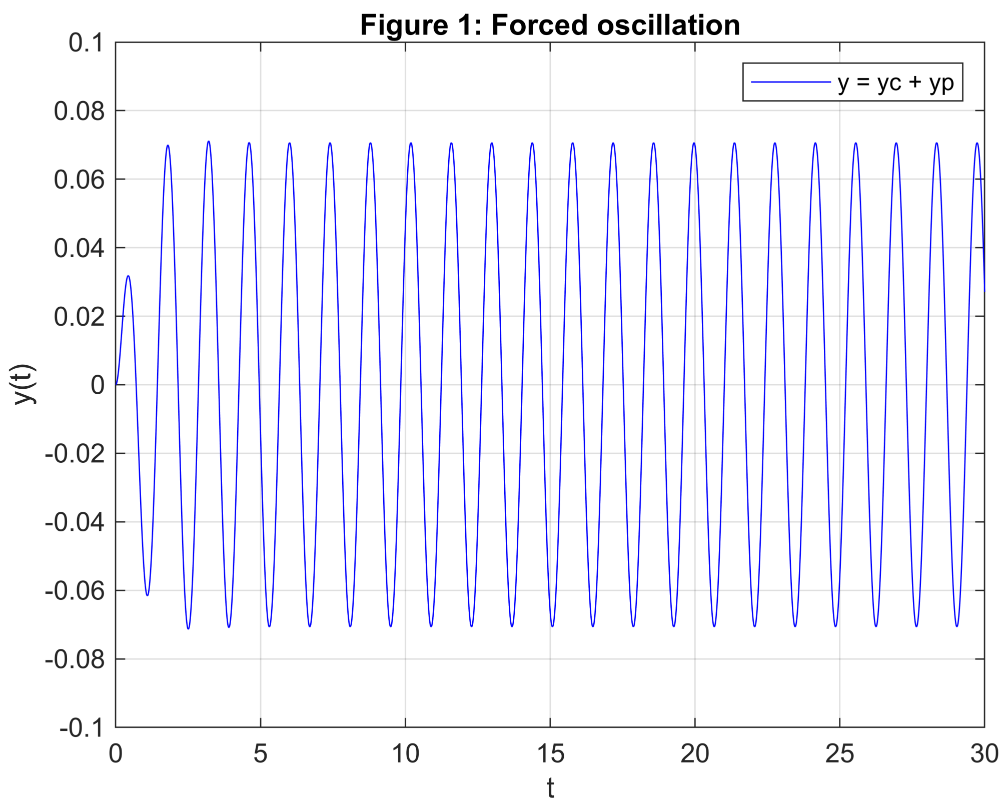
MAT 275 - Lab 6 Bo-Chang, Lin

Forced Equations and Resonance

$$m \cdot x'' + c \cdot x' + k \cdot x = F_0 \cos(\omega t)$$

Exercise 1

```
LAB06ex1
```




```
Computed amplitude of forced oscillation = 0.070655
Theoretical amplitude = 0.070656
```

Part (a)

```
% Graphically - finding the average time interval between each peaks
[~, i]=findpeaks(y); format short    % locate peaks
T = [t(i); 0]-[0; t(i)];             % calculate time intervals (periods)
T = mean(T(2:end-1))                 % remove first and last data
```

```
T = 1.3957
```

```
T = mean(diff(t(i)))                 % or using diff() : Differences/Derivatives
```

```
T = 1.3957
```

```
% Analytically - Period: T = 2*pi/omega
```

```
T = 2*pi/omega
```

```
T = 1.3963
```

The period of the forced oscillation is about 1.4 second.

```
alpha = pi + atan(c*omega/(omega0^2-omega^2))    % w0 < w
```

```
alpha = 1.8758
```

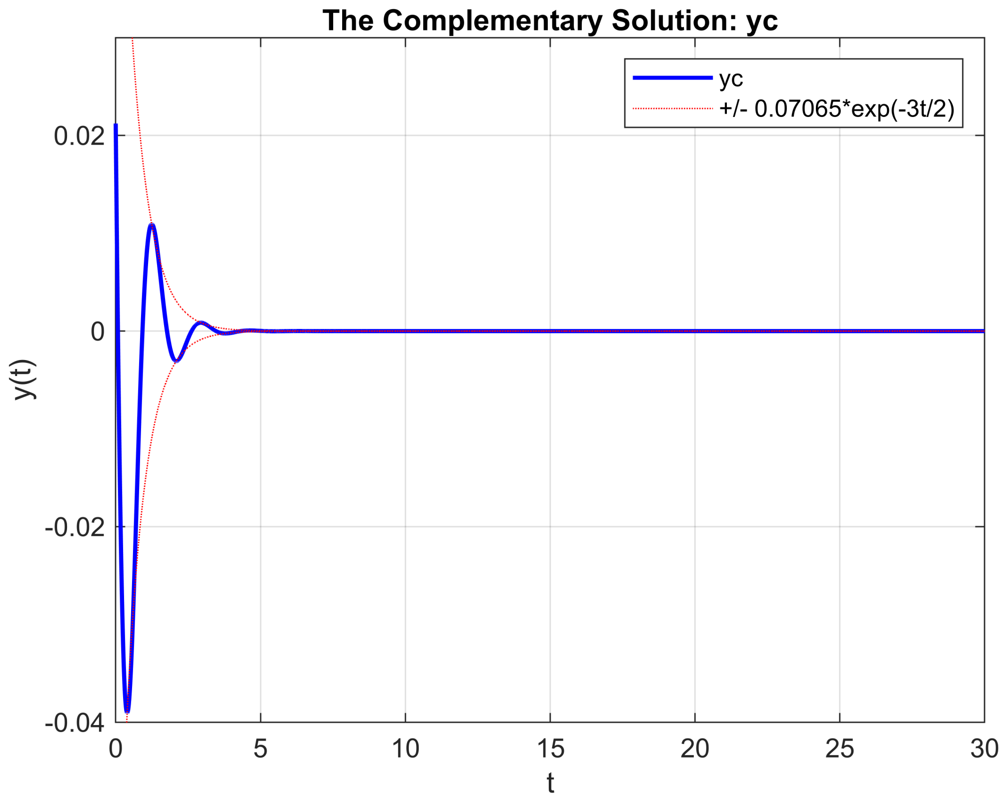
The numerical value of the angle α is 1.8758 (rad).

Part (b)

```
type LAB06ex1_b.m
```

```
clear all;
omega0 = 4; c = 3; omega = 4.5;
param = [omega0,c,omega];
t0 = 0; y0 = 0; v0 = 0; Y0 = [y0;v0]; tf = 30;
options = odeset('AbsTol',1e-10,'relTol',1e-10);
[t,Y] = ode45(@f,[t0,tf],Y0,options,param);
y = Y(:,1); v = Y(:,2);

alpha = pi + atan(c*omega/(omega0^2-omega^2));         % w0 < w
Ctheory = 1/sqrt((omega0^2-omega^2)^2+(c*omega)^2);    % c < 2w0
yc = y - Ctheory*cos(omega*t-alpha);

figure
plot(t,yc,'b-',LineWidth=1.5);
xlabel('t'); ylabel('y(t)'); grid on;
ylim([-.04 .03]); hold on; plot(t,0.07065*exp(-3*t/2),'r:',t,-0.07065*exp(-3*t/2),'r:')
title("The Complementary Solution: yc"); legend("yc","+/- 0.07065*exp(-3t/2)")


%-----------------------------------------------------------------
function dYdt = f(t,Y,param)
y = Y(1); v = Y(2);
omega0 = param(1); c = param(2); omega = param(3);
dYdt = [ v ; cos(omega*t)-omega0^2*y-c*v ];
end
```

```
LAB06ex1_b
```



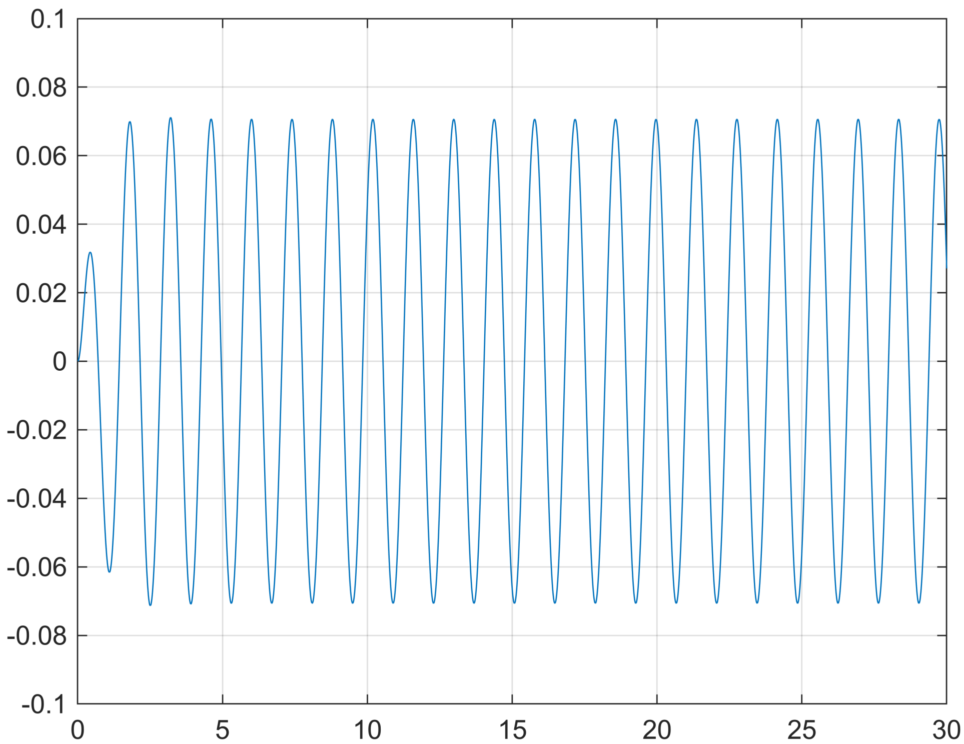
It dose look like an exponentially decreasing oscillation. The complementary solution represents the natural behavior of the system in the absence of the external force.

```
% For reference
syms y(t)
ode = diff(y,2)+3*diff(y)+16*y == cos(4.5*t);
dy = diff(y,t); cond = [y(0) == 0; dy(0) == 0];
ySol(t) = dsolve(ode,cond); ySol = simplify(ySol)
```

ySol(t) =

$$\frac{216\sin\left(\frac{9\,t}{2}\right)}{3205}-\frac{68\cos\left(\frac{9\,t}{2}\right)}{3205}+\frac{68\,\mathrm{e}^{-\frac{3\,t}{2}}\cos\left(\frac{\sqrt{55}\,t}{2}\right)}{3205}-\frac{348\,\sqrt{55}\,\mathrm{e}^{-\frac{3\,t}{2}}\sin\left(\frac{\sqrt{55}\,t}{2}\right)}{35255}$$

```
figure; fplot(ySol,[0 30]); ylim([-.1 .1]); grid on
```

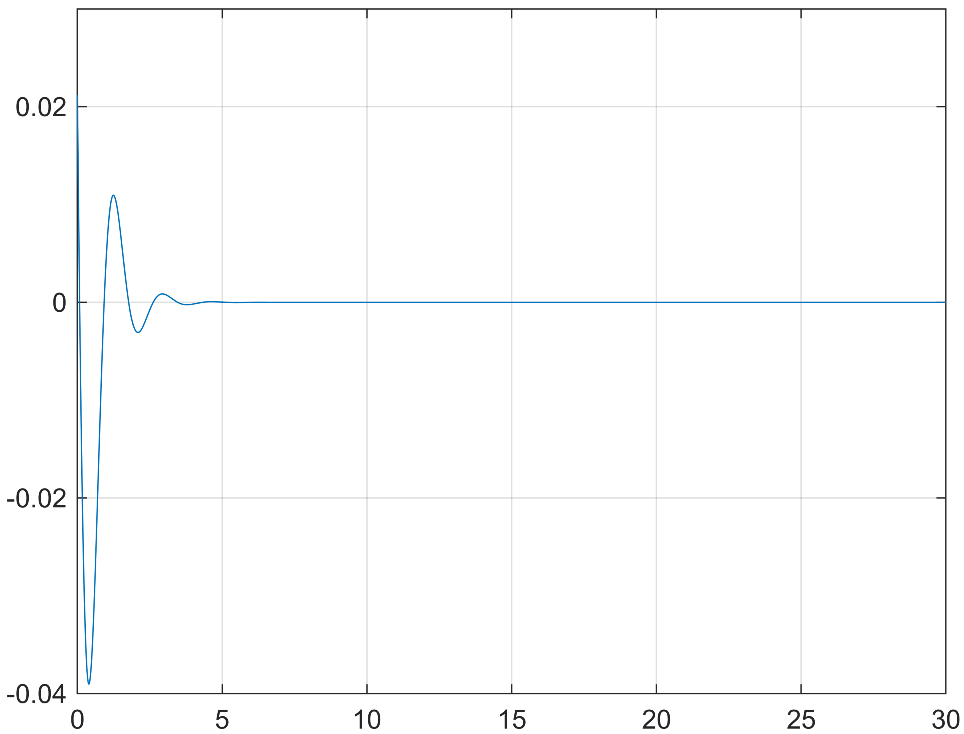



```
yc = subs(subs(ySol,sin(9*t/2),0),cos(9*t/2),0)
```

yc(t) =

$$\frac{68\,e^{-\frac{3t}{2}}\cos\left(\frac{\sqrt{55}\,t}{2}\right)}{3205} - \frac{348\,\sqrt{55}\,e^{-\frac{3t}{2}}\sin\left(\frac{\sqrt{55}\,t}{2}\right)}{35255}$$

```
figure; fplot(yc,[0 30]); ylim([-.04 .03]); grid on
```



Exercise 2

Part (a)

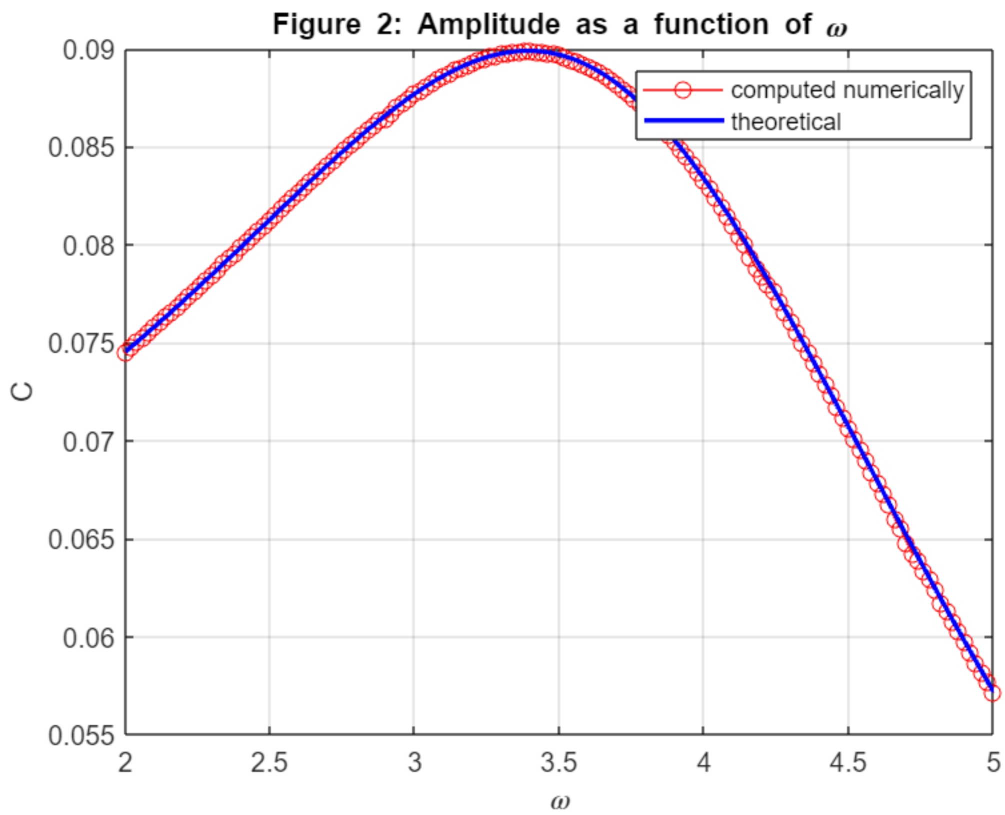
NOTE: *Fill in the missing parts in LAB06ex2.m. Then print it and run it. Delete this note upon submission.*

```
type LAB06ex2.m
```

```
clear all;
omega0 = 4; c = 3;
OMEGA =2:0.01:5;
C = zeros(size(OMEGA));
Ctheory = zeros(size(OMEGA));
t0 = 0; y0 = 0; v0 = 0; Y0 = [y0;v0]; tf =30; t1 = 9;

for k = 1:length(OMEGA)
   omega = OMEGA(k);
   param = [omega0,c,omega];
   [t,Y] = ode45(@f,[t0,tf],Y0,[],param);
   i = find(t>t1);
   C(k) = (max(Y(i,1))-min(Y(i,1)))/2;
   Ctheory(k) = 1/sqrt((omega0^2-OMEGA(k)^2)^2+(c*OMEGA(k))^2);
end

figure
plot(OMEGA,C,'r-o','MarkerIndices',1:2:length(OMEGA)); grid on;
hold on; plot(OMEGA,Ctheory,'b-',LineWidth=1.7)
xlabel('\omega'); ylabel('C');
```

```
legend('computed numerically','theoretical')
title('Figure 2: Amplitude as a function of \omega')


%-----------------------------------------------------------
function dYdt = f(t,Y,param)
y = Y(1); v = Y(2);
omega0 = param(1); c = param(2); omega = param(3);
dYdt = [ v ; cos(omega*t)-omega0^2*y-c*v ];
end
```

```
LAB06ex2 % Here I reduce the marker density to 1/2 for better showing the other data
```



Part (b)

```
[maxC,i] = max(C)
```

```
maxC = 0.0899
i = 139
```

```
theOMEGA = OMEGA(i)
```

```
theOMEGA = 3.3800
```

Value of ω that gives the practical resonance frequency: 3.38

Coresponding maximum value of the amplitude C: 0.0899

Part (c)

Compute analytically the value of ω that gives the practical resonance frequency by differentiating C:

```
syms w; assume(w > 0)
C = 1/sqrt((omega0^2-w^2)^2+(c*w)^2)
```

C =

$$\frac{1}{\sqrt{(w^2-16)^2+9\,w^2}}$$

```
dCdw = diff(C,w)
```

dCdw =

$$-\frac{18\,w+4\,w\,(w^2-16)}{2\,\left((w^2-16)^2+9\,w^2\right)^{3/2}}$$

```
w = solve(dCdw==0)
```

w =

$$\frac{\sqrt{46}}{2}$$

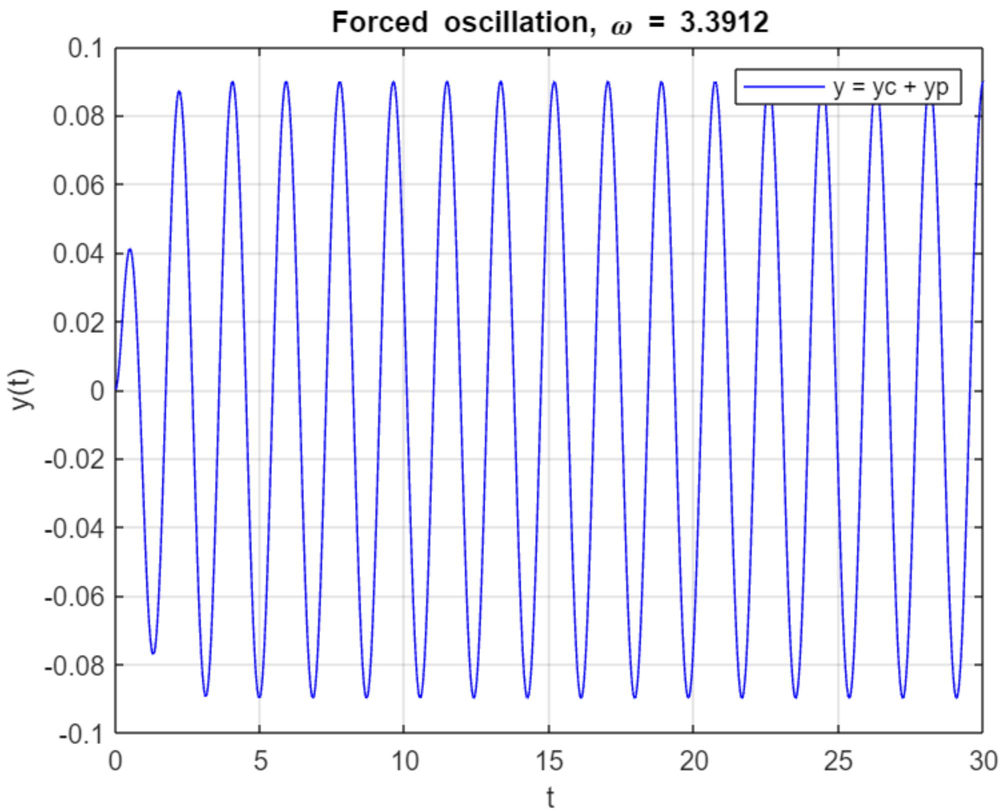
```
w = double(w)
```

w = 3.3912

The theoretica value (3.3912) is slightly larger than computed result (3.3800).

Part (d)

```
LAb06ex1_d
```

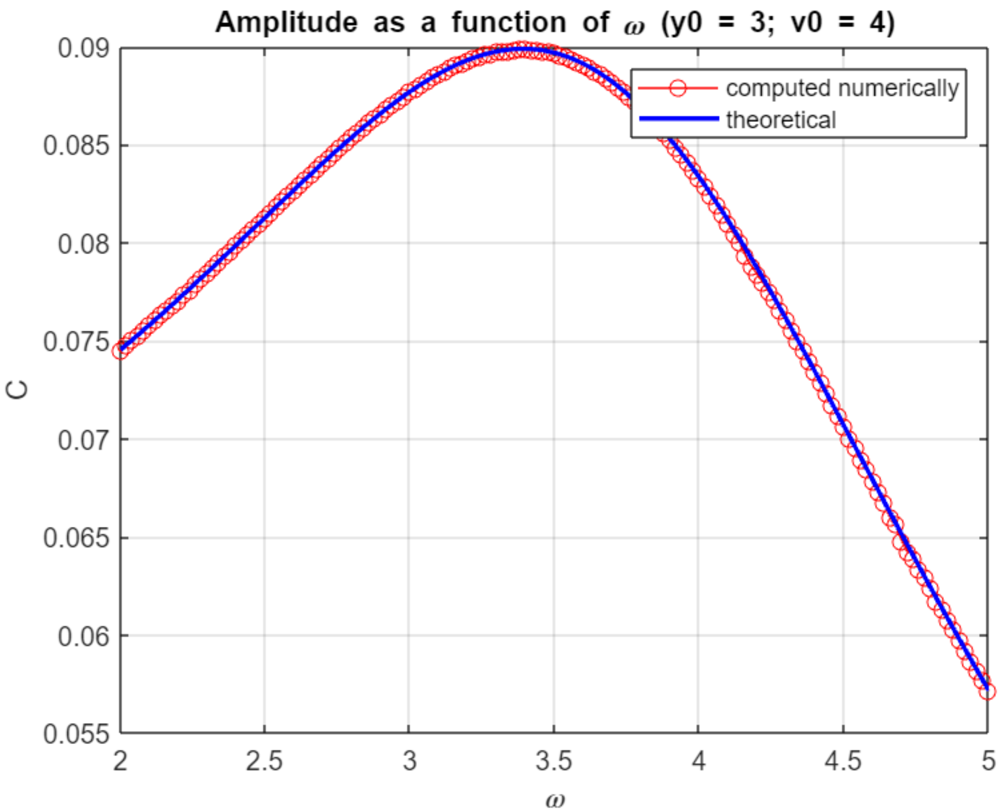



```
Computed amplitude of forced oscillation = 0.089893
Theoretical amplitude = 0.089893
```

Amplitude of the forced oscillation: 0.089893. The amplitude of the forced oscillation found in problem 1 is smaller, and no other value of ω could lead to larger amplitude of the solution.

Part (e)

```
LAB06ex2_e
```



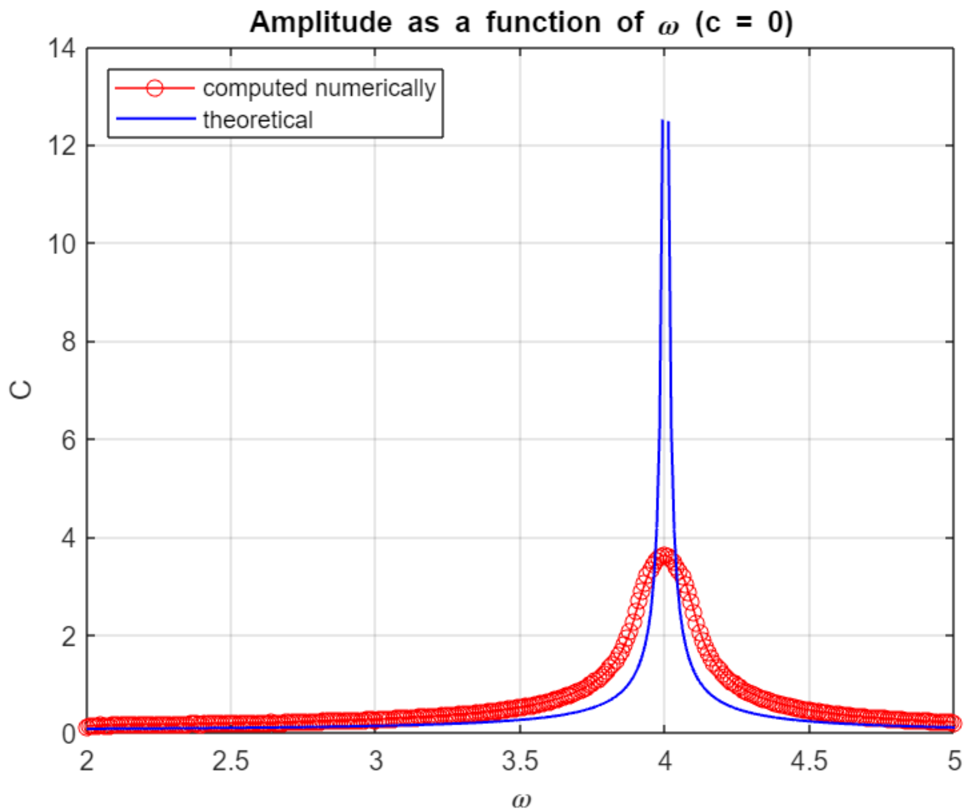
$$C = \frac{1}{\sqrt{(\omega_0^2 + \omega^2)^2 + c^2\omega^2}}$$

The amplitude of the forced oscillation, C, is never affected by the initial conditions, since they are not parts of the fomula.

Exercise 3

Set $c = 0$ in LAB06ex2.m and run it

```
LAB06ex3
```



Part (a)

```
[maxC,i] = max(C)
```

```
maxC = 3.6247
i = 201
```

```
theOMEGA = OMEGA(i)
```

```
theOMEGA = 4
```

The maximal amplitude is 3.6247. The value of ω yielding the maximal amplitude in the forced oscillation is 4, and it match the value of ω_0

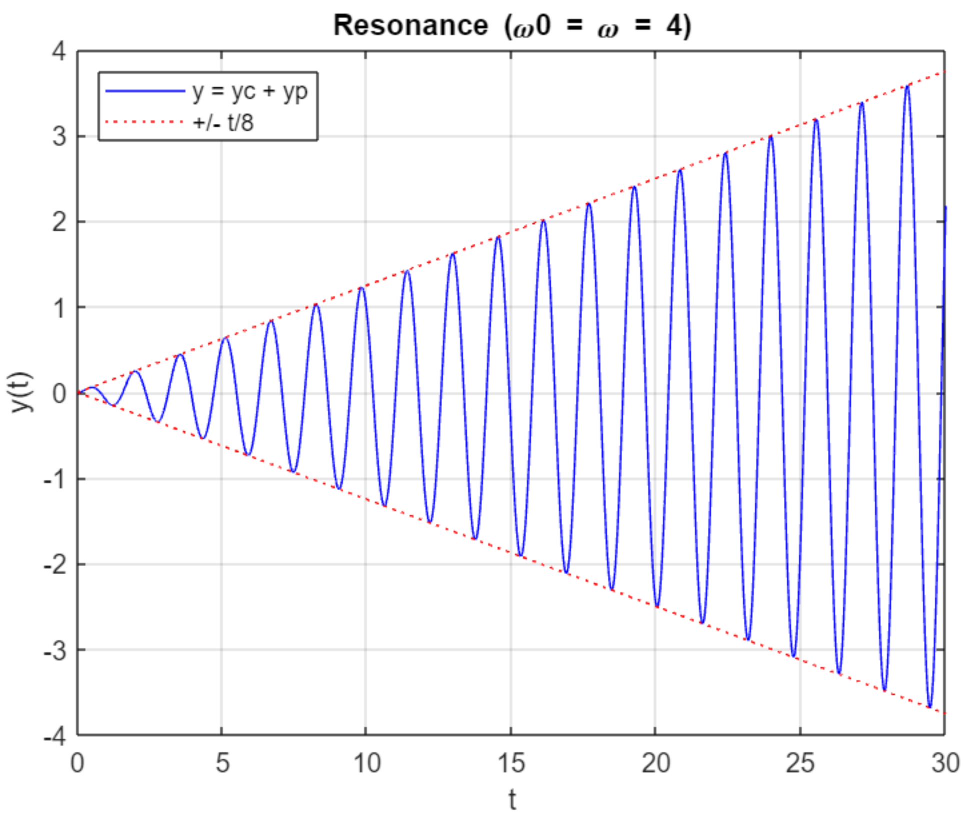
Part (b)

```
% For reference
syms y(t)
ode = diff(y,2)+16*y == cos(4*t);
dy = diff(y,t); cond = [y(0) == 0; dy(0) == 0];
ySol(t) = dsolve(ode,cond); ySol = simplify(ySol) % (F/2*w)*t*sin(w*t) =
(1/2*4)*t*sin(4*t)
```

```
ySol(t) =
```

$$\frac{t \sin(4\,t)}{8}$$

```
LAB06ex3_b
```

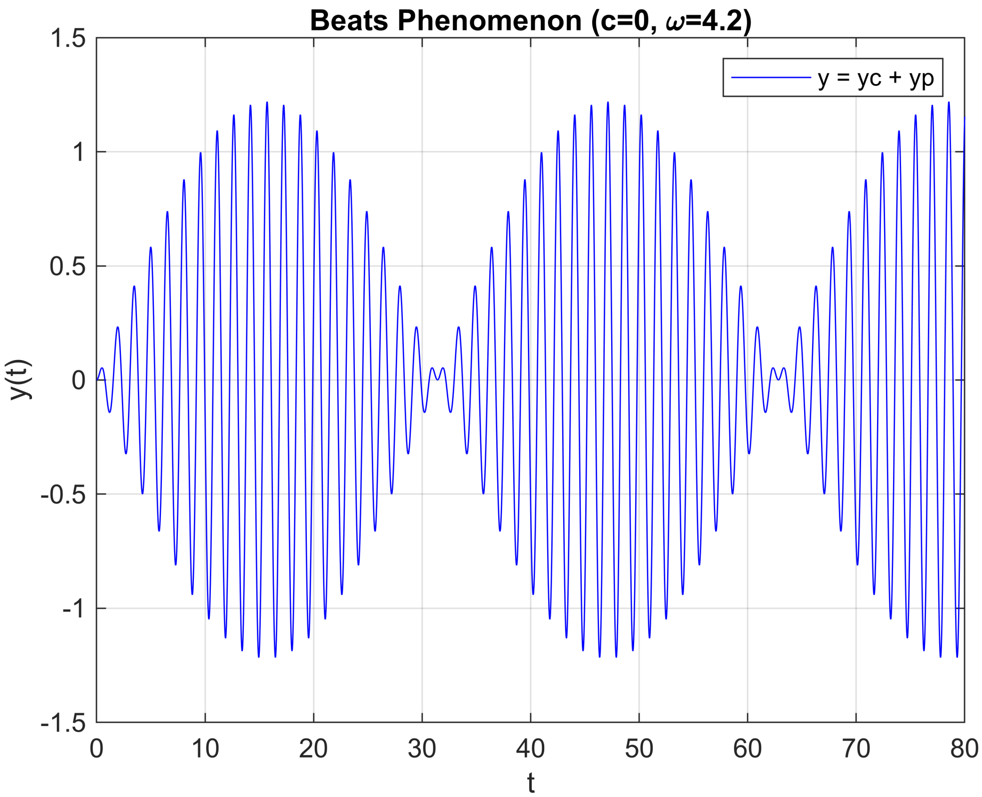



```
Computed amplitude of forced oscillation = 3.6326
Theoretical amplitude = Inf
```

The amplitude is inceeasing linearly and endless with the time.

Exercise 4

```
LAB06ex4
```



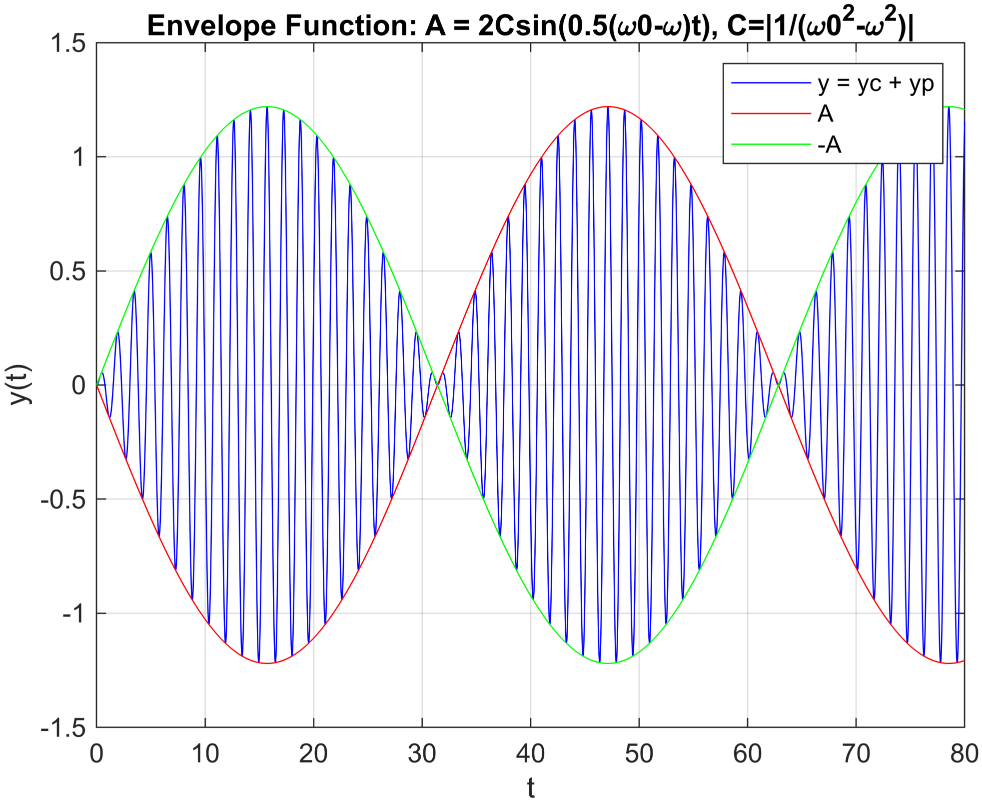
Part (a)

Define the "envelope" functions A and $-A$ in LAB06ex4.m

```
type LAB06ex4_a.m
```

```
clear all;
omega0 = 4; c = 0; omega = 4.2;
param = [omega0,c,omega];
t0 = 0; y0 = 0; v0 = 0; Y0 = [y0;v0]; tf = 80;
options = odeset('AbsTol',1e-10,'relTol',1e-10);
[t,Y] = ode45(@f,[t0,tf],Y0,options,param);
y = Y(:,1); v = Y(:,2);

C = abs(1/(omega0^2-omega^2));
A = 2*C*sin(0.5*(omega0-omega)*t);

figure
plot(t,y,'b-'); xlabel('t'); ylabel('y(t)'); grid on;
hold on; plot(t,A,'r-',t,-A,'g-')
title("Envelope Function: A = 2Csin(0.5(\omega0-\omega)t), C=|1/(\omega0^2-\omega^2)|")
legend("y = yc + yp","A","-A")


%----------------------------------------------------------------
function dYdt = f(t,Y,param)
y = Y(1); v = Y(2);
omega0 = param(1); c = param(2); omega = param(3);
dYdt = [ v ; cos(omega*t)-omega0^2*y-c*v ];
end
```

```
LAB06ex4_a
```



Part (b)

Period of the rapidly oscillating function:

```
T = 4*pi/(omega0+omega)
```

```
T = 1.5325
```

Part (c)

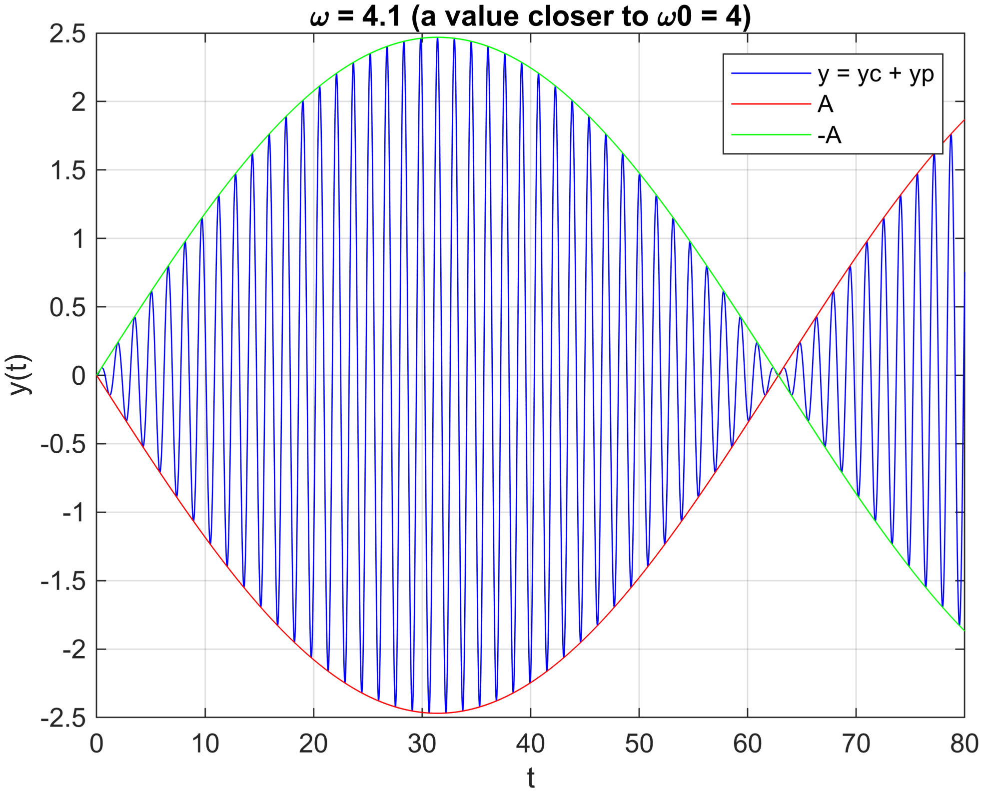
Length of beats:

```
LB = 2*pi/abs(omega0-omega)
```

```
LB = 31.4159
```

Part (d)

```
LAB06ex4_d1
```



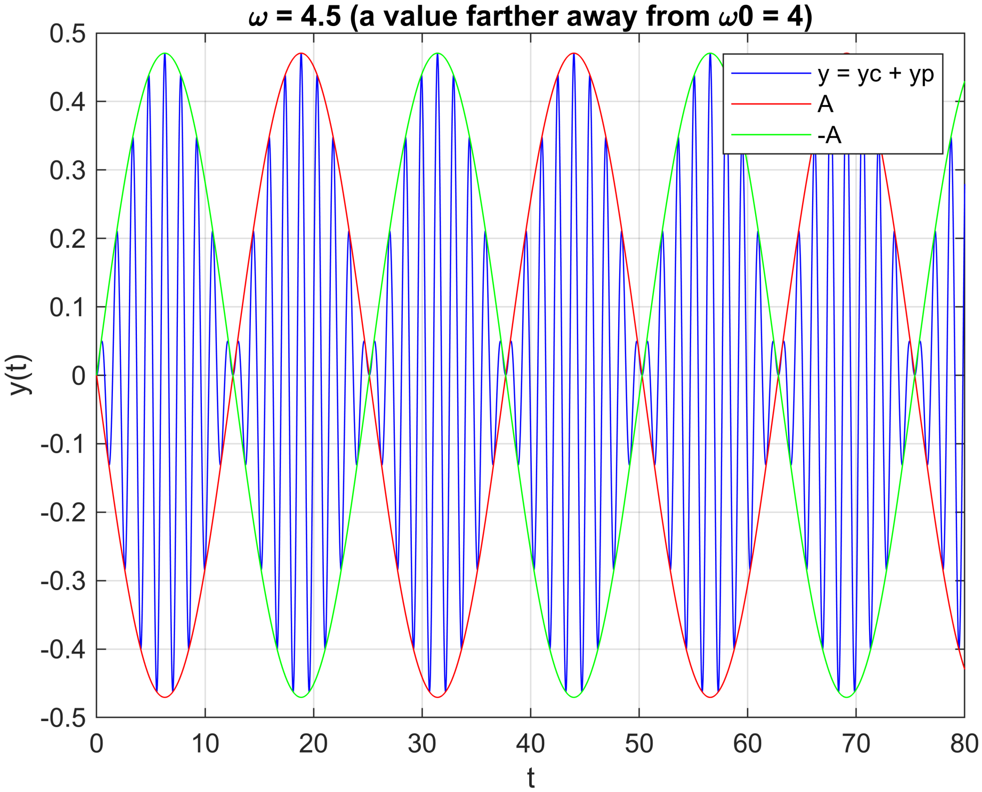

```
T = 4*pi/(omega0+omega)
```

```
T = 1.5514
```

```
LB = 2*pi/abs(omega0-omega)
```

```
LB = 62.8319
```

```
LAB06ex4_d2
```




```
T = 4*pi/(omega0+omega)
```

T = 1.4784

```
LB = 2*pi/abs(omega0-omega)
```

LB = 12.5664

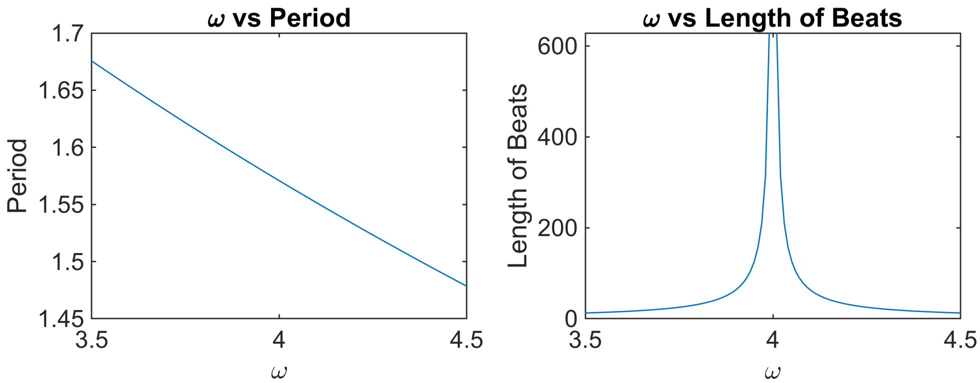
```
T1 = array2table(horzcat([1.5325, 1.5514, 1.4784]',[31.4159, 62.8319, 12.5664]',
[4.2-4, 4.1-4, 4.5-4]'));
T1.Properties.VariableNames(1:3) = {'Period','Length of Beats','|w0-w|'}
```

T1 = 3×3 table

	Period	Length of Beats	|w0-w|
1	1.5325	31.4159	0.2000
2	1.5514	62.8319	0.1000
3	1.4784	12.5664	0.5000

```
% For W0 = 4
subplot(2,2,1)
omega = 3.5:0.01:4.5; T = 4*pi./(4+omega);
plot(omega,T); title('\omega vs Period'); xlabel("\omega"); ylabel("Period")
subplot(2,2,2)
```

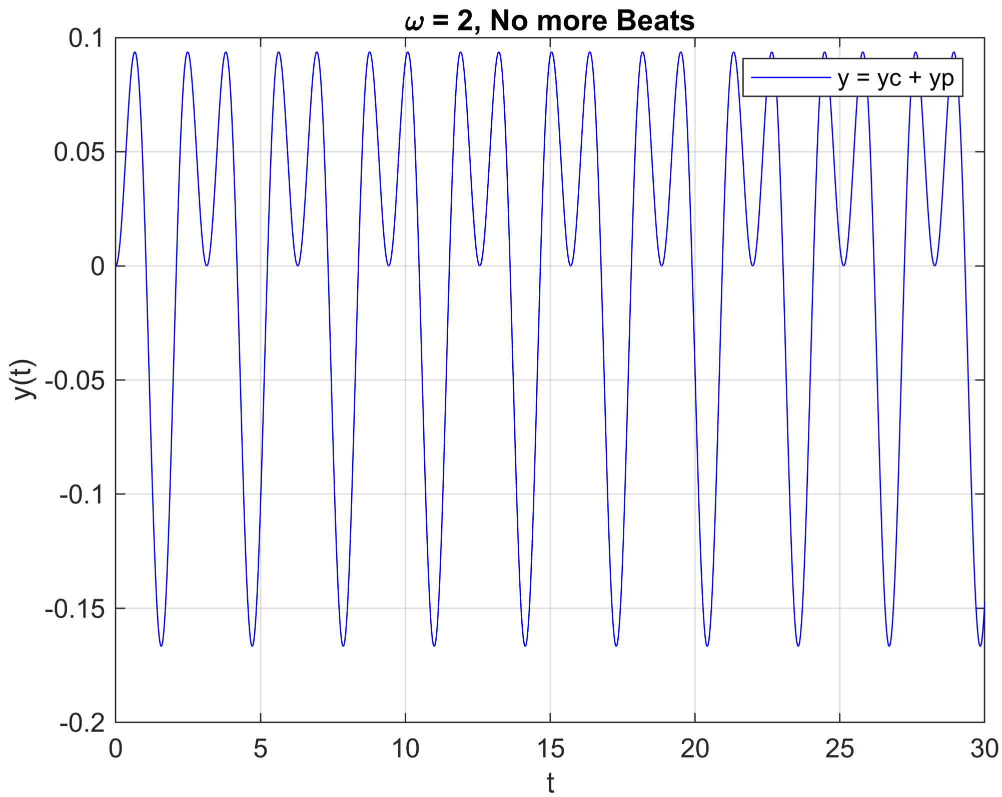
```
LB = 2*pi./abs(4-omega);
plot(omega,LB); title('\omega vs Length of Beats'); xlabel("\omega");
ylabel("Length of Beats")
```



The period will decrease as ω gets farter away from ω_0 if $\omega > \omega_0$ but increase as ω gets farter away from ω_0 if $\omega < \omega_0$. On the other hand, the length of beats will decrease rapidly as ω gets farter away from ω_0.

Part (e)

```
LAB06ex4_e
```



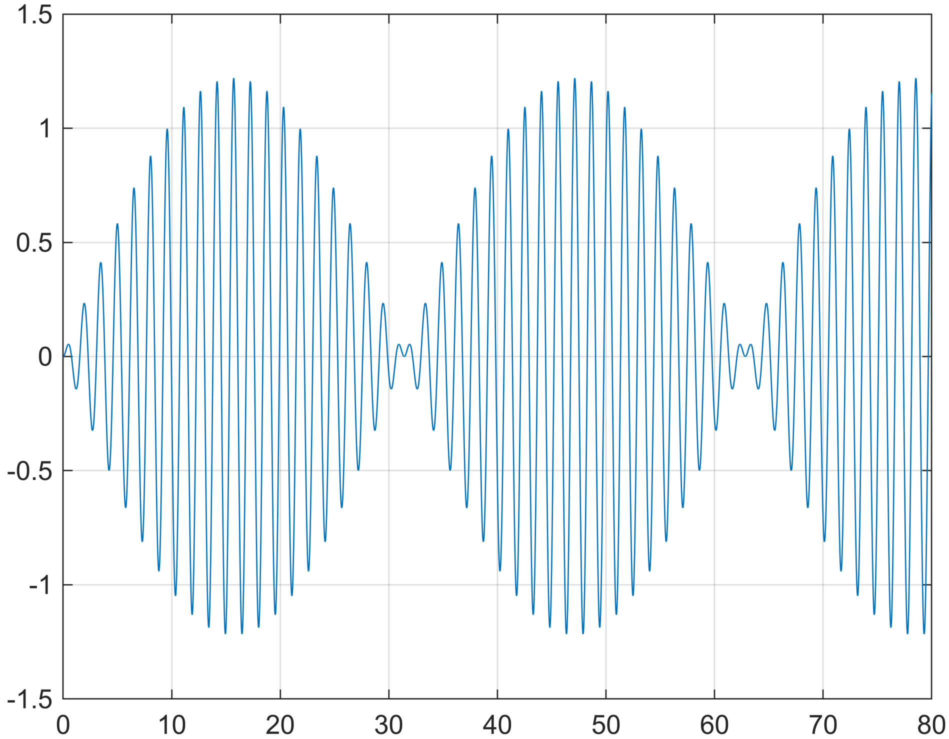
The beats phenomemon seems disappear probably because ω gets too farter away from ω_0.

```
% For reference
syms y(t)
ode = diff(y,2)+16*y == cos(4.2*t);
dy = diff(y,t); cond = [y(0) == 0; dy(0) == 0];
ySol(t) = dsolve(ode,cond); ySol = simplify(ySol)
```

ySol(t) =

$$\frac{25\cos(4\,t)}{41} - \frac{25\cos\left(\frac{21\,t}{5}\right)}{41}$$

```
figure; fplot(ySol,[0 80]); ylim([-1.5 1.5]); grid on
```

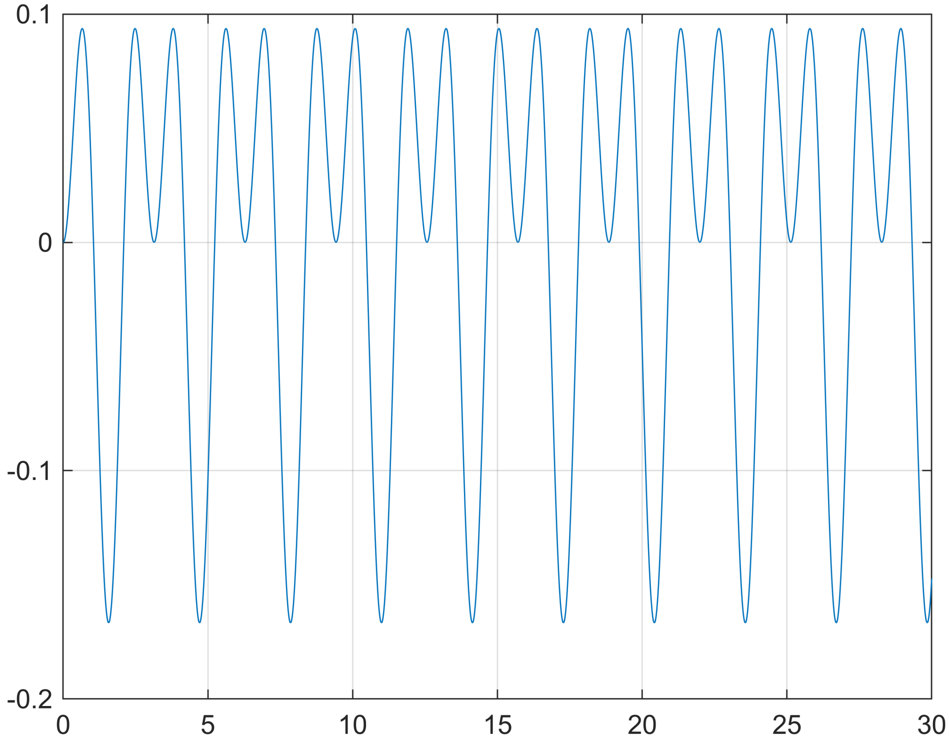



```
syms y(t)
ode = diff(y,2)+16*y == cos(2*t);
dy = diff(y,t); cond = [y(0) == 0; dy(0) == 0];
ySol(t) = dsolve(ode,cond); ySol = simplify(ySol)
```

ySol(t) =

$$-\frac{\sin(t)^2\ (4\sin(t)^2-3)}{6}$$

```
figure; fplot(ySol,[0 30]); ylim([-.2 .1]); grid on
```


0.1
0
-0.1
-0.2
0
5
10
15
20
25
30