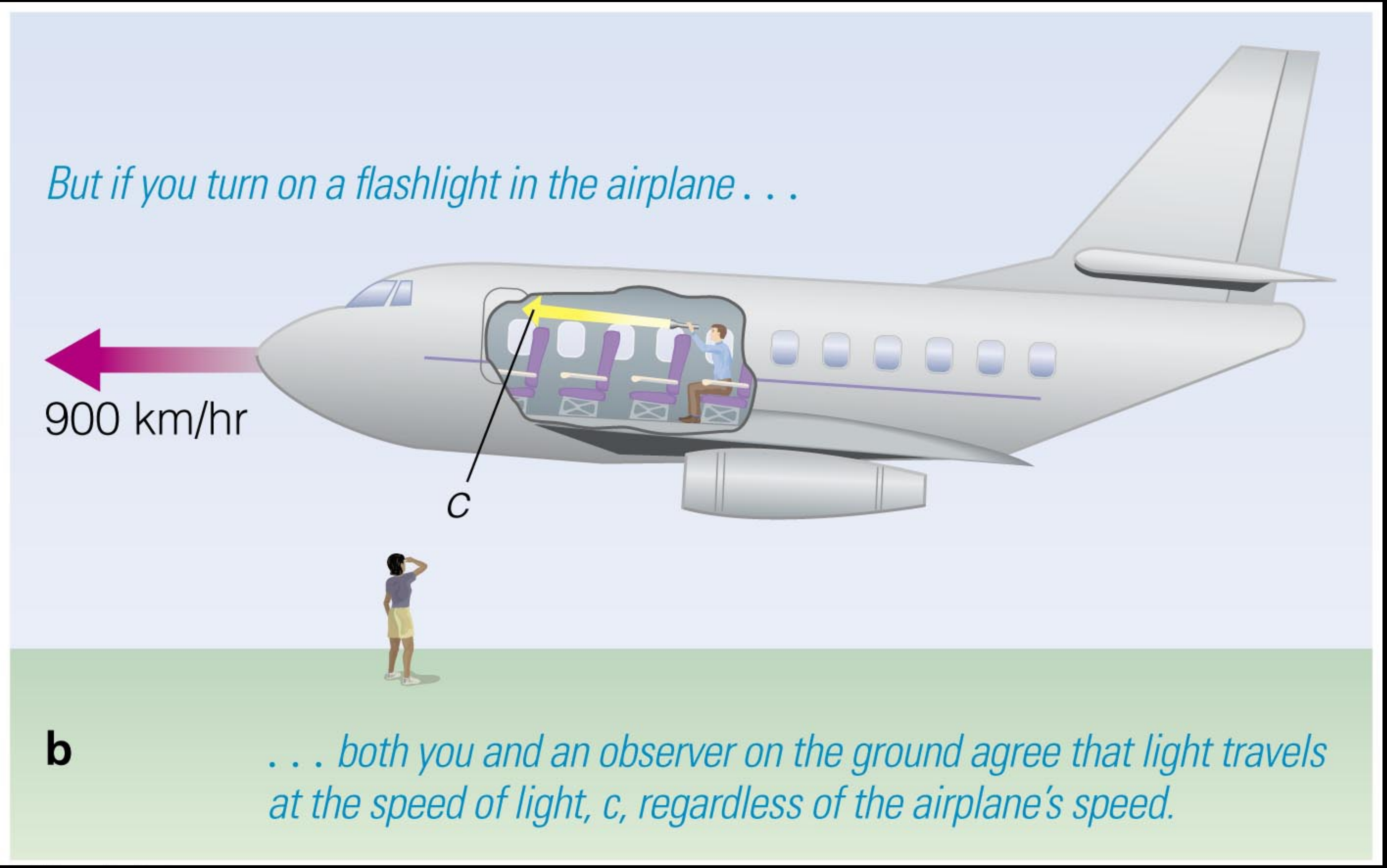
Recap: reference frames



We must define all motion with respect to a particular frame of reference

(motion is relative)

Recap: speed of light



Einstein hypothesized that light should move at exactly the same speed (c) in all reference frames

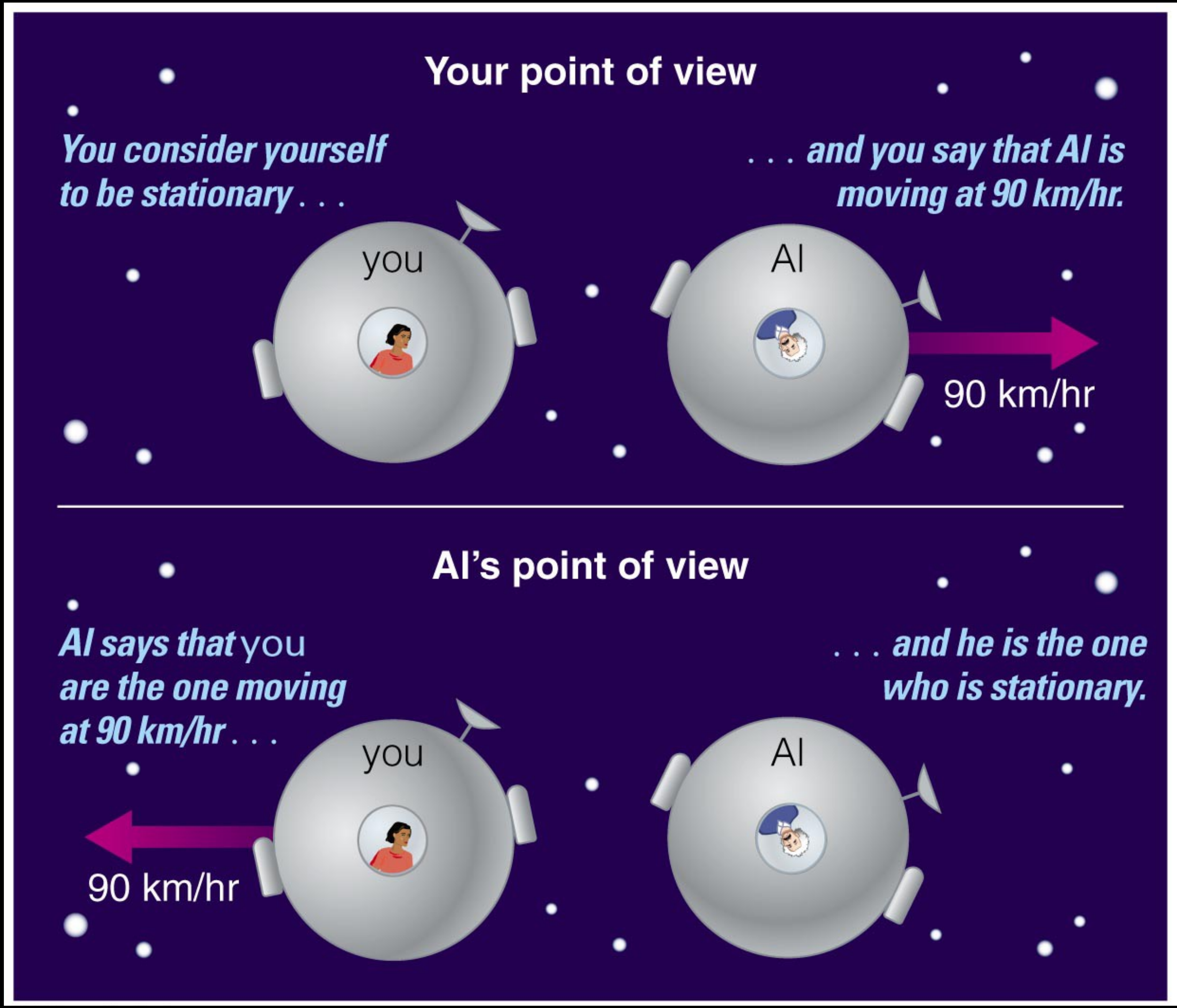
(speed of light is absolute)

What have we learned (so far)?

- **What is “relative” about relativity?**
  - Motion is relative
  - We must define motion relative to some reference frame
- **What is absolute about relativity?**
  - The speed of light is absolute (always the same, regardless of motion)
  - The laws of physics are always the same

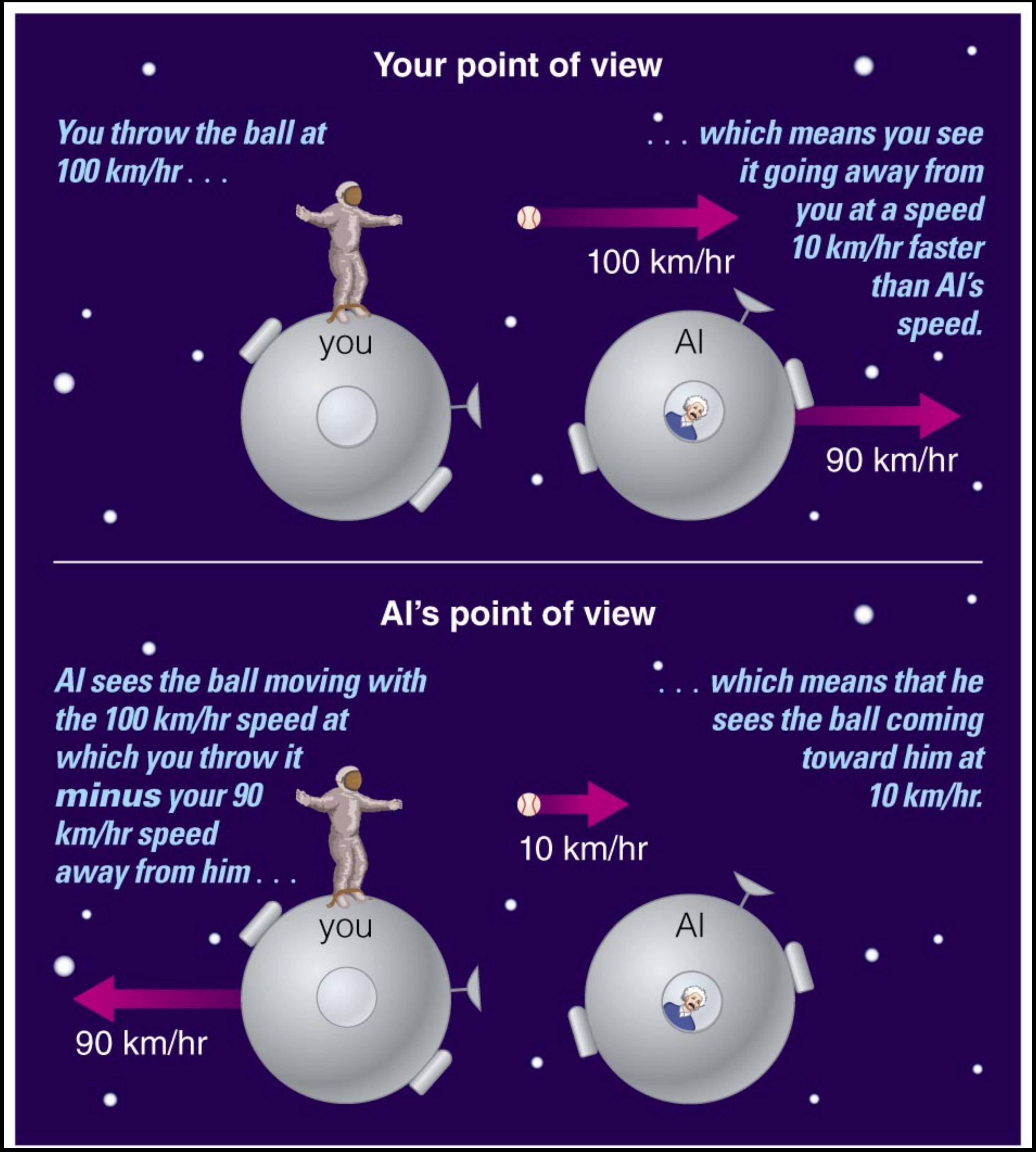
Thought experiments

Motion is relative

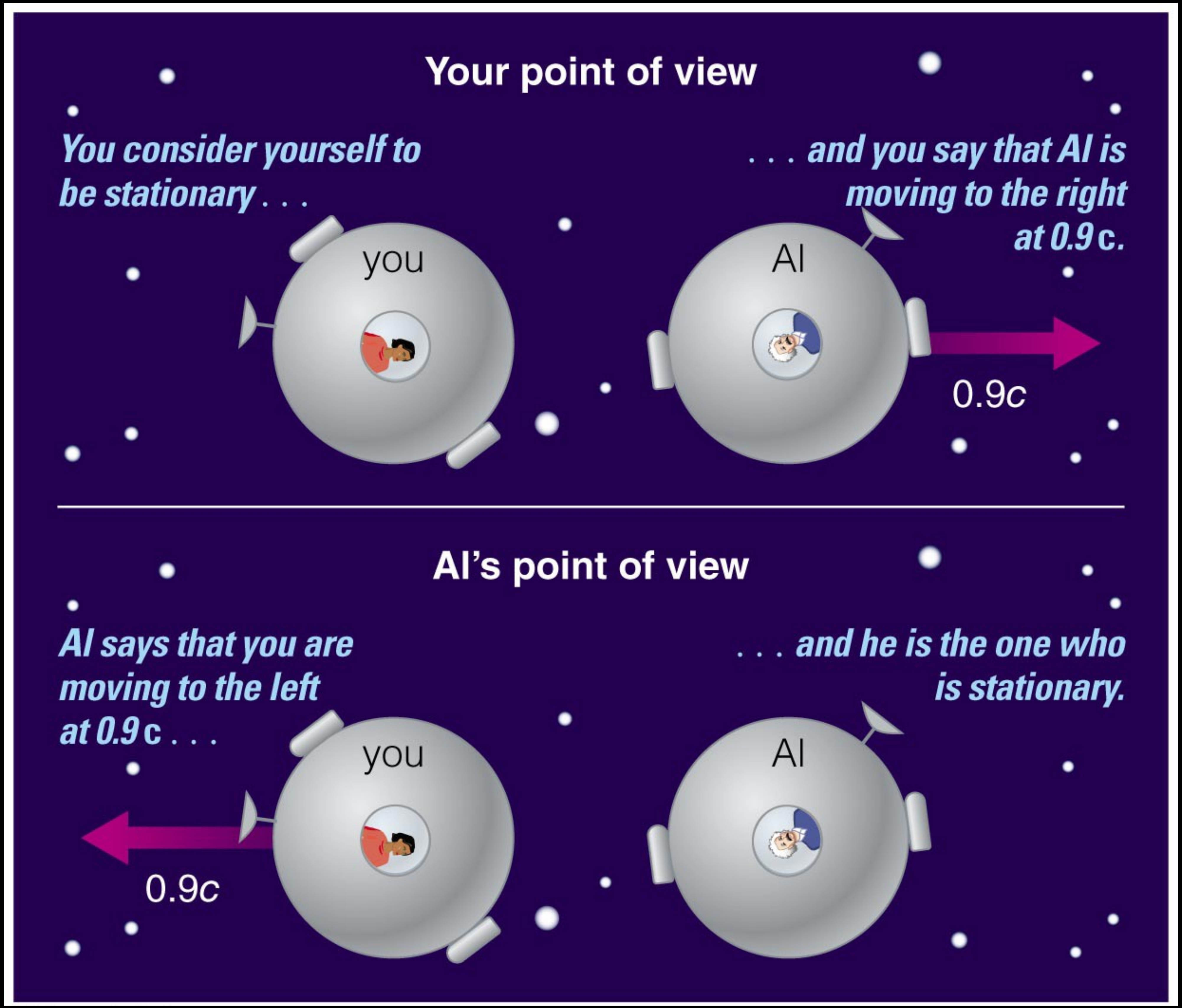


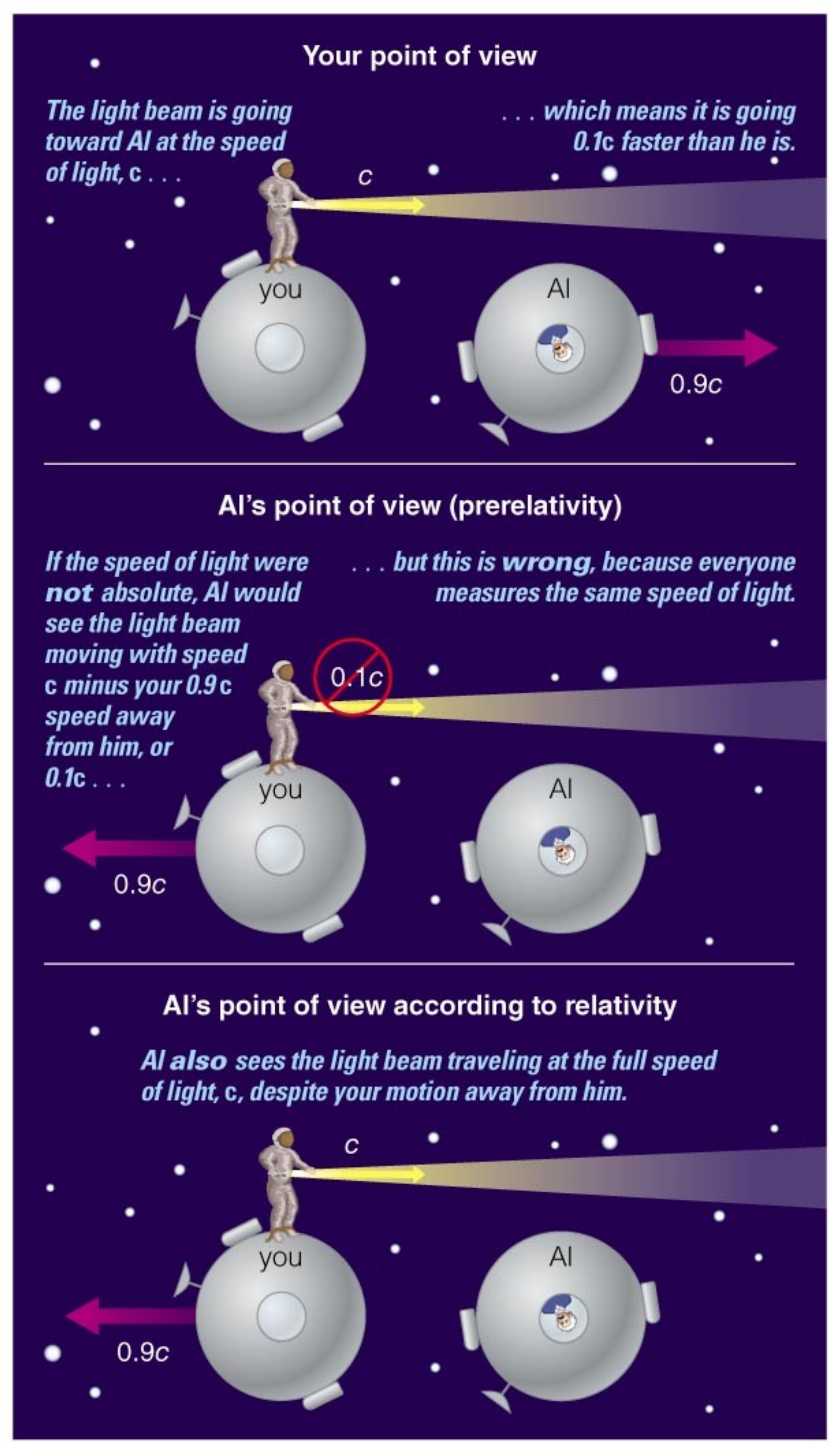
"Point of view" = reference frame

Motion is relative



Motion at high speed (approaching c)



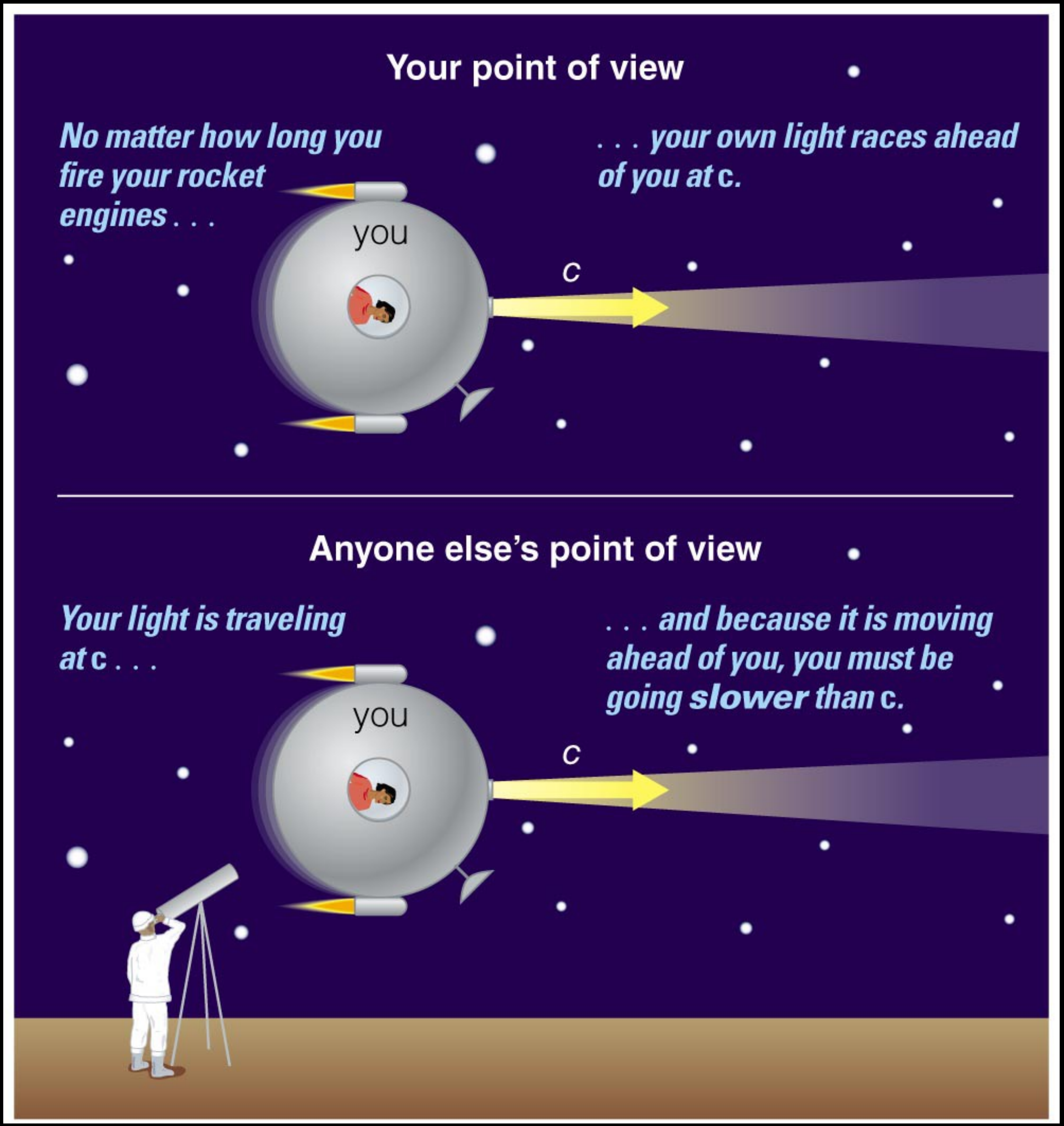


Light speed is absolute

$c - 0.9c = c$!?!

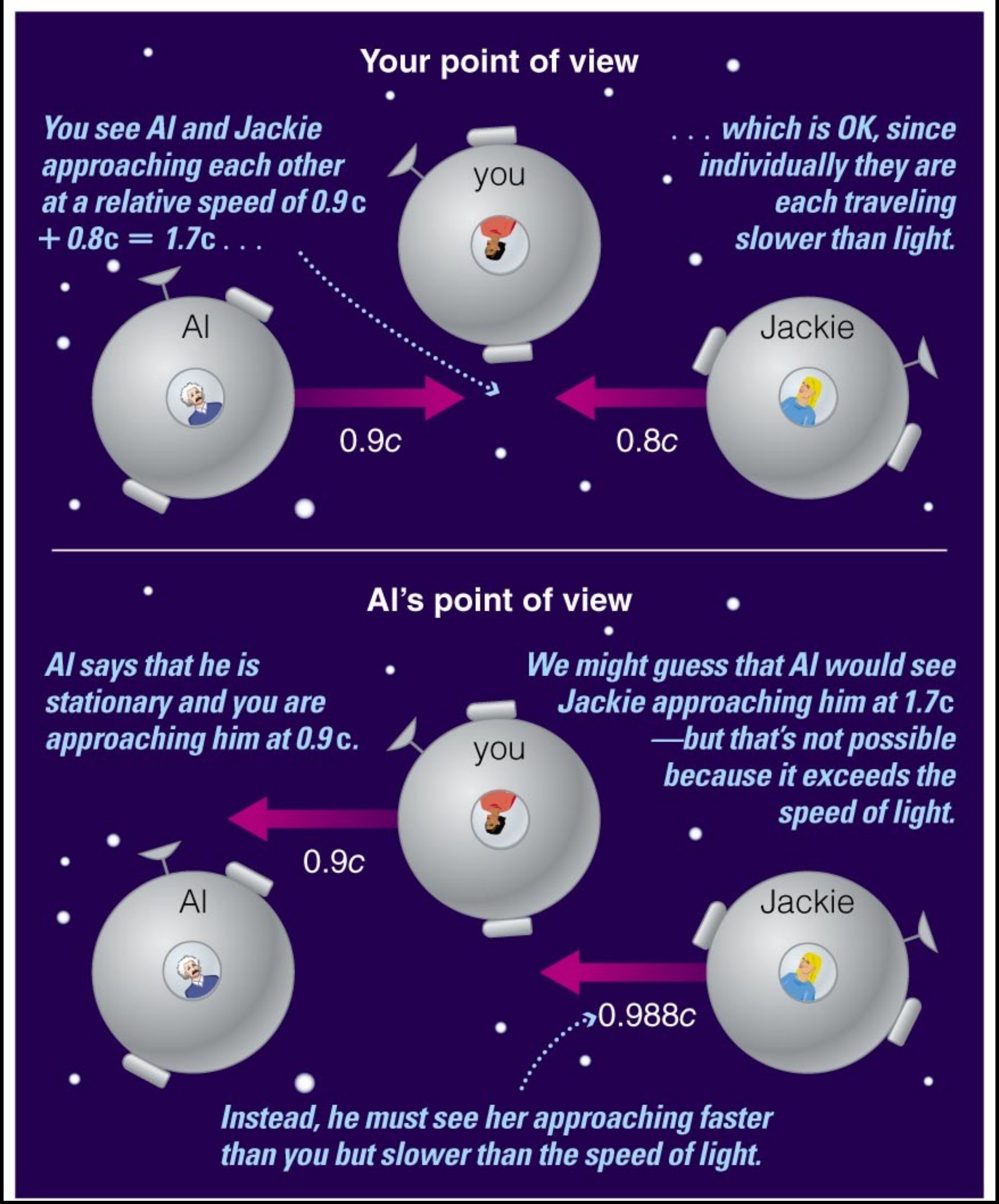
This appears to be true, based on our measurements

Consequence: we cannot move at the speed of light



- Suppose you tried to catch up to your own headlight beams
- You'd always see them moving away at speed *c*
- Anyone else would also see the light moving ahead of you
  - Therefore, you are always moving slower than light

Wait, how does this work??



"Normal" math:

- We would normally say that Jackie is moving toward Al at $v = 0.9c + 0.8c = 1.7c$
- But this would mean Jackie is moving faster than light, which is not allowed

Relativity math:

- Velocities add in a funny way: $v = \dfrac{v_1 + v_2}{1 + \dfrac{v_1 v_2}{c^2}}$
- So "0.9c + 0.8c" gives 0.988c
- Everyone is moving slower than c

Quick recap: what have we learned?

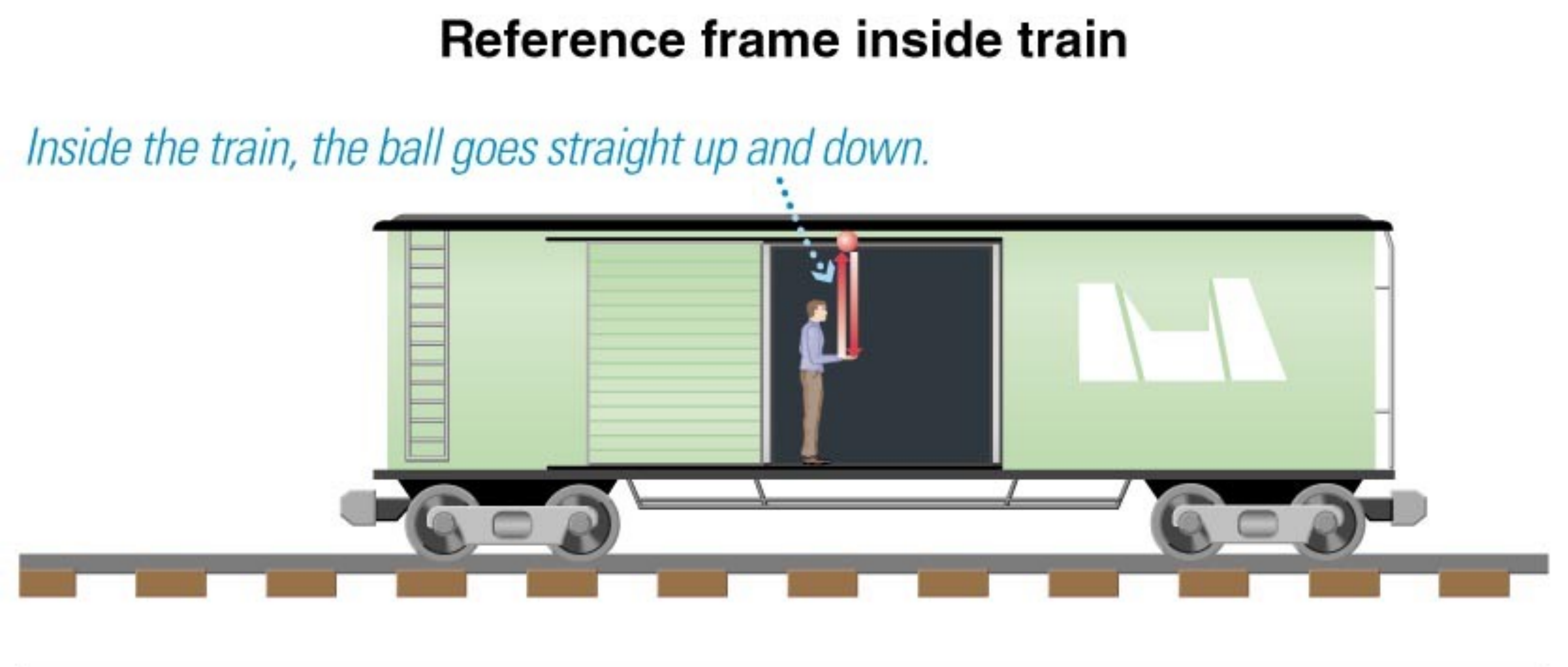
- **What is surprising about the absoluteness of the speed of light?**
  - Velocities in different reference frames do not add up like we expect them to, because the speed of light must be the same for everyone
  - (Relativity involves weird math)

- **Why can't we reach the speed of light?**
  - No matter how fast we go, light always appears to move away from us, so we are always moving slower than light

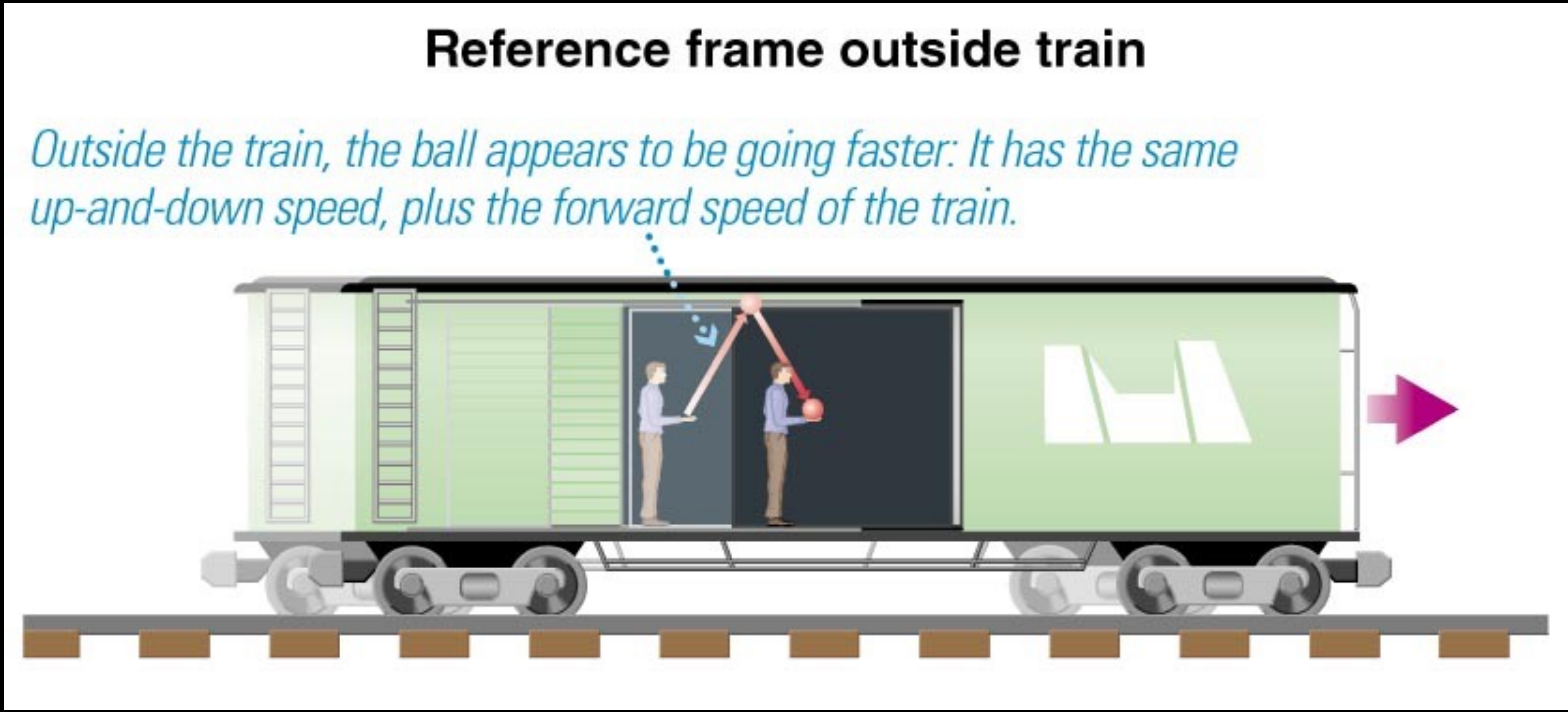
How can this make sense???

We need to change the way we think about space and time

Path of a ball in **stationary** train

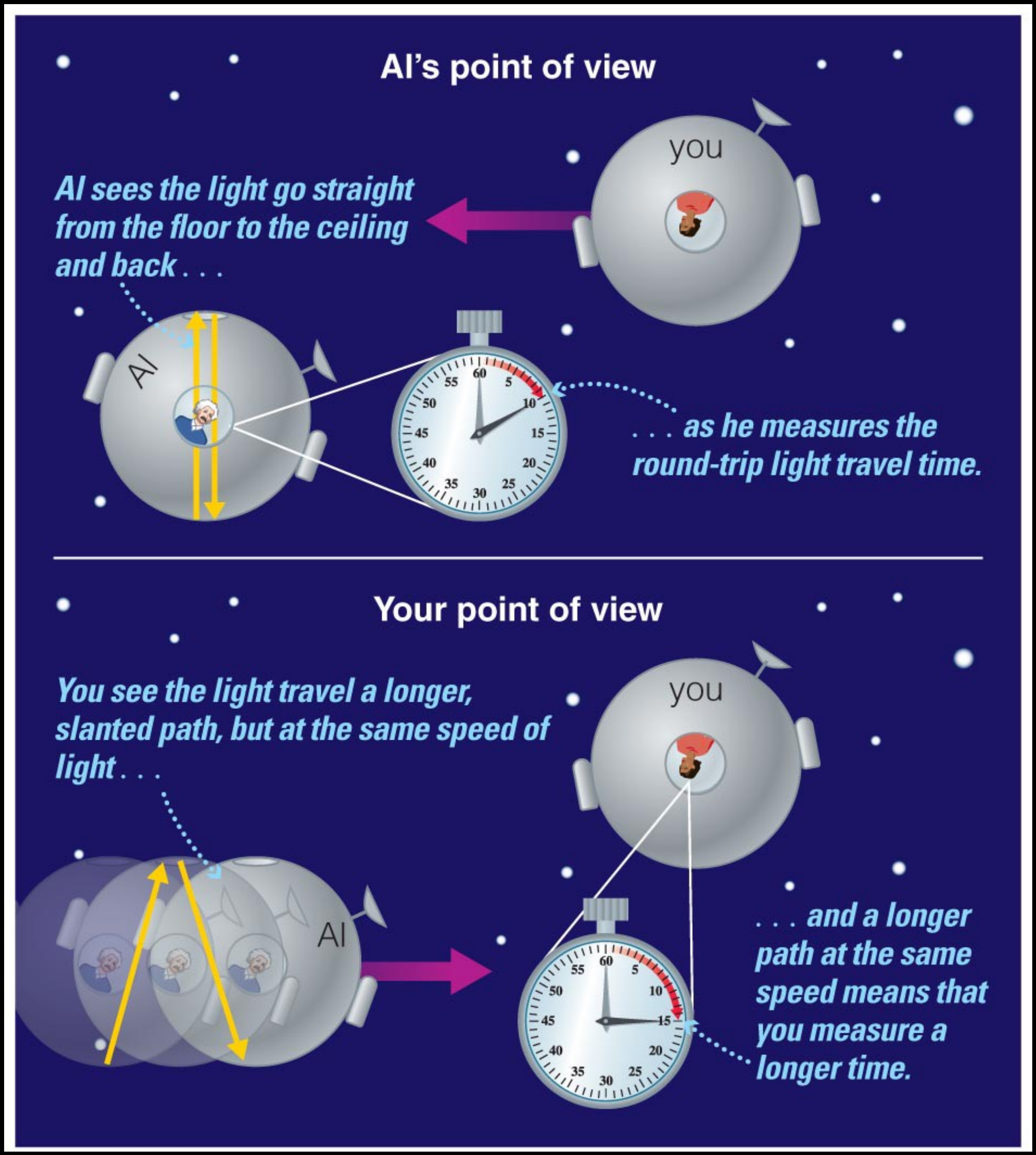


Path of a ball in **moving** train



- Someone outside the train would see the ball travel a longer distance in one up-down cycle
- The faster the train is moving, the longer that path would be

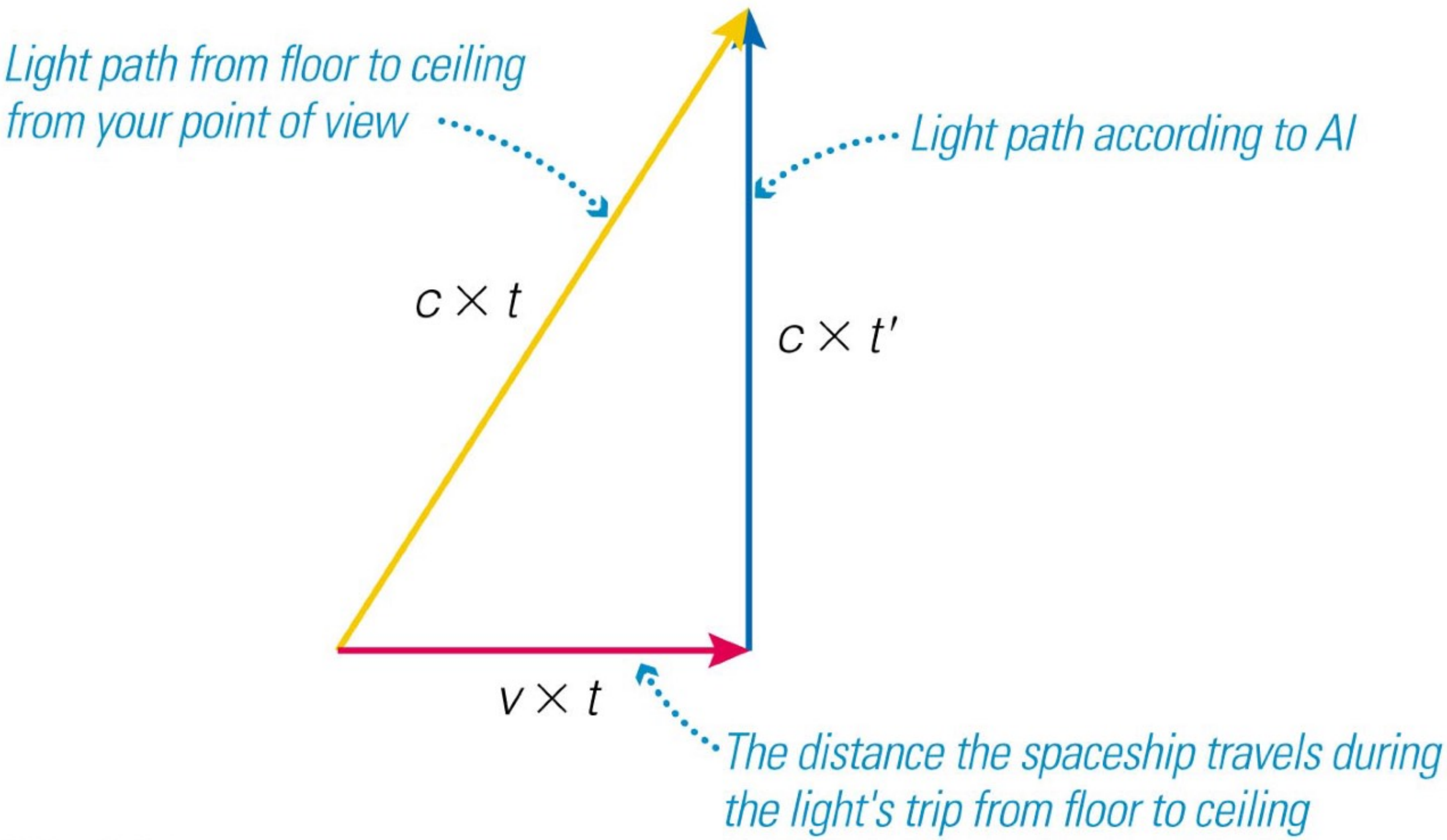
Path of light in a moving spaceship



Time Dilation

- We can perform a thought experiment with a light beam replacing the ball
- The light beam must be moving at speed *c*
  - … but it travels a longer distance in a moving object
  - Distance = speed × time
  - Inside the moving spaceship, distance is smaller, so amount of time must be smaller!
- **Time passes more slowly in moving objects!**

The Time Dilation Formula

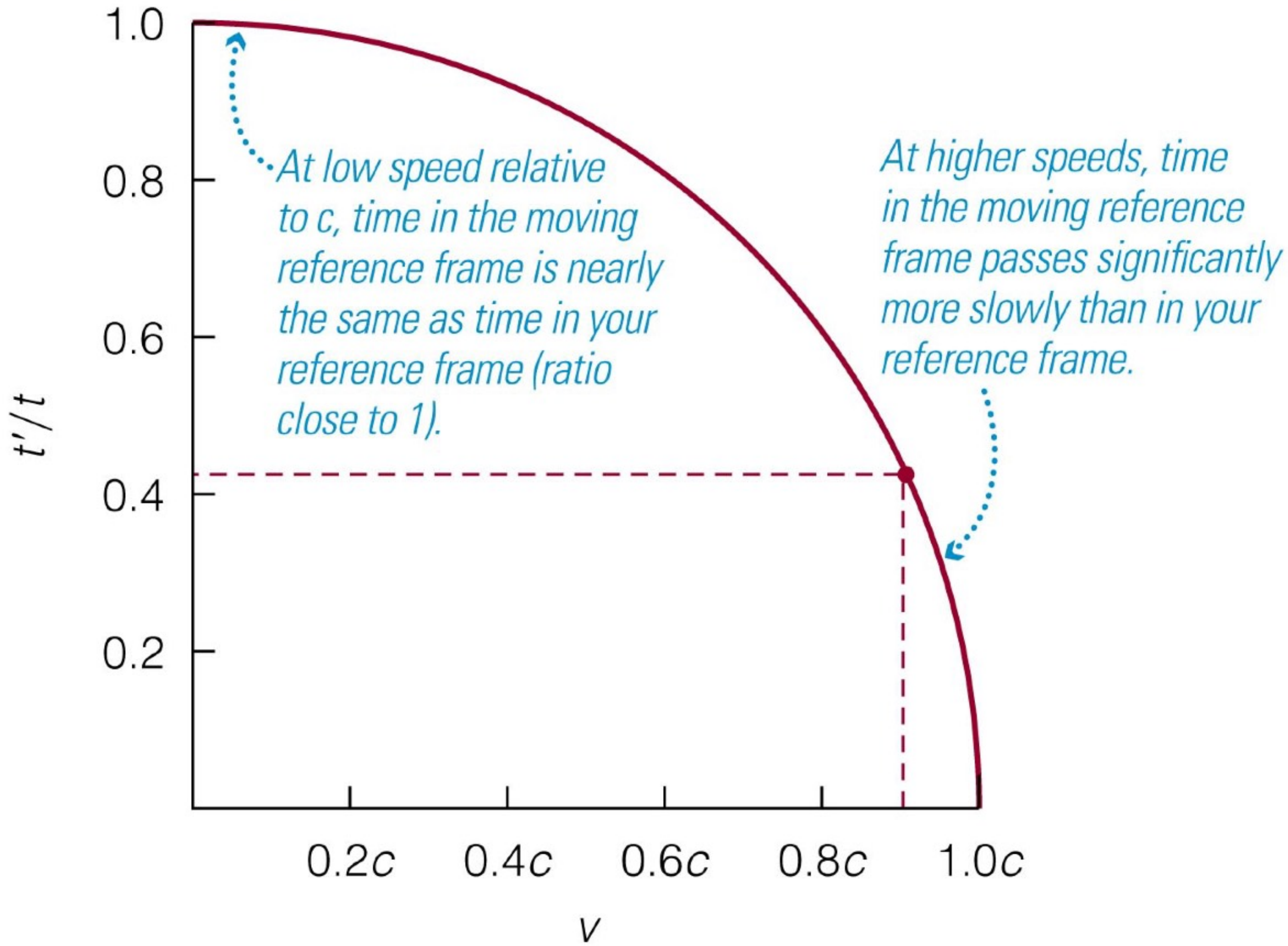


$$c^2 t'^2 + v^2 t^2 = c^2 t^2$$

$$t'^2 = t^2 - \frac{v^2}{c^2} t^2$$

$$t' = t\sqrt{1 - \left(\frac{v^2}{c^2}\right)}$$

The Time Dilation Formula

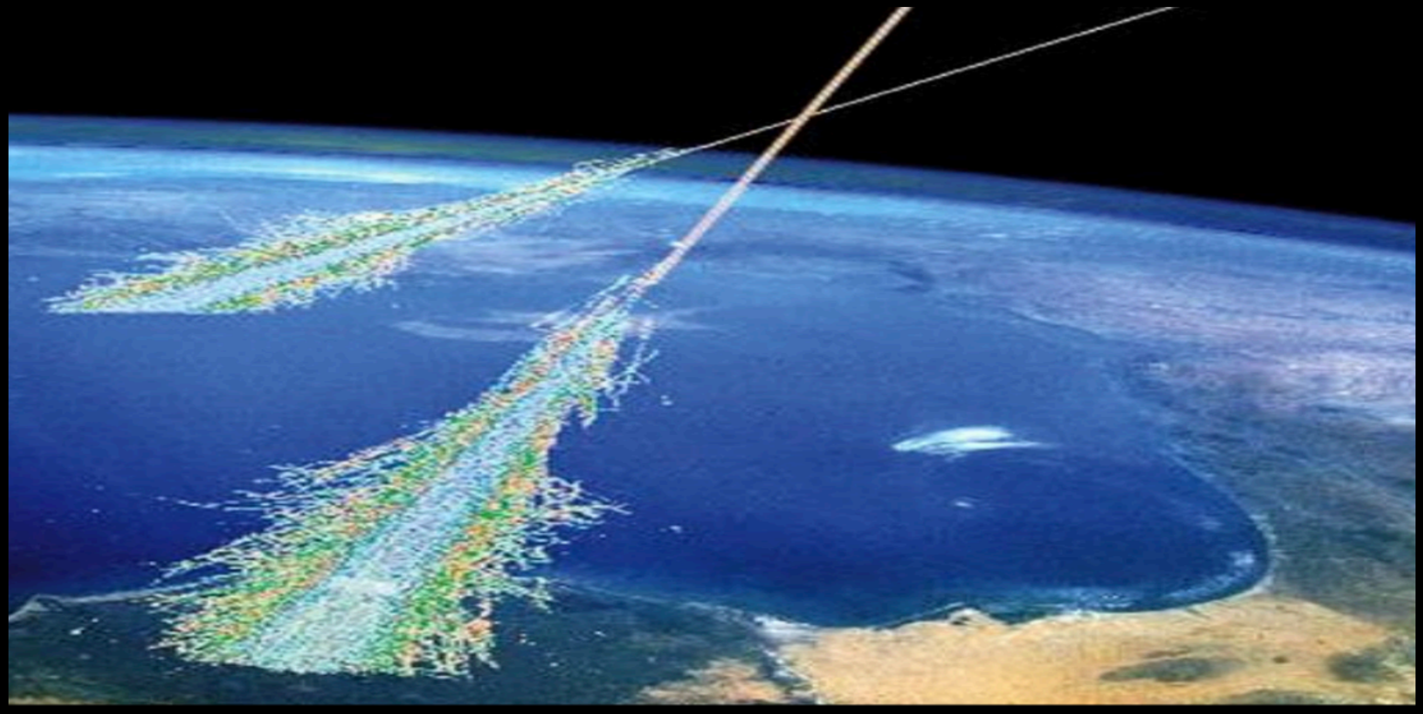


- Time will appear to pass more slowly in a moving object by an amount depending on its speed.
- Time almost halts for objects nearing the speed of light.

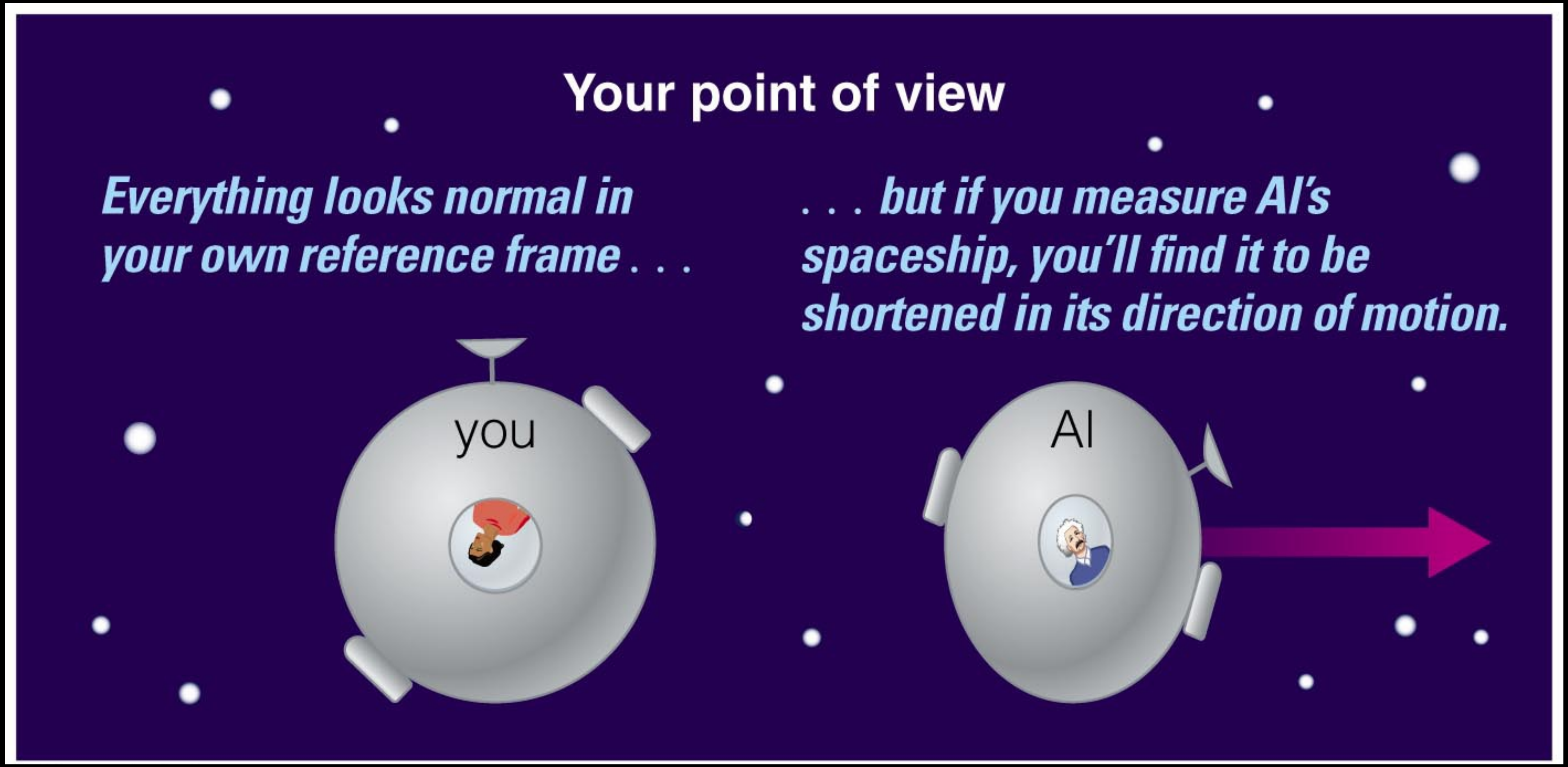
Time dilation!

This really happens!

- Example: muons (type of particle, like a heavy electron) are produced in the upper atmosphere, about 15 km above sea level
- Muon lifetime is only ~2 microsecond
  - Even at the speed of light, it should only travel ~1 km…
  - … but we see lots of muons at sea level which must have traveled for 10-20 km! How??
- **Time passes slowly at high speeds.** Muons are moving so fast that time moves ~40x slower. Their lifetime appears 40x longer to us, and they can travel 40x farther!

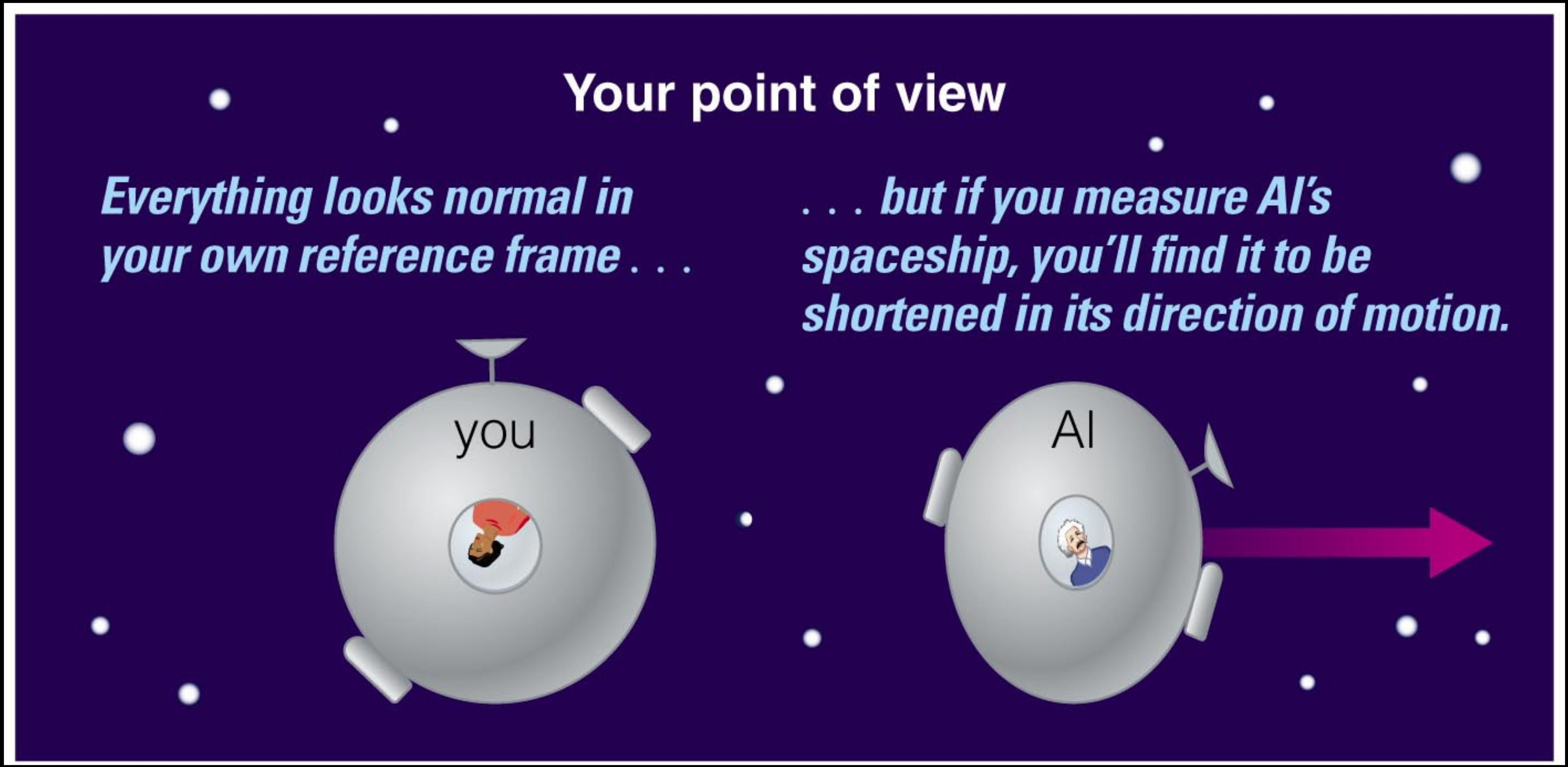


Length contraction!



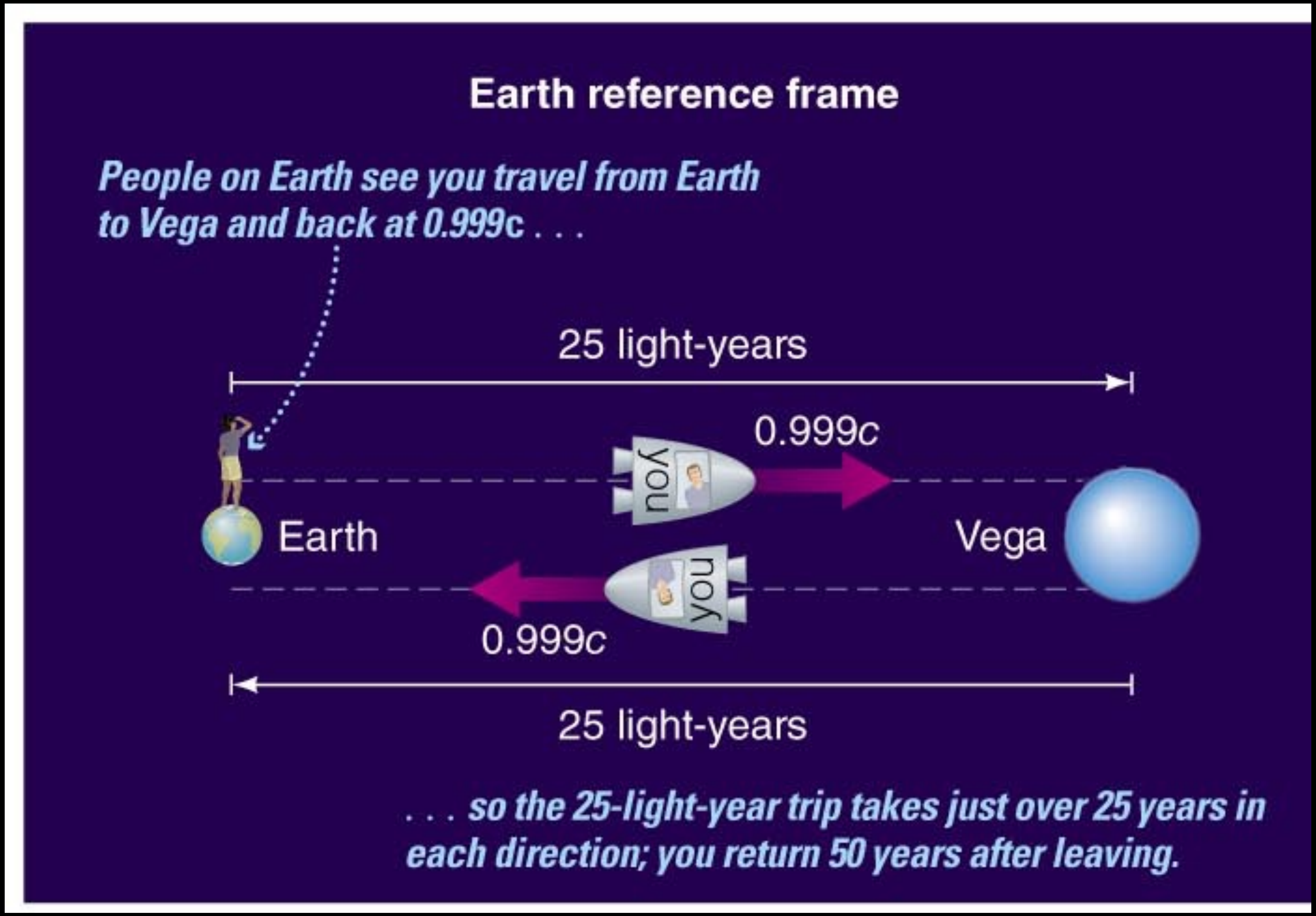
- We just saw that time passes more slowly at higher speeds
- Similar thought experiments tell us that an object's length appears to become **shorter** in it's direction of motion: **length contraction**

Length contraction!



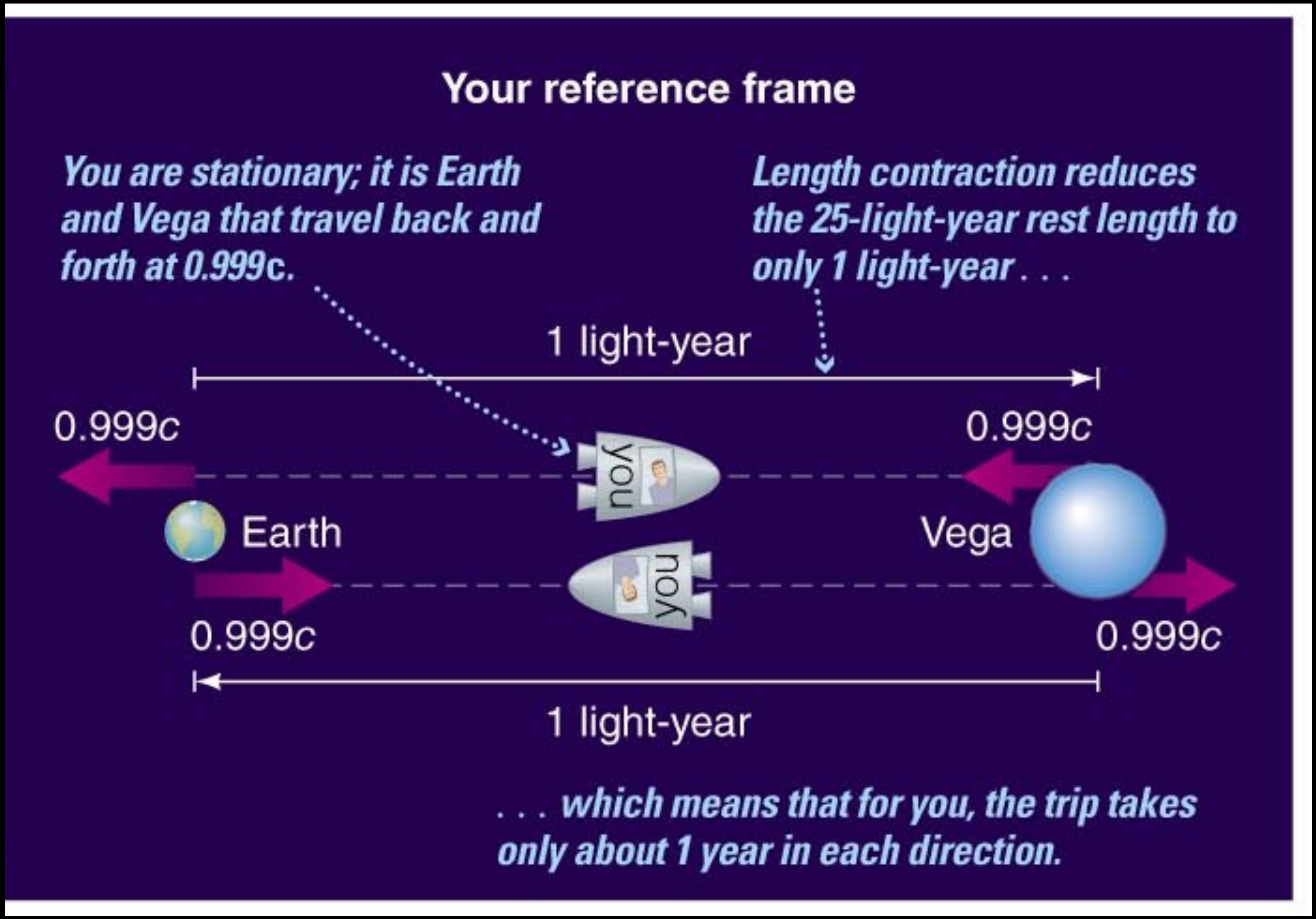
- Basic idea: suppose you observe Al traveling a distance **D** in what appears to you to be 100 seconds.
- Al's clock appears to be 10x slower due to time dilation: only 10 seconds pass on his spaceship
  - So, the same distance appears 10x shorter to Al

A ticket to the stars



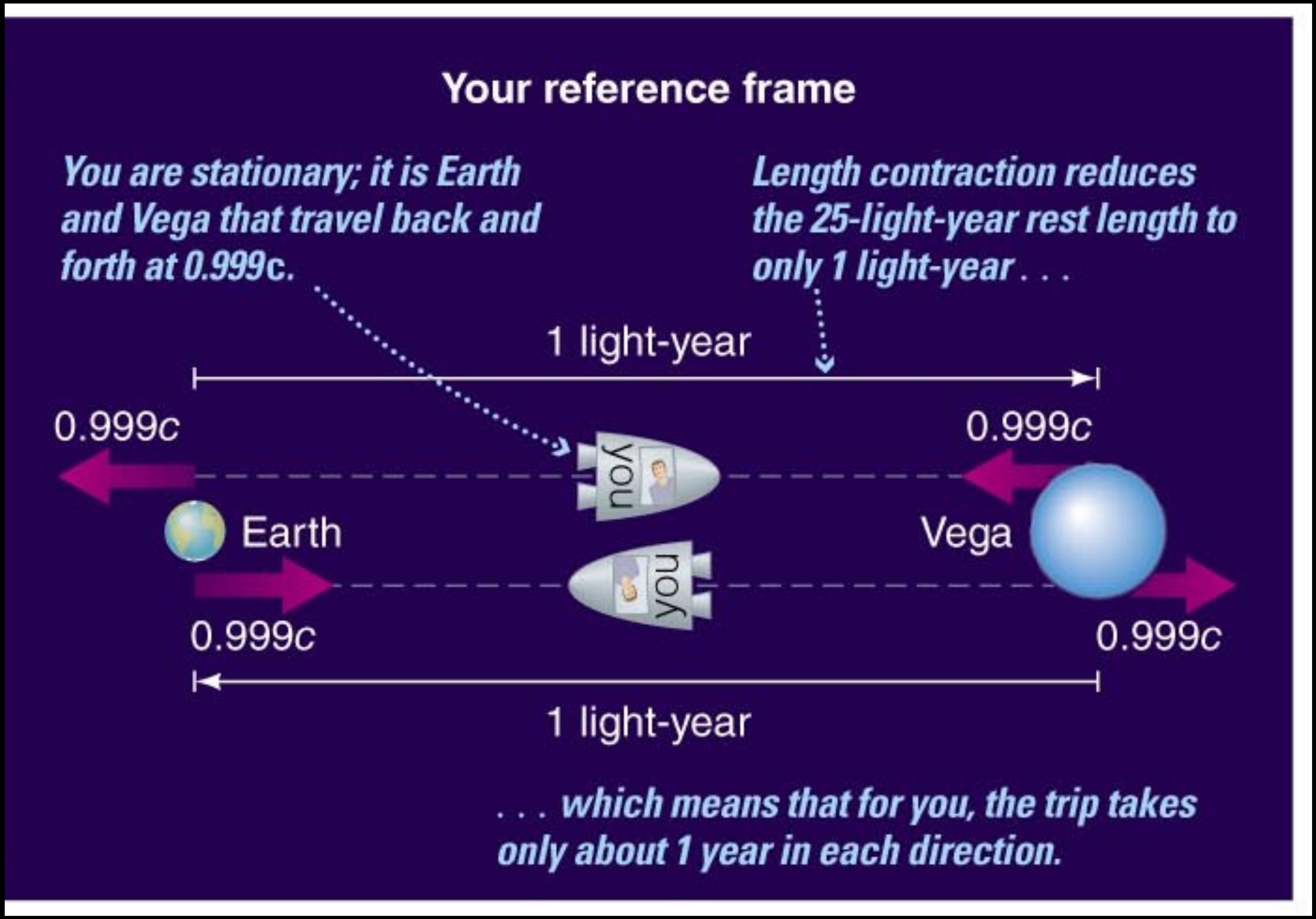
- Suppose you want to visit the star Vega, which is 25 light-years away
- … but suppose that you can travel at a speed of 0.999*c*
- According to an observer on Earth, it takes you about 25+25 = 50 years to complete the round-trip journey

A ticket to the stars



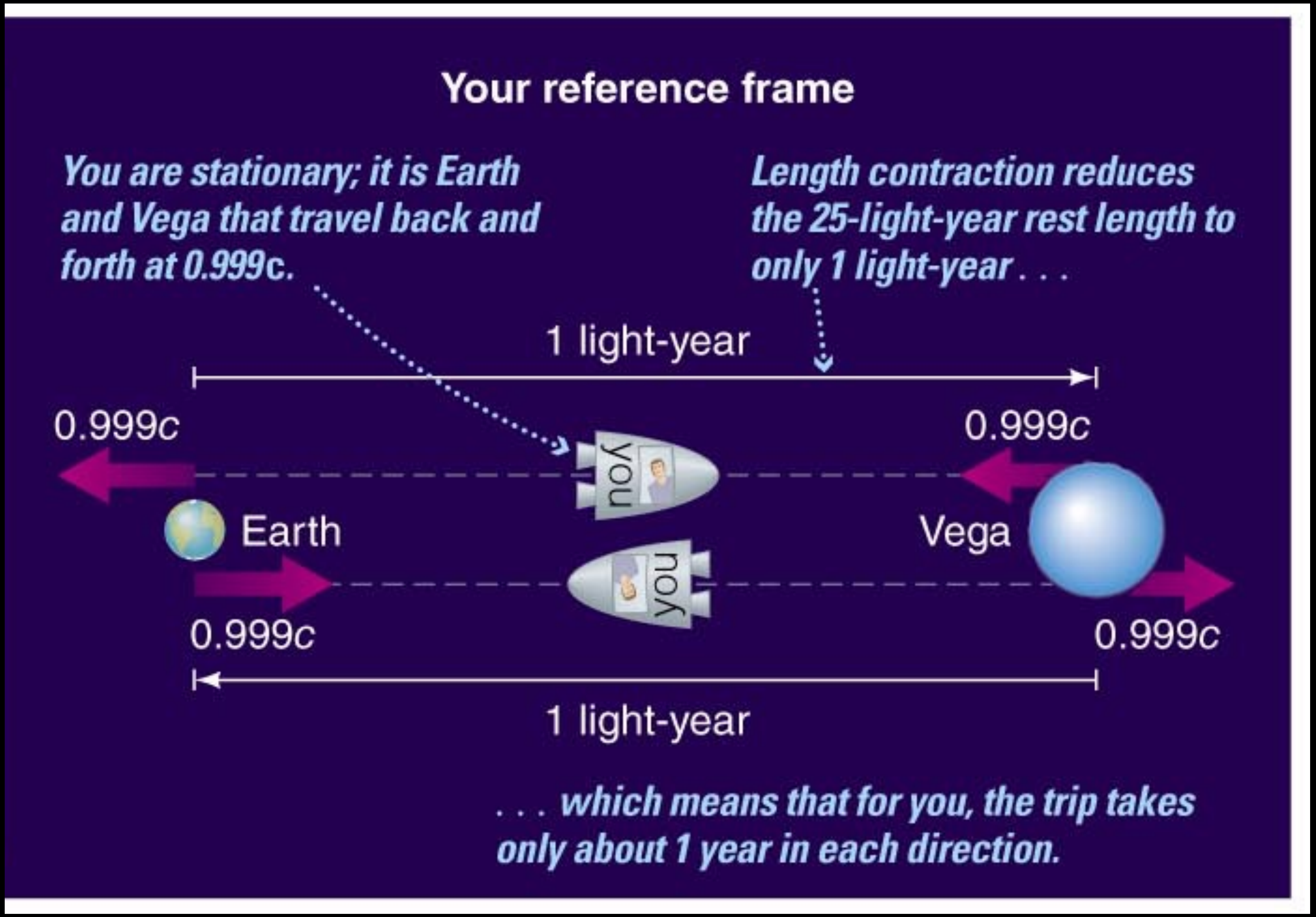
- Suppose you want to visit the star Vega, which is 25 light-years away
- … but suppose that you can travel at a speed of 0.999*c*
  - According to you, time dilates so much that the journey appears to take only 1 year (and the distance contracts to only 1 light-year)
  - Going faster would make the trip seem even shorter!

A ticket to the stars



- Suppose you want to visit the star Vega, which is 25 light-years away
- … but suppose that you can travel at a speed of 0.999*c*
  - You might also say: Vega is moving toward you at 0.999*c*, and it takes one year for it to reach you at this speed. So it appears that the distance to Vega was about 1 light-year.

A ticket to the stars



- Suppose you want to visit the star Vega, which is 25 light-years away
- … but suppose that you can travel at a speed of 0.999*c*
  - *You can go to Vega and back in only 2 years*
  - But your friends on Earth would be 50 years older when you return, while you would only be 2 years older

Ok, so, when you're in motion…

- Time appears to slow down
- Distances appear shorter (in the direction of motion)
- The amount of "time dilation" and "length contraction" is such that light appears to travel at the same speed, no matter how fast you are moving

Quick recap: what have we learned?

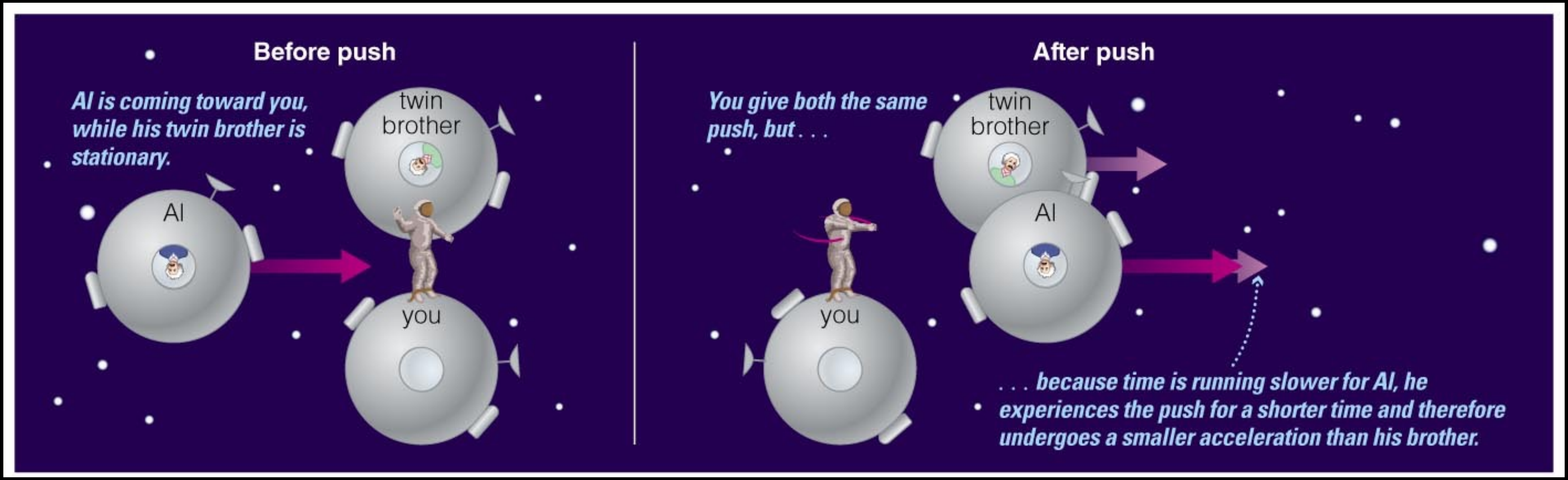
- **How can we make sense of relativity?**
  - “Everything you thought you knew is wrong”
  - We need abandon our notions of space and time as absolute, and adopt new a new common sense in which time and space depend on your perspective
  - Time and space are relative!

- **How does special relativity offer us a ticket to the stars?**
  - When moving at nearly light speed, distances appear shorter because of length contraction, so it appears to take a shorter time (for you) to travel interstellar distances

Question

- According to you, time slows down in a moving spaceship
- According to someone on that spaceship, your time slows down
- Who is right?
- You are **both** right, because time is not absolute but depends on your perspective!

Ok, so what about force and acceleration and kinetic energy?



- A force applied to a rapidly moving object produces *less acceleration* compared to the same force on a stationary object
  - But, **F = m a**
  - Same force, different acceleration… why?
  - When an object is moving faster, it appears to have *more mass*!
  - (This is how we get the equation $E = mc^2$)

Deriving $E = mc^2$

$$m = \frac{m_0}{\sqrt{1 - \left(\frac{v^2}{c^2}\right)}} \approx m_0\left(1 + \frac{1}{2}\frac{v^2}{c^2}\right) \quad \text{for small } v$$

$$\text{Total energy} = mc^2 \approx \underbrace{m_o c^2}_{\textbf{Mass-energy of object at rest}} + \underbrace{\frac{1}{2} m_o v^2}_{\textbf{kinetic energy}}$$

This all sounds crazy.
Can it really be correct?

Recall our discussion of the scientific method, and what is required for a successful scientific theory.

We must be able to make predictions which can be experimentally tested.

Tests of special relativity

- First evidence for absoluteness of speed of light came from the *Michelson-Morley experiment* performed in 1887, and has been verified many times over
- Time dilation happens routinely to subatomic particles that approach the speed of light in accelerators, in our atmosphere, and in space
- Time dilation has also been verified through precision measurements in airplanes moving at much slower speeds
- Prediction that $E = mc^2$ is what powers nuclear reactors and makes the Sun shine

Recap: what have we learned?

- **What are the major ideas of special relativity?**
  - No object can exceed the speed of light
  - We must revise our notions of space, time, and mass when dealing with moving objects (especially near light speed)
- **What is “relative” about relativity?**
  - Motion is relative (so is time, space, and mass)
  - We must define motion relative to some reference frame
- **What is absolute about relativity?**
  - The speed of light
  - The laws of physics