MAT 275 - LAB 4 Bo-Chang Lin

Exercise 1

Part (a)

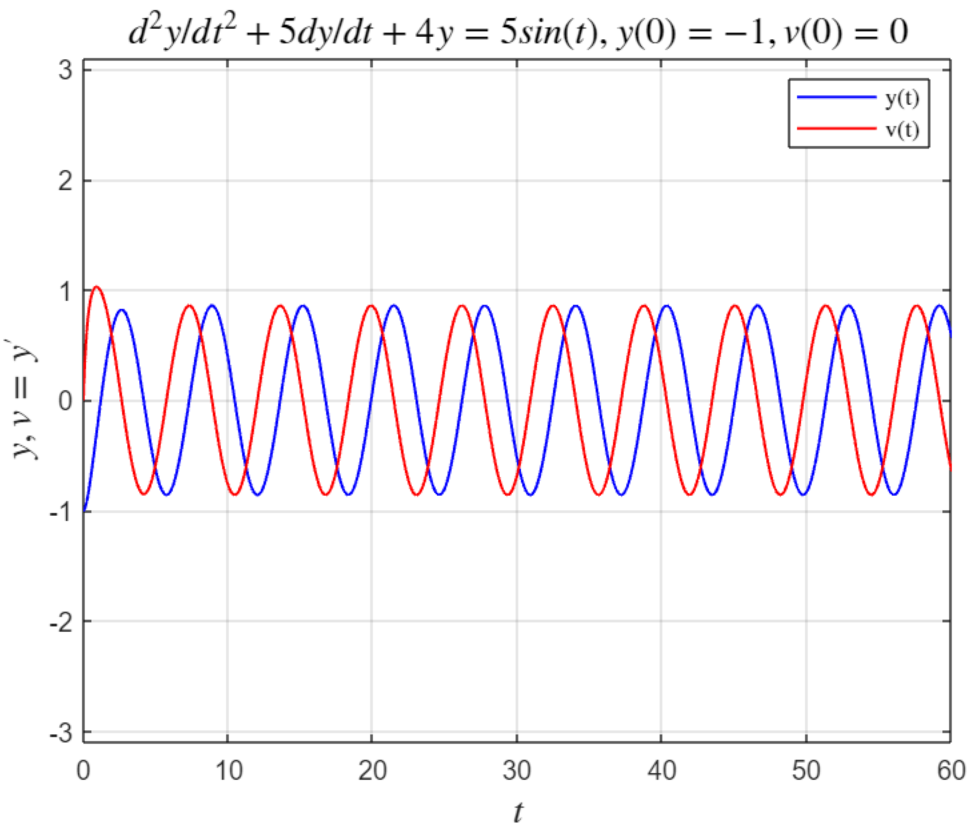
Solve the IVP

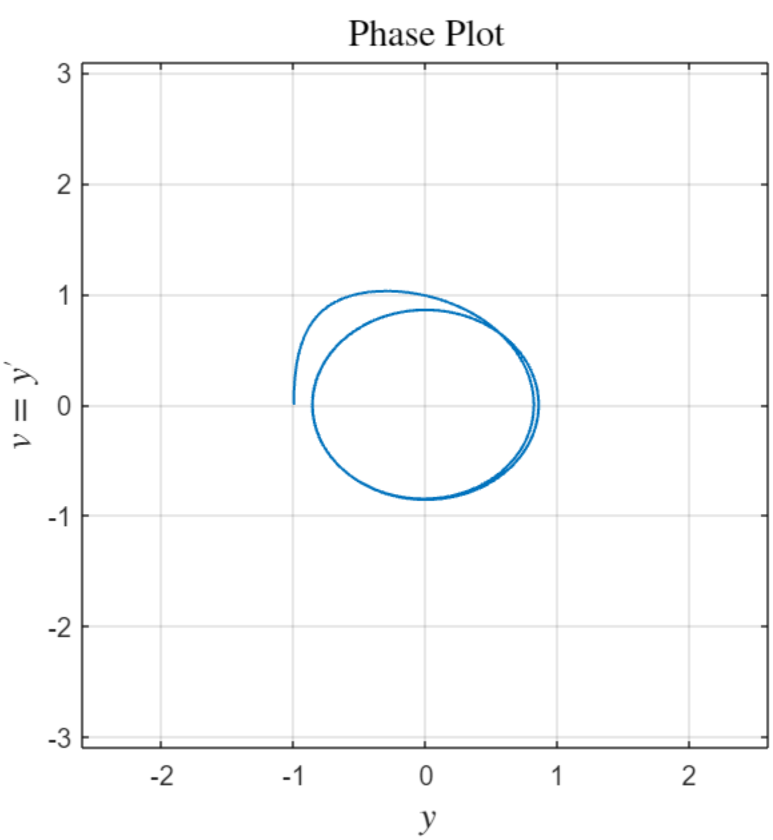
$$\frac{d^2y}{dt^2} + 5\frac{dy}{dt} + 4y = 5\sin(t),\ y(0) = -1,\ v(0) = 0$$ **Eqn. (4)**

```
type 'LAB04ex1.m'
```

```
t0 = 0; tf = 60;
Y0 = [-1;0]; % [y(0); v(0)]

[t,Y] = ode45(@f,[t0,tf],Y0,[]);
y = Y(:,1); v = Y(:,2);    % Y in output has 2 columns corresponding to y and v

figure(1)
plot(t,y,'b',t,v,'r',LineWidth=1)
ylim([-3.1,3.1])
grid on

xlabel('$t$',"Interpreter","latex","FontSize",14);
ylabel("$y,v=y'$","Interpreter","latex","FontSize",14)
legend('y(t)','v(t)',"Interpreter","latex")
title("$d^2y/dt^2+5dy/dt+4y=5sin(t), y(0)=-1, v(0)=0$","Interpreter","latex","FontSize",14)

figure(2)
plot(y,v,LineWidth=1);   % plot the phase plot
axis square
ylim([-3.1,3.1])
xlim([-2.6,2.6])
grid on

xlabel('$y$',"Interpreter","latex","FontSize",14);
ylabel("$v=y'$","Interpreter","latex","FontSize",14)
title("Phase Plot","Interpreter","latex","FontSize",14)

%----------------------------------------------------------------------
function dYdt = f(t,Y)
y=Y(1); v=Y(2);
dYdt = [v; 5*sin(t)-5*v-4*y];
end
```

```
LAB04ex1                        % Fig. (7)
```





Part (b)

Find (approximately) the last three values of **t** in the interval **[0,60]** at which **y** reaches a **local maximum**.

```
% Method 1 (local max only)
format short
[pks,locs] = findpeaks(y);
Data = [t(locs(end-2:end)),pks(end-2:end)];
T1 = array2table(Data);
T1.Properties.VariableNames(1:2) = {'t','y(t)'}
```

T1 = 3×2 table

	t	y(t)
1	46.5562	0.8572
2	52.8947	0.8572
3	59.1202	0.8571

```
% Method 2 (local max & min)
DATA1 = [t, Y(:,1), Y(:,2)];
DATA2 = zeros(length(pks)*2,3);
j = 1;
for i = 1:length(Y)-1
    if Y(i,2)*Y(i+1,2) < 0
        DATA2(j,:)=DATA1(i,:);
        j=j+1;
        DATA2(j,:)=DATA1(i,:);
    end
end
DATA2 = DATA2(end-6:end-1,:);
T2 = array2table(DATA2);
T2.Properties.VariableNames(1:3) = {'t','y(t)','v(t)'}
```

T2 = 6×3 table

	t	y(t)	v(t)
1	43.4333	-0.8575	-0.0073
2	46.5562	0.8572	0.0233
3	49.7049	-0.8573	-0.0173
4	52.8344	0.8571	0.0276
5	55.9975	-0.8575	-0.0092
6	59.1202	0.8571	0.0254

```
% Method 2 modified (local max only)
data1 = [t, Y(:,1), Y(:,2)];
data2 = zeros(length(pks),3);
j = 1;
for i = 1:length(Y)-1
    if (Y(i,2)*Y(i+1,2) < 0) && (Y(i,2) > Y(i+1,2))
        data2(j,:)=DATA1(i,:);
        j=j+1;
        data2(j,:)=DATA1(i,:);
    end
```

```
end
data2 = data2(end-3:end-1,:);
T3 = array2table(data2);
T3.Properties.VariableNames(1:3) = {'t','y(t)','v(t)'}
```

T3 = 3×3 table

	t	y(t)	v(t)
1	46.5562	0.8572	0.0233
2	52.8344	0.8571	0.0276
3	59.1202	0.8571	0.0254

Part (c)

What seems to be the long term behavior of y?

The long term behavior of y values seems to be in the range of -0.857 to 0.857 as the phase plot can tell, it turn out like a sort of circle with radius about 0.86

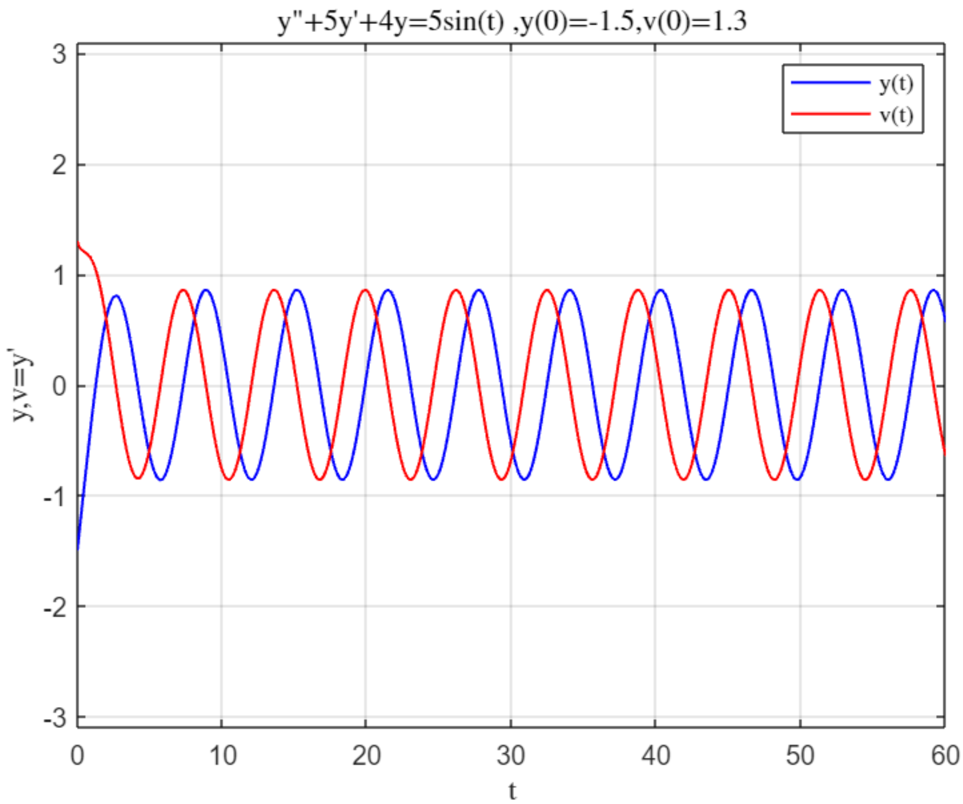
Part (d)

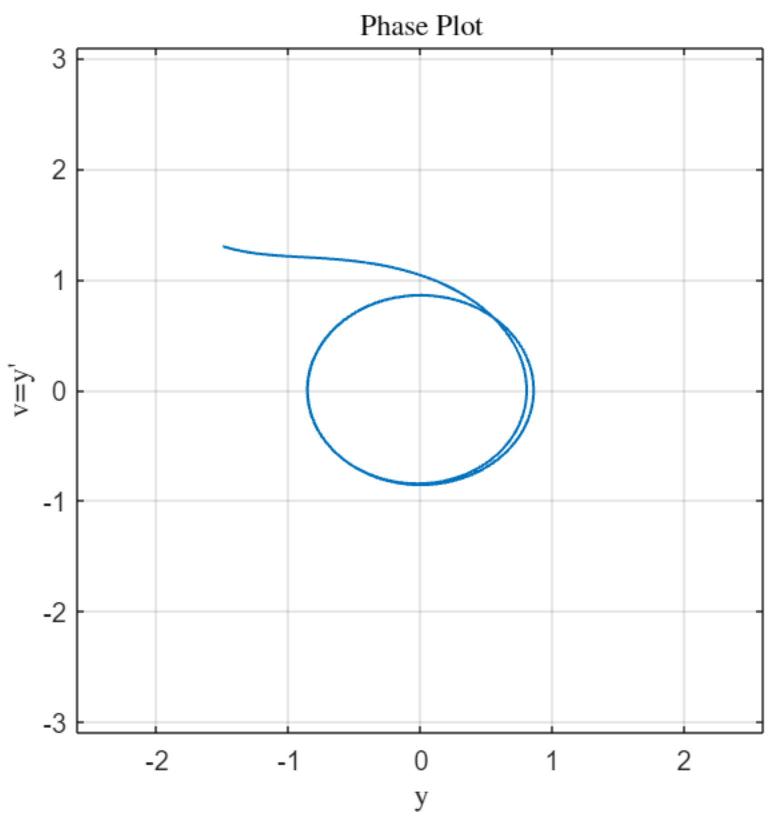
Modify the initial conditions to **y(0) = -1:5, v(0) = 1:3**

```
type LAB04ex1d
```

```
t0 = 0; tf = 60;
Y0 = [-1.5;1.3];

[t,Y] = ode45(@f,[t0,tf],Y0,[]);
y = Y(:,1); v = Y(:,2);    % Y in output has 2 columns corresponding to y and v

figure
plot(t,y,'b',t,v,'r',LineWidth=1)
ylim([-3.1,3.1])
grid on

xlabel('t',"Interpreter","latex"); ylabel("y,v=y'","Interpreter","latex")
legend('y(t)','v(t)',"Interpreter","latex")
title(strcat('y"',"+5y'+4y=5sin(t)",' ,y(0)=-1.5,v(0)=1.3'),"Interpreter","latex")

figure
plot(y,v,LineWidth=1);   % plot the phase plot
axis square
ylim([-3.1,3.1])
xlim([-2.6,2.6])
grid on

xlabel('y',"Interpreter","latex"); ylabel("v=y'","Interpreter","latex")
title("Phase Plot","Interpreter","latex")

%----------------------------------------------------------------------
function dYdt = f(t,Y)
y=Y(1); v=Y(2);
dYdt = [v; 5*sin(t)-5*v-4*y];
end
```

```
LAB04ex1d
```





Based on the new graphs, determine whether the long term behavior of the solution changes.

The long term behavior of y values will remain the same regardless of the change of initial conditions as the phase plot can tell, it turn out like a sort of circle with radius about 0.86

Exercise 2

Part (a)

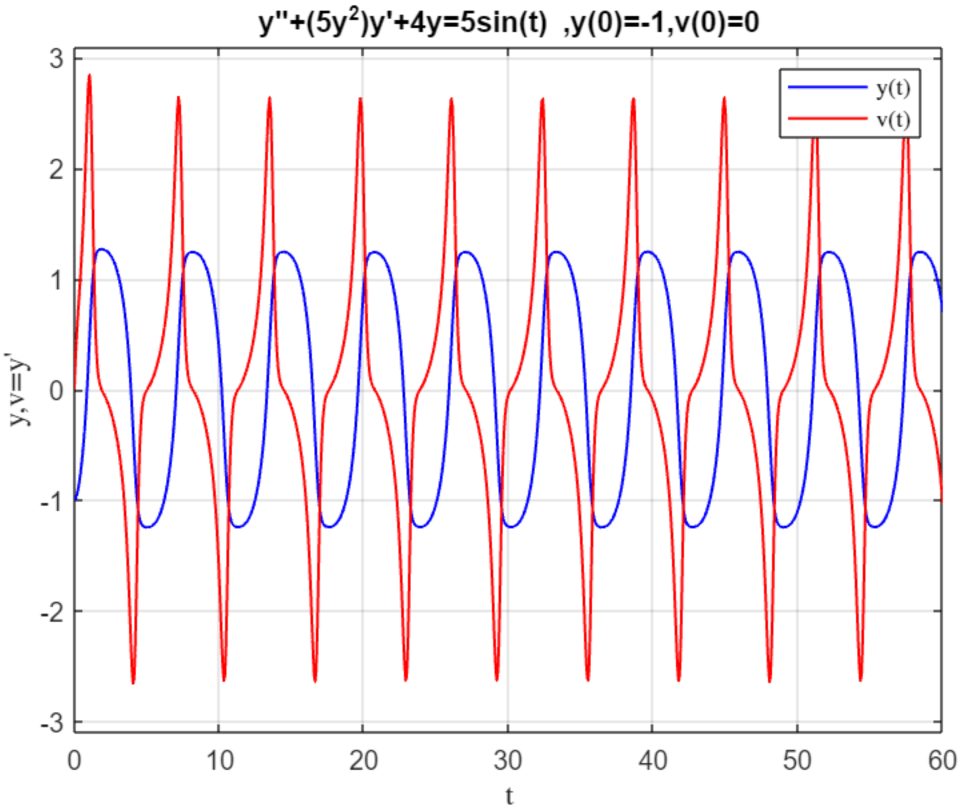
Modified IVP

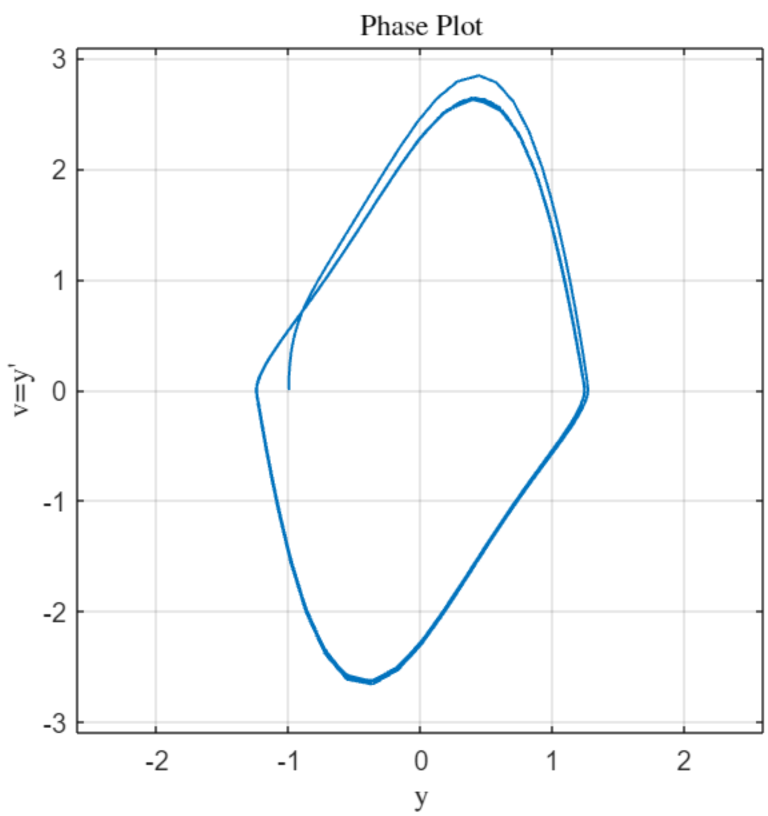
$\frac{d^2y}{\mathrm{dt}^2} + 5y^2\frac{\mathrm{dy}}{\mathrm{dt}} + 4y = 5\sin(t),\ y(0) = -1,\ v(0) = 0$ **Eqn. (7)**

```
type  LAB04ex2
```

```
t0 = 0; tf = 60;
Y0 = [-1;0];

[t,Y] = ode45(@f,[t0,tf],Y0,[]);
y = Y(:,1); v = Y(:,2);    % Y in output has 2 columns corresponding to y and v

figure
plot(t,y,'b',t,v,'r',LineWidth=1)
ylim([-3.1,3.1])
grid on

xlabel('t',"Interpreter","latex"); ylabel("y,v=y'","Interpreter","latex")
legend('y(t)','v(t)',"Interpreter","latex")
title(strcat('y"',"+(5y^2)y'+4y=5sin(t)",' ,y(0)=-1,v(0)=0'))

figure
plot(y,v,LineWidth=1);   % plot the phase plot
axis square
ylim([-3.1,3.1])
xlim([-2.6,2.6])
grid on

xlabel('y',"Interpreter","latex"); ylabel("v=y'","Interpreter","latex")
title("Phase Plot","Interpreter","latex")

%----------------------------------------------------------------------
function dYdt = f(t,Y)
y=Y(1); v=Y(2);
dYdt = [v; 5*sin(t)-5*y^2*v-4*y];
end
```

```
LAB04ex2                        % Fig. (8)
```





Part (b)

Compare the output of Figs. (7) and (8). Describe the changes in the behavior of the solution in

the short term.

Comparing the two v(t), the later one has more drastically chage and the larger amplitude.

Part (c)

Compare the long term behavior of both problems Eqn. (4) and Eqn. (7), in particular the amplitude of oscillations.

For problem 7, it has the amplitude larger than 1 while for problem is about 0.86, on the other hand, the phase plot for p.7 is less symmetrical.

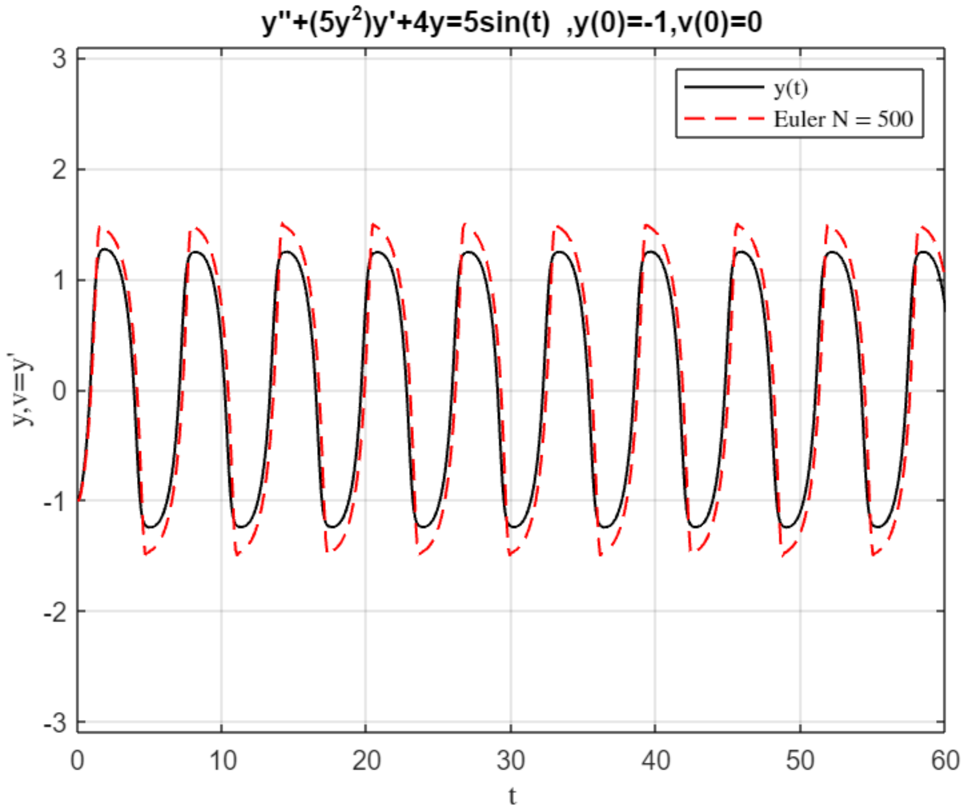
Part (d)

Solves the ODE IVP using Euler's method with **N = 500**, and compare the result with **ode45**.

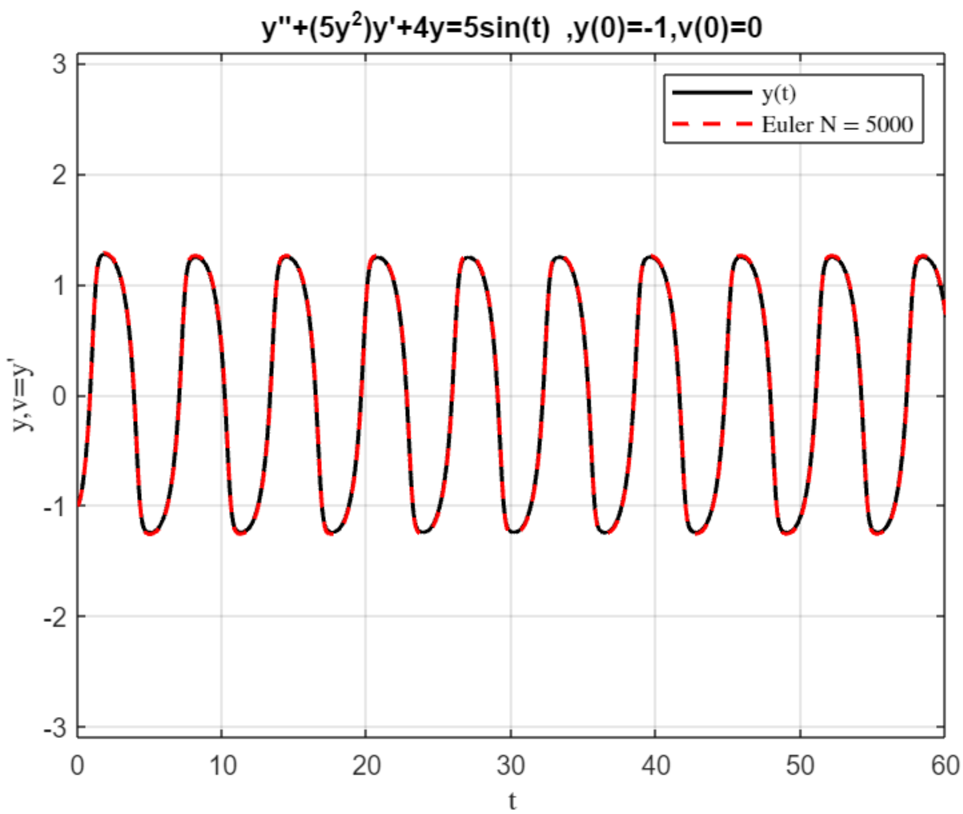
```
type LAB04ex2d
```

```
t0 = 0; tf = 60;
Y0 = [-1;0];

[t,Y] = ode45(@f,[t0,tf],Y0,[]);
y = Y(:,1); v = Y(:,2);    % Y in output has 2 columns corresponding to y and v

[te,Ye] = euler(@f,[t0,tf],Y0,500);  % solve the ODE using Euler's 500 steps

figure
plot(t,y,'k',te,Ye(:,1),'r--',LineWidth=1)
ylim([-3.1,3.1])
grid on

xlabel('t',"Interpreter","latex"); ylabel("y,v=y'","Interpreter","latex")
legend('y(t)','Euler N = 500',"Interpreter","latex")
title(strcat('y"',"+(5y^2)y'+4y=5sin(t)",' ,y(0)=-1,v(0)=0'))



%----------------------------------------------------------------------
function dYdt = f(t,Y)
y=Y(1); v=Y(2);
dYdt = [v; 5*sin(t)-5*y^2*v-4*y];
end
```

```
LAB04ex2d
```




```
% Supplementary figure
LAB04ex2dnew
```



Are the solutions identical? What happens if we increase the value of N?

With Eular's N=500 approxmation, the solution is close but not quite identical in this case, but the accuracy will growth with the number of steps N increase.

Exercise 3

Another modified IVP

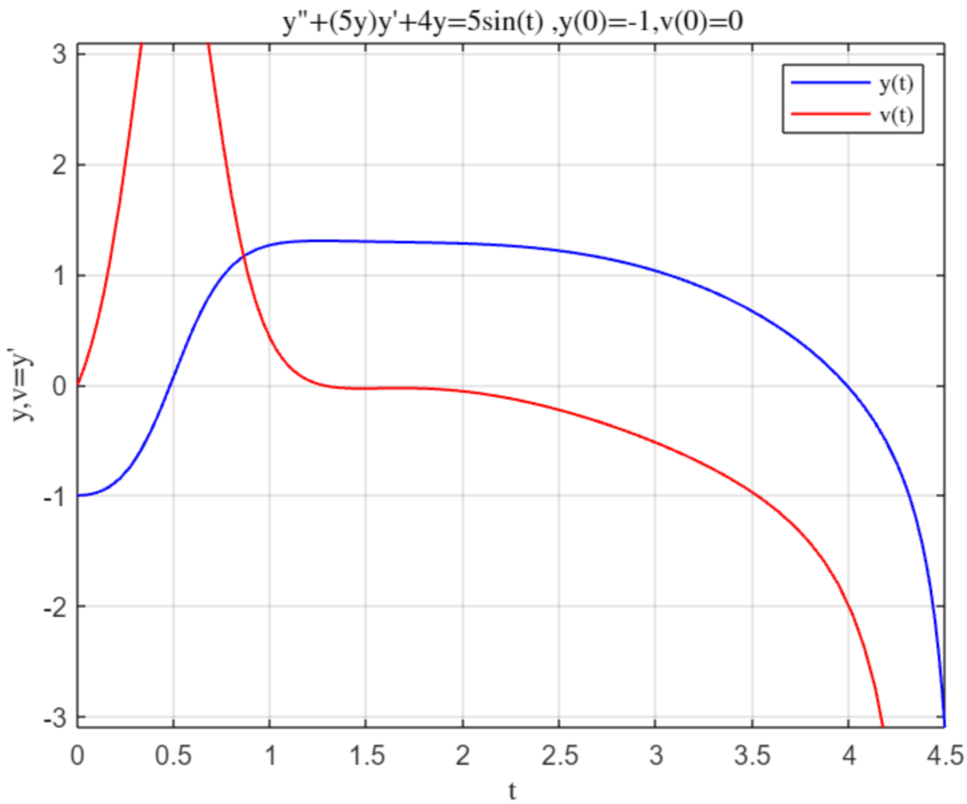
$$\frac{d^2y}{\mathrm{dt}^2} + 5y\frac{\mathrm{dy}}{\mathrm{dt}} + 4y = 5\sin(t),\ y(0) = -1,\ v(0) = 0$$

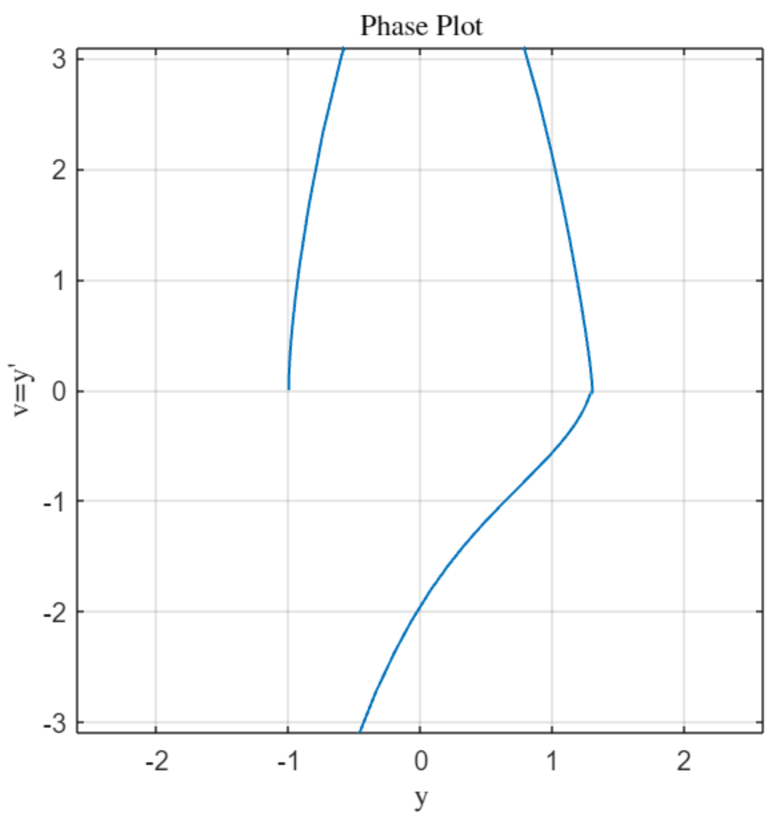
```
type LAB04ex3
```

```
t0 = 0; tf = 60;
Y0 = [-1;0];

[t,Y] = ode45(@f,[t0,tf],Y0,[]);
y = Y(:,1); v = Y(:,2);    % Y in output has 2 columns corresponding to y and v

figure(1)
plot(t,y,'b',t,v,'r',LineWidth=1)
ylim([-3.1,3.1])
grid on

xlabel('t',"Interpreter","latex"); ylabel("y,v=y'","Interpreter","latex")
legend('y(t)','v(t)',"Interpreter","latex")
title(strcat('y"',"+(5y)y'+4y=5sin(t)",' ,y(0)=-1,v(0)=0'),"Interpreter","latex")

figure(2)
plot(y,v,LineWidth=1);   % plot the phase plot
axis square
ylim([-3.1,3.1])
xlim([-2.6,2.6])
grid on

xlabel('y',"Interpreter","latex"); ylabel("v=y'","Interpreter","latex")
title("Phase Plot","Interpreter","latex")

%----------------------------------------------------------------------
function dYdt = f(t,Y)
y=Y(1); v=Y(2);
dYdt = [v; 5*sin(t)-5*y*v-4*y];
end
```

```
LAB04ex3
```

```
Warning: Failure at t=4.620602e+00.  Unable to meet integration tolerances without reducing the step size
below the smallest value allowed (1.421085e-14) at time t.
```


y''+(5y)y'+4y=5sin(t) ,y(0)=-1,v(0)=0
y(t)
v(t)
y,v=y'
t
3
2
1
0
-1
-2
-3
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5


Phase Plot
v=y'
y
3
2
1
0
-1
-2
-3
-2
-1
0
1
2

Exercise 4

A Third-Order Problem

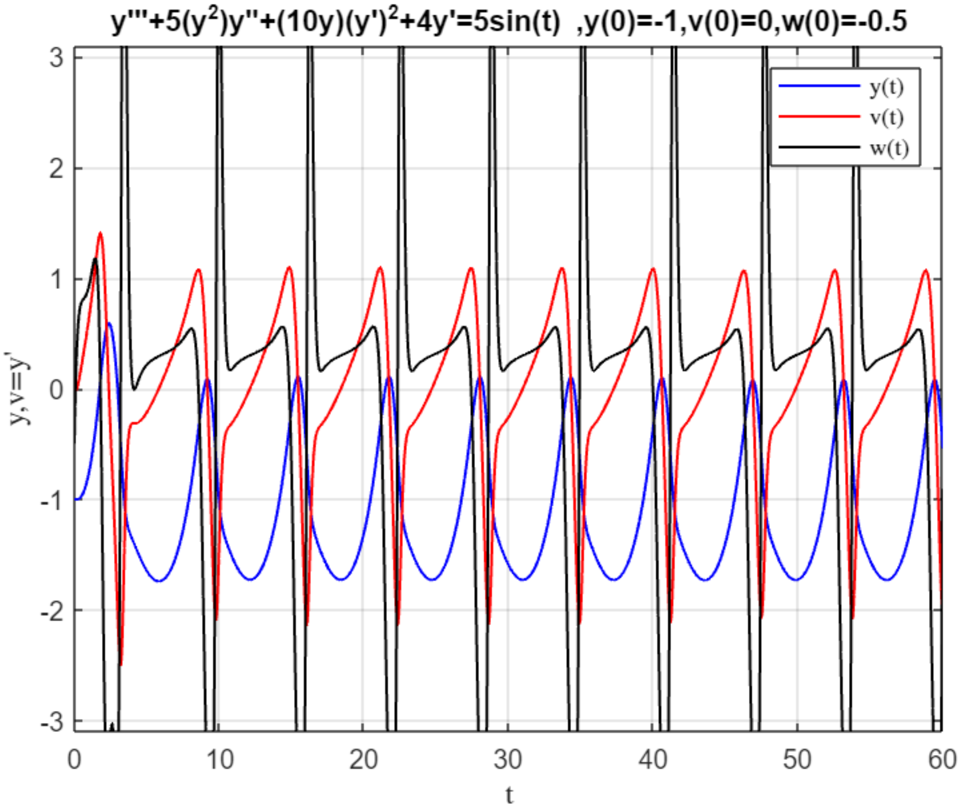
Consider the third-order IVP

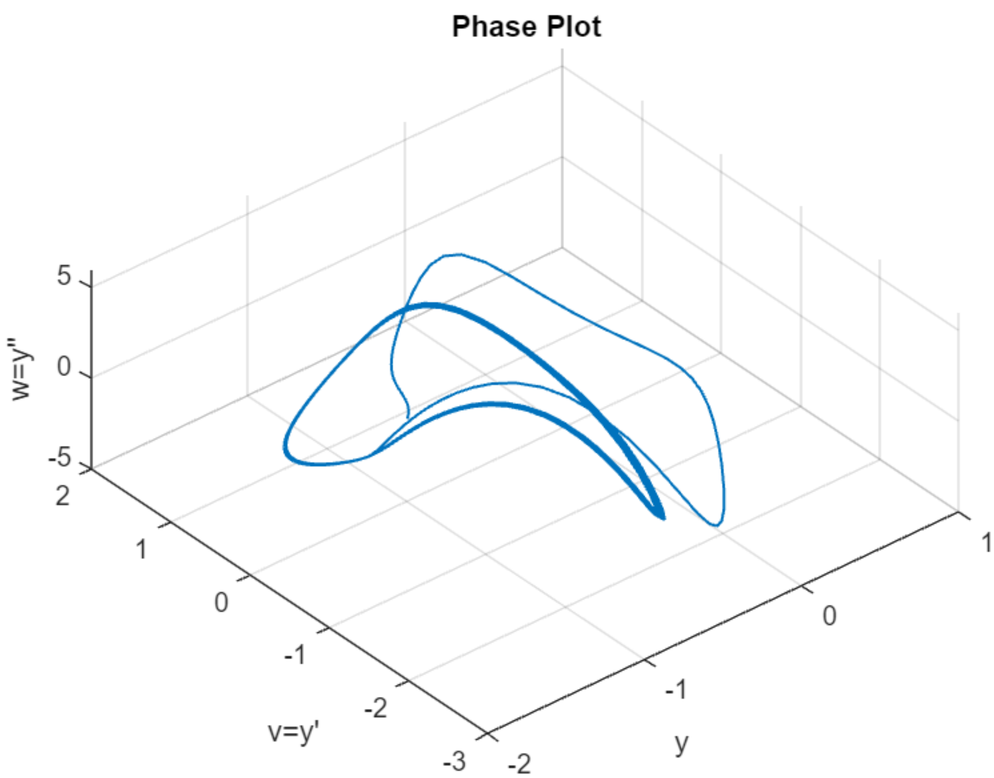
$$\frac{d^3y}{dt^3} + 5y^2\frac{d^2y}{dt^2} + 10y\left(\frac{dy}{dt}\right)^2 + 4\frac{dy}{dt} = 5\cos(t), y(0) = -1, v(0) = 0, w(0) = -0.5$$

```
type LAB04ex4
```

```
t0 = 0; tf = 60;
Y0 = [-1;0;-0.5]; % [y(0); v(0); w(0)]

[t,Y] = ode45(@f,[t0,tf],Y0,[]);
y = Y(:,1); v = Y(:,2); w = Y(:,3);   % Y in output has 3 columns corresponding to y, v, and w

figure
plot(t,y,'b',t,v,'r',t,w,'k',LineWidth=1)
ylim([-3.1,3.1])
grid on

xlabel('t',"Interpreter","latex"); ylabel("y,v=y'","Interpreter","latex")
legend('y(t)','v(t)','w(t)',"Interpreter","latex")
title(strcat("y'''",'+5(y^2)y"',"+(10y)(y')^2+4y'=5sin(t)",' ,y(0)=-1,v(0)=0,w(0)=-0.5'))

figure
plot3(y,v,w,LineWidth=1);   % plot the phase plot
hold on;
view([-40, 60])  % Azimuth=-40 and Elevation=60 (Angle)
xlabel('y'); ylabel('v=y'''); zlabel('w=y''''');
grid on

title("Phase Plot")

%----------------------------------------------------------------------
function dYdt = f(t,Y)
y=Y(1); v=Y(2); w=Y(3);
dYdt = [v; w; 5*cos(t)-5*y^2*w-10*y*v^2-4*v];
end
```

```
LAB04ex4
```


y'''+5(y^2)y''+(10y)(y')^2+4y'=5sin(t) ,y(0)=-1,v(0)=0,w(0)=-0.5
y(t)
v(t)
w(t)
y,v=y'
t
0
10
20
30
40
50
60
3
2
1
0
-1
-2
-3


Phase Plot
w=y''
v=y'
y
5
0
-5
2
1
0
-1
-2
-3
-2
-1
0
1